Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 697151, 6 pages
http://dx.doi.org/10.1155/2013/697151

Research Article

A Note on Various Classes of Compatible-Type Pairs of Mappings and Common Fixed Point Theorems

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Received 8 September 2013; Accepted 27 September 2013

Academic Editor: Wei-Shih Du

Several authors have introduced various conditions which can be used in order to prove common fixed point results. However, it became clear recently that some of these conditions, though formally distinct from each other, actually coincide in the case when the given mappings have a unique point of coincidence. Hence, in fact, new common fixed point results cannot be obtained in this way. We make a review of such connections and results in this paper.

1. Introduction

The simplest common fixed point results for mappings $A, S : X \to X$ can be obtained if A and S commute (Jungck, [1]). Obviously, this condition is too strong, and so it is natural to seek for weaker assumptions. Hence, several authors have introduced various other conditions (we will call them compatible-type conditions) which can be used in order to prove common fixed point results. Some of these conditions were given in [2–19]. These (and other) conditions were used in other papers cited in the references. A review of the relationship between various compatible-type conditions introduced until 2001 was given in [20].

However, it became clear recently that some of these conditions, though formally distinct from each other, actually coincide in the case when the given mappings have a unique point of coincidence. Hence, in fact, new common fixed point results cannot be obtained in this way. We make a review of such connections and results in this paper.

2. Definitions and Relations between Various Types of Pairs

Most of the notions and results that follow can be formulated and proved in various types of spaces—metric, symmetric, cone metric, b-metric, probabilistic metric, and so forth. For the sake of simplicity, we will stay within the framework of metric spaces.

Let (X, d) be a metric space, and let $A, S : X \to X$. We will denote by $C(A, S)$ the set of coincidence points (CP) of A and S, that is,

$$C(A, S) = \{x \in X \mid Ax = Sx\}, \tag{1}$$

by $PC(A, S)$ the set of points of coincidence (POC) of A and S, that is,

$$PC(A, S) = \{y \in X \mid y = Ax = Sx, \text{ for some } x \in X\}, \tag{2}$$

and by $\mathscr{L}(A, S)$ the set of sequences (x_n) in X satisfying $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n$, that is,

$$\mathscr{L}(A, S) = \left\{(x_n) \mid \lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n\right\}. \tag{3}$$

The following are definitions of some of the multitude of compatible-type conditions, introduced and used for establishing common fixed point results in recent decades.

TABLE 1

If	Properties that trivially hold	Properties that cannot hold
$C(A,S)=\emptyset$	(7), (9), (19)	(8), (14), (15), (16), (17), (20)
$\mathscr{L}(A,S)=\emptyset$	(3), (6), (11), (13), (18) (7), (9), (10), (19)	(2), (4), (5), (12) (8), (14), (15), (16), (17), (20)
$C(A,S)\neq\emptyset$	(7) $\Rightarrow$ (8), (8) $\Leftrightarrow$ (9), (10) $\Rightarrow$ (8) (8) $\Rightarrow$ (17), (19) $\Rightarrow$ (20) (2), (3) $\Rightarrow$ (6), (5) $\Leftrightarrow$ (6) (11) $\Rightarrow$ (12), (12) $\Leftrightarrow$ (13)	
$\mathscr{L}(A,S)\neq\emptyset$	(2), (3) $\Rightarrow$ (6), (5) $\Leftrightarrow$ (6) (11) $\Rightarrow$ (12), (12) $\Leftrightarrow$ (13)	

Definition 1. It is said that the pair (A,S) is

(1) weakly commuting [2] if, for all $x\in X$, $d(ASx,SAx)\le d(Ax,Sx)$;

(2) said to satisfy the property (E.A) [7] if $\mathscr{L}(A,S)\neq\emptyset$ (i.e., if there exists a sequence (x_n) in X such that $\lim_{n\to\infty}Ax_n=\lim_{n\to\infty}Sx_n$);

(3) compatible [3] if, for all (x_n), $(x_n)\in\mathscr{L}(A,S)$ implies that $\lim_{n\to\infty}d(ASx_n,SAx_n)=0$;

(4) noncompatible [4] if, for some $(x_n)\in\mathscr{L}(A,S)$, $\lim_{n\to\infty}d(ASx_n,SAx_n)\neq 0$ or does not exist;

(5) subcompatible [15] if, for some $(x_n)\in\mathscr{L}(A,S)$, $\lim_{n\to\infty}d(ASx_n,SAx_n)=0$;

(6) conditionally compatible [6] if $\mathscr{L}(A,S)\neq\emptyset$ implies that for some $(x_n)\in\mathscr{L}(A,S)$, $\lim_{n\to\infty}d(ASx_n,SAx_n)=0$;

(7) weakly compatible [5] if, for all $x\in X$, $Ax=Sx$ implies that $ASx=SAx$ (i.e., if, for all $x\in X$, $x\in C(A,S)$ implies that $Ax\in C(A,S)$);

(8) occasionally weakly compatible [8] (see also [21–23]) if, for some $x\in X$, $Ax=Sx$ and $ASx=SAx$ (i.e., if, for some $x\in C(A,S)$, $Ax\in C(A,S)$);

(9) conditionally commuting [9] if $C(A,S)\neq\emptyset$ implies that there exists $\emptyset\neq Y\subseteq C(A,S)$ such that, for all $y\in Y$, $ASy=SAy$;

(10) faintly compatible [16] if it is conditionally compatible and conditionally commuting (i.e., (6) and (9) hold);

(11) reciprocally continuous [13] if, for all $(x_n)\in\mathscr{L}(A,S)$, $\lim_{n\to\infty}Ax_n=\lim_{n\to\infty}Sx_n=z$ implies that $\lim_{n\to\infty}ASx_n=Az$ and $\lim_{n\to\infty}SAx_n=Sz$;

(12) subsequentially continuous [15] if, for some $(x_n)\in\mathscr{L}(A,S)$, $\lim_{n\to\infty}Ax_n=\lim_{n\to\infty}Sx_n=z$ and $\lim_{n\to\infty}ASx_n=Az$, $\lim_{n\to\infty}SAx_n=Sz$;

(13) conditionally reciprocal continuous [19] if $\mathscr{L}(A,S)\neq\emptyset$ implies that, for some $(x_n)\in\mathscr{L}(A,S)$, $\lim_{n\to\infty}Ax_n=\lim_{n\to\infty}Sx_n=z$, $\lim_{n\to\infty}ASx_n=Az$, and $\lim_{n\to\infty}SAx_n=Sz$;

(14) a $\mathscr{P}$-operator pair [10] if, for some $x\in C(A,S)$, $d(x,Ax)\le\operatorname{diam}C(A,S)$;

(15) a $\mathscr{J}\mathscr{H}$-operator pair [11] if, for some $x\in C(A,S)$, $d(x,Ax)\le\operatorname{diam}PC(A,S)$;

(16) a generalized $\mathscr{J}\mathscr{H}$-operator pair of order $n\in\mathbb{N}$ [12] if, for some $x\in C(A,S)$, $d(x,Ax)\le(\operatorname{diam}PC(A,S))^n$;

(17) a $\mathscr{P}\mathscr{D}$-operator pair [14] if, for some $x\in C(A,S)$, $d(ASx,SAx)\le\operatorname{diam}PC(A,S)$;

(18) S-biased [17] if, for all $(x_n)\in\mathscr{L}(A,S)$, $\alpha d(Sx_n,SAx_n)\le\alpha d(Ax_n,ASx_n)$, where $\alpha=\limsup$ or $\alpha=\liminf$;

(19) weakly S-biased [17] if, for all $x\in X$, $x\in C(A,S)$ implies that $d(SAx,Sx)\le d(ASx,Ax)$;

(20) occasionally weakly S-biased [11, 18] if, for some $x\in C(A,S)$, $d(SAx,Sx)\le d(ASx,Ax)$;

Note that the conditions (7), (8), and (9) are purely set-theoretical and do not depend on the metric structure of (X,d). All other conditions are metrical and could change if the metric of the space is changed (or some other structure of the space is applied).

In Table 1, we state which of these properties trivially hold (or can never hold) if one (or both) of the sets $C(A,S)$ and $\mathscr{L}(A,S)$ is empty or nonempty. Also, some of implications between these conditions obviously hold in some of these cases. Note that $\mathscr{L}(A,S)=\emptyset\Rightarrow C(A,S)=\emptyset$ (and hence, $C(A,S)\neq\emptyset\Rightarrow\mathscr{L}(A,S)\neq\emptyset$).

Counter examples for some of the reverse implications are given in [8, Example in Section 2], [14, Example 3.1], [21, Example], and [24, Example 2.12].

The following are some other implications (mostly clear from definitions) that hold between the introduced notions. When it is not obvious that the reverse implication does not hold, a reference for a counterexample is given.

(1) $\Rightarrow$ (3). For the reverse implication see [3, Examples 2.1 and 2.2].

(4) $\Rightarrow$ (2). For the reverse implication see [25, Example 1].

(3) $\wedge$ (2) $\Rightarrow$ (5) and (6) $\wedge$ (2) $\Rightarrow$ (5). For the reverse implications see [26, Example 2.3].

(3) $\Rightarrow$ (6). For the reverse implication see [16, Example 1.2].

(5) $\Rightarrow$ (6) (for the reverse implication take arbitrary mappings satisfying $\mathscr{L}(A,S)=\emptyset$) and (5) $\Rightarrow$ (2).

(2) and (7) are independent of each other (see [27, Examples 2.1-2.2]).

(3) ⇒ (7). For the reverse implication see, for example, [28, Example 2.3] (note that a part of the definition of the mapping A in this example is missing; it should be, for example, $Ax = (x+5)/5$ for $5 < x \le 20$).

(6) does not imply (7) (see [16, Example 1.3]).

(3) and (8) are independent (in one direction take any pair with $\mathscr{L}(A,S) = \emptyset$, and for the reverse see [9, Example 1.1]).

(4) and (8) are independent of each other (see [9, Examples 1.1–1.3]).

(8) ⇒ (5). For the reverse implication see [15, Example 1.2].

(8) ⇒ (6) and (8) ⇒ (9). For the reverse implications take arbitrary mappings satisfying $\mathscr{L}(A,S) = \emptyset$ (and hence, $C(A,S) = \emptyset$).

(9) does not imply (7) (see [9]).

(3) ⇒ (10). For the reverse implication see [16, Example 1.4].

(4) and (10) are independent of each other. An example of a noncompatible pair which is not faintly compatible is [16, Example 1.5] (see also [9, Example 1.1]). Any pair of commuting that maps with $C(A,S) \neq \emptyset$ is faintly compatible and not noncompatible.

(8) ⇒ (10). For the reverse implication see [16, Example 1.7] or [29, Example 2.1].

(11) and (12) are independent conditions, (in one direction, take arbitrary pair satisfying $\mathscr{L}(A,S) = \emptyset$, and in the other see [15, Example 1.4]).

(11) ⇒ (13) and (12) ⇒ (13). For the reverse implications see [19, Example 6].

(7) ⇒ (14) and (8) ⇒ (14). Indeed, it follows from (8) that there exists $x \in C(A,S)$ such that $ASx = SAx$; that is, $Ax = Sx \in C(A,S)$. But then $d(x, Ax) \le \operatorname{diam} C(A,S)$.

(14) and (15) (together) do not imply either (7), or (8) (see [10, Example A]), or (17) (see [14, Example 3.3]).

(15) ⇒ (16) (for the reverse implication and that (16) does not imply (8) see [12, Example 3.2]).

(17) does not imply (14), and (17) does not imply (15) (see [14, Example 3.4]).

(3) ⇒ (18). For the reverse implication see [17].

(18) ⇒ (19). For the reverse implication see [17].

(7) ⇒ (19) and (8) ⇒ (20). For the reverse implications see [30, Example 2.3] and [18, Example 3.2].

(19) does not imply (8) (see [30, Example 2.4]) and (20) does not imply (19) (see [18, Example 3.2]).

Finally, if $PC(A,S)$ is a singleton, then

(8) ⇒ (7) [31, Proposition 2.2],

(5) ⇒ (7) [32, Proposition 2.22], and

(20) ⇒ (19) [24, Proposition 2.11].

3. Reducing Common Fixed Point Results to the Case of Weak Compatibility

The following simple result can be used to show that several common fixed point theorems obtained recently are actually not generalizations of previously known results.

Proposition 2. *Let (X,d) be a metric space, and let $A, S : X \to X$. Let the pair (A,S) have exactly one point of coincidence; that is,*

$$PC(A,S) = \{w\}. \tag{4}$$

Then conditions (7), (8), (9), and (17) are equivalent, and equivalent with the condition that the pair (A,S) has a unique common fixed point.

Proof. Note first that $PC(A,S) \neq \emptyset$ implies that $C(A,S) \neq \emptyset$ and $\mathscr{L}(A,S) \neq \emptyset$ (just take $x_n = x$, where $x \in C(A,S)$, for all $n \in \mathbb{N}$) Consider the following.

(7) ⇒ (8) holds because $C(A,S) \neq \emptyset$.

(8) ⇒ (7) follows by [31, Proposition 2.2].

(8) ⇔ (9) holds because $C(A,S) \neq \emptyset$.

(7) ⇒ (17) was proved in [14], and (17) ⇒ (8) follows because $\operatorname{diam} PC(A,S) = \operatorname{diam}\{w\} = 0$.

In the case that (A,S) has a unique POC, it was proved in [22] that condition (8) implies that (A,S) has a unique common fixed point. The converse is obvious. □

An easy example of mappings $Ax = 3x - 2$ and $Sx = x^2$ on $X = [1, +\infty)$, when $PC(A,S)$ has two elements, shows that the condition that $PC(A,S)$ is a singleton cannot be removed from the previous proposition, neither does this proposition hold when $PC(A,S) = \emptyset$, as [19, Example 6] shows.

When considering two pairs of mappings, the following is a direct consequence of Proposition 2.

Corollary 3. *Let (A,S) and (B,T) be two pairs of self-maps on a metric space (X,d), satisfying*

$$PC(A,S) = PC(B,T) = \{w\}. \tag{5}$$

Then the following conditions are equivalent.

(i) *(A,S) and (B,T) both satisfy condition (7).*

(ii) *(A,S) and (B,T) both satisfy condition (8).*

(iii) *(A,S) and (B,T) both satisfy condition (17).*

(iv) *A, B, S, and T have a unique common fixed point.*

Applying Proposition 2 or Corollary 3, it is easy to show that a lot of the results of papers cited in the references are actually not generalizations of previously known ones.

As a sample, consider Theorems 2.1 and 2.2 of [16]. $PC(A,S)$ in these assertions is a singleton. Further, mappings A and S have a unique common fixed point. By Proposition 2, it follows that the pair (A,S) is weakly compatible (condition (4)). Hence, using formally weaker assumption (10) does not produce a more general assertion.

Similarly, in Theorems 2.1 and 2.2 and Corollaries 2.1–2.6 of [33], applying Corollary 3, we get that the pairs (f, S) and (g, T) are weakly compatible.

In the same way, it can be concluded that the following results are actually not generalizations of previously known ones: [6, Theorems 1.4 and 1.5]; [8, Theorem 2.1]; [19, Theorems 1–3 of Section 2]; [11, Theorems 2.8–2.12]; [14, Theorems 4.1, 4.4, 4.6, 4.8, 4.10, 4.12, and 5.1]; [18, Theorems 4.1, 4.6, 5.6, 6.2; Corollaries 4.3, 4.4]; [22, Lemma 1; Theorems 1–5; Corollaries 1–5]; [30, Theorem 2.5; Corollary 2.7]; [34, Theorems 1–4 and Corollaries 1–3]; [35, Theorems 3.1, 3.4, and 3.6; Corollaries 3.9, 3.10, and 3.11]; [36, Theorems 2.2 and 2.6]; [37, Theorems 2.1–2.3 and 3.1–3.4; Corollary 2.1]; [38, Theorem 1.1]; [39, Theorems 2.1 and 2.3; Corollaries 2.2, 2.4 and 2.5]; [40, Theorems 3.1–3.3]; [41, Theorems 2.1 and 2.2; Corollaries 2.1–2.3]; [42, Theorems 3.1–3.3 and 4.1–4.3]; [43, Theorems 2.1 and 2.2]; [44, Theorem 3.2; Corollaries 3.1 and 3.2]; [45, Theorems 2.3]; [46, Theorem 11; Corollary 13]; [47, Theorems 2.2 and 2.3]; [48, Theorems 2.1–2.5]; [49, Theorems 2.1, 2.4, and 2.10]; [50, Theorems 4.1, 4.2, and 5.1–5.5].

A different kind of conclusions can be made for the results from [28, 51, 52].

Again, as a sample, consider [52, Theorem 2.1], which (abbreviated) reads as follows.

Let f and g be two pseudoreciprocal continuous self-mappings of a complete metric space (X, d) such that $fX \subseteq gX$ and satisfying certain contractive condition. If the pair (f, g) is conditionally sequential absorbing, then f and g have a unique common fixed point.

It can be reformulated as follows.

Under the previous conditions, the pair (f, g) is weakly compatible (i.e., satisfies condition (7)).

Indeed, the proof of [52, Theorem 2.1] shows that f and g have a unique common fixed point. The contractive condition easily implies that they also have a unique POC (i.e., $PC(f, g)$ is a singleton). Then, Proposition 2 implies that (f, g) is weakly compatible (and occasionally weakly compatible, of the type PD, conditionally commuting). Hence, weak compatibility is again a natural (and the weakest possible) assumption for this kind of results.

Similar conclusions can be made for the following results: [28, Corollary 2.1]; [51, Theorems 1–3]; [52, Theorems 2.2 and 2.3].

Several very interesting results were also obtained for multivalued mappings. We just note [53–55].

It is interesting that in the case of hybrid pairs of mappings (one single-valued and one multivalued) conclusions similar to those from this paper cannot be made. Namely, it was shown in [31, Example 2.5] that in this case Proposition 2 no longer holds. Hence, for example, results from the papers [56–59] cannot be directly obtained from previously known ones.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

The first and the second authors are grateful to the Ministry of Education, Science and Technological Development of Serbia. The third author acknowledges with thanks the Deanship of Scientific Research (DSR), King Abdulaziz University Jeddah, for financial support.

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