Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2013, Article ID 659251, 10 pages
http://dx.doi.org/10.1155/2013/659251

Research Article

Exponential L_2-L_∞ Filtering for a Class of Stochastic System with Mixed Delays and Nonlinear Perturbations

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Received 12 August 2013; Revised 21 November 2013; Accepted 28 November 2013

Academic Editor: Weihai Zhang

The delay-dependent exponential L_2-L_∞ performance analysis and filter design are investigated for stochastic systems with mixed delays and nonlinear perturbations. Based on the delay partitioning and integral partitioning technique, an improved delay-dependent sufficient condition for the existence of the L_2-L_∞ filter is established, by choosing an appropriate Lyapunov-Krasovskii functional and constructing a new integral inequality. The full-order filter design approaches are obtained in terms of linear matrix inequalities (LMIs). By solving the LMIs and using matrix decomposition, the desired filter gains can be obtained, which ensure that the filter error system is exponentially stable with a prescribed L_2-L_∞ performance γ. Numerical examples are provided to illustrate the effectiveness and significant improvement of the proposed method.

1. Introduction

Time delays are quite often encountered in various practical engineering systems, and they are regarded as one of the main sources causing instability and degrading performance of control systems [1–3]. Over the past decades, numerous results and various approaches on delay systems have been reported in the literatures. Many researchers have focused on the stability analysis, stabilization, and filtering for time-delay systems; see [4–9] and the references therein. Time delays are usually classified into discrete delays and distributed delays. In the existing literatures, discrete time-delay system [10–12], distributed time-delay system [13, 14], and mixed (including both discrete and distributed time delays) system [15–17] are considered.

Since certain unavoidable stochastic perturbations are widely existing in many engineering systems, stochastic systems have gained considerable research attention over the past few years [18–20]. Stochastic dynamic modeling has come to play an important role in many fields of science and engineering. In the past years, many researchers have focused on the problems of stability and stabilization of stochastic time-delay systems. For instance, robust stabilization for a class of large-scale stochastic systems was investigated in [21], delay-dependent stability results for stochastic systems were presented in [22–26], and H_∞ state feedback control and H_∞ dynamic output feedback control for uncertain stochastic time-delay systems were investigated in [27, 28], respectively.

In the field of stochastic dynamic system with time delays, the filtering problem, which is to estimate the unavailable state of variables of a given control system, is also an important issue. Kalman filtering scheme is a well-known effective way to deal with the filtering problem. However, it has some limitations in practical applications due to the fact that it assumes that the system and its disturbances are exactly known, that is, stationary Gaussian noised with known statistics. Under this view, recently, H_∞ filtering, mixed H_2/H_∞ filtering and L_2-L_∞ filtering for stochastic time-delay systems have been widely studied [8, 9, 29–38]. In H_2/H_∞ filtering, and L_2-L_∞ filtering problems, the external disturbances are assumed to be bounded. In H_2/H_∞ filtering problem, it requires that the filtering error systems satisfy not only a prescribed H_∞ disturbance attenuation level but also the H_2 performance (minimum of the H_2 norm of transfer function of the filter error systems), while in L_2-L_∞ filtering problem, it requires that the filtering error systems satisfy a prescribed L_2-L_∞ disturbance attenuation level. H_∞ filtering and mixed H_2/H_∞ filtering problems of nonlinear stochastic systems are investigated in [30, 31]. In [32], a delay-independent robust L_2-L_∞ filtering design approach for uncertain stochastic time-delay system is investigated. It is well known that the delay-independent results are generally more conservative than the delay-dependent ones. Authors in [33–35] developed delay-dependent filtering for stochastic time-delay systems. Authors in [36] proposed a delay-dependent L_2-L_∞ filter design approach for stochastic time-delay systems, based on a delay partitioning technique presented in [37]. As the results showed, delay-partitioning can reduce conservatism to some extent. Authors in [38] investigated the problem of robust L_2-L_∞ filtering for stochastic systems with both discrete and distributed delays. Although the filtering problems for stochastic systems with time delays have been well investigated in the aforementioned literatures, most of them are dealing with linear stochastic time-delay systems. To the authors' knowledge, the L_2-L_∞ filtering problems of stochastic time-delay systems with nonlinear perturbation are still insufficient. This motivates the authors to deal with the L_2-L_∞ filtering problem of a class of nonlinear stochastic time-delay systems.

This paper focuses on the problems of delay-dependent L_2-L_∞ filtering for stochastic systems with mixed delays and nonlinear perturbations. By Lyapunov-Krasovskii approach based on the delay partitioning and integral partitioning technique, we first develop a delay-dependent sufficient condition for L_2-L_∞ performance analysis. And then, an improved delay-dependent sufficient condition is obtained for the existence of desired filter in the form of linear matrix inequalities (LMIs). The L_2-L_∞ performance analysis and filter design of linear stochastic system with mixed delays are also investigated. Finally, numerical examples are provided to show that the proposed method is effective and less conservative than some existing literatures.

Notations. Throughout this paper, $X > 0$ $(X < 0)$ means that the matrix X is positive definite (negative definite). $\mathbf{R}^n$ denotes the n-dimensional Euclidean space; $\mathbf{R}^{m\times n}$ is the set of all $m \times n$ real matrices; $\mathbf{L}_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$. The superscript "T" represents the transpose; "$*$" denotes the symmetric terms in a matrix; diag( ) denotes a block-diagonal matrix; $\lambda_{\max}(\,)$ and $\lambda_{\min}(\,)$ denote the maximum eigenvalue and minimum eigenvalue, respectively. $\mathrm{sym}(X) = X + X^T$; $|\cdot|$ denotes the Euclidean vector norm; $\|\cdot\|_2$ stands for the usual $\mathbf{L}_2[0,\infty)$ norm. $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$ is a probability space with Ω the sample space, $\mathbf{F}$ the σ-algebra of subsets of $\mathbf{\Omega}$, and $\mathbf{P}$ the probability measure on $\mathbf{F}$. $\mathbf{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathbf{P}$. $\mathbf{0}$ and $\mathbf{I}$ represent zero matrix and identity matrix with appropriate dimensions, respectively, unless we say otherwise.

2. Problem Formulation

Consider the following stochastic systems with mixed delays and nonlinear perturbations:

$$\begin{aligned} dx(t) &= \Big[Ax(t) + A_1 x(t-h) \\ &\quad + A_2 \int_{t-d}^{t} x(s)\,ds + A_3 f(x(t), x(t-h), t) + A_v v(t)\Big] dt \\ &\quad + g(x(t), x(t-h), t)\, d\omega(t), \\ dy(t) &= \Big[Cx(t) + C_1 x(t-h) + C_2 \int_{t-d}^{t} x(s)\,ds + C_v v(t)\Big] dt, \\ z(t) &= Lx(t), \\ x(t) &= \varphi(t), \quad \forall t \in [-\tau, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state; $y(t) \in \mathbf{R}^m$ is the measured output; $z(t) \in \mathbf{R}^p$ is the signal to be estimated; $v(t) \in \mathbf{R}^q$ is the disturbance input which belongs to $\mathbf{L}_2[0,\infty)$, which is the space of square-integrable vector functions; $\omega(t)$ is a one-dimensional Brownian motion defined on a complete probability space $(\mathbf{\Omega}, \mathbf{F}, \mathbf{P})$ satisfying $\mathbf{E}\{d\omega(t)\} = 0$ and $\mathbf{E}\{d\omega^2(t)\} = dt$; $\varphi(t)$ is an initial function that is continuous on $[-\tau, 0]$ with $\tau = \max\{h, d\}$. h and d are discrete and distributed constant delays, respectively. $f(\cdot,\cdot,\cdot)$: $\mathbf{R}^n \times \mathbf{R}^n \times \mathbf{R} \rightarrow \mathbf{R}^n$ is a nonlinear function, which satisfies

$$|f(x, y, t)|^2 \le |F_1 x|^2 + |F_2 y|^2, \qquad f(0,0,0) = 0, \tag{2}$$

where $F_1 \in \mathbf{R}^{n\times n}$ and $F_2 \in \mathbf{R}^{n\times n}$ are known constant matrices; $g(\cdot,\cdot,\cdot)$: $\mathbf{R}^n \times \mathbf{R}^n \times \mathbf{R} \rightarrow \mathbf{R}^n$ is a nonlinear perturbance input function, satisfying

$$|g(x, y, t)|^2 \le |G_1 x|^2 + |G_2 y|^2, \qquad g(0,0,0) = 0, \tag{3}$$

where $G_1 \in \mathbf{R}^{n\times n}$ and $G_2 \in \mathbf{R}^{n\times n}$ are known constant matrices.

For system (1), we are interested in constructing the following full-order linear filter:

$$\begin{aligned} dx_f(t) &= A_f x_f(t)\,dt + B_f dy(t), \\ z_f(t) &= C_f x_f(t), \end{aligned} \tag{4}$$

where $x_f(t) \in \mathbf{R}^n$ is the filter state; A_f, B_f, and C_f are filter matrices to be determined.

Define

$$\xi^T(t) = \left[x^T(t),\ x_f^T(t)\right]^T, \qquad e(t) = z(t) - z_f(t). \tag{5}$$

Then, the filtering error system can be written as

$$\begin{aligned} d\xi(t) = & \left[\overline{A}\xi(t) + \overline{A}_1 H\xi(t-h) + \overline{A}_2 H\int_{t-d}^{t}\xi(s)\,ds \right. \\ & \left. + \overline{A}_3\overline{f}(\xi(t),\xi(t-h),t) + \overline{A}_v v(t)\right] dt \\ & + \overline{g}(\xi(t),\xi(t-h),t)\,d\omega(t) \\ e(t) = & \overline{L}\xi(t), \end{aligned} \tag{6}$$

where

$$\begin{gathered} \overline{A} = \begin{bmatrix} A & 0 \\ B_f C & A_f \end{bmatrix}, \qquad \overline{A}_1 = \begin{bmatrix} A_1 \\ B_f C_1 \end{bmatrix}, \\ \overline{A}_2 = \begin{bmatrix} A_2 \\ B_f C_2 \end{bmatrix}, \qquad \overline{A}_3 = \begin{bmatrix} A_3 & 0 \\ 0 & 0 \end{bmatrix}, \\ \overline{A}_v = \begin{bmatrix} A_v \\ B_f C_v \end{bmatrix}, \qquad H = [I_n \ \ 0_n], \\ \overline{f}(\xi(t),\xi(t-h),t) = \begin{bmatrix} f(x(t),x(t-h),t) \\ 0 \end{bmatrix}, \\ \overline{g}(\xi(t),\xi(t-h),t) = \begin{bmatrix} g(x(t),x(t-h),t) \\ 0 \end{bmatrix}, \\ \overline{L} = [L \ \ -C_f]. \end{gathered} \tag{7}$$

The objective of this paper is to design full-order L_2-L_∞ filter (4) for the stochastic time-delay system (1) such that the filtering error system (6) satisfies the following two requirements:

(i) the filtering error system (6) with $v(t) = 0$ is exponentially stable [39];

(ii) under the zero initial condition, the filtering error system (6) is stochastically asymptotically stable and achieves a prescribed L_2-L_∞ attenuation level γ. The filtering error $e(t)$ satisfies

$$\|e(t)\|_{E_\infty} < \gamma\|v(t)\|_2, \tag{8}$$

with $\|e(t)\|_{E_\infty} = \sup_t \sqrt{\mathbf{E}\{|e(t)|^2\}}$, $\|v(t)\|_2 = \sqrt{\int_0^\infty v^T(t)v(t)dt}$ for any nonzero $v(t) \in \mathbf{L}_2[0,\infty]$.

Before presenting the main results of this paper, we introduce the following lemmas, which will be essential to our derivation.

Lemma 1 (see [40]). *For a given symmetrical matrix* $S = \begin{pmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{pmatrix}$, *where* $S_{11} = S_{11}^T$, *and* $S_{22} = S_{22}^T$, *the linear matrix inequality* $S < 0$ *is equivalent to*

$$\begin{gathered} S_{11} < 0, \qquad S_{22} - S_{12}S_{11}^{-1}S_{12}^T < 0, \\ \text{or} \\ S_{22} < 0, \qquad S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0. \end{gathered} \tag{9}$$

Lemma 2 (see [1]). *For any positive symmetric matrix* $W \in \mathbf{R}^{n\times n}$, *scalars* δ_1 *and* δ_2 *satisfying* $\delta_1 < \delta_2$, *a vector function* $x : [\delta_1, \delta_2] \to \mathbf{R}^n$, *one has*

$$\begin{aligned} & \int_{\delta_1}^{\delta_2} x^T(s)Wx(s)\,ds \\ & \geq \frac{1}{(\delta_2-\delta_1)}\left(\int_{\delta_1}^{\delta_2} x(s)\,ds\right)^T W \left(\int_{\delta_1}^{\delta_2} x(s)\,ds\right). \end{aligned} \tag{10}$$

Lemma 3 (see [14]). *For any positive symmetric matrix* $W \in \mathbf{R}^{n\times n}$, *scalars* a *and* b *satisfying* $a < b \leq 0$, *a vector function* $x : [a,b] \to \mathbf{R}^n$, *one has*

$$\begin{aligned} & \int_a^b \int_{t+\lambda}^{t} x^T(s)Wx(s)\,ds\,d\lambda \\ & \geq \frac{2}{a^2-b^2}\left(\int_a^b \int_{t+\lambda}^{t} x(s)\,ds\,d\lambda\right)^T W\left(\int_a^b \int_{t+\lambda}^{t} x(s)\,ds\,d\lambda\right). \end{aligned} \tag{11}$$

3. Filtering Performance Analysis

In this section, a new delay-dependent condition of the L_2-L_∞ filtering performance analysis for system (1) will be presented. A Lyapunov-Krasovskii functional is constructed; based on the idea of delay partitioning and integral partitioning, the conservatism will be reduced. For the convenience of expression, assume that the filter matrices (A_f, B_f, and C_f) are known.

Theorem 4. *Consider the stochastic time-delay system* (1). *For given scalars* $\gamma > 0$, $h > 0$, $d > 0$, $\rho > 0$, *and* $\varepsilon > 0$ *and integers* $r_1 \geq 1$ *and* $r_2 \geq 1$, *there exists a linear filter* (4) *such that the filtering error system* (6) *is stochastically asymptotically stable with a guaranteed* L_2-L_∞ *performance* γ, *if there exist symmetrical positive definite matrices* $P \in \mathbf{R}^{2n\times 2n}$, $Q_i \in \mathbf{R}^{n\times n}$, $R_i \in \mathbf{R}^{n\times n}$ $(i = 1, 2, \ldots, r_1)$, $W_j \in \mathbf{R}^{n\times n}$, *and* $Z_j \in \mathbf{R}^{n\times n}$ $(j = 1, 2, \ldots, r_2)$ *and matrix* $M \in \mathbf{R}^{n\times n}$ *satisfying*

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & P\overline{A}_3 & P\overline{A}_v & \overline{A}^T H^T M \\ * & \Phi_{22} & 0 & 0 & 0 & \Phi_{26} \\ * & * & \Phi_{33} & 0 & 0 & \Phi_{36} \\ * & * & * & -\varepsilon I & 0 & \overline{A}_3^T H^T M \\ * & * & * & * & -I & \overline{A}_v^T H^T M \\ * & * & * & * & * & \Phi_{66} \end{bmatrix} < 0, \tag{12}$$

$$P \leq \rho I, \tag{13}$$

$$\Gamma = \begin{bmatrix} P & \overline{L}^T \\ * & \gamma^2 I \end{bmatrix} > 0, \tag{14}$$

where

$$\Phi_{11} = P\overline{A} + \overline{A}^T P + H^T Q_1 H - H^T R_1 H - \sum_{j=1}^{r_2} H^T \left(\frac{2}{2j-1} Z_j\right) H + \left(\frac{d}{r_2}\right)^2 \sum_{j=1}^{r_2} H^T W_j H + \rho H^T G_1^T G_1 H + \varepsilon H^T F_1^T F_1 H,$$

$$\Phi_{12} = H^T R_1 K + P\overline{A}_1 K_{r_1},$$

$$\Phi_{13} = P\overline{A}_2 K_{r_2} + H^T \overline{Z},$$

$$\overline{Z} = \frac{2m}{d}\left[Z_1 \quad \frac{1}{3}Z_2 \quad \cdots \quad \frac{1}{2r_2-1}Z_{r_2}\right],$$

$$\Phi_{22} = \begin{bmatrix} Q_2 - Q_1 - R_2 - R_1 & R_2 & \cdots & 0 & 0 \\ * & Q_3 - Q_2 - R_3 - R_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & Q_{r_1} - Q_{r_1-1} - R_{r_1} - R_{r_1-1} & R_{r_1} \\ * & * & \cdots & * & \overline{\Phi}_{22} \end{bmatrix},$$

$$\overline{\Phi}_{22} = -Q_{r_1} - R_{r_1} + \rho G_2^T G_2 + \varepsilon F_2^T F_2,$$

$$\Phi_{26} = K_{r_1}^T \overline{A}_1^T H^T M,$$

$$\Phi_{33} = \operatorname{diag}\left(-W_1 - \frac{2m^2}{d^2}Z_1, -W_2 - \frac{2m^2}{3d^2}Z_2, \ldots, -W_{r_2} - \frac{2m^2}{(2r_2-1)d^2}Z_{r_2}\right),$$

$$\Phi_{36} = K_{r_2}^T \overline{A}_2^T H^T M,$$

$$\Phi_{66} = \left(\frac{h}{r_1}\right)^2 \sum_{i=1}^{r_1} R_i + \left(\frac{d}{r_2}\right)^2 \sum_{j=1}^{r_2} \frac{2j-1}{2} Z_j - M - M^T,$$

$$K = \left[I_n \quad 0_{n\times(r_1-1)n}\right],$$

$$K_{r_1} = \left[0_{n\times(r_1-1)n} \quad I_n\right],$$

$$K_{r_2} = \left[I \quad I \quad \cdots \quad I\right]_{n\times r_2 n}. \tag{15}$$

Proof. First, show the asymptotic stability of system (6) with $v(t) = 0$. For simplicity of notations, rewrite the filtering error system (6) as

$$d\xi(t) = u(t)\,dt + \pi(t)\,d\omega(t), \tag{16}$$

where

$$\begin{aligned} u(t) &:= \overline{A}\xi(t) + \overline{A}_1 H\xi(t-h) + \overline{A}_2 H \int_{t-d}^{t} \xi(s)\,ds \\ &\quad + \overline{A}_3 \overline{f}(\xi(t), \xi(t-h), t) + \overline{A}_v v(t), \\ \pi(t) &:= \overline{g}(\xi(t), \xi(t-h), t). \end{aligned} \tag{17}$$

Next, denote $\eta(t)dt = d\xi(t)$, and choose the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(\xi_t, t) &= \xi^T(t) P\xi(t) + \sum_{i=1}^{r_1} \int_{t-(i/r_1)h}^{t-((i-1)/r_1)h} \xi^T(s) H^T Q_i H\xi(s)\,ds \\ &\quad + \sum_{i=1}^{r_1} \frac{h}{r_1} \int_{-(i/r_1)h}^{-((i-1)/r_1)h} \int_{t+\theta}^{t} \eta^T(s) H^T R_i H\eta(s)\,ds\,d\theta \\ &\quad + \sum_{j=1}^{r_2} \frac{d}{r_2} \int_{-(j/r_2)d}^{-((j-1)/r_2)d} \int_{t+\theta}^{t} \xi^T(s) H^T W_j H\xi(s)\,ds\,d\theta \\ &\quad + \sum_{j=1}^{r_2} \int_{-(j/r_2)d}^{-((j-1)/r_2)d} \int_{\theta}^{0} \int_{t+\beta}^{t} \eta^T(s) H^T Z_j H\eta(s)\,ds\,d\beta\,d\theta. \end{aligned} \tag{18}$$

Then, by Itô differential formula, the stochastic differential along the trajectories of system (6) is

$$dV(\xi_t, t) = \mathbf{L}V(\xi_t)\,dt + 2\xi(t) P\pi(t)\,d\omega(t), \tag{19}$$

where

$$\begin{aligned} \mathbf{L}V(\xi_t, t) &= 2\xi^T(t) Pu(t) + \operatorname{trace}\left(\pi^T(t) P\pi(t)\right) \\ &\quad + \sum_{i=1}^{r_1} \xi^T\left(t - \frac{i-1}{r_1}h\right) H^T Q_i H\xi\left(t - \frac{i-1}{r_1}h\right) \\ &\quad - \sum_{i=1}^{r_1} \xi^T\left(t - \frac{i}{r_1}h\right) H^T Q_i H\xi\left(t - \frac{i}{r_1}h\right) \\ &\quad + \left(\frac{h}{r_1}\right)^2 \sum_{i=1}^{r_1} \eta^T(t) H^T R_i H\eta(t) \\ &\quad - \sum_{i=1}^{r_1} \frac{h}{r_1} \int_{t-(i/r_1)h}^{t-((i-1)/r_1)h} \eta^T(s) H^T R_i H\eta(s)\,ds \\ &\quad + \left(\frac{d}{r_2}\right)^2 \sum_{j=1}^{r_2} \xi^T(t) H^T W_i H\xi(t) \\ &\quad - \sum_{j=1}^{r_2} \frac{d}{r_2} \int_{t-(j/r_2)d}^{t-((j-1)/r_2)d} \xi^T(s) H^T W_j H\xi(s)\,ds \\ &\quad + \left(\frac{d}{r_2}\right)^2 \sum_{j=1}^{r_2} \frac{2j-1}{2} \eta^T(t) H^T Z_j H\eta(t) \\ &\quad - \sum_{j=1}^{r_2} \int_{-(j/r_2)d}^{-((j-1)/r_2)d} \int_{t+\theta}^{t} \eta^T(t) H^T Z_j H\eta(t)\,ds\,d\theta. \end{aligned} \tag{20}$$

By Lemma 2, we have

$$-\sum_{i=1}^{r_1}\frac{h}{r_1}\int_{t-(i/r_1)h}^{t-((i-1)/r_1)h}\eta^T(s)H^TR_iH\eta(s)\,ds \le -\sum_{i=1}^{r_1}\left(\int_{t-(i/r_1)h}^{t-((i-1)/r_1)h}\eta(s)\,ds\right)^T \times H^TR_iH\left(\int_{t-(i/r_1)h}^{t-((i-1)/r_1)h}\eta(s)\,ds\right) = -\sum_{i=1}^{r_1}\left(\xi\left(t-\frac{i-1}{r_1}h\right)-\xi\left(t-\frac{i}{r_1}h\right)\right)^T \times H^TR_iH\left(\xi\left(t-\frac{i-1}{r_1}h\right)-\xi\left(t-\frac{i}{r_1}h\right)\right), \tag{21}$$

$$-\sum_{j=1}^{r_2}\frac{d}{r_2}\int_{t-(j/r_2)d}^{t-((j-1)/r_2)d}\xi^T(s)H^TW_jH\xi(s)\,ds \le -\sum_{j=1}^{r_2}\left(\int_{t-(j/r_2)d}^{t-((j-1)/r_2)d}\xi(s)\,ds\right)^T \times H^TW_jH\left(\int_{t-(j/r_2)d}^{t-((j-1)/r_2)d}\xi(s)\,ds\right). \tag{22}$$

By Lemma 3, we have

$$-\sum_{j=1}^{r_2}\int_{-(j/r_2)d}^{-((j-1)/r_2)d}\int_{t+\theta}^{t}\eta^T(t)H^TZ_jH\eta(t)\,ds\,d\theta \le -\sum_{j=1}^{r_2}\frac{2}{2j-1}\left(\frac{r_2}{d}\right)^2\left(\int_{-(j/r_2)d}^{-((j-1)/r_2)d}\int_{t+\theta}^{t}\eta(s)\,ds\,d\theta\right)^T \times H^TZ_jH\left(\int_{-(j/r_2)d}^{-((j-1)/r_2)d}\int_{t+\theta}^{t}\eta(s)\,ds\,d\theta\right) = -\sum_{j=1}^{r_2}\frac{2}{2j-1}\left(\frac{r_2}{d}\right)^2\left(\frac{d}{r_2}\xi(t)-\int_{t-(j/r_2)d}^{t-((j-1)/r_2)d}\xi(s)\,ds\right)^T \times H^TZ_jH\left(\frac{d}{r_2}\xi(t)-\int_{t-(j/r_2)d}^{t-((j-1)/r_2)d}\xi(s)\,ds\right). \tag{23}$$

From (16), for any appropriately dimensioned matrix M, we have

$$2\eta^T(t)H^TM^TH\left[u(t)\,dt+\pi(t)\,d\omega(t)-\eta(t)\,dt\right]=0. \tag{24}$$

On the other hand, (2) implies that there exists $\varepsilon>0$ such that

$$\xi^T(t)\varepsilon H^TF_1^TF_1H\xi(t) + \xi^T(t-h)\varepsilon H^TF_2^TF_2H\xi(t-h)-\varepsilon\overline{f}^T\overline{f}\ge 0, \tag{25}$$

where we take $\overline{f}$ for $\overline{f}(x(t),x(t-h),t)$, for simplicity of notation.

Notice the fact of (3), and from (13), we have

$$\operatorname{trace}\left(\pi^T(t)P\pi(t)\right) \le \xi^T(t)\rho H^TG_1^TG_1H\xi(t)+\xi^T(t-h)\rho H^TG_2^TG_2H\xi(t-h). \tag{26}$$

Combine (20)–(26); then

$$\mathbf{L}V(x_t,t)\le\zeta^T(t)\overline{\Phi}\zeta(t)+2\eta^T(t)H^TM^TH\pi(t)\,d\omega(t), \tag{27}$$

where

$$\zeta^T(t)=\left[\xi_{p1}^T(t)\ \ \xi_{p2}^T(t)\ \ \overline{f}(t)\ \ \eta^T(t)H^T\right],$$
$$\xi_{p1}^T=\left[\xi^T(t)\ \ \xi^T\left(t-\frac{1}{r_1}h\right)H^T\ \ \xi^T\left(t-\frac{2}{r_1}h\right)H^T\ \cdots\ \xi^T(t-h)H^T\right],$$
$$\xi_{p2}^T(t)=\left[\int_{t-(1/r_2)d}^{t}\xi^T(s)H^Tds\ \ \int_{t-(2/r_2)d}^{t-(1/r_2)d}\xi^T(s)H^Tds\ \cdots\ \int_{t-d}^{t-((r_2-1)/r_2)d}\xi^T(s)H^Tds\right], \tag{28}$$
$$\overline{\Phi}=\begin{bmatrix}\Phi_{11}&\Phi_{12}&\Phi_{13}&P\overline{A}_3&\overline{A}^TH^TM\\ *&\Phi_{22}&0&0&\Phi_{26}\\ *&*&\Phi_{33}&0&\Phi_{36}\\ *&*&*&-\varepsilon I&\overline{A}_3^TH^TM\\ *&*&*&*&\Phi_{66}\end{bmatrix}.$$

Thus,

$$\mathbf{E}\left\{\mathbf{L}V(\xi_t,t)\right\}\le\mathbf{E}\left\{\zeta^T(t)\overline{\Phi}\zeta(t)\right\}. \tag{29}$$

By Schur complement lemma, it is easy to show that $\Phi<0$ implies $\overline{\Phi}<0$. Combined with (29), these imply that, for any $\zeta(t)\ne 0$, we have $\mathbf{E}\{\mathbf{L}V(\xi_t,t)\}<0$.

By Dynkin's formula, there exists $\beta > 0$, such that

$$e^{\beta t}\mathbf{E}V(\xi_t, t) \le \mathbf{E}V(\xi_0, 0). \tag{30}$$

Recalling the Lyapunov-Krasovskii functional in (18), notice the fact that there always exists $\kappa > 0$ satisfying

$$|\eta(t)|^2 \le \kappa|\xi(t)|^2 \tag{31}$$

for any $-\tau \le t \le 0$ such that

$$\mathbf{E}V(\xi_0, 0) \le \sum_{i=1}^{5}\alpha_i \sup_{-\tau\le s\le 0}\mathbf{E}|\xi(s)|^2, \tag{32}$$

where $\alpha_1 = \lambda_{\max}(P)$, $\alpha_2 = h\max_i\{\|Q_i\|\}$, $\alpha_3 = (\kappa h^3/2r_1)\max_i\{\|R_i\|\}$, $\alpha_4 = (d^3/2r_2)\max_j\{\|W_j\|\}$, and $\alpha_5 = (\kappa d^3/6)\max_j\{\|Z_j\|\}$.

On the other hand, from (18)

$$\mathbf{E}V(\xi_t, t) \ge \lambda_{\min}(P)\,\mathbf{E}|\xi(t)|^2. \tag{33}$$

From (32) and (33), it can be easily obtained that

$$\mathbf{E}|\xi(t;\tilde{\varphi})|^2 \le \alpha e^{-\beta t}\sup_{-\tau\le s\le 0}\mathbf{E}|\xi(s)|^2, \tag{34}$$

where $\alpha = \sum_{i=1}^{5}\alpha_i/\lambda_{\min}(P)$ and $\tilde{\varphi}$ is the initial condition of filtering error system (6). Then by exponential stability definition of stochastic systems [39], the filtering error system (6) with $v(t) = 0$ is exponentially stable in the sense of mean square.

Now, we will establish the L_2-L_∞ performance for the filtering error system (6). To this end, we assume the zero initial condition $\zeta(t) = 0$ for $t \in [-\tau, 0]$. Under the initial condition, it is easy to see that, for any $t > 0$,

$$\mathbf{E}\{V(\xi_t, t)\} = \mathbf{E}\left\{\int_0^t \mathbf{L}V(\xi_s, s)\,ds\right\}. \tag{35}$$

Define

$$J(t) = \mathbf{E}\{V(\xi_t, t)\} - \int_0^t v^T(s)v(s)\,ds. \tag{36}$$

Then, for any nonzero $v(t) \in \mathbf{L}_2[0,\infty)$ and $t > 0$, combined with (29), (35)-(36), we have

$$\begin{aligned} J(t) &= \mathbf{E}\left\{\int_0^t \left[\mathbf{L}V(\xi_s, s) - v^T(s)v(s)\right]ds\right\} \\ &\le \mathbf{E}\left\{\int_0^t \vartheta^T(s)\Phi\vartheta(s)\,ds\right\}, \end{aligned} \tag{37}$$

where $\vartheta^T(t) = \left[\xi_{p1}^T(t)\ \ \xi_{p2}^T(t)\ \ \overline{f}^T\ \ v^T(t)\ \ \eta^T(t)H^T\right]$. $\Phi < 0$ ensuring that $J(t) \le 0$. Thus,

$$\mathbf{E}\left\{\xi^T(t)P\xi(t)\right\} \le \mathbf{E}\{V(\xi_t, t)\} \le \int_0^t v^T(s)v(s)\,ds. \tag{38}$$

Moreover, by Schur complement, (14) holds if and only if

$$\overline{L}^T\overline{L} < \gamma^2 P. \tag{39}$$

It follows from (38) and (39) that

$$\begin{aligned} \mathbf{E}\left\{|e(t)|^2\right\} &= \mathbf{E}\left\{\xi^T(t)\overline{L}^T\overline{L}\xi(t)\right\} < \gamma^2\mathbf{E}\left\{\xi^T(t)P\xi(t)\right\} \\ &\le \gamma^2\mathbf{E}\{V(\xi_t, t)\} \le \gamma^2\int_0^t v^T(s)v(s)\,ds. \end{aligned} \tag{40}$$

Therefore, if (12)–(14) hold, the filtering error system (6) is mean-square exponentially stable with a prescribed L_2-L_∞ performance γ under zero initial condition. This completes the proof. □

In system (1), if $A_3 = \mathbf{0}$ and $g(x(t), x(t-h), t) = Bx(t) + B_1x(t-h) + B_2\int_{t-d}^{t}x(s)ds + B_v v(t)$, then the linear stochastic system with mixed delays can be written as

$$\begin{aligned} dx(t) = {} & \Big[Ax(t) + A_1x(t-h) \\ & + A_2\int_{t-d}^{t}x(s)\,ds + A_v v(t)\Big]dt \\ & + \Big[Bx(t) + B_1x(t-h) \\ & + B_2\int_{t-d}^{t}x(s)\,ds + B_v v(t)\Big]d\omega(t), \\ dy(t) = {} & \Big[Cx(t) + C_1x(t-h) + C_2\int_{t-d}^{t}x(s)\,ds + C_v v(t)\Big]dt, \\ z(t) = {} & Lx(t), \\ x(t) = {} & \varphi(t), \quad \forall t \in [-\tau, 0] \end{aligned} \tag{41}$$

which is the same as the system in [38] with constant delays. Thus, following the similar lines in Theorem 4, a sufficient condition can be obtained guaranteeing that there exists a linear filter (4) such that the filtering error system is exponentially stable and achieves a prescribed L_2-L_∞ performance γ.

Corollary 5. *Consider the stochastic time-delay system* (41). *For given scalars $\gamma > 0$, $h > 0$, and $d > 0$ and integers $r_1 \ge 1$ and $r_2 \ge 1$, there exists a linear filter* (4) *such that the corresponding filtering error system is exponentially stable with a guaranteed L_2-L_∞ performance γ, if there exist symmetrical positive definite matrices $P \in \mathbf{R}^{2n\times 2n}$, $Q_i \in \mathbf{R}^{n\times n}$, $R_i^T \in \mathbf{R}^{n\times n}$ $(i = 1, 2, \ldots, r_1)$, $W_j \in \mathbf{R}^{n\times n}$, and $Z_j \in \mathbf{R}^{n\times n}$ $(j = 1, 2, \ldots, r_2)$ and matrix $M \in \mathbf{R}^{n\times n}$ satisfying* (14) *and*

$$\widetilde{\Phi} = \begin{bmatrix} \widetilde{\Phi}_{11} & \Phi_{12} & \Phi_{13} & P\overline{A}_v & \overline{A}^T H^T M \\ * & \widetilde{\Phi}_{22} & 0 & 0 & \Phi_{26} \\ * & * & \Phi_{33} & 0 & \Phi_{36} \\ * & * & * & -I & \overline{A}_v^T H^T M \\ * & * & * & * & \Phi_{66} \end{bmatrix} + B_\xi^T P B_\xi < 0, \tag{42}$$

where $\widetilde{\Phi}_{11} = \Phi_{11} - \rho H^T G_1^T G_1 H - \varepsilon H^T F_1^T F_1 H$, $\widetilde{\Phi}_{22} = \Phi_{22} - K_{r_1}^T(\rho G_2^T G_2 + \varepsilon F_2^T F_2)K_{r_1}$, and $B_\xi = \left[\overline{B}\ \ \overline{B}_1 K_{r_1}\ \ \overline{B}_2 K_{r_2}\ \ \overline{B}_v\right]$.

4. Filter Design

In this section, we will focus on the design of L_2-L_∞ filter for stochastic system (1). Based on Theorem 4, a delay-dependent sufficient condition will be obtained in the forms of LMI, which ensures that the filtering error system (6) is stochastically asymptotically stable and achieves a prescribed L_2-L_∞ performance γ.

Theorem 6. *Consider the stochastic time-delay system* (1). *For given scalars $\gamma > 0$, $h > 0$, $d > 0$, $\rho > 0$, and $\varepsilon > 0$ and integers $r_1 \geq 1$ and $r_2 \geq 1$, there exists a linear filter* (4) *such that the filtering error system* (6) *is stochastically asymptotically stable with a prescribed L_2-L_∞ performance γ, if there exist symmetrical positive definite matrices $X \in \mathbf{R}^{n\times n}$, $Y \in \mathbf{R}^{n\times n}$, $Q_i \in \mathbf{R}^{n\times n}$, $R_i \in \mathbf{R}^{n\times n}$ $(i = 1, 2, \ldots, r_1)$, $W_j \in \mathbf{R}^{n\times n}$, and $Z_j \in \mathbf{R}^{n\times n}$ $(j = 1, 2, \ldots, r_2)$ and matrices $M \in \mathbf{R}^{n\times n}$, $\widehat{A}_f \in \mathbf{R}^{n\times m}$, $\widehat{B}_f \in \mathbf{R}^{n\times n}$, and $\widehat{C}_f \in \mathbf{R}^{n\times p}$ satisfying*

$$\Upsilon = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14} & \Upsilon_{15} & \Upsilon_{16} \\ * & \Phi_{22} & 0 & 0 & 0 & \Upsilon_{26} \\ * & * & \Phi_{33} & 0 & 0 & \Upsilon_{36} \\ * & * & * & -\varepsilon I & 0 & A_3^T M \\ * & * & * & * & -I & A_v^T M \\ * & * & * & * & * & \Phi_{66} \end{bmatrix} < 0, \quad (43)$$

$$\begin{bmatrix} X - \rho I & Y \\ Y & (1-\rho) Y \end{bmatrix} < 0, \quad (44)$$

$$\Pi_2 \leq I, \quad (45)$$

$$\Lambda = \begin{bmatrix} X & Y & L^T \\ * & Y & -\widehat{C}_f^T \\ * & * & \gamma^2 I \end{bmatrix} > 0, \quad (46)$$

where

$$\Upsilon_{11} = \begin{bmatrix} \Upsilon_{11}^1 & \widehat{A}_f + A^T Y + C^T \widehat{B}_f^T \\ * & \widehat{A}_f + \widehat{A}_f^T \end{bmatrix},$$

$$\Upsilon_{12} = \begin{bmatrix} R_1 K_1 + \left(X A_1 + \widehat{B}_f C_1\right) K_{r_1} \\ \left(Y A_1 + \widehat{B}_f C_1\right) K_{r_1} \end{bmatrix},$$

$$\Upsilon_{13} = \begin{bmatrix} \left(X A_2 + \widehat{B}_f C_2\right) K_{r_2} + \overline{Z} \\ \left(Y A_2 + \widehat{B}_f C_2\right) K_{r_2} \end{bmatrix},$$

$$\Upsilon_{11}^1 = \operatorname{sym}\left(XA + \widehat{B}_f C\right) + Q_1 - R_1 + \left(\frac{d}{r_2}\right)^2 \sum_{j=1}^{r_2} W_j - \sum_{j=1}^{r_2} \frac{2}{2j-1} Z_j + \rho G_1^T G_1 + \varepsilon F_1^T F_1,$$

$$\Upsilon_{14} = \begin{bmatrix} X A_3 \\ Y A_3 \end{bmatrix}, \qquad \Upsilon_{15} = \begin{bmatrix} X A_v + \widehat{B}_f C_v \\ Y A_v + \widehat{B}_f C_v \end{bmatrix},$$

$$\Upsilon_{16} = \begin{bmatrix} A^T M \\ 0_{n,n} \end{bmatrix}, \qquad \Upsilon_{26} = K_{r_1}^T A_1^T M, \qquad \Upsilon_{36} = K_{r_2}^T A_2^T M. \quad (47)$$

In this case, the parameters of a desired filter in the form of (4) *can be taken as*

$$A_f = \Pi_1^{-1} \widehat{A}_f \Pi_1^{-T} \Pi_2, \qquad B_f = \Pi_1^{-1} \widehat{B}_f, \qquad C_f = \widehat{C}_f \Pi_1^{-T} \Pi_2, \quad (48)$$

where Π_1 and Π_2 are nonsingular matrices satisfying $0 < \Pi_2 = \Pi_2^T \leq I$, $0 < Y = \Pi_1 \Pi_2^{-1} \Pi_1^T$, and $X > Y > 0$.

Proof. From (46), it can be seen that $\left[\begin{smallmatrix} X & Y \\ Y & Y \end{smallmatrix}\right] > 0$, and $X > Y > 0$. For any positive definite and symmetric matrix Y, one can always find nonsingular matrix $\Pi_1 \in \mathbf{R}^{n\times n}$ and $0 < \Pi_2 = \Pi_2^T \in \mathbf{R}^{n\times n}$, such that $Y = \Pi_1 \Pi_2^{-1} \Pi_1^T$.

Set

$$P = \begin{bmatrix} X & \Pi_1 \\ \Pi_1^T & \Pi_2 \end{bmatrix}. \quad (49)$$

Then $X - \Pi_1 \Pi_2^{-1} \Pi_1^T = X - Y > 0$, and $P > 0$.

Define

$$\Pi = \begin{bmatrix} I_n & 0_{n,n} \\ 0_{n,n} & \Pi_2^{-1} \Pi_1^T \end{bmatrix}. \quad (50)$$

Then

$$\Pi^{-1} = \begin{bmatrix} I_n & 0_{n,n} \\ 0_{n,n} & \Pi_1^{-T} \Pi_2 \end{bmatrix}. \quad (51)$$

Substitute $\widehat{A}_f = \Pi_1 A_f \Pi_2^{-1} \Pi_1^T$, $\widehat{B}_f = \Pi_1 B_f$, $\widehat{C}_f = C_f \Pi_2^{-1} \Pi_1^T$, and $Y = \Pi_1 \Pi_2^{-1} \Pi_1^T$ into (43), (44), and (46) and then pre- and postmultiply (43) by $\operatorname{diag}\{\Pi^{-T}, I_{r_1 n}, I_{r_2 n}, I_n, I_n, I_n\}$ and its transpose, respectively. Premultiply and postmultiply (46) by $\operatorname{diag}\{\Pi^{-T}, I_p\}$ and its transpose, respectively. Noticing that $P = \left[\begin{smallmatrix} X & \Pi_1 \\ \Pi_1^T & \Pi_2 \end{smallmatrix}\right]$, using Schur complement Lemma, one can obtain (12) and (14).

On the other hand, (44) implies

$$\begin{bmatrix} X & Y \\ Y & Y \end{bmatrix} < \rho \begin{bmatrix} I & 0 \\ 0 & Y \end{bmatrix}. \quad (52)$$

Pre- and postmultiply (52) by Π^{-T} and Π^{-1}, respectively. Notice that $Y = \Pi \Lambda^{-1} \Pi^T$, one can obtain

$$\begin{bmatrix} X & \Pi_1 \\ \Pi_1^T & \Pi_2 \end{bmatrix} < \rho \begin{bmatrix} I & 0 \\ 0 & \Pi_2 \end{bmatrix}. \quad (53)$$

By (45), it is easy to see that

$$\begin{bmatrix} X & \Pi_1 \\ \Pi_1^T & \Pi_2 \end{bmatrix} < \rho \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \quad (54)$$

So, (13) is satisfied.

Therefore, by Theorem 4, the suitable filter parameters can be constructed by (48), which ensures the filtering error system (6) to be stochastically asymptotically stable with L_2-L_∞ performance γ. This completes the proof. □

Remark 7. When deriving the results in Theorem 6 based on Theorem 4, considering dealing with the LMI (13), we give a method to avoid nonlinear terms emerging. Using Matlab LMI toolbox, one can solve linear matrix inequalities (43)-(44) and (46). Then, by matrix diagonalization approach, one can easily find that diagonally positive matrix Π_2 and nonsingular matrix Π_1 satisfy $\Pi_2 = \Pi_1^T Y^{-1} \Pi_1$. If the obtained matrix Π_2 does not satisfy (45), one can take Π_2 for $\Pi_2/\max\{\text{eig}(\Pi_2)\}$ and Π_1 for $\sqrt{\max\{\text{eig}(\Pi_2)\}}\Pi_1$. Thus, the desired filter parameters can be obtained by (48).

Following the similar method in Theorem 6, one can obtain a result of filter design for linear stochastic time-delay system (41).

Corollary 8. *Consider the stochastic time-delay system* (41). *For given scalars $\gamma > 0$, $h > 0$, and $d > 0$ and integers $r_1 \geq 1$ and $r_2 \geq 1$, there exists a linear filter* (4) *such that the corresponding filtering error system is stochastically asymptotically stable with a prescribed L_2-L_∞ performance γ, if there exist symmetrical positive definite $X \in \mathbf{R}^{n\times n}$, $Y \in \mathbf{R}^{n\times n}$, $Q_i \in \mathbf{R}^{n\times n}$, $R_i \in \mathbf{R}^{n\times n}$ $(i = 1, 2, \ldots, r_1)$, $W_j \in \mathbf{R}^{n\times n}$, and $Z_j \in \mathbf{R}^{n\times n}$ $(j = 1, 2, \ldots, r_2)$ and matrices $M \in \mathbf{R}^{n\times n}$, $\widehat{A}_f \in \mathbf{R}^{n\times m}$, $\widehat{B}_f \in \mathbf{R}^{n\times n}$, and $\widehat{C}_f \in \mathbf{R}^{n\times p}$ satisfying* (46) *and*

$$\widetilde{\Upsilon} = \begin{bmatrix} \widetilde{\Upsilon}_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{15} & \Upsilon_{16} & \Upsilon_{17} \\ * & \widetilde{\Phi}_{22} & 0 & 0 & \Upsilon_{26} & \Upsilon_{27} \\ * & * & \Phi_{33} & 0 & \Upsilon_{36} & \Upsilon_{37} \\ * & * & * & -I & A_v^T M & \Upsilon_{47} \\ * & * & * & * & \Phi_{66} & 0 \\ * & * & * & * & * & \Upsilon_{77} \end{bmatrix} < 0, \quad (55)$$

where

$$\widetilde{\Upsilon}_{11} = \begin{bmatrix} \widetilde{\Upsilon}_{11}^1 & \widehat{A}_f + A^T Y + C^T \widehat{B}_f^T \\ * & \widehat{A}_f + \widehat{A}_f^T \end{bmatrix},$$

$$\widetilde{\Upsilon}_{11}^1 = \Upsilon_{11}^1 - \rho G_1^T G_1 - \varepsilon F_1^T F_1, \qquad \Upsilon_{17} = \begin{bmatrix} B^T X & B^T Y \\ 0_{n,n} & 0_{n,n} \end{bmatrix},$$

$$\Upsilon_{27} = K_{r_1}^T \left[B_1^T X \;\; B_1^T Y \right], \qquad \Upsilon_{37} = K_{r_2}^T \left[B_2^T X \;\; B_2^T Y \right],$$

$$\Upsilon_{47} = \left[B_v^T X \;\; B_v^T Y \right], \qquad \Upsilon_{77} = \begin{bmatrix} -X & -Y \\ * & -Y \end{bmatrix}. \quad (56)$$

Remark 9. The results presented in Theorem 6 and Corollary 8 can be easily extended to the systems with only discrete or distributed delays and also to the robust performance analysis for uncertain stochastic systems with mixed delays.

5. Numerical Examples

Example 1. Consider the stochastic time-delay system (1) with parameters

$$A = \begin{bmatrix} -1.5 & 0.5 \\ -1 & -3 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -0.8 & 0.2 \\ 0.2 & -0.5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$A_v = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.2 \end{bmatrix}, \qquad C = \begin{bmatrix} 2 & -0.5 \\ -1.5 & 0.5 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.15 & 0.1 \\ -0.1 & 0.1 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.5 & -0.2 \\ 0.6 & 0 \end{bmatrix},$$

$$C_v = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.03 \end{bmatrix}, \qquad L = \begin{bmatrix} 0.1 & -0.2 \\ 0 & 0.1 \end{bmatrix},$$

$$\varepsilon = 1, \qquad \rho = 7. \quad (57)$$

Table 1: The upper bound of $d_{\max}$ for $h = 1$ and $\gamma = 0.2$.

Methods	$d_{\max}$
Theorem 6 ($r_1 = 1, r_2 = 1$)	7.481
Theorem 6 ($r_1 = 2, r_2 = 2$)	8.190
Theorem 6 ($r_1 = 3, r_2 = 3$)	8.317
Theorem 6 ($r_1 = 5, r_2 = 5$)	8.379

Moreover, for the nonlinear functions, we let $G_1 = G_2 = 0.1I$ and $F_1 = F_2 = 0.1I$. Given $h = 1$ and $\gamma = 0.2$, from Theorem 6, one can obtain the upper bound of time delay d, which is listed in Table 1.

In the case of $r_1 = 2$ and $r_2 = 2$, the desired filter parameters can be obtained:

$$A_f = \begin{bmatrix} -6.1460 & 2.2644 \\ 0.6168 & -4.5679 \end{bmatrix}, \qquad B_f = \begin{bmatrix} -3.8749 & 1.8868 \\ 0.7066 & -0.1266 \end{bmatrix},$$

$$C_f = \begin{bmatrix} -0.0568 & 0.0052 \\ 0.0202 & 0.0057 \end{bmatrix}. \quad (58)$$

Example 2. Consider the stochastic time-delay system (36)–(39) with parameters

$$A = \begin{bmatrix} -1.5 & 0.5 \\ -1 & -3 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -0.8 & 0.2 \\ 0.2 & -0.5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad A_v = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.2 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.8 & 0.2 \\ 0.5 & -0.5 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.3 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad B_v = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad (59)$$

$$C = \begin{bmatrix} 2 & -0.5 \\ -1.5 & 0.5 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0.15 & 0.1 \\ -0.1 & 0.1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.5 & -0.2 \\ 0.6 & 0 \end{bmatrix}, \qquad C_v = \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.03 \end{bmatrix},$$

$$L = \begin{bmatrix} 0.1 & -0.2 \\ 0 & 0.1 \end{bmatrix}.$$

Table 2: The upper bound of $h_{\max}$ for $d = 1$ and $\gamma = 0.2$.

Methods	$h_{\max}$
[38]	1.725
Corollary 8 ($r_1 = 1, r_2 = 1$)	3.755
Corollary 8 ($r_1 = 2, r_2 = 1$)	5.054
Corollary 8 ($r_1 = 2, r_2 = 2$)	5.111
Corollary 8 ($r_1 = 3, r_2 = 3$)	5.688
Corollary 8 ($r_1 = 5, r_2 = 5$)	5.920

Given $d = 1$ and $\gamma = 0.2$, from Corollary 8, one can obtain the upper bound of time delay h. Table 2 lists the results of Corollary 8 and [38] with constant delays. It is easy to see that the proposed filter design method in this paper is less conservative than [38].

From Corollary 8, in the case of $r_1 = 2$ and $r_2 = 2$, the desired filter parameters can be obtained:

$$A_f = \begin{bmatrix} -0.0000 & -4.1570 \\ -0.0000 & -2.5527 \end{bmatrix}, \qquad B_f = \begin{bmatrix} 10.1507 & -11.1236 \\ 6.0149 & -6.1484 \end{bmatrix},$$
$$C_f = \begin{bmatrix} 0.0000 & 0.0065 \\ -0.0000 & -0.0027 \end{bmatrix}. \tag{60}$$

Remark 3. It can be seen from the results that the conservatism can be reduced with the increase of partition integers. However, it is necessary to point out that the less conservatism is at the cost of a higher computational complexity.

6. Conclusions

In this paper, a new approach has been developed to investigate the problems of delay-dependent L_2-L_∞ filter design for stochastic system with mixed delays and nonlinear perturbations. Based on the idea of delay partitioning and integral partitioning, using Lyapunov-Krasovskii functional approach, a delay-dependent sufficient condition has been established that ensures the filtering error system is exponentially stable with L_2-L_∞ performance γ. By solving the LMIs, one can get the desired filter gain matrices. The results also depend on the partition integers with the increase of partition integers, the conservatism can be decreased. Finally, numerical examples are presented to demonstrate the effectiveness of the proposed approach.

Acknowledgments

The project is supported by Nature Science Foundation of China (NSFC, Grant no. 51277022). The authors would also like to thank the editor and the anonymous reviewers for their valuable comments and suggestions which helped to improve the quality of this paper.

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