Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 616289, 13 pages
http://dx.doi.org/10.1155/2013/616289

Research Article

Synchronization of Coupled Networks with Uncertainties

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Received 25 September 2013; Accepted 5 November 2013

Academic Editor: Qiankun Song

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Asymptotic synchronization for a class of coupled networks with nondelayed and delayed couplings is investigated. A distinct feature of the network is that all the dynamical nodes are affected by uncertain nonlinear nonidentical perturbations. In order to synchronize the network onto a given isolate trajectory, a novel adaptive controller is designed to overcome the effects of the nonidentical uncertain nonlinear perturbations. The designed controller has better robustness than classical adaptive controller, since it can realize the synchronization goal whether the nodes have these perturbations or not. Based on the Lyapunov stability theory and the Barbalat lemma, sufficient conditions guaranteeing the asymptotic synchronization of the coupled network are derived. Two examples with numerical simulations are given to illustrate the effectiveness of the theoretical results. Simulations also demonstrate that our adaptive controller has better robustness than existing ones.

1. Introduction

A coupled network or complex network is a set of interconnected nodes, where each node is a dynamical system. In fact, many natural and man-made systems, such as the Internet networks [1], biological networks [2], epidemic spreading networks [3], collaborative networks [4], and social networks [5], can be described by coupled networks. Since the seminal work of Wu and Chua [6], much attention has been payed to the dynamical behaviors in an array of coupled networks [7–9]. A coupled network can exhibit complicated dynamical behaviors or collective behaviors which may be absolutely different from those of a single node. Hence, investigating the dynamical behaviors of coupled networks is of great importance.

One of the important collective behaviors of coupled networks is synchronization. Actually, many natural phenomena can be well explained by the synchronization of coupled networks. Moreover, synchronization has potential applications in many fields such as secure communication [10, 11] and information processing [12]. Therefore, in recent years, an increasing number of researchers have been devoted to the investigation of synchronization in an array of coupled networks [6, 7, 9, 10, 13–22]. It is well known that sometimes the coupled networks may not be synchronized when a controller is not added into the infrastructure of individual nodes [7]. Thus the controlled synchronization of coupled networks is believed to be a rather significant topic in both theoretical research and practical applications. Consequently, many control methods have been proposed to realize chaos synchronization of coupled networks, for instance, intermittent control [7], state feedback control [9], and adaptive control [23]. Particularly, adaptive control technique is an effective method due to its good robustness.

In the literature, there were many results concerning synchronization of coupled networks via adaptive control [14–16, 19–22]. Cao et al. [14] investigated the complete synchronization in an array of linearly stochastically coupled identical neural networks with delays. In [15], Chen and Zhou studied the synchronization of coupled nondelayed networks under the adaptive control. By designing a simple adaptive controller, authors of [20] investigated the locally and globally adaptive synchronization of an uncertain coupled dynamical network. Recently, authors of [21] considered the local and global synchronization of coupled networks without delays via adaptive pinning control. Authors of [16, 19, 22] investigated the global synchronization of the coupled networks with nondelayed and delayed couplings by utilizing adaptive control technique.

It should be noted that most of the above results are valid only for the coupled networks without perturbation. However, in practice, chaotic systems are inevitably subject to many types of uncertain or even nonlinear perturbations, such as unknown parameters, exogenous disturbances, and artificial factors [23]. For instance, it is reported that the famous Lorenz system is derived from partial differential equations after a series of approximations [24]. Therefore, there usually exist modelling errors between the deterministic model and practical system. One can also find that dynamical nodes are difficult to be identical all the time since the parameters of dynamical nodes may be variant due to some environmental changes [25]. Another example is the social network [5], in which a person is a node and the relationship between persons is the edge. A person's mood may change according to the evolution of the time and environments and hence many uncertainties may interfere with the individual's behaviors. Furthermore, in the process of signal transmitting, the states of subsystems in coupled networks are unavoidably subject to some uncertain perturbations. Thus, synchronization of coupled chaotic systems with uncertain perturbations is essential and useful in both theoretical research and practical applications. Although the usual adaptive technique used in [14–17, 19–22] has good robustness when synchronizing coupled networks without perturbation, it is not always effective to synchronize coupled networks with various types of uncertain nonlinear perturbations. Simulations of this paper show that the adaptive technique used in [14–17, 19–22] cannot effectively synchronize coupled dynamical systems with some uncertain perturbations onto a given trajectory. Note that the uncertain perturbations of this paper are not stochastic as those in [7, 9, 19]. A distinct feature of the uncertain perturbations in this paper is that their effects cannot disappear even after the synchronization has been realized.

On the other hand, time delays usually exist in spreading due to the finite speeds of transmission as well as traffic congestions. Therefore, it is significant to investigate the synchronization in an array of coupled dynamical systems with delayed couplings. In [14, 16, 22], synchronization in an array of coupled dynamical systems with delayed couplings was studied. However, to the best of our knowledge, result on synchronization in an array of coupled systems with nondelayed and delayed couplings and uncertain nonlinear perturbations is seldom.

Motivated by the above analysis, in this paper, a general model of coupled networks with nondelayed and delayed couplings as well as uncertain nonlinear perturbations is proposed. Our model is applicable to most of known chaotic systems with or without uncertainties. The coupling configuration matrices are not assumed to be symmetric. Global asymptotic synchronization of the proposed model is studied. A new simple but robust adaptive controller is designed to overcome the effects of uncertainties and nonlinear external perturbations. The designed controller can synchronize the considered network to a given isolate trajectory. Moreover, our controller includes controllers used in [14–17, 19–22] as a special case. The designed adaptive controller can also synchronize the coupled dynamical network without any uncertainty and perturbation. Based on the Lyapunov stability theory and the Barbalat lemma, sufficient conditions are obtained to guarantee the realization of the synchronization of the coupled network. Numerical simulations verify the effectiveness of the theoretical results. Simulations also demonstrate that our adaptive controller has better robustness than those used in [14–17, 19–22].

The rest of this paper is organized as follows. In Section 2, a class of general coupled networks with uncertainties is proposed. Some necessary assumptions and lemmas are also given in this section. In Section 3, synchronization of the coupled networks with or without uncertainties is studied. Then, in Section 4, numerical simulations are given to show the effectiveness of our results. Section 5 concludes the investigation and expresses the acknowledgements. Future research field is also discussed in this section.

Notations. In the sequel, if not explicitly stated, matrices are assumed to have compatible dimensions. I_N stands for the identity matrix of N-dimension. $\mathbb{R}^n$ is the Euclidean space of n-dimension and $\mathbb{R}^+$ is the set of nonnegative real numbers. For vector $x \in \mathbb{R}^n$, the Euclidean norm is $\|x\| = \sqrt{x^T x}$, where T denotes transposition. $A = (a_{ij})_{m\times m}$ denotes a matrix of m-dimension, $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$, $A^s = (1/2)(A + A^T)$. $A > 0$ or $A < 0$ means that the matrix A is symmetric and positive or negative definite matrix. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are the minimum and maximum eigenvalues of the matrix A, respectively.

2. Model Formulation and Preliminaries

The general coupled network consisting of N identical nodes with nondelayed and delayed couplings as well as uncertain nonlinear perturbations is described as follows:

$$\begin{aligned}\dot{x}_i(t) &= f_1(x_i(t)) + f_2(x_i(t-\tau(t))) \\ &\quad + \sum_{j=1}^{N} u_{ij}\Phi x_j(t) + \sum_{j=1}^{N} v_{ij}\Upsilon x_j(t-\tau(t)) \\ &\quad + \sigma_i(t, x_i(t), x_i(t-\tau(t))) + R_i, \quad i = 1, 2, \ldots, N,\end{aligned} \tag{1}$$

where $x_i(t) = (x_{i1}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ represents the state vector of the ith node of the network at time t and $f_1(\cdot)$ and $f_2(\cdot)$ are continuous vector functions. $R_i \in \mathbb{R}^n$ is the control input. $\tau(t) > 0$ is time-varying delay and $\Phi, \Upsilon \in \mathbb{R}^{n\times n}$ are inner coupling matrices of the network, which describe the individual coupling between two subsystems. Matrices $U = (u_{ij})_{N\times N}$ and $V = (v_{ij})_{N\times N}$ are outer couplings of the whole network satisfying the following diffusive conditions:

$$u_{ij} \geq 0 \quad (i \neq j), \qquad u_{ii} = -\sum_{j=1, j\neq i}^{N} u_{ij}, \quad i, j = 1, 2, \ldots, N,$$

$$v_{ij} \geq 0 \quad (i \neq j), \qquad v_{ii} = -\sum_{j=1, j\neq i}^{N} v_{ij}, \quad i, j = 1, 2, \ldots, N. \tag{2}$$

Vector $\sigma_i(t, x_i(t), x_i(t-\tau(t))) = (\sigma_{i1}(t, x_i(t), x_i(t-\tau(t))), \ldots, \sigma_{in}(t, x_i(t), x_i(t-\tau(t))))^T \in \mathbb{R}^n$ describes the uncertain nonlinear perturbation of ith node of the coupled systems at time t. In this paper, we always assume that the derivative of $\tau(t)$ satisfies $\dot{\tau}(t) \leq h_\tau < 1$, where h_τ is constant.

We assume that (1) has a unique continuous solution for any initial condition in the following form:

$$x_i(s) = \varphi_i(s), \quad -\tau \leq s \leq 0, \; i = 1, 2, \ldots, N, \tag{3}$$

where τ is the maximum of $\tau(t)$. For convenience of writing, in the sequel, we denote $\sigma_i(t, x_i(t), x_i(t-\tau(t)))$ as $\sigma_i(t)$.

The aim of this paper is to synchronize all the states of the coupled network (1) to the following manifold:

$$x_1(t) = x_2(t) = \cdots = x_N(t) = z(t), \tag{4}$$

where $z(t)$ is the state of an isolate node system without perturbation, which is described as follows:

$$\dot{z}(t) = f_1(z(t)) + f_2(z(t-\tau(t))), \tag{5}$$

and $z(t)$ can be any desired state: equilibrium point, a non-trivial periodic orbit, or even a chaotic orbit.

The following assumptions are needed in this paper.

(H_1) $f_1(0) = f_2(0) \equiv 0$ and there exist positive constants h_1 and h_2 such that

$$\begin{gathered} \|f_1(u) - f_1(v)\| \leq h_1 \|u - v\|, \\ \|f_2(u) - f_2(v)\| \leq h_2 \|u - v\|, \\ \text{for any } u, v \in \mathbb{R}^n. \end{gathered} \tag{6}$$

(H_2) Solutions of all the nodes in network (1) without control are bounded; that is, there exists a positive constant D such that $\|x_i(t)\| \leq D$, $i = 1, 2, \ldots, N$.

(H_3) $\sigma_{ik}(t, 0, 0) \equiv 0$, and for any positive constants D and D' such that $\|u\| \leq D$ and $\|v\| \leq D'$, $u, v \in \mathbb{R}^n$, there exists positive constant M_{ik} such that $|\sigma_{ik}(t, u, v)| \leq M_{ik}$, $i = 1, 2, \ldots, N$, $k = 1, 2, \ldots, n$.

Remark 1. It follows from (H_1) that system (5) unifies many well-known chaotic systems with or without delays such as Chua system [26], Lorenz system [27], Rössler system [28], Chen system [29], delayed chaotic neural networks [14], and delayed Chua system [30, 31]. A similar condition to (H_1) is also used in [32–34] for neural networks. Hence, results of this paper are general.

Remark 2. Conditions (H_2) and (H_3) are reasonable. We do not impose the usual condition such as Lipschitz condition or differentiability on the uncertain perturbation function $\sigma_i(t, x_i(t), x_i(t-\tau(t)))$. It can be discontinuous or even impulsive function. If the state of each node system in (1) is an equilibrium point or a nontrivial periodic orbit, the condition (H_2) can be easily satisfied. If the state of each node in system (1) is a chaotic orbit, the condition (H_2) can also be satisfied. Since chaotic system has strange attractors, there exists a bounded region containing all attractors of it such that every orbit of the system never leaves them. Anyway, condition (H_2) can be satisfied for equilibrium point, a nontrivial periodic orbit, and a chaotic orbit. For such three cases, all the solutions of (1) are bounded and (H_3) is satisfied. Moreover, we shall subsequently prove that, with suitable robust adaptive controller, the coupled network (1) can be synchronized to the trajectory of (5) even without knowing the exact values of h_1, h_2, and M_{ik}, $i = 1, 2, \ldots, N$, $k = 1, 2, \ldots, n$. However, examples at the end of this paper show that the usual adaptive controller used in [14–17, 19–22] is not sufficient to achieve this goal.

Lemma 3 ([35, Barbalat lemma]). *If $f(t) : \mathbb{R} \to \mathbb{R}^+$ is a uniformly continuous function for $t \geq 0$ and if the limit of the integral*

$$\lim_{t \to \infty} \int_0^t f(s)\, \mathrm{d}s \tag{7}$$

exists and is finite, then $\lim_{t\to\infty} f(t) = 0$.

3. Synchronization with Uncertain Nonlinear Perturbations

In this section, an adaptive controller is designed such that the controlled network (1) can be asymptotically synchronized onto the isolate system (5). Corresponding results are also given for the network in the two cases: only partial dynamical nodes are perturbed and none of the dynamical nodes is perturbed. The advantages of the designed adaptive controller over those of existing ones are discussed in Remark 7.

Let $e_i(t) = x_i(t) - z(t)$. Subtracting (5) from (1) yields the following error dynamical system:

$$\begin{aligned} \dot{e}_i(t) = {} & g_1(e_i(t)) + g_2(e_i(t-\tau(t))) \\ & + \sum_{j=1}^{N} u_{ij} \Phi e_j(t) + \sum_{j=1}^{N} v_{ij} \Upsilon e_j(t-\tau(t)) \\ & + \sigma_i(t) + R_i, \quad i = 1, 2, \ldots, N, \end{aligned} \tag{8}$$

where $g_1(e_i(t)) = f_1(x_i(t)) - f_1(z(t))$, $g_2(e_i(t)) = f_2(x_i(t)) - f_2(z(t))$.

From (H_1) and (H_3) we know that (8) admits a trivial solution $e_i(0) \equiv 0$, $i = 1, 2, \ldots, N$. Obviously, to reach goal (4), we only need to prove that the trivial solution of system (8) is asymptotically stable. Theorem 4 is our main result.

Theorem 4. *Under the assumption conditions* (H_1)–(H_3), *the trivial solution of the error system* (8) *is asymptotically stable with the following adaptive controllers:*

$$R_i = -\varepsilon_i(t)e_i(t) - \omega\beta_i(t)\operatorname{sign}(e_i(t)), \quad i = 1,2,\ldots,N,$$
$$\dot{\varepsilon}_i(t) = p_i e_i^T(t)e_i(t), \quad i = 1,2,\ldots,N,$$
$$\dot{\beta}_i(t) = \xi_i\sum_{k=1}^{n}|e_{ik}(t)|, \quad i = 1,2,\ldots,N, \tag{9}$$

where $\operatorname{sign}(e_i(t)) = (\operatorname{sign}(e_{i1}(t)),\ldots,\operatorname{sign}(e_{in}(t)))^T$, $\omega > 1$, $p_i > 0$, *and* $\xi_i > 0$ *are arbitrary constants, respectively.*

Proof. It follows from (H_2) that there exists positive constant D such that $\|x_i(t)\| \le D$ for $t \in [-\tau,+\infty)$ $(i = 1,2,\ldots,N)$. In view of (H_3), there exists positive constant M_{ik} such that $|\sigma_{ik}(t,x_i(t),x_i(t-\tau(t)))| \le M_{ik}$ $(i = 1,2,\ldots,N, k = 1,2,\ldots,n)$. Take $M_i = \max\{M_{ik}, k = 1,2,\ldots,n\}$ $(i = 1,2,\ldots,N)$ and define the Lyapunov function as

$$V(t) = V_1(t) + V_2(t), \tag{10}$$

where

$$V_1(t) = \frac{1}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\frac{(\varepsilon_i(t)-k_i)^2}{p_i} + \frac{1}{2}\sum_{i=1}^{N}\frac{(M_i-\beta_i(t))^2}{\xi_i}, \tag{11}$$
$$V_2(t) = \int_{t-\tau(t)}^{t}\eta^T(s)Q\eta(s)\,ds,$$

in which $\eta(t) = (\|e_1(t)\|,\|e_2(t)\|,\ldots,\|e_N(t)\|)^T$, Q is symmetric positive definite matrix, k_i is positive constant, and k_i and Q are to be determined.

Differentiating $V_1(t)$ along the solution of (8), one obtains from (H_1) and (H_2) that

$$\begin{aligned}
\dot{V}_1(t) &= \sum_{i=1}^{N}e_i^T(t)\dot{e}_i(t) + \sum_{i=1}^{N}(\varepsilon_i(t)-k_i)e_i^T(t)e_i(t) - \sum_{i=1}^{N}(M_i-\beta_i(t))\sum_{k=1}^{n}|e_{ik}(t)| \\
&= \sum_{i=1}^{N}e_i^T(t)\Bigg[g_1(e_i(t)) + g_2(e_i(t-\tau(t))) + \sum_{j=1}^{N}u_{ij}\Phi e_j(t) + \sum_{j=1}^{N}v_{ij}\Upsilon e_j(t-\tau(t))\Bigg] \\
&\quad + \sum_{i=1}^{N}e_i^T(t)\sigma_i(t) - \sum_{i=1}^{N}k_ie_i^T(t)e_i(t) - \omega\sum_{i=1}^{N}\sum_{k=1}^{n}\beta_i(t)|e_{ik}(t)| - \sum_{i=1}^{N}\sum_{k=1}^{n}(M_i-\beta_i(t))|e_{ik}(t)| \\
&\le \sum_{i=1}^{N}h_1\|e_i(t)\|^2 + \sum_{i=1}^{N}h_2\|e_i(t)\|\|e_i(t-\tau(t))\| - \sum_{i=1}^{N}k_i\|e_i(t)\|^2 + \sum_{i=1}^{N}\lambda_{\min}(\Phi^s)u_{ii}\|e_i(t)\|^2 \\
&\quad + \sum_{i=1}^{N}\sum_{j=1,j\ne i}^{N}u_{ij}\|\Phi\|\|e_i(t)\|\|e_j(t)\| + \sum_{i=1}^{N}\sum_{j=1}^{N}|v_{ij}|\|\Upsilon\|\|e_i(t)\|\|e_j(t-\tau(t))\| \\
&= \eta^T(t)\left(h_1I_N + \|\Phi\|\widehat{U}^s - K\right)\eta(t) + \eta^T(t)(h_2I_N + \|\Upsilon\||V|)\eta(t-\tau(t)) \\
&\le \eta^T(t)\left[\left(h_1+\frac{1}{2}\right)I_N + \|\Phi\|\widehat{U}^s - K\right]\eta(t) + \eta^T(t-\tau(t))P^TP\eta(t-\tau(t)),
\end{aligned} \tag{12}$$

where $K = \operatorname{diag}(k_1,k_2,\ldots,k_N)$, $\widehat{U} = (\widehat{u}_{ij})_{N\times N}$, $\widehat{u}_{ij} = u_{ij}$, $i \ne j$, $\widehat{u}_{ii} = (\lambda_{\min}(\Phi^s)/\|\Phi\|)u_{ii}$, $P = h_2I_N + \|\Upsilon\||V|$, $|V| = (|v_{ij}|)_{N\times N}$, and we have used the following deduction:

$$\begin{aligned}
&\sum_{i=1}^{N}e_i^T(t)\sigma_i(t) - \omega\sum_{i=1}^{N}\sum_{k=1}^{n}\beta_i(t)|e_{ik}(t)| - \sum_{i=1}^{N}\sum_{k=1}^{n}[M_i-\beta_i(t)]|e_{ik}(t)| \\
&\le \sum_{i=1}^{N}\sum_{k=1}^{n}[|e_{ik}(t)|M_{ik} - M_i|e_{ik}(t)| - (\omega-1)\beta_i(t)|e_{ik}(t)|] \\
&\le -\sum_{i=1}^{N}\sum_{k=1}^{n}(\omega-1)\beta_i(t)|e_{ik}| \le 0.
\end{aligned} \tag{13}$$

Differentiating $V_2(t)$, one has

$$\begin{aligned}
\dot{V}_2(t) &= \eta^T(t)Q\eta(t) - (1-\dot{\tau}(t))\eta^T(t-\tau(t))Q\eta(t-\tau(t)) \\
&\le \eta^T(t)Q\eta(t) - (1-h_\tau)\eta^T(t-\tau(t))Q\eta(t-\tau(t)).
\end{aligned} \tag{14}$$

Taking $Q = (1/(1-h_\tau))P^T P$, one gets from the definition of $V(t)$ that

$$\dot{V}(t) \le \eta^T(t)\left[\left(h_1 + \frac{1}{2}\right)I_N + \|\Phi\|\widehat{U}^s + \frac{1}{1-h_\tau}P^T P - K\right]\eta(t). \quad (15)$$

Take $k_i = \lambda_{\max}[(h_1+1/2)I_N + \|\Phi\|\widehat{U}^s + (1/(1-h_\tau))P^T P] + 1$. Then, one can easily derive from the above inequality that

$$\dot{V}(t) \le -\eta^T(t)\eta(t). \quad (16)$$

Integrating both sides of the above equation from 0 to t yields

$$V(0) \ge V(t) + \sum_{i=1}^{N}\int_0^t \|e_i(s)\|^2 ds \ge \sum_{i=1}^{N}\int_0^t \|e_i(s)\|^2 ds. \quad (17)$$

Therefore,

$$\lim_{t\to\infty}\sum_{i=1}^{N}\int_0^t \|e_i(s)\|^2 ds \le V(0). \quad (18)$$

It is obvious that $e_i(t)$ $(i = 1,2,\ldots,N)$ is uniformly continuous for $t \ge 0$; hence $\sum_{i=1}^N \|e_i(t)\|^2$ is uniformly continuous for $t \ge 0$ according to the continuity of norm. Moreover, the above inequality means that $\lim_{t\to\infty}\sum_{i=1}^N \int_0^t \|e_i(s)\|^2 ds$ exists and is finite. In view of Lemma 3, one can easily get

$$\lim_{t\to\infty}\sum_{i=1}^{N}\|e_i(t)\|^2 = 0, \quad (19)$$

which in turn means

$$\lim_{t\to\infty}\|e_i(t)\| = 0, \quad i = 1,2,\ldots,N. \quad (20)$$

Therefore, the trivial solution of the error system (8) is asymptotically stable. This completes the proof. □

If some nodes are effected by uncertain perturbations while others are not perturbed, the trivial solution of the error system (8) can also be asymptotically stabilized by the adaptive controllers (9). Without loss of generality, rearrange the order of the nodes in the network, and let the first l nodes have uncertainties; that is, $\sigma_i(t) \ne 0$, $i = 1,2,\ldots,l$, $\sigma_i(t) \equiv 0$, $i = l+1, l+2,\ldots,N$. We have Theorem 5.

Theorem 5. *Suppose $\sigma_i(t) \ne 0$, $i = 1,2,\ldots,l$, $\sigma_i(t) \equiv 0$, $i = l+1, l+2,\ldots,N$. Then, under the assumption conditions (H_1)–(H_3), the trivial solution of the error system* (8) *is asymptotically stable with the adaptive controllers* (9).

Proof. Define the Lyapunov function as

$$\overline{V}(t) = \overline{V}_1(t) + V_2(t), \quad (21)$$

where

$$\overline{V}_1(t) = \frac{1}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\frac{(\varepsilon_i(t)-k_i)^2}{p_i} + \frac{1}{2}\sum_{i=1}^{l}\frac{(M_i-\beta_i(t))^2}{\xi_i}, \quad (22)$$

$V_2(t)$ is defined as that in the proof of Theorem 4.

Differentiating $\overline{V}(t)$ along the solution of (8) and noting that the following inequality holds

$$\begin{aligned}
&\sum_{i=1}^{l}e_i^T(t)\sigma_i(t) - \omega\sum_{i=1}^{l}\sum_{k=1}^{n}\beta_i(t)|e_{ik}(t)| \\
&\quad - \sum_{i=1}^{l}\sum_{k=1}^{n}[M_i - \beta_i(t)]|e_{ik}(t)| - \omega\sum_{i=l+1}^{N}\sum_{k=1}^{n}\beta_i(t)|e_{ik}(t)| \\
&\le \sum_{i=1}^{l}\sum_{k=1}^{n}[|e_{ik}(t)|M_{ik} - M_i|e_{ik}(t)| - (\omega-1)\beta_i(t)|e_{ik}(t)|] \\
&\quad - \omega\sum_{i=l+1}^{N}\sum_{k=1}^{n}\beta_i(t)|e_{ik}(t)| \\
&\le -\sum_{i=1}^{l}\sum_{k=1}^{n}(\omega-1)\beta_i(t)|e_{ik}(t)| \\
&\quad - \omega\sum_{i=l+1}^{N}\sum_{k=1}^{n}\beta_i(t)|e_{ik}(t)| \le 0,
\end{aligned} \quad (23)$$

by the same procedure of the proof of Theorem 4, one can easily finish the proof. This completes the proof. □

A special case of Theorem 5 is that none of the nodes is perturbed. In this case, the designed adaptive controller (9) can obviously realize the synchronization goal. Moreover, we have more general result than the adaptive controllers [14–16, 19–22], which is presented in Corollary 6 and Remark 7.

Corollary 6. *Suppose $\sigma_i(t) \equiv 0$, $i = 1,2,\ldots,N$, and the assumption condition (H_1) holds. Then the trivial solution of the error system* (8) *is asymptotically stable with the adaptive controller* (9). *Moreover, the scalar ω can be relaxed to any nonnegative constant.*

Proof. Define the Lyapunov function as

$$\widetilde{V}(t) = \widetilde{V}_1(t) + V_2(t), \quad (24)$$

where $V_2(t)$ is defined as that in the proof of Theorem 4:

$$\widetilde{V}_1(t) = \frac{1}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\frac{(\varepsilon_i(t)-k_i)^2}{p_i}. \quad (25)$$

Differentiating $\widetilde{V}(t)$ along the solution of (8) and noting that $-\omega\sum_{i=1}^N\sum_{k=1}^n \beta_i(t)|e_{ik}(t)| \le 0$, by the same procedure of the proof of Theorem 4, one can easily finish the proof. This completes the proof. □

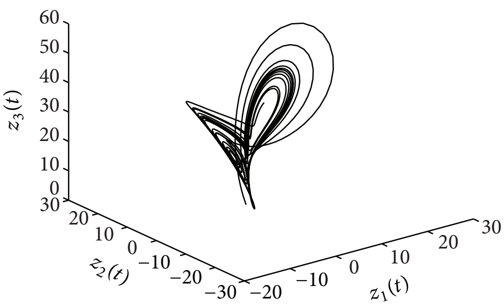


Figure 1: Chaotic trajectory of system (28).

Remark 7. The new adaptive controller of this paper has better property of robustness than that used in [14–16, 19–22], which is

$$R_i = -\varepsilon_i(t) e_i(t), \quad i = 1, 2, \ldots, N,$$
$$\dot{\varepsilon}_i(t) = p_i e_i(t)^T e_i(t), \quad i = 1, 2, \ldots, N. \tag{26}$$

The adaptive controller (26) is a special case of (9) when $\omega = 0$. Although controller (26) has robustness, it is not sufficient to synchronize networks with nonlinear perturbations. From Corollary 6, one sees that both adaptive controller (9) and (26) can synchronize a coupled network without any external perturbation to a given isolate system. Since uncertain perturbations to coupled networks are unavoidable in practice, our adaptive controller (9) is more practical than (26). Numerical simulations of this paper verify that new adaptive controller (9) has better robustness than (26).

Remark 8. Note that controller (9) are discontinuous and the phenomenon of chattering will appear [36, 37]. In order to eliminate the chattering, controller (9) can be modified as

$$R_i = -\varepsilon_i(t) e_i(t) - \omega \beta_i(t) S_i, \quad i = 1, 2, \ldots, N,$$
$$\dot{\varepsilon}_i(t) = p_i e_i(t)^T e_i(t), \quad i = 1, 2, \ldots, N, \tag{27}$$
$$\dot{\beta}_i(t) = \xi_i \sum_{k=1}^{n} |e_{ik}(t)|, \quad i = 1, 2, \ldots, N,$$

where $S_i = ((e_{i1}(t)/(|e_{i1}(t)| + \zeta_i)), \ldots, (e_{in}(t)/(|e_{in}(t)| + \zeta_i)))^T$, ζ_i, $i = 1, 2, \ldots, N$, are sufficiently small positive constants and $\omega > 1$, $p_i > 0$, and $\xi_i > 0$, are arbitrary constants, respectively.

Remark 9. If only some nodes of (1) are effected by uncertain nonlinear perturbations and matrix U is irreducible, we can also consider adaptive pinning control scheme (see [16, 21, 22]) with the new adaptive controller to synchronize coupled networks (1). Moreover, we can also consider stochastic perturbations [14] and the Markovian jump (see [18, 38]) in (1) to get more general results. This is our next research topic.

4. Numerical Examples

In this section, we first consider a network composed of three Lorenz systems with uncertain perturbations. Then one WS small-world model with partially perturbed nodes, in which the node systems are delayed neural networks, is given. Numerical simulations demonstrating better robustness of the designed adaptive controller than existing one are also given.

Example 1. The Lorenz system is described as [23, 27]

$$\dot{z}(t) = f_1(z(t)) = Cz(t) + f(z(t)), \tag{28}$$

where $z(t) = (z_1(t), z_2(t), z_3(t))^T$, $f(z(t)) = (0, -z_1(t)z_3(t), z_1(t)z_2(t))^T$, and

$$C = \begin{pmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -\frac{8}{3} \end{pmatrix}. \tag{29}$$

When initial values are taken as $z_1(0) = 0.8$, $z_2(0) = 2$, and $z_3(0) = 2.5$, chaotic trajectory of (28) can be seen in Figure 1.

Now consider the following controlled network with nondelayed coupling, where each node system is the above Lorenz system with uncertainties:

$$\dot{x}_i(t) = f_1(x_i(t)) + \alpha \sum_{j=1}^{3} u_{ij} \Phi x_j(t) + \sigma_i(t) + R_i, \quad i = 1, 2, 3, \tag{30}$$

where $\alpha = 0.5$, $\Phi = I_3$, $\sigma_1(t) = [0.1x_{11}^2(t), 0.2x_{12}(t), 0.2x_{13}(t)]^T$, $\sigma_2(t) = [0.1x_{21}(t), 0.05x_{22}^2(t), \sin x_{23}(t)]^T$, $\sigma_3(t) = [0.1x_{31}(t), (\sin x_{32}(t))^2, \sin x_{33}(t)]^T$, and

$$U = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}. \tag{31}$$

Numerical simulations show that the states of (30) without control are bounded; see Figure 2. Therefore, (H_2) is satisfied.

It is obvious that $f_1(0) = 0$. Let $e_i(t) = x_i(t) - z(t) = (e_{i1}(t), e_{i2}(t), e_{i3}(t))^T$; then

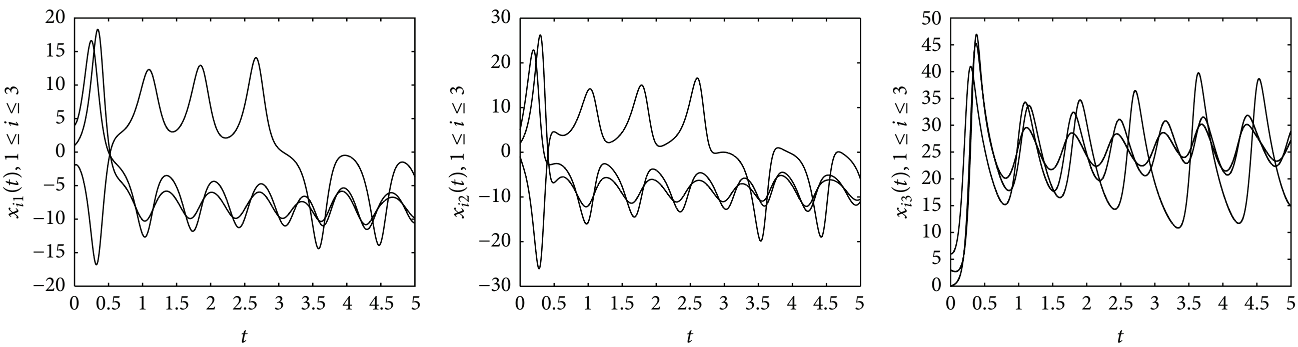
$$\begin{aligned} &\|f_1(x_i(t)) - f_1(z(t))\| \\ &\quad = \|Ce_i(t) + f(x_i(t)) - f(z(t))\| \\ &\quad \le \|C\| \|e_i(t)\| + \|f(x_i(t)) - f(z(t))\|. \end{aligned} \tag{32}$$

On the other hand, one can easily get

$$\begin{aligned} &(f(x_i(t)) - f(z(t)))^T (f(x_i(t)) - f(z(t))) \\ &\quad = (x_{i1}e_{i3} + z_3 e_{i1})^2 + (x_{i1}e_{i2} + z_2 e_{i1})^2 \\ &\quad \le a e_{i1}^2(t) + b e_{i2}^2(t) + c e_{i3}^2(t), \end{aligned} \tag{33}$$

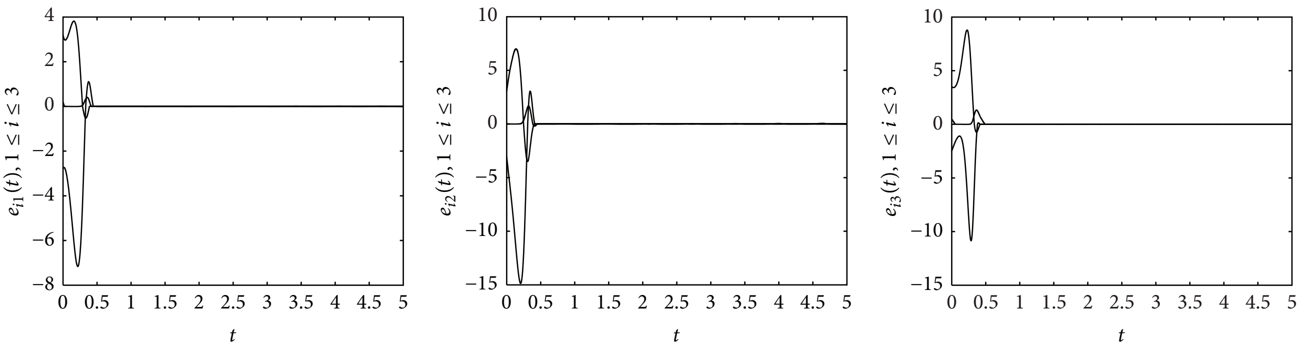
where $a = z_3^2(t) + z_2^2(t) + |x_{i1}(t)z_3(t)| + |x_{i1}(t)z_2(t)|$, $b = x_{i1}^2(t) + |x_{i1}(t)z_2(t)|$, and $c = x_{i1}^2(t) + |x_{i1}(t)z_3(t)|$. From Figures 1 and 2, it is obvious that $x_{ij}(t)$, $i, j = 1, 2, 3$, and $z_j(t)$, $j = 1, 2, 3$, are all bounded; hence a, b, and c are all bounded. Take $d = \max\{a, b, c\}$; then

$$\|f(x_i(t)) - f(z(t))\| \le d^{1/2} \|e_i(t)\|. \tag{34}$$



(a) $x_{i1}(t)$, $1 \le i \le 3$ (b) $x_{i2}(t)$, $1 \le i \le 3$ (c) $x_{i3}(t)$, $1 \le i \le 3$

Figure 2: State trajectories of (30) without control: (a) $x_{i1}(t)$, $1 \le i \le 3$; (b) $x_{i2}(t)$, $1 \le i \le 3$; (c) $x_{i3}(t)$, $1 \le i \le 3$.



(a) $e_{i1}(t)$, $1 \le i \le 3$ (b) $e_{i2}(t)$, $1 \le i \le 3$ (c) $e_{i3}(t)$, $1 \le i \le 3$

Figure 3: Synchronization errors between (30) and (28) with adaptive controllers (9).

One gets from (32) and (34) that

$$\left\| f\left(x_i(t)\right) - f\left(z(t)\right)\right\| \le \left(\|C\| + d^{1/2}\right)\left\|e_i(t)\right\|. \tag{35}$$

Take $h_1 = \|C\| + d^{1/2}$; then (H_1) is satisfied. Since $x_{ij}(t)$, $i, j = 1, 2, 3$, are bounded, every component of $\sigma_i(t)$, $i = 1, 2, 3$, is bounded, and (H_3) is satisfied. According to Theorem 4, the coupled network (30) can be asymptotically synchronized onto (28) with adaptive controller (9).

In the simulations, the forward Euler method is used to simulate in Matlab (math works). The initial conditions of the numerical simulations are as follows: step = 0.0005, $x_1(0) = (-2, -1, 0)^T$, $x_2(0) = (1, 2, 3)^T$, $x_3(0) = (4, 5, 6)^T$, $\omega = 4$, $\varepsilon_i(0) = 1$, $\beta_i(0) = 1$, $p_i = \xi_i = 0.05$, $i = 1, 2, 3$. Figure 3 describes the synchronization errors $e_{ij}(t) = x_{ij}(t) - z_j(t)$ $(i, j = 1, 2, 3)$ between (30) and (28) by using controllers (9), from which one can see that the synchronization errors quickly turn to zero as time goes. Figure 4 presents the time response of the feedback gain parameters $\varepsilon_i(t)$ and $\beta_i(t)$, $1 \le i \le 3$, which reach constants eventually. Numerical simulations show that when all nodes are perturbed by uncertainties, the new adaptive controller (9) can synchronize the coupled networks onto a given trajectory, which verify the effectiveness of Theorem 4.

In order to show the better robustness and advantage of our new controller, we now control (30) with the adaptive controller (26), where all the values of parameters are the same as those above. Figure 5 describes the trajectories of synchronization errors between (30) and (28) by using adaptive controller (26), from which one can see that $e_{ij}(t) = x_{ij}(t) - z_j(t)$ $(i, j = 1, 2, 3)$ do not turn to zero as time goes. Hence, controllers (26) cannot synchronize (30) onto the trajectory of (28).

Example 2. In [39], Watts and Strogatz proposed a small-word network (WS small-world) model, which is described as follows.

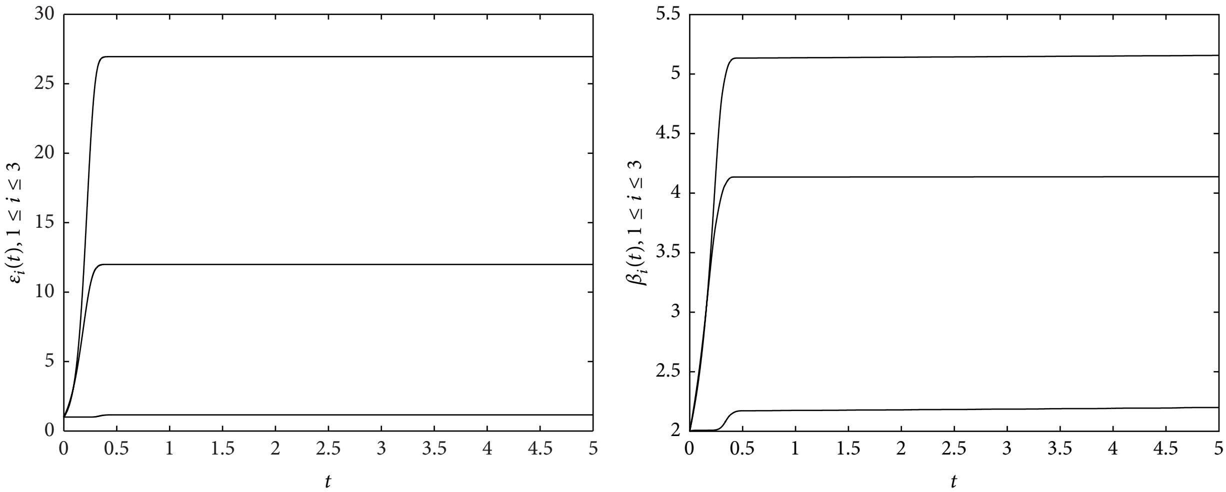
(1) *Initialization*. Starting with a regular ring lattice of N nodes, each node is connected to K (even number) nearest neighbors by undirected links.

(2) *Rewiring*. Randomly rewire each link of the network with probability p such that self-connected and duplicated links are excluded. Here, rewiring means reconnecting one end of a selected link to another randomly chosen node.

The delayed neural network is described as [14]

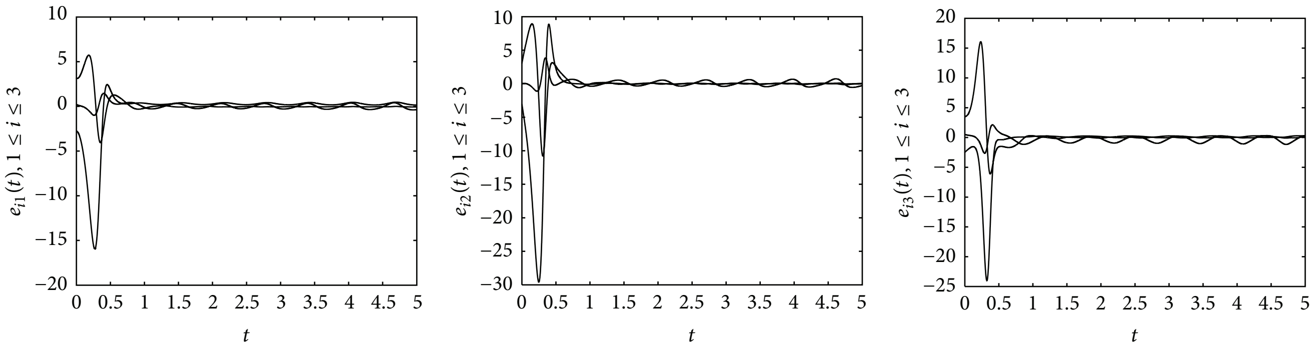
$$\dot{z}(t) = f_1(z(t)) + f_2(z(t - \tau(t))), \tag{36}$$

where $z(t) = (z_1(t), z_2(t))^T$, $f_1(z(t)) = Cz(t) + Af(z(t))$, $f_2(z(t - \tau(t))) = Bg(z(t - \tau(t)))$, $f(z(t)) =$



(a) $\varepsilon_i(t)$, $1 \le i \le 3$ (b) $\beta_i(t)$, $1 \le i \le 3$

FIGURE 4: Feedback gain parameters $\varepsilon_i(t)$ (a) and $\beta_i(t)$ (b) ($1 \le i \le 3$) of (30) with adaptive controller (9).



(a) $e_{i1}(t)$, $1 \le i \le 3$ (b) $e_{i2}(t)$, $1 \le i \le 3$ (c) $e_{i3}(t)$, $1 \le i \le 3$

FIGURE 5: Synchronization errors between (30) and (28) by using adaptive controllers (26).

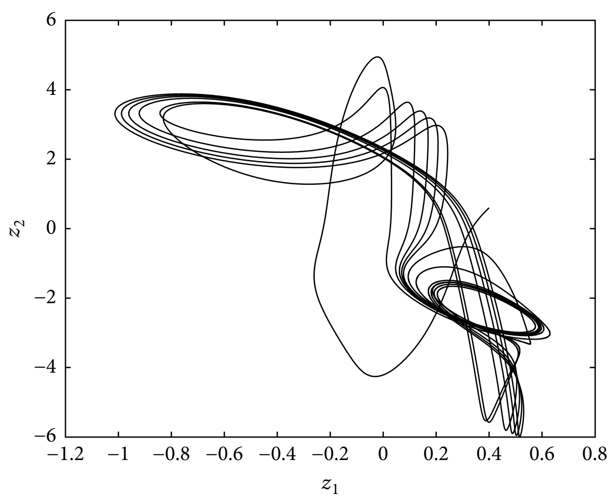


FIGURE 6: Trajectory of the delayed neural network (36).

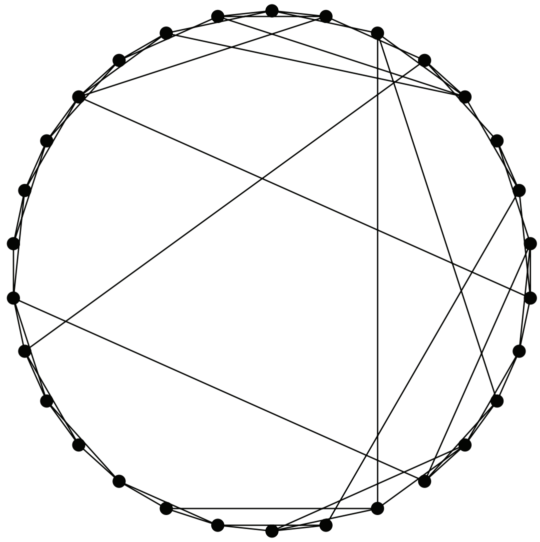
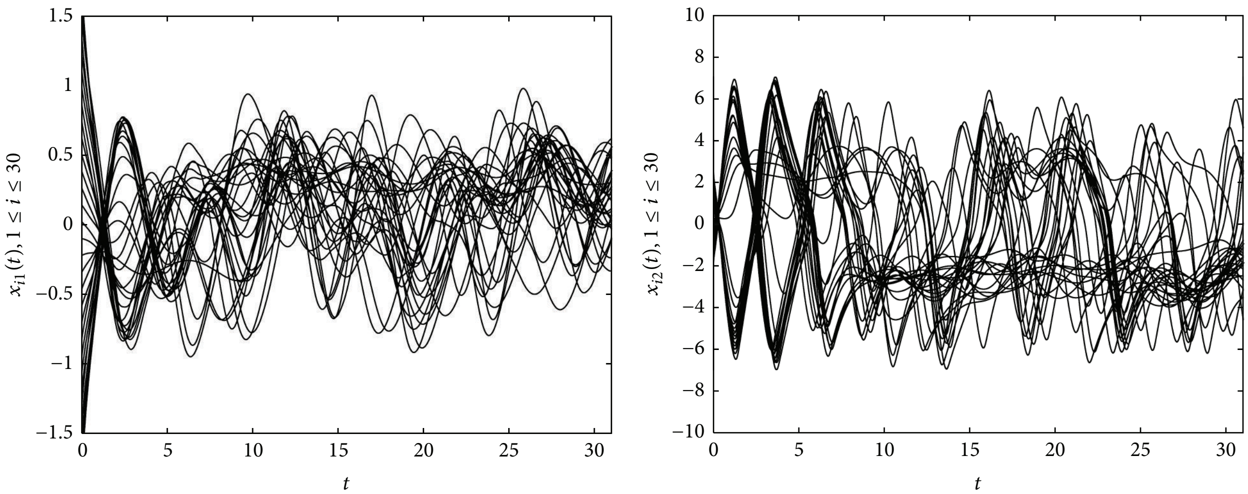


Figure 7: The structure of WS small-world network with $N = 30$, $K = 4$, and $p = 0.2$.



(a) $x_{i1}(t)$ $(1 \le i \le 30)$

(b) $x_{i2}(t)$ $(1 \le i \le 30)$

Figure 8: State trajectories of (39) without control: (a) $x_{i1}(t)$ $(1 \le i \le 30)$, (b) $x_{i2}(t)$ $(1 \le i \le 30)$.

$(\tanh(z_1(t)), \tanh(z_2(t)))^T$, $g(z(t-\tau(t))) = (\tanh(z_1(t-\tau(t))), \tanh(z_2(t-\tau(t))))^T$, $\tau(t) = 1$,

$$C = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad A = \begin{pmatrix} 2 & -0.1 \\ -5 & 4.5 \end{pmatrix}, \qquad B = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -4 \end{pmatrix}. \tag{37}$$

When the initial values are taken as $z(s) = (0.4, 0.6)^T$, $\forall s \in [-1, 0]$, system (36) is chaotic as shown in Figure 6.

It can be easily verified that $f_1(0) = f_2(0) = 0$, $\|C\| = 1$, $\|A\| = 6.9099$, $\|B\| = 4.0094$, and $\|f(x) - f(y)\| \le \|x - y\|$ for any $x, y \in \mathbb{R}^2$. Hence

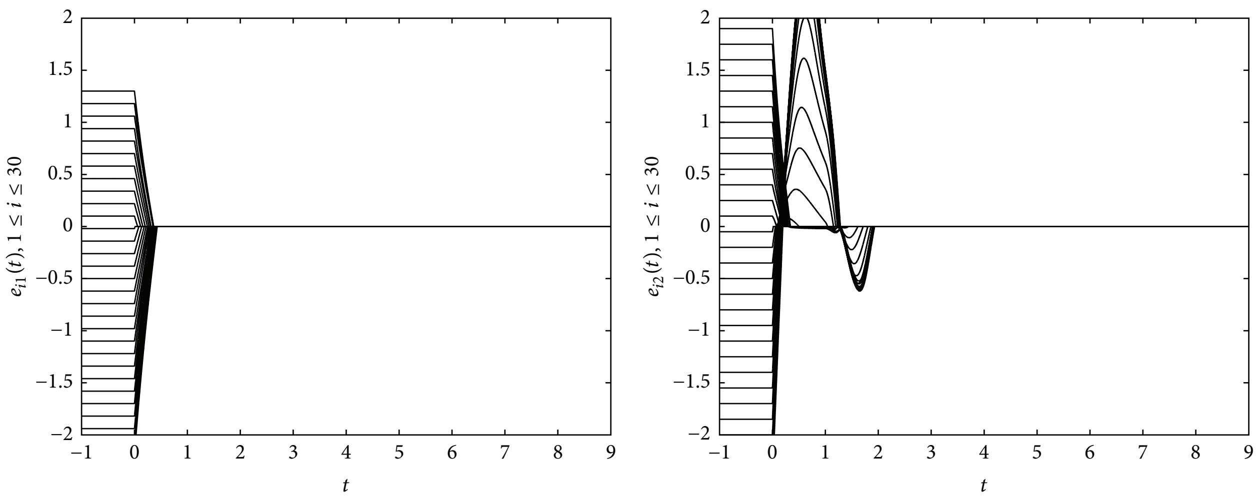
$$\begin{aligned} \left\|f_1(x) - f_1(y)\right\| &\le (\|C\| + \|A\|)\left\|x - y\right\| \\ &= 7.9099\left\|x - y\right\|, \\ \left\|f_2(x) - f_2(y)\right\| &\le \|B\|\left\|x - y\right\| = 4.0094\left\|x - y\right\|. \end{aligned} \tag{38}$$

Therefore, (H_1) is satisfied with $h_1 = 7.9099$ and $h_2 = 4.0094$.

Next, consider the following coupled network subject to uncertainties and control inputs:

$$\begin{aligned} \dot{x}_i(t) = {} & Cx_i(t) + Af(x_i(t)) + Bg(x_i(t-\tau(t))) \\ & + c_1 \sum_{j=1}^{N} u_{ij}\Phi x_j(t) + c_2 \sum_{j=1}^{N} v_{ij}\Upsilon x_j(t-\tau(t)) \\ & + \sigma_i(t, x_i(t), x_i(t-\tau(t))) + R_i, \quad i = 1, 2, \ldots, N, \end{aligned} \tag{39}$$

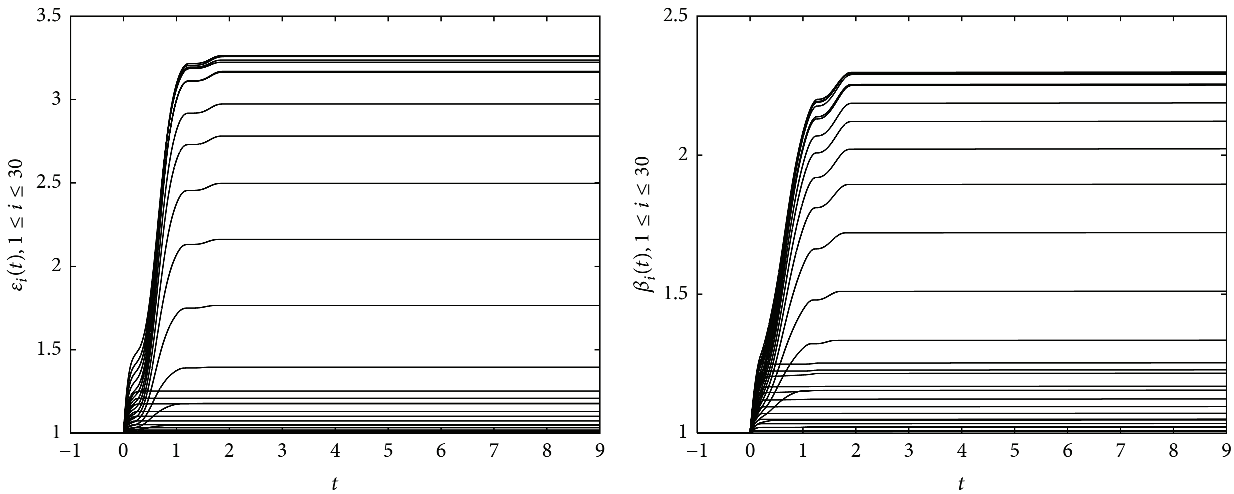
where $\Phi = \Upsilon = I_2$, $c_1 = 0.05$, and $c_2 = 0.005$. Suppose that the network is connected in a small-world topology. If there is



(a) $e_{i1}(t)$, $1 \le i \le 30$

(b) $e_{i2}(t)$, $1 \le i \le 30$

FIGURE 9: Synchronization errors between (39) and (36) with adaptive controllers (9).



(a) Feedback gain parameters $\varepsilon_i(t)$, $1 \le i \le 30$

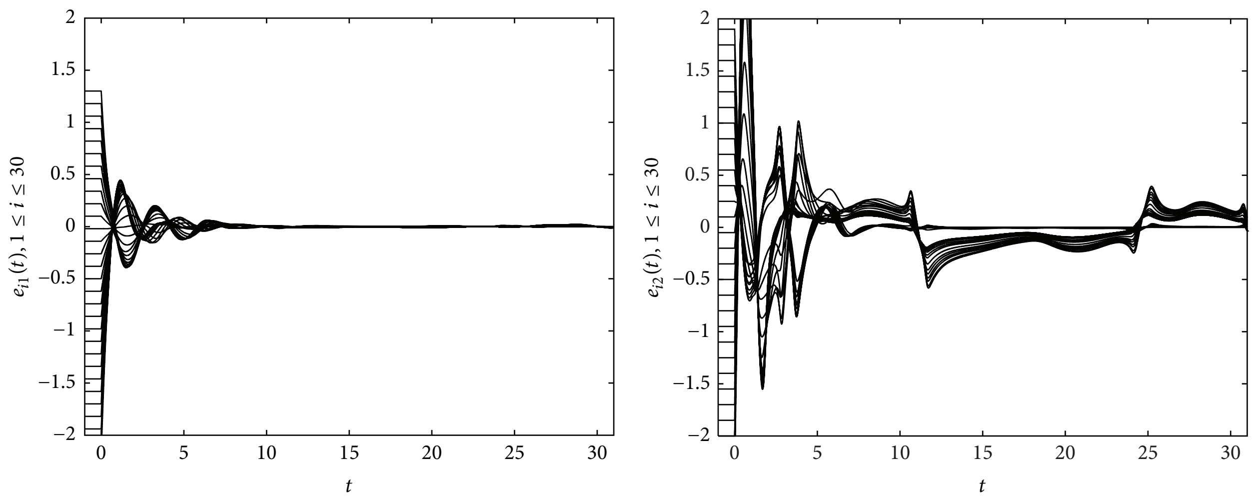
(b) Feedback gain parameters $\beta_i(t)$, $1 \le i \le 30$

FIGURE 10: Feedback gain parameters $\varepsilon_i(t)$ (a) and $\beta_i(t)$ (b) ($1 \le i \le 30$) of (39) with controller (9).

a connection between nodes i and j ($j \neq i$), then $u_{ij} = v_{ij} = 1$; otherwise, $u_{ij} = v_{ij} = 0$. Taking $N = 30, K = 4$, and $p = 0.2$, we get structure of WS small world shown in Figure 7.

Suppose the uncertain perturbations of the nodes $i = 1, 2, \ldots, 10$ are $\sigma_i(t, x_i(t), x_i(t - \tau(t))) = (-0.1x_{i1}(t), 0.2x_{i2}(t))^T$ ($1 \le i \le 10$), the uncertain perturbations of the nodes $i = 11, 12, \ldots, 20$ are $\sigma_i(t, x_i(t), x_i(t - \tau(t))) = (0.05(x_{i1}(t - \tau(t)))^2, 0.2x_{i2}(t))^T$ ($11 \le i \le 20$), and the nodes $i = 21, 22, \ldots, 30$ have no perturbation. The state trajectories $x_{i1}(t)$ and $x_{i2}(t)$ ($1 \le i \le 30$) of the perturbed network (39) without control are shown in Figure 8, from which one can see that $x_{i1}(t)$ and $x_{i2}(t)$ ($1 \le i \le 30$) are all bounded. Obviously, conditions (H_2) and (H_3) are also satisfied. According to Theorem 5, the coupled network (39) can be synchronized to (36) with adaptive controller (9).

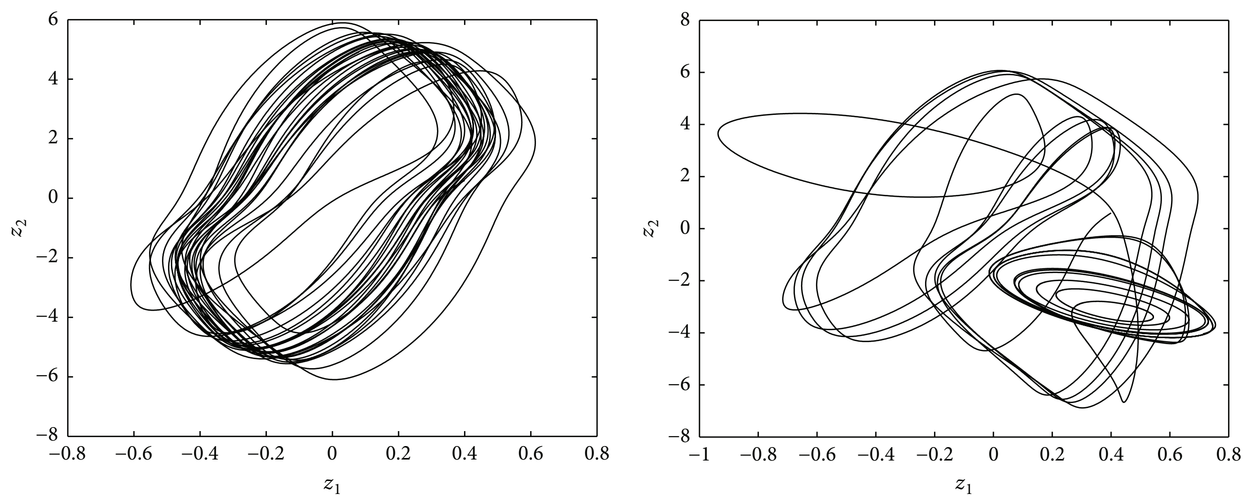
Again, the forward Euler method is used to simulate in Matlab. The initial conditions of the numerical simulations are as follows: step = 0.0001, the initial values of $x_i(t)$ ($i = 1, 2, \ldots, 30$) are arbitrary in $[-2, 2]$, $\omega = 1.5$, $\varepsilon_i(t) = \beta_i(t) = 1$, $t \in [-1, 0]$, and $p_i = \xi_i = 0.5$ ($i = 1, 2, \ldots, 30$). Figure 9 describes the synchronization errors $e_{ij}(t) = x_{ij}(t) - z_j(t)$ ($i = 1, 2, \ldots, 30$, $j = 1, 2, 3$) between (39) and (36) by using adaptive controller (9). One can see from Figure 9 that synchronization errors quickly turn to zero as time goes. Figure 10 shows the feedback gain parameters $\varepsilon_i(t)$ and $\beta_i(t)$ ($1 \le i \le 30$), which all reach constants eventually. Numerical simulations show that when only partial nodes are perturbed by uncertainties, the new adaptive controller (9) can also synchronize the coupled networks onto a given trajectory, which verify the effectiveness of Theorem 5.



(a) $e_{i1}(t)$, $1 \le i \le 30$

(b) $e_{i2}(t)$, $1 \le i \le 30$

FIGURE 11: Synchronization errors between (39) and (36) by using adaptive controllers (26).



(a) Trajectory of system (40) with $\sigma(t, z(t), z(t-\tau(t))) = (-0.1z(t), 0.2z(t))^T$

(b) Trajectory of system (40) with $\sigma(t, z(t), z(t-\tau(t))) = (0.05(z(t-\tau(t)))^2, 0.2z(t))^T$

FIGURE 12: Trajectories of system (40) under different perturbations.

However, with the same parameter values, controller (26) cannot synchronize (39) to the trajectory of (36). The synchronization errors between (39) and (36) by using adaptive controller (26) are shown in Figure 11.

Remark 10. Figure 11(a) shows that the usual adaptive controller has robustness; however, one can see from Figure 11(b) that its robustness is limited. Nevertheless, by comparing Figures 5 and 11 with Figures 3 and 9, respectively, one can easily see that the new adaptive controller (9) has better robustness than the usual one (26). Therefore, the new adaptive controller proposed in this paper is more effective and more robust than that in [14–16, 19–22] for coupled networks with uncertain perturbations.

Remark 11. Since uncertain perturbations to coupled networks in real life are unavoidable, the new adaptive controller (9) is important. Actually, external uncertain perturbations may have key effects on the dynamics of node system. For example, consider the system (36) with perturbation:

$$\begin{aligned} \dot{z}(t) = {} & f_1(z(t)) + f_2(z(t-\tau(t))) \\ & + \sigma(t, z(t), z(t-\tau(t))). \end{aligned} \tag{40}$$

Take the same initial values as those in Figure 6; that is, $z(s) = (0.4, 0.6)^T$, $\forall s \in [-1, 0]$. Figure 12 presents the different trajectories of system (40) under different perturbations, which are completely different from those of system (36); see Figure 6. However, under these perturbations, the coupled network (39) can still be synchronized onto trajectory of (36) by the designed controller (9).

5. Conclusion

Uncertainties for systems are unavoidable in practice. Therefore, in this paper, we introduced a class of coupled networks with delays and uncertain nonlinear perturbations. A simple but robust adaptive controller is designed to synchronize the coupled networks onto an isolate node even without knowing priori the bounds of such perturbations. Results of this paper are also applicable to complex networks with asymmetric coupling configuration matrix. The designed controller enhances the robustness and reduces fragility of coupled networks; hence, it has great practical significance. Two types of coupled network model with uncertain perturbations, asymmetric coupled Lorenz network and WS small-world type of complex network with delayed neural network as node system, are employed to verify the effectiveness of the theoretical results. Numerical simulations also show that the designed adaptive controller is more robust and more effective in synchronizing a coupled network than the usual adaptive controller used in the references.

Obviously, it is optimal to synchronize complex networks with delays and uncertain perturbations in finite time. However, the existing controllers cannot synchronize a delayed complex network in finite time. Therefore, our next research is to design simple and effective controllers to realize finite-time synchronization of complex networks with delays and uncertain perturbations, which is challenging.

Acknowledgments

This work was jointly supported by the National Natural Science Foundation of China under Grant no. 61263020, the Scientific Research Fund of Chongqing Municipal Education Commission under Grant no. KJ130613, and the Scientific Research Fund of Chongqing Normal University under Grants no. 12XLB031 and no. 940115.

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