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Research Article

Sliding Mode Control with State Derivative Output Feedback in Reciprocal State Space Form

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This paper investigates the novel sliding mode control design with state derivative output feedback in nontraditional reciprocal state space (RSS) form. The concepts and the need of RSS form are comprehensively reviewed and explained. Novel switching function and approaching condition based on the derivative of sliding surface are introduced. In addition, a sufficient condition for finding the upper bound of system uncertainty to guarantee the stability in sliding surface is developed for robustness analysis. A compact sliding mode controller utilizing only state derivative related output feedback is proposed for systems with system uncertainty, matched input uncertainty, and matched external disturbance. Simulation results for a circuit system successfully verify the validities of the proposed algorithms. Our derivation is basically parallel to that for systems in standard state space form. Therefore, those who understand the concepts of sliding mode control can easily apply our method to handle more control problems without being involved in complex mathematics.

1. Introduction

The solutions of output feedback control designs are not always available for systems. However, when they are available, comparing with full state feedback controllers and full estimated state feedback controllers, the output feedback controllers usually have the most compact structures and the lowest implementation costs. Therefore, in many cases, output feedback controllers are the designers' first choice as long as the closed loop system can be stabilized. In some applications, the system outputs are not related to states but to state derivatives. For this reason, the state derivative output feedback algorithms are needed. However, in the past, state derivative output feedback algorithms were rarely investigated because the closed loop systems are complex in state space form. In this paper, a sliding mode controller utilizing state derivative related output feedback in novel reciprocal state space (RSS) form is proposed.

In general, a dynamic linear continuous time invariant system using state variables with physical meanings can be naturally expressed in the following equation under the names of generalized state space form [1] or descriptor form [2] or singular system form [3]:

$$E\dot{x} = Fx + Nu, \tag{1}$$

where $x_{n\times 1}$ and $u_{m\times 1}$ are state vector and control vector, respectively, and $E_{n\times n}$, $F_{n\times n}$, and $N_{n\times m}$ are known constant system matrices. Controllability and observability of generalized state space systems were investigated in [2, 3]. The following is the characteristic equation of open loop system of (1):

$$\det(sE - F) = 0. \tag{2}$$

The degree d of characteristic equation in (2) is the number of system's finite eigenvalues, while $n - d$ is the number of system's eigenvalues at infinity [4].

If E in (1) is nonsingular, the system has no eigenvalue at infinity but can have zero eigenvalues. The system can be expressed as the following standard state space system:

$$\dot{x} = E^{-1}Fx + E^{-1}Nu = \overline{A}x + \overline{B}u. \tag{3}$$

For the above state space systems, if $\overline{A}$ is nonsingular, every state derivative variable can be expressed in terms of state variables and control inputs. Most of the control algorithms developed for state space systems are related to state feedback such as full state feedback, state related output feedback, and estimated state feedback when estimators are implemented. However, in many applications, the sensors directly measure state derivatives rather than states. For instance, accelerometers [5] in many micro- and nanoelectromechanical systems (M/NEMS) and structural applications [6, 7] are such cases, because acceleration signals can only be modeled as state derivatives when physical dynamic second order systems are expressed in state space form [6–8]. Consequently, abundant control algorithms with state related feedback developed for standard state space systems cannot be readily applicable in this situation. Additional integrators which may increase the cost and complexity of the implementation are needed. Mathematically speaking, state derivative related feedback designs cannot be carried out as straightforward as state feedback for systems expressed in standard state space form. For example, if we apply the following full state derivative feedback control law:

$$u = -K\dot{x} \tag{4}$$

to the state space system in (3), the closed loop system becomes

$$\dot{x} = \left(I + \overline{B}K\right)^{-1}\overline{A}x. \tag{5}$$

In (5), since gain K is inside the inverse matrix $(I + \overline{B}K)^{-1}$ which is further coupled with the open loop system matrix $\overline{A}$ by multiplication, it is obvious that advanced mathematics is needed to design gain K in (4). Therefore, in the past, the developed algorithms of state derivative related feedback for systems in state space form were very few and rarely used to control the system alone [8]. In a word, standard state space system in (1) is the best system form for open loop systems without poles at infinity in designing state related feedback control algorithms. However, standard state space system is not the most suitable form to develop state derivative related feedback control algorithms and cannot handle the systems with open loop poles at infinity.

If E in (1) is singular, the system has poles at infinity and is called generalized state space system. In the past, the majority of control designs for system with poles at infinity were directly developed in generalized state space form in (1).

Extensive applications of generalized state space systems arise in many areas of engineering such as electrical networks [9], aerospace systems [10], smart structures [7, 11], and chemical processes [12]. Generalized state space systems also exist in other areas such as the dynamic Leontief model for economic production sectors [13] and biological complex systems [14]. A comprehensive review is available in [11]. In this paper, generalized state space system is used as the name to represent such systems. In previous studies, generalized state space systems are further categorized as impulse-free ones [11] and with impulse mode ones in analysis. If a generalized state space system has impulse mode, further investigations of impulse controllable and the impulse mode elimination [15] have to be analyzed in control designs. Therefore, this kind of generalized state space system is considered to be difficult in control designs. On the other hand, control designs for impulse-free generalized state space systems can be handled in easier ways and have been an active area of research. Mathematically speaking, the available control design algorithms which are carried out in augmented systems and require feedbacks of both state and state derivative variables for generalized state space systems [15–19] are much more complex than those for the standard state space forms. Consequently, there are difficulties for engineers without strong mathematical background to apply those sophisticated control algorithms.

As mentioned before, when the state derivative coefficient matrix E in (1) is nonsingular, the system can be expressed in standard state space form in (3). If the system is controllable, applying state feedback alone is sufficient to control the system. Similarly, it is natural to ask if applying state derivative feedback alone is sufficient to control the system when the state coefficient matrix F in (1) is nonsingular. To answer this question and to provide supplementary design algorithms of state derivative feedback, a direct state derivative feedback control scheme was developed using the "reciprocal state space" (RSS) methodology [7, 20–24] by the first author of this paper as follows:

$$x = F^{-1}E\dot{x} - F^{-1}Nu = A\dot{x} + Bu. \tag{6}$$

For the above reciprocal state space (RSS) systems, if A is nonsingular, every state variable can be expressed in terms of state derivative variables and control inputs. After applying full state derivative feedback control law in (4), the closed loop system becomes

$$x = (A - BK)\,\dot{x} = A_C\dot{x}. \tag{7}$$

The concept of RSS is based on a fact that, for a nonsingular matrix, the eigenvalues of its inverse matrix must be the reciprocals of its eigenvalues. Therefore, the eigenvalues of A_C in (7) are the reciprocals of the closed loop system poles. To address this nature, the name of reciprocal state space framework was given. If state derivative feedback gain K can be designed such that real parts of all eigenvalues of A_C in (7) are strictly negative, the closed loop system in RSS form in (7) can achieve global asymptotic stability. When a controllable system has no open loop pole at zero, it can be expressed in RSS form to carry out state derivative related feedback control designs.

Here is an example for quick understanding of why expressing system in RSS form and applying state derivative feedback can easily accomplish control designs for some systems that were once thought to be difficult in control designs. For the following generalized state space system with impulse mode [25], its state coefficient matrix is invertible. Therefore, the open loop system has no open loop pole at zero and the system can be expressed in RSS form:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\dot{x} = \begin{bmatrix} 0.5 & 0 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & 0 \end{bmatrix}x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}u. \tag{8}$$

Suppose that we want to assign the closed loop poles at −2, −2.5, and −5; we can first express the system in RSS framework as follows:

$$x = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ -2 & -1 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} u = A\dot{x} + Bu. \quad (9)$$

Then, apply the state derivative feedback law $u = -K\dot{x}$ to assign −0.5, −0.4, and −0.2 (the reciprocals of −2, −2.5, and −5, resp.) as the eigenvalues of matrix $(A - BK)$. Using "place" command of Matlab, one can easily obtain $K = [-1.63 \;\; -0.2 \;\; 0.02]$. Therefore, for the systems without open loop pole at zero, including difficult systems to be controlled such as generalized state space systems with impulse mode in this example, they can be expressed in RSS form in (6) and properly controlled by applying state derivative alone. Usually, handling the same problem in generalized state space system form, both state feedback and state derivative feedback are needed [25].

The controllability and observability analyses for system in RSS form have been investigated in [23, 24]. They show that they turn out to be the same as their counterparts in state space form. They also show that state derivative feedback designs are as straightforward in RSS form as state feedback designs in standard state space form in pole placement, eigenstructure assignment, and linear quadratic regulator (LQR) designs [20–24].

To streamline the design processes and keep the controller as compact as possible, the following control design procedure is suggested. For an open loop system, if it has no pole at infinity, one can express the system in state space form and apply state related feedback to control it. If it has poles at infinity but has no pole at zero, one can express the system in RSS form and apply state derivative related feedback to control it. If it has neither pole at infinity nor pole at zero, based on the type of available sensors (state related sensors or state derivative related sensors), one can make choice between state space form and RSS form to carry out control design. Generalized state space system form and control laws applying both state feedback and state derivative feedback might be considered as the last resort to handle the system with both poles at infinity and poles at zero. In a nutshell, RSS form fills in the gap between standard state space system and generalized state space system and provides additional flexibility in control designs.

In recent years, robust control is one of the most popular topics in control area. One of the famous methods is the so-called sliding mode control [26–28], which has been proven as an effectively robust control technology with many practical applications. The main idea of sliding mode control is to design a controller rendering the trajectory of states trapped on a predetermined sliding surface and remaining on it thereafter. Sliding mode control utilizes a high-speed switching control law to drive the state trajectory staying on this sliding surface for all subsequence times such that the robust stability of the system is assured. In the present, sliding mode control (SMC) is a highly active area of research. Finite-time convergence due to discontinuous control law, low sensitivity to plant parameter uncertainty and/or external perturbation, and greatly reduced-order modeling of plant dynamics are the main advantages of SMC. Therefore, based on SMC, many works in state space form have been developed [29–33].

Since the majority of available SMC algorithms and the corresponding switching conditions for linear systems involve only state variables or state related output, they are not suitable for systems with state derivative or state derivative related output. The main purpose of this paper is to combine the advantages of both RSS and SMC to develop sliding mode control in reciprocal state space form so that state derivate output feedback can be systematically applied in SMC to handle a wider range of control problems.

This paper is organized as follows. In Section 2, stability analysis in RSS form is reviewed. Comprehensive SMC design approach in RSS form with considerations of system uncertainty, input uncertainty and disturbance, selection of sliding surface with modified transfer matrix method in RSS form, and a novel switching function which is a function of the derivative of the sliding surface are presented in Section 3. In addition, algorithm of finding upper bound of system uncertainty has also been developed for robustness analysis in Section 3. In Section 4, a numerical example of RLC circuit system that verifies the proposed controller is provided. Finally, conclusions are drawn in Section 5.

2. Stability Analysis in RSS Form

Since Lyapunov stability is the fundamental of sliding mode control, in this section, Lyapunov stability analysis in RSS form is presented.

For a linear time invariant system, it is globally asymptotically stable if the real parts of all system poles are strictly negative. Therefore, such system must have no pole at infinity or pole at zero. Consequently, a globally asymptotically stable system can be expressed in both state space form and RSS form as follows:

$$\dot{x} = \overline{A}x, \quad (10)$$

$$x = A\dot{x}, \quad (11)$$

where $\overline{A} = A^{-1}$ and both A and $\overline{A}$ are nonsingular. Furthermore, the eigenvalues of A are the reciprocals of the eigenvalues of $\overline{A}$ which are the system poles.

Based on the above discussion, the following Lyapunov equation can also test the stability of RSS systems in (11):

$$PA + A^T P = -Q. \quad (12)$$

The solution of P in Lyapunov equation (12) must be symmetric positive definite to ensure that RSS system matrix A is globally asymptotically stable when Q is a symmetric and positive matrix.

3. Novel Approaching Condition and Sliding Mode Control Design for System in RSS Form

In general, sliding mode control process consists of two parts. The first part involves the selection of an appropriate sliding surface and the second part is the design of a controller to meet the approaching condition.

The purpose of satisfying the approaching condition [27, 28] is to force the system toward the predetermined sliding surface $s(t)$ which can stabilize the system. According to the matrix sizes specified in (1), suppose that the sliding surface $s(t)$ is selected by

$$s(t) = cx(t) = 0, \tag{13}$$

where $s \in R^{m\times 1}$ and $c \in R^{m\times n}$.

Approaching condition is briefly explained as follows. Define a Lyapunov function candidate as

$$V = \frac{1}{2}s^T s. \tag{14}$$

The derivative of V with respect to time is given by

$$\dot{V} = s^T(t)\cdot \dot{s}(t). \tag{15}$$

For RSS form and state derivative related feedback used in design, given a positive constant α, the following novel approaching condition is proposed:

$$\dot{V} = s^T(t)\cdot \dot{s}(t) < -\alpha\|\dot{s}\| < 0, \tag{16}$$

where $\|\|$ denotes norm in this paper.

Consider the following system with system uncertainty, input uncertainty, and disturbance in RSS form:

$$x(t) = (A + \Delta A(t))\dot{x}(t) + (B + \Delta B(t))u(t) + d(t), \tag{17}$$

$$y(t) = C\dot{x}(t), \tag{18}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^p$, and $d(t) \in R^n$ are system's states, control inputs, outputs, and external disturbance, respectively. The triple pair (A, B, C) is known and the dimensions are $A \in R^{n\times n}$, $B \in R^{n\times m}$, and $C \in R^{p\times n}$. In (17), $\Delta A(t) \in R^{n\times n}$ is the system uncertainty, $\Delta B(t)$ is the matched uncertainty in the input, and $d(t)$ is the matched external disturbance. Therefore, we have

$$\Delta B(t) = B\cdot D_B(t), \tag{19}$$

$$d(t) = B\cdot d_r(t), \tag{20}$$

where the term $d_r(t) \in R^{m\times 1}$.

The differential sliding surface will be

$$\dot{s}(t) = Gy(t) = GC\dot{x}, \tag{21}$$

where $s \in R^{m\times 1}$ and $G \in R^{m\times p}$.

The details of selecting G will be discussed later in the paper.

In addition, an $m \times m$ matrix $d_B(t)$ which is a particular similarity transformation of $D_B(t)$ will be used in design and is defined as follows:

$$d_B(t) = (GCB)D_B(t)(GCB)^{-1}. \tag{22}$$

Furthermore, there are three positive scalars δ_A, δ_B, and δ_d such that

$$\|\Delta A(t)\| \le \delta_A, \tag{23}$$

$$\|d_B(t)\| \le \delta_B < 1, \quad \|d_r(t)\| \le \delta_d. \tag{24}$$

Therefore, according to the above descriptions, (17) can be rewritten as follows:

$$x(t) = [A + \Delta A(t)]\dot{x}(t) + B\cdot[u(t) + D_B(t) + d_r(t)]. \tag{25}$$

Similarly, using the SMC method to achieve the asymptotic stability for the system (17) involves two major steps. The first step of SMC design is to find a sliding surface $s(t)$ which is described as

$$s(t) = GCx(t). \tag{26}$$

When the system is in sliding mode, the system dynamic with an applicable control law will satisfy the following newly proposed approaching condition:

$$\dot{s}^T(t)\cdot s(t) < -\alpha\|\dot{s}\| < 0. \tag{27}$$

The second step of the SMC process is to design a SMC control law so that the above approaching condition can occur; consequently, the system can reach the sliding surface and keep itself on the close neighborhood of the sliding surface.

3.1. The Selection of Sliding Surface. In this section, we would like to design the sliding surface in order to develop a new variable structure controller for system (17) such that the sliding motion is asymptotically stable.

If matrix B is partitioned into

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \tag{28}$$

where B_1 is $(n-m)\times m$ and B_2 is $m \times m$, one can define the following transfer matrix:

$$T = \begin{bmatrix} I_{(n-m)\times(n-m)} & -B_1B_2^{-1} \\ 0_{m\times(n-m)} & I_{m\times m} \end{bmatrix} \tag{29}$$

such that $T\cdot B = \begin{bmatrix} 0 & B_2 \end{bmatrix}^T$.

Please note that, for a controllable system, one can always obtain a B matrix with an invertible submatrix B_2 by properly defining the state variables and consequently obtain T.

Applying

$$q = Tx, \tag{30}$$

(17) and (18) can be transferred into

$$q_1 = (A_{11} + \Delta A_{11})\dot{q}_1 + (A_{12} + \Delta A_{12})\dot{q}_2, \quad (31)$$

$$q_2 = (A_{21} + \Delta A_{21})\dot{q}_1 + (A_{22} + \Delta A_{22})\dot{q}_2 + B_2 u + B_2 \cdot d_r, \quad (32)$$

$$y = C_1\dot{q}_1 + C_2\dot{q}_2 = C\dot{x} = \overline{C}\dot{q}, \quad (33)$$

where

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \in \begin{bmatrix} R^{n-m} \\ R^m \end{bmatrix},$$
$$TAT^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},$$
$$T\Delta AT^{-1} = \begin{bmatrix} \Delta A_{11} & \Delta A_{12} \\ \Delta A_{21} & \Delta A_{22} \end{bmatrix}, \quad (34)$$
$$\overline{C} = CT^{-1} = [C_1 \;\; C_2].$$

Now, considering the differential of sliding surface formed with state derivative output in (18) and (33)

$$\dot{s} = Gy = GC\dot{x} = G\overline{C}\dot{q} = GC_1\dot{q}_1 + GC_2\dot{q}_2 = 0, \quad (35)$$

the sliding surface is consequently obtained as follows:

$$s = GC_1q_1 + GC_2q_2 = GCx = G\overline{C}q = 0, \quad (36)$$

where G is an $m \times p$ matrix to be designed.

Assuming that the matrix GC_2 is nonsingular, from (35), one can have

$$\dot{q}_2 = -(GC_2)^{-1}(GC_1)\dot{q}_1. \quad (37)$$

Substituting (37) into (31), we obtain

$$q_1 = (A_{11} + \Delta A_{11})\dot{q}_1 + (A_{12} + \Delta A_{12})\left[(GC_2)^{-1}(GC_1)\dot{q}_1\right]. \quad (38)$$

From (38), the nominal system ($\Delta A_{11} = \Delta A_{12} = 0$) in sliding surface ($\because s = 0$, and $\dot{s} = 0$) is represented as

$$q_1 = (A_{11} - A_{12}FC_1)\dot{q}_1, \quad (39)$$

where

$$F = (GC_2)^{-1}G. \quad (40)$$

Therefore, (39) can be considered as a usual linear RSS output feedback problem [24]. Consequently, the design of sliding surface involves finding an output feedback gain F to stabilize the system (39).

For the matrices $F \in R^{m\times p}$ in (39) and $C_2 \in R^{p\times m}$ in (33), if F is a left inverse of C_2, we have

$$FC_2 = (GC_2)^{-1}GC_2 = I_{m\times m}. \quad (41)$$

When (41) holds, following the similar discussions in [34], the necessary and sufficient condition for the solution of $G \in R^{m\times p}$ in (36) can be selected as follows:

$$G = kF, \quad (42)$$

where k is a constant design parameter.

Substituting (42) into (41), one can easily verify the correctness of (42).

Since the proposed controller is based on the state derivative related output feedback, the solution exists only when one can find a matrix F which is a left inverse of C_2 such that (39) is stabilized. When this happens, one can select $G = kF$ and consequently both the differential sliding surface in (35) and the sliding surface in (36) are determined.

3.2. Sufficient Condition for Finding the Upper Bound of System Uncertainty to Guarantee the Stability in Sliding Surface. In this subsection, we will provide a sufficient condition to determine the upper bound of uncertainty ΔA so that the stability in sliding surface still can be guaranteed.

Using (38), (39), and (40), the mismatched uncertain system in sliding surface can be expressed as

$$q_1 = A_e\dot{q}_1 + \Delta A_e\dot{q}, \quad (43)$$

where

$$A_e = A_{11} - A_{12}FC_1, \quad \Delta A_e = \Delta A_{11} - \Delta A_{12}FC_1. \quad (44)$$

Theorem 1. *Assuming that A_e is a stable matrix and the time-varying uncertainty matrix ΔA_e in (43) has a bounded value ζ such that $\|\Delta A_e\| < \zeta$, one has*

$$\|\Delta A_e\| < \zeta = \frac{\min\{\eta_i\}}{2\lambda_{\max}(P_e)}, \quad i = 1, 2, \ldots, (n-m), \quad (45)$$

where η_i are all positive diagonal elements in Q_e, while Q_e and P_e are symmetric positive definite matrices in the following Lyapunov equation:

$$A_e^T P_e + P_e A_e = -Q_e. \quad (46)$$

Proof. Define the Lyapunov functional:

$$V = q_1^T P_e q_1, \quad (47)$$

where P_e is a symmetric positive definite matrix. It can be easily verified that V is a positive function. The time derivative of V along the trajectory of the system (43) is expressed as

$$\begin{aligned} \dot{V} &= \dot{q}_1^T P_e q_1 + q_1^T P_e \dot{q}_1 \\ &= \dot{q}_1^T P_e \left[A_e\dot{q}_1 + \Delta A_e\dot{q}_1\right] + \left[A_e\dot{q}_1 + \Delta A_e\dot{q}_1\right]^T P_e\dot{q}_1 \\ &= \dot{q}_1^T P_e A_e\dot{q}_1 + \dot{q}_1^T P_e\Delta A_e\dot{q}_1 + \dot{q}_1^T A_e^T P_e\dot{q}_1 + \dot{q}_1^T\Delta A_e^T P_e\dot{q}_1 \\ &= \dot{q}_1^T\left[P_eA_e + A_e^TP_e\right]\dot{q}_1 + 2\dot{q}_1^TP_e\Delta A_e\dot{q}_1. \end{aligned} \quad (48)$$

Then, substituting (46) into (48), one obtains

$$\dot{V} = \dot{q}_1^T[-Q_e]\dot{q}_1 + 2\dot{q}_1^TP_e\Delta A_e\dot{q}_1. \quad (49)$$

From (49), when the following condition is satisfied, we have $\dot{V} < 0$:

$$\dot{q}_1^TQ_e\dot{q}_1 > 2\dot{q}_1^TP_e\Delta A_e\dot{q}_1. \quad (50)$$

By Rayleigh's principle [35], the lower bound of $\dot{q}_1^T Q_e \dot{q}_1$ in (50) can be obtained as follows:

$$\begin{aligned} \dot{q}_1^T Q_e \dot{q}_1 &\geq \lambda_{\min}(Q_e)\, \dot{q}_1^T \dot{q}_1 = \lambda_{\min}(Q_e) \left\| \dot{q}_1 \right\|^2 \\ &= \min\{\eta_i\} \left\| \dot{q}_1 \right\|^2. \end{aligned} \tag{51}$$

From (50), the following inequality can also be obtained:

$$\begin{aligned} 2\dot{q}_1^T P_e \Delta A_e \dot{q}_1 &\leq 2\left\|\Delta A_e\right\| \lambda_{\max}(P_e)\, \dot{q}_1^T \dot{q}_1 \\ &= 2\left\|\Delta A_e\right\| \lambda_{\max}(P_e) \left\|\dot{q}_1\right\|^2. \end{aligned} \tag{52}$$

Thus, substituting (51) and (52) into (50), we have

$$2\left\|\Delta A_e\right\| \lambda_{\max}(P_e) \left\|\dot{q}_1\right\|^2 < \min\{\eta_i\} \left\|\dot{q}_1\right\|^2. \tag{53}$$

Therefore, if the following inequality holds, (50) holds, and, consequently, we have $\dot{V} < 0$:

$$\left\|\Delta A_e\right\| < \frac{\min\{\eta_i\}}{2\lambda_{\max}(P_e)} = \zeta, \quad i = 1, 2, \ldots, (n-m). \tag{54}$$

□

In a word, when (54) holds, the system with mismatched time-varying uncertainty ΔA_e in (43) in the sliding surface is asymptotically stable. Next, we have to provide another condition to guarantee that the system with the mismatched uncertainty ΔA is asymptotically stable in the sliding mode.

Theorem 2. *Let the transform matrix T in* (30) *be partitioned as*

$$T = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \qquad T^{-1} = \begin{bmatrix} R_1 & R_2 \end{bmatrix}, \tag{55}$$

where $L_1 \in R^{(n-m)\times n}$, $L_2 \in R^{m\times n}$, $R_1 \in R^{n\times(n-m)}$, and $R_2 \in R^{n\times m}$.

If the following condition holds:

$$\left\|\Delta A\right\| \leq \frac{\min\{\eta_i\}}{2\left(\left\|L_1\right\| \cdot \left\|R_1\right\| + \left\|L_1\right\| \cdot \left\|R_2\right\| \cdot \left\|F\right\| \cdot \left\|C_1\right\|\right) \lambda_{\max}(P_e)}, \tag{56}$$

where P_e and Q_e are defined in (46) *in Theorem 1, the RSS system with mismatched uncertainty ΔA in* (17) *is stable in the sliding surface.*

Proof. Since the transform matrix T in (29) can be partitioned as $T = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$ and $T^{-1} = \begin{bmatrix} R_1 & R_2 \end{bmatrix}$, the uncertain matrices ΔA_{11} and ΔA_{12} in (31) can be expressed as

$$\Delta A_{11} = L_1 \Delta A R_1, \qquad \Delta A_{12} = L_1 \Delta A R_2. \tag{57}$$

So the uncertainty ΔA_e given in (44) can be rewritten as

$$\Delta A_e = L_1 \Delta A R_1 - L_1 \Delta A R_2 F C_1. \tag{58}$$

Taking the norm of (58), one can obtain the following inequality:

$$\begin{aligned} \left\|\Delta A_e\right\| &\leq \left\|L_1 \Delta A R_1\right\| + \left\|L_1 \Delta A R_2 F C_1\right\| \\ &\leq \left\|\Delta A\right\| \left(\left\|L_1\right\| \cdot \left\|R_1\right\| + \left\|L_1\right\| \cdot \left\|R_2\right\| \cdot \left\|F\right\| \cdot \left\|C_1\right\|\right) \\ &< \frac{\min\{\eta_i\}}{2\lambda_{\max}(P_e)}. \end{aligned} \tag{59}$$

Consequently, the upper bound of ΔA is obtained as follows:

$$\left\|\Delta A\right\| < \frac{\min\{\eta_i\}}{2\left(\left\|L_1\right\| \cdot \left\|R_1\right\| + \left\|L_1\right\| \cdot \left\|R_2\right\| \cdot \left\|F\right\| \cdot \left\|C_1\right\|\right) \lambda_{\max}(P_e)}, \quad i = 1, 2, \ldots, (n-m). \tag{60}$$

□

From the above proof, it is clear to find that if the condition (60) in Theorem 2 and the condition (45) in Theorem 1 are satisfied, the system with the mismatched uncertainty ΔA is stable in the sliding mode. Since it is a sufficient condition, if (60) does not hold, it does not mean that the system will definitely become unstable. The above procedure is analogous to that in [34].

3.3. Design the State Derivative Related Output Feedback Controller. In this subsection, a sliding mode controller is designed to drive the system to sliding surface. Once the sliding surface is reached, the controller can keep the system inside the differential sliding layer without causing "chattering phenomenon." The only information that the controller needs is the state derivative related output signals.

Theorem 3. *Considering the dynamics system in* (17) *and* (18), *if the following output feedback controller $u(t)$ is applied, the system will be kept inside the differential sliding layer of $|\dot{s}| \leq \varepsilon$ and inside bounded sliding layer as well. Consider*

$$u(t) := -(GCB)^{-1} \left(\frac{\left\|\widetilde{w}_1\right\| \cdot \left\|y\right\| + \left\|\widetilde{w}_2\right\|}{\eta} + \alpha \right) \operatorname{sat}(\dot{s}(t), \varepsilon), \tag{61}$$

where $\widetilde{w}_1$, $\widetilde{w}_2$, and η are positive scalars such that

$$\begin{aligned} \widetilde{w}_1 &= \|G\| \cdot \|A\| + \|G\|\, \delta_A, \\ \widetilde{w}_2 &= \|G\| \cdot \|C\| \cdot \|B\|\, \delta_d, \\ \dot{s}(t) &= Gy = GC\dot{x}(t), \\ \eta &= 1 - \delta_B > 0, \end{aligned} \tag{62}$$

while ε is a selected small positive value, α is a selected positive scalar, and sat is a saturation function described as follows:

$$\operatorname{sat}(\dot{s}, \varepsilon) = \begin{cases} 1 & \dot{s} > \varepsilon \\ \dfrac{\dot{s}}{\varepsilon} & |\dot{s}| \leq \varepsilon \\ -1 & \dot{s} < -\varepsilon \end{cases} = \begin{cases} \operatorname{sign}(\dot{s}) & |\dot{s}| > \varepsilon \\ \dfrac{\dot{s}}{\varepsilon} & |\dot{s}| \leq \varepsilon. \end{cases} \tag{63}$$

Please note that $y = C\dot{x}(t)$ and $\dot{s}(t) = Gy = GC\dot{x}(t)$. Therefore, only state derivative related output feedback information is required for applying control law in (61).

Remark 4. The control law (61) cannot eliminate the external disturbance completely, but it can diminish the influence of the external disturbance and force both state derivatives and states in bounded layers.

Proof. According to (63), at first, we consider the following controller $u(t)$ for the case when $|\dot{s}| > \varepsilon$. Consider

$$u(t) := -(GCB)^{-1}\left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{\eta} + \alpha\right)\operatorname{sign}(\dot{s}(t), \varepsilon), \tag{64}$$

where sign is a sign function described as follows:

$$\operatorname{sign}(\dot{s}) = \frac{\dot{s}}{\|\dot{s}\|} = \begin{cases} 1 & \dot{s} > 0 \\ 0 & \dot{s} = 0 \\ -1 & \dot{s} < 0. \end{cases} \tag{65}$$

Then, substituting (17), (35), (64), and (65) into $s(t)$, the following equation is obtained:

$$\begin{aligned} s(t) &= GCA\dot{x}(t) + GC\Delta A\dot{x}(t) + GCB(1 + D_B) \\ &\quad \times \left[-(GCB)^{-1}\left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{\eta} + \alpha\right)\operatorname{sign}(\dot{s}(t))\right] \\ &\quad + GCd(t). \end{aligned} \tag{66}$$

Premultiplying $\dot{s}^T(t)$ on both sides of (66) and applying (22) and (62), we have

$$\begin{aligned} &\dot{s}^T(t) \cdot s(t) \\ &= \dot{s}^T(t)\left\{GCA\dot{x}(t) + GC\Delta A\dot{x}(t) + GCB(1 + D_B)\right. \\ &\qquad \times \left[-(GCB)^{-1}\left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{\eta} + \alpha\right)\right. \\ &\qquad \left.\left.\times \operatorname{sign}(\dot{s}(t))\right] + GCd(t)\right\} \\ &= \dot{s}^T(t) \cdot [GCA\dot{x}(t)] + \dot{s}^T(t) \cdot [GC\Delta A\dot{x}(t)] \\ &\quad - \left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{\eta} + \alpha\right)\|\dot{s}(t)\| \\ &\quad - \left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{\eta} + \alpha\right)\dot{s}^T(t)\, d_B \cdot \frac{\dot{s}(t)}{\|\dot{s}(t)\|} \\ &\quad + \dot{s}^T(t) \cdot GCBd_r(t) \\ &\leq -\alpha\|\dot{s}(t)\| - \left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{\eta}\right) \cdot \|\dot{s}(t)\| \\ &\quad - \left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{\eta} + \alpha\right) \cdot (-\delta_B)\|\dot{s}(t)\| \\ &\quad + \left\|(\|GCA\| + \|GC\|\,\delta_A)\,\dot{x}(t)\right\| \cdot \|\dot{s}(t)\| \\ &\quad + \delta_d \cdot \|GCB\| \cdot \|\dot{s}(t)\|. \end{aligned} \tag{67}$$

In (67), one can obtain the following inequalities.

For $\|d_B(t)\| \leq \delta_B < 1$ and $\eta = 1 - \delta_B > 0$, one obtains

$$\begin{aligned} &-\left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{\eta} + \alpha\right)\dot{s}^T(t)\, d_B \cdot \frac{\dot{s}(t)}{\|\dot{s}(t)\|} \\ &\quad \leq -\left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{\eta} + \alpha\right) \cdot (-\delta_B)\|\dot{s}(t)\|. \end{aligned} \tag{68}$$

For $\|\Delta A(t)\| \leq \delta_A$ and $\|d_r(t)\| \leq \delta_d$, on can have

$$\begin{aligned} &\dot{s}^T(t) \cdot [GCA\dot{x}(t)] + \dot{s}^T(t) \cdot [GC\Delta A\dot{x}(t)] \\ &\qquad \leq \left\|(\|GCA\| + \|GC\|\,\delta_A)\,\dot{x}(t)\right\| \cdot \|\dot{s}(t)\|, \\ &\quad \dot{s}^T(t) \cdot GCBd_r(t) \leq \delta_d \cdot \|GCB\| \cdot \|\dot{s}(t)\|. \end{aligned} \tag{69}$$

From (67), we can further obtain

$$\begin{aligned} &\dot{s}^T(t) \cdot s(t) \\ &\leq -\alpha\|\dot{s}(t)\| - \left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{\eta}\right) \cdot \|\dot{s}(t)\| \\ &\quad - \left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{\eta} + \alpha\right) \cdot (-\delta_B)\|\dot{s}(t)\| \\ &\quad + \left\|(\|GCA\| + \|GC\|\,\delta_A)\,\dot{x}(t)\right\| \cdot \|\dot{s}(t)\| \\ &\quad + \delta_d \cdot \|GCB\| \cdot \|\dot{s}(t)\| \\ &= -\left(\frac{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}{1 - \delta_B}\right) \cdot (1 - \delta_B) \cdot \|\dot{s}(t)\| \\ &\quad - (1 - \delta_B)\,\alpha\|\dot{s}(t)\| + \left(\left\|(\|GCA\| + \|GC\|\,\delta_A)\,\dot{x}(t)\right\|\right. \\ &\qquad \left. + \delta_d \cdot \|GCB\|\right) \cdot \|\dot{s}(t)\| \\ &= -(1 - \delta_B)\,\alpha\|\dot{s}(t)\| - \left(\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|\right)\|\dot{s}(t)\| \\ &\quad \cdot \left(1 - \frac{\left\|(\|GCA\| + \|GC\|\,\delta_A)\,\dot{x}(t)\right\| + \delta_d \cdot \|GCB\|}{\|\widetilde{w}_1\| \cdot \|y\| + \|\widetilde{w}_2\|}\right) \\ &\leq -(1 - \delta_B)\,\alpha\|\dot{s}(t)\| = -\beta\|\dot{s}(t)\| < 0, \end{aligned} \tag{70}$$

where

$$\begin{aligned}(1-\delta_B)\,\alpha &= \beta > 0,\\ \|\widetilde{w}_1\|\cdot\|y\| &= \big\|\|G\|\cdot\|A\| + \delta_A\|G\|\big\|\cdot\|C\dot{x}(t)\| \\ &\geq \big\|(\|GCA\| + \|GC\|\,\delta_A)\,\dot{x}(t)\big\|,\\ \|\widetilde{w}_2\| &= \|G\|\,\|C\|\,\|B\|\,\delta_d \geq \|(GCB)\|\,\delta_d,\end{aligned} \tag{71}$$

and, consequently,

$$\left(1-\frac{\big\|(\|GCA\|+\|GC\|\,\delta_A)\,\dot{x}(t)\big\| + \delta_d\cdot\|GCB\|}{\|\widetilde{w}_1\|\cdot\|y\| + \|\widetilde{w}_2\|}\right) \geq 0. \tag{72}$$

Therefore, from (70), the approaching condition for the case of $|\dot{s}| > \varepsilon$ is assured.

When $|\dot{s}| \leq \varepsilon$, the controller becomes

$$u(t) := -(GCB)^{-1}\left(\frac{\|\widetilde{w}_1\|\cdot\|y\| + \|\widetilde{w}_2\|}{\eta} + \alpha\right)\cdot\frac{\dot{s}}{\varepsilon}. \tag{73}$$

Since $|\dot{s}/\varepsilon| < 1$, through a similar derivation, one can find that the approaching condition may not always happen inside the differential sliding layer of $|\dot{s}| \leq \varepsilon$ and may result in the increase of $|\dot{s}|$. However, when $|\dot{s}|$ is increased to be bigger than ε, the controller will switch to (64) to push the system back to the differential sliding layer of $|\dot{s}| \leq \varepsilon$.

One may wonder that if $\dot{s}$ is bounded inside the differential sliding layer $|\dot{s}| \leq \varepsilon$, can the amplitude of its integral s still keep increasing as time goes by and finally become diverged? The answer is negative because when $\dot{s} = GC\dot{x}$ is bounded, so is $\dot{x}$ due to the fact that G and C are constant matrices. When $\dot{x}$ is bounded, from the system equation in (17) and controller in (61), x which can be expressed in terms of $\dot{x}$ must be bounded too. Consequently, $s = GCx$ must also be bounded. Similarly, through the system constraint in (17), when the approaching condition does not happen inside the differential sliding layer of $|\dot{s}| \leq \varepsilon$, both $|s|$ and $|x|$ will be increased, so will $|\dot{s}|$ and $|\dot{x}|$. When $\dot{s}$ finally reaches to the condition of $|\dot{s}| > \varepsilon$, the controller will switch to (64) to push the system back to the differential sliding layer of $|\dot{s}| \leq \varepsilon$. In this manner, the controller can keep the $\dot{s}$ inside the differential sliding layer of $|\dot{s}| \leq \varepsilon$ in steady state. Consequently, through the system constraint, s and x can also be bounded in steady state. □

Ideally, the controller can drive the system to reach and stay on the sliding surface. However, it may cause "chattering phenomenon" because its switching frequency is required to be infinite. To avoid this problem, "sign" function is replaced by "sat" function in our proposed controller in (61) to bound the system inside the differential sliding layer of $|\dot{s}| \leq \varepsilon$ in steady state. Although it may cause the deterioration of accuracy and robustness, wasting small accuracy is still worthier than causing "chattering phenomenon." This concludes the proof.

Other than sliding mode control, an expert in control can easily apply Lyapunov approach [36] to stabilize the system in (22)-(23). However, for a novice in control, it is not always apparent to him that the Lyapunov function should be chosen in designs. Therefore, there are difficulties for him to apply Lyapunov approach to stabilize the system in (22)-(23). On the contrary, the structure of the proposed controller in (61) is clear and simple and only state derivative related output feedback information is required. Therefore, a novice in control can easily handle the implementation of the proposed controller. Our derivation is basically parallel to that of systems in standard state space form. Therefore, the proposed method can be easily adopted by many control designers.

4. Numerical Example

The following example is provided to justify the proposed algorithm of SMC design with state derivative output feedback in RSS form.

Example 1. Considering the following RLC circuit system in Figure 1, let x_1, x_2, x_3, and u be the current of inductor L, the voltage of capacitor C_2, the voltage of capacitor C_1, and the control input voltage, respectively.

Applying Kirchhoff's current law at node P, one can obtain x_2 as follows:

$$C_1\dot{x}_3 = C_2\dot{x}_2 + \frac{x_2}{R}, \qquad x_2 = RC_1\dot{x}_3 - RC_2\dot{x}_2. \tag{74}$$

Applying Kirchhoff's current law at node Q and (74), one can obtain x_1 as follows:

$$x_1 = C_2\dot{x}_2 + \frac{x_2}{R} = C_2\dot{x}_2 + C_1\dot{x}_3 - C_2\dot{x}_2 = C_1\dot{x}_3. \tag{75}$$

Applying (74) and Kirchhoff's voltage law to the left-hand side loop, x_3 is solved as

$$\begin{aligned}u &= x_2 + x_3 + L\dot{x}_1,\\ x_3 &= u - x_2 - L\dot{x}_1 = -RC_1\dot{x}_3 + RC_2\dot{x}_2 - L\dot{x}_1 + u.\end{aligned} \tag{76}$$

The system of (74)–(76) in RSS form is obtained as follows:

$$\begin{aligned}x = \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} &= \begin{bmatrix}0 & 0 & C_1\\ 0 & -RC_2 & RC_1\\ -L & RC_2 & -RC_1\end{bmatrix}\begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix}u,\\ y = x_2 &= \begin{bmatrix}L & -RC_2 & RC_1\end{bmatrix}\cdot\begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\end{bmatrix}.\end{aligned} \tag{77}$$

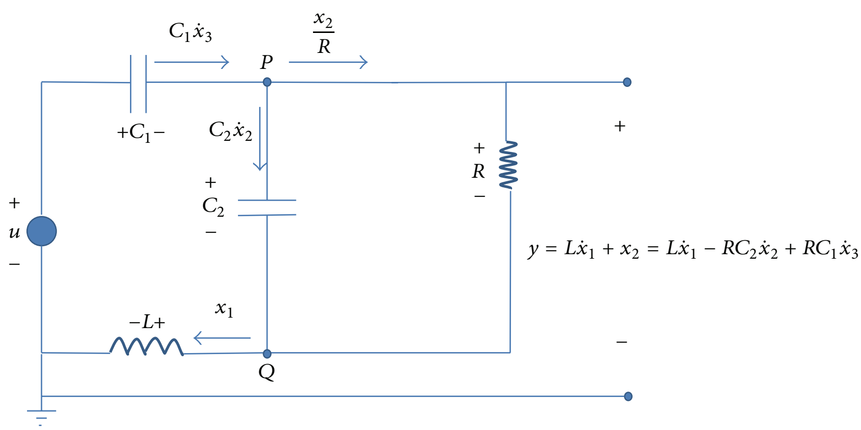

Figure 1: RLC circuit system.

Let $R = 100$, $C_1 = 0.1$, $C_2 = 0.1$, and $L = 1000$; the nominal system in RSS form is as follows:

$$\begin{aligned} x &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.1 \\ 0 & -10 & 10 \\ -1000 & 10 & -10 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ &= A\dot{x} + Bu, \\ y &= \begin{bmatrix} 1000 & -10 & 10 \end{bmatrix} \cdot \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = C\dot{x}, \\ A &= \begin{bmatrix} 0 & 0 & 0.1 \\ 0 & -10 & 10 \\ -1000 & 10 & -10 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad B_2 = [1], \\ C &= \begin{bmatrix} 1000 & -10 & 10 \end{bmatrix}. \end{aligned} \tag{78}$$

From (29), we have

$$\begin{aligned} T &= \begin{bmatrix} I_{(n-m)\times(n-m)} & -B_1 B_2^{-1} \\ 0_{m\times(n-m)} & I_{m\times m} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ T^{-1} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{79}$$

Therefore, the system is already in the form of (31)–(33) and we have

$$\begin{aligned} \overline{C} &= CT^{-1} = \begin{bmatrix} C_1 & C_2 \end{bmatrix} = \begin{bmatrix} 1000 & -10 & 10 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 1000 & -10 \end{bmatrix}, \\ C_2 &= 10, \\ T \cdot B &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ B_2 \end{bmatrix}, \qquad B_2 = 1, \\ A_{11} &= \begin{bmatrix} 0 & 0 \\ 0 & -10 \end{bmatrix}, \qquad A_{12} = \begin{bmatrix} 0.1 \\ 10 \end{bmatrix}, \end{aligned} \tag{80}$$

for

$$q_1 = \left(A_{11} - A_{12} F C_1\right) \dot{q}_1. \tag{81}$$

Since $C_2 = 10$ in this example, from (41), we have $F = 0.1$. When $F = 0.1$, q_1 in (81) is stable. Therefore, $F = 0.1$ is a workable output feedback gain for (81).

When constant k in (42) is selected as $k = 1$, the following sliding surface is obtained for simulation:

$$\begin{aligned} s &= GCx = kFCx = 0.1 \cdot 1 \cdot \begin{bmatrix} 1000 & -10 & 10 \end{bmatrix} x \\ &= \begin{bmatrix} 100 & -1 & 1 \end{bmatrix} x. \end{aligned} \tag{82}$$

To illustrate the performance of the proposed controller, suppose that the system has the following uncertainties and disturbance:

$$\begin{aligned} \Delta A &= \begin{bmatrix} 0 & 0 & 10 \\ 0 & 10 & 10 \\ 10 & 10 & 10 \end{bmatrix} \sin(t), \\ \Delta B &= \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix} \cos(5t), \\ d &= \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \sin(3t). \end{aligned} \tag{83}$$

Based on them, the design parameters are selected as follows.

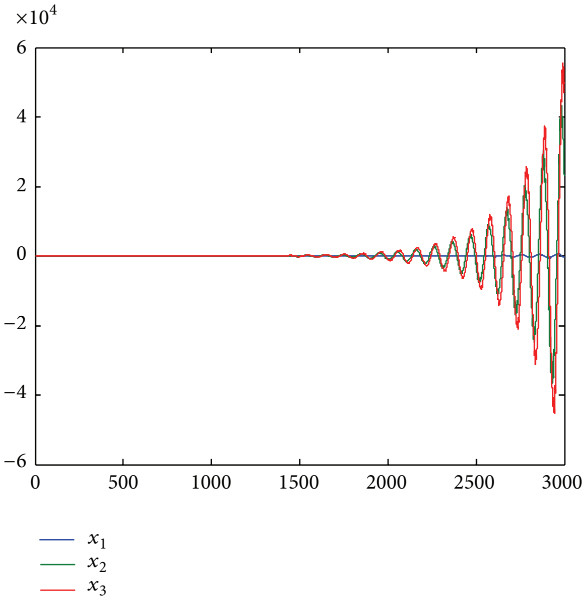

FIGURE 2: State responses for uncontrolled system subject to uncertainties and disturbance.

Since the norm of ΔA is 22.4698, we select $\delta_A = 23$ in design:

$$\because \Delta B = BD_B(t) = \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix} \cos(5t),$$
$$\therefore D_B(t) = 0.2\cos(5t), \tag{84}$$
$$d_B(t) = (GCB)\,D_B(t)\,(GCB)^{-1} = 0.2\cos(5t).$$

Therefore, $\delta_B = 0.2$ is selected in design:

$$\because d = Bd_r(t) = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \sin(3t), \tag{85}$$
$$\therefore d_r(t) = 3\sin(3t), \qquad \delta_d = 3.$$

When external disturbance and system uncertainties are considered, the system is unstable if no control is applied. All states are diverged as shown in Figure 2.

To test the correctness of proposed approaching condition in (27) and the selected sliding surface, simulations were performed. For simulation, $\varepsilon = 0.005$ and $\alpha = 5$ are selected in the SMC control law of (61). Applying this control law, the time responses of sliding surface, states, and control effort are plotted in Figures 3, 4, and 5, respectively.

From Figure 3, we can find that the sliding surface response does converge to zero; that is, both approaching phase and sliding phase occur. Therefore, when the control is applied, it is not surprising to see that all states and control effort are converged in Figures 4 and 5, respectively. From the simulation result, we conclude that approaching condition in (27), the SMC controller design of state derivative output feedback in (61), and the proposed novel saturation switching function do work effectively for the system in RSS form.

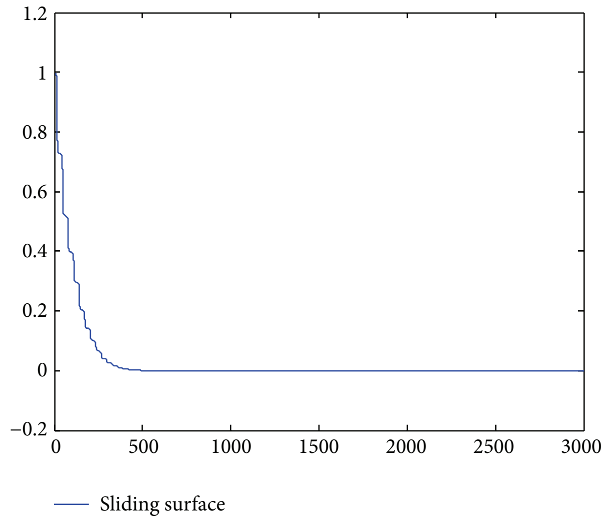

FIGURE 3: Sliding surface with system uncertainties and control.

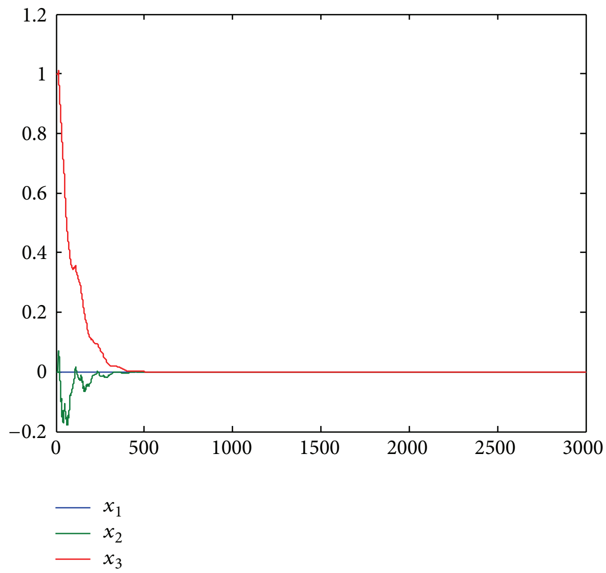

FIGURE 4: State responses with system uncertainties and control.

5. Conclusion

The needs for reciprocal state space (RSS) form have been addressed. Novel sliding mode control design approach with state derivative output feedback in reciprocal state space framework has been presented. Nontraditional switching function utilizing the derivative of sliding surface is proposed and proven to satisfy the approaching condition of sliding mode. In addition, algorithm of finding upper bound of system uncertainty has been developed for robustness analysis. Simulation results of the RLC circuit system successfully verify the proposed algorithms. Since our derivation is basically parallel to that of systems in standard state space form, the contribution of this paper is to provide SMC design approach by applying direct state derivative related output feedback in nontraditional RSS form so that people

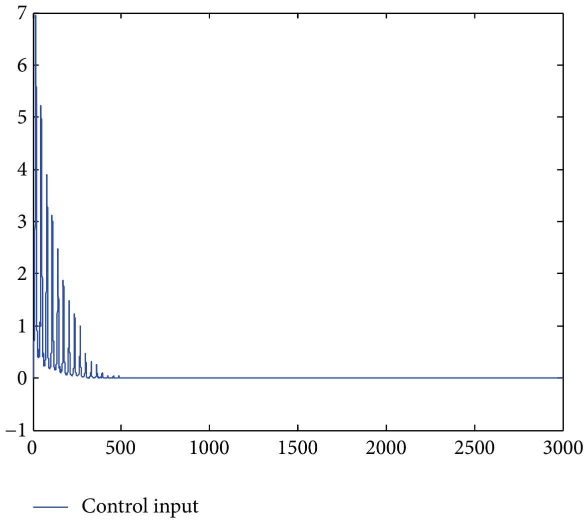

FIGURE 5: Control effort.

can handle more control problems without too much of a mathematical overhead.

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