Research Article

Optimal Control Policies of Pests for Hybrid Dynamical Systems

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We improve the traditional integrated pest management (IPM) control strategies and formulate three specific management strategies, which can be described by hybrid dynamical systems. These strategies can not only effectively control pests but also reduce the abuse of pesticides and protect the natural enemies. The aim of this work is to study how the factors, such as natural enemies optimum choice in the two kinds of different pests, timings of natural enemy releases, dosages and timings of insecticide applications, and instantaneous killing rates of pesticides on both pests and natural enemies, can affect the success of IPM control programmes. The results indicate that the pests outbreak period or frequency largely depends on the optimal selective feeding of the natural enemy between one of the pests and the control tactics. Ultimately, we obtain the only pest x_2 needs to be controlled below a certain threshold while not supervising pest x_1.

1. Introduction

A pest is an insect which is detrimental to humans or human concerns (as agriculture or livestock production). In its broadest sense, a pest is a competitor of humanity. Often insects are regarded as pests as they cause damage to agriculture by feeding on crops or parasitizing livestock, such as codling moth on apples or boll weevil on cotton. An animal could also be a pest when it causes damage to a wild ecosystem or carries germs within human habitats. Examples of these include those organisms which are vector-borne human diseases, such as rats and fleas which carry the plague disease, mosquitoes which are vector-borne malaria, and ticks which carry Lyme disease. The most serious pests (in the order of economic importance) are insects. Pesticides are chemicals and other agents (e.g., beneficial microorganisms) that are used to control or protect other organisms from insect pests. To control these insect pests, farmers rely strongly on intervention with chemical pesticides, which remain a significant component of the cost of production and ecological problems from pesticide resistance in key pests. In order to address the issue, researchers are increasingly embracing more components of the integrated pest management (IPM) [1–6] systems approach that is always applied in ecology.

With the rise of interdisciplinary research, the mathematical ecology has also emerged and developed rapidly. A variety of mathematical methods can be used in ecological science. There have been numerous publications [7–15] over the last ten years using ecological mathematical model to research IPM strategy (spraying pesticides and introducing additional natural enemy into a pest-natural enemy system). When we study the dynamic property between the pest and its natural enemy (predator-prey), one of the most important components of the predator-prey relationship is the so-called functional responses. In [7–15], the Holling functional responses and Beddington-DeAngelis functional response are introduced. The Holling type extends the range of values of x and y over which the feeding term is realistic. However, in some situations, the increase of the feeding rate is not proportional to the increase of the predator density, as a result of mutual interference between predators, which decreases the efficiency of predation [16]. In addition, it is shown that most of the mathematical models on IPM include only one pest and one natural enemy. In [1], Finch and Collier's study concerning IPM strategies in field vegetable crops focuses on two key pests, the cabbage root fly (*Delia radicum*) and the carrot fly (*Psila rosae*), the two major root feeding pests. Thus, we consider a continuous three-level food web model with

Beddington-DeAngelis functional response, which consists of two competing pests (x_1 and x_2) and a natural enemy (y), and it can be represented as follows [17]:

$$\frac{dx_1}{dt} = a_1 x_1 \left(1 - \frac{x_1}{K_1} - \frac{c_1 x_2}{K_1} \right) - F_1 \left(x_1, x_2, y \right) y,$$

$$\frac{dx_2}{dt} = a_2 x_2 \left(1 - \frac{x_2}{K_2} - \frac{c_2 x_1}{K_2} \right) - F_2 \left(x_1, x_2, y \right) y,$$

$$\frac{dy}{dt} = \omega_1 F_1 \left(x_1, x_2, y \right) y + \omega_2 F_2 \left(x_1, x_2, y \right) y - dy,$$

$$x_i \left(0 \right) \geq 0, \quad y \left(0 \right) \geq 0, \quad i = 1, 2,$$

where $F_i(x_1, x_2, y) = \alpha_i x_i / (r + x_1 + bx_2 + cy)$, $i = 1, 2$. a_i, K_i, ω_i, c_i, and d are positive ($i = 1, 2$). The prey x_i grows with intrinsic growth rate a_i and carries capacity K_i in the absence of the predator. The constant c_i is the interspecific competition rate between the two prey species. The predator y consumes the prey x_i with a functional response of Beddington-DeAngelis type $F_i(x_1, x_2, y)$ and contributes to its growth with rate $\omega_i F_i(x_1, x_2, y)$. The constant d is the death rate of the predator, and the term, cy, measures the mutual interference between predators. The constants α_i ($i = 1, 2$) show the predation capacity of predators, and r is the saturating functional response parameters. The parameter b is the predator's relative preference on x_2 with respect to x_1.

There are numbers of diagnostic tools to detect the qualitative behavior of the system (1). We give a simple investigation of the dynamics of the system (1) through constructing a bifurcation diagram using the software XPPAUT [18]. To construct the bifurcation diagram, we use a numerical integration with varying a key parameter and keeping other parameters fixed; our choice of parameters is guided by two assumptions: first, the system has to be biologically feasible, and second, natural enemies have food preference phenomenon [19]; that is, $\alpha_1 > \alpha_2$ or $\alpha_1 < \alpha_2$ in this paper. The bifurcation diagram as a function of α_1 in the range $0.3 < \alpha_1 < 0.7$ is drawn in Figure 1. As shown in Figure 1, the thick black lines represent the stable limit cycle, the black solid curves represent the stable equilibrium state, and the black dashed curves represent the unstable equilibrium state. When $\alpha_1 = 0.4601$, system (1) undergoes a Hopf bifurcation, where the stable focus becomes unstable and a stable limit cycle emanates from it. When the parameter α_1 increases further, system (1) undergoes a Hopf bifurcation again at $\alpha_1 = 0.5644$, where an unstable focus becomes stable and the other unstable focus is still unstable. When $\alpha_1 = 0.5698$, a transcritical bifurcation appears, where the stability of two focuses occurs in exchange. In Figure 2, we fixed the parameter $\alpha_1 = 0.6000$, and the other parameters are the same as those in Figure 1. As we can see in Figure 2, the pest x_1 tends to be extinct eventually due to the impact of the food preference phenomenon of natural enemies before recurrence of the next generation of pests, while the pest x_2 and natural enemies can coexist through stabilizing the boundary equilibrium.

In order to manage pests through spraying pesticides and introducing additional natural enemies (IPM strategy),

we apply the impulsive differential equations (IDE) [20–22] to integrate system (1). In [13], the authors constructed a predator-prey impulsive system to show the process of releasing natural enemies periodically and spraying pesticides twice at different fixed times in a period. The authors, obviously, avoid the side effects of pesticides on natural enemies existing in [7–9]. But only two pulses in a period cannot effectively control pests. And most of the research is single species of pests while few papers have discussed multispecies of pests. However, most real pests communities are more complex than the community previously analysed by them. In the present paper, we make the following improvements: (a) in the modeling, we introduce a one-natural enemy and two-pest model, and the natural enemy shows optimal foraging between pest x_1 and pest x_2, which is a well-known behavior of many predators [23]; (b) in the control strategy, we can control two pests more selectively by controlling pulse frequencies appropriately.

2. Model Formulation and Auxiliary Lemmas

Considering the previous factors, firstly, two models of different control strategies are discussed as follows.

Case 1. We suppose pesticides are sprayed several times in a release period, and the kill rates of pesticides to pests (δ_k^i, $i = 1, 2$) and natural enemies (δ_k^3) are different in different impulsive moments in a release period. That is, we consider the following model:

$$\frac{dx_1}{dt} = a_1 x_1 \left(1 - \frac{x_1}{K_1} - \frac{c_1 x_2}{K_1} \right)$$

$$- \frac{\alpha_1 x_1 y}{r + x_1 + bx_2 + cy},$$

$$\frac{dx_2}{dt} = a_2 x_2 \left(1 - \frac{x_2}{K_2} - \frac{c_2 x_1}{K_2} \right)$$

$$- \frac{\alpha_2 x_2 y}{r + x_1 + bx_2 + cy},$$

$$\frac{dy}{dt} = \frac{\omega_1 \alpha_1 x_1 y}{r + x_1 + bx_2 + cy}$$

$$+ \frac{\omega_2 \alpha_2 x_2 y}{r + x_1 + bx_2 + cy} - dy,$$

$$t \neq \tau_{nk}, \quad t \neq nT,$$

$$x_1 \left(\tau_{nk}^+ \right) = \left(1 - \delta_k^1 \right) x_1 \left(\tau_{nk} \right),$$

$$x_2 \left(\tau_{nk}^+ \right) = \left(1 - \delta_k^2 \right) x_2 \left(\tau_{nk} \right),$$

$$y \left(\tau_{nk}^+ \right) = \left(1 - \delta_k^3 \right) y \left(\tau_{nk} \right),$$

$$t = \tau_{nk},$$

$$x_1 \left(nT^+ \right) = 0,$$
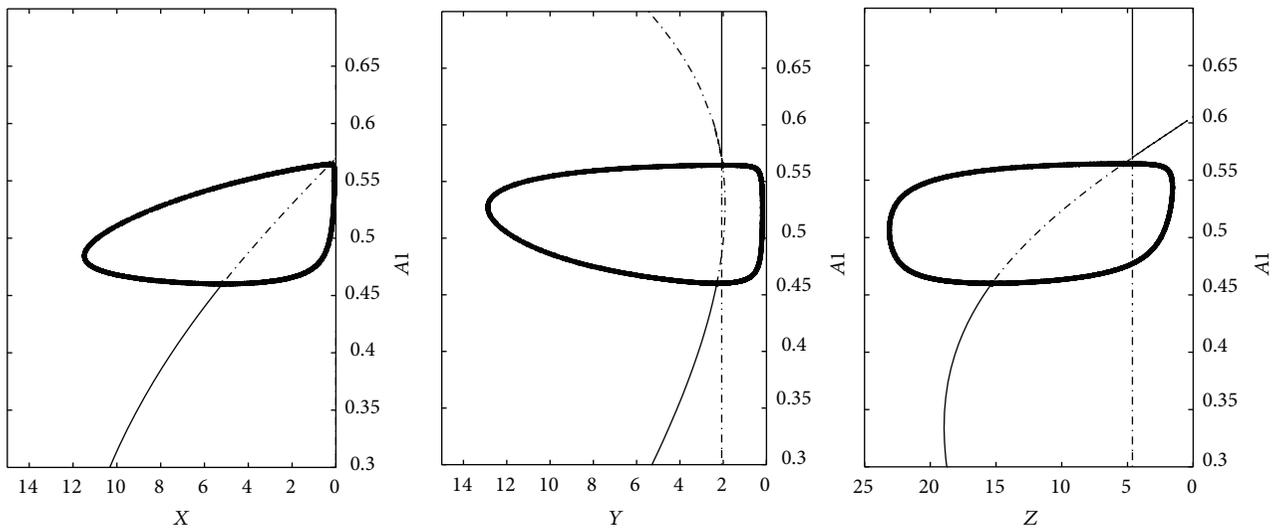
$$x_2 \left(nT^+ \right) = 0,$$

FIGURE 1: Bifurcation diagram of system (1) for $0.3 < \alpha_1 < 0.7$, where $X = x_1$, $Y = x_2$, $Z = y$, and $A1 = \alpha_1$ and the initial value $x_1 = 1.3$; $x_2 = 1.5$; and $y = 5$ with parameters as follows: $a_1 = 0.9$, $a_2 = 0.8$, $\alpha_2 = 0.5$, $c_1 = 0.55$, $K_1 = 20$, $r = 1.2$, $b = 0.25$, $c = 0.3$, $\alpha_2 = 0.5$, $K_2 = 30$, $c_2 = 0.55$, $W_1 = 0.85$, $W_2 = 0.75$, and $d = 0.25$.
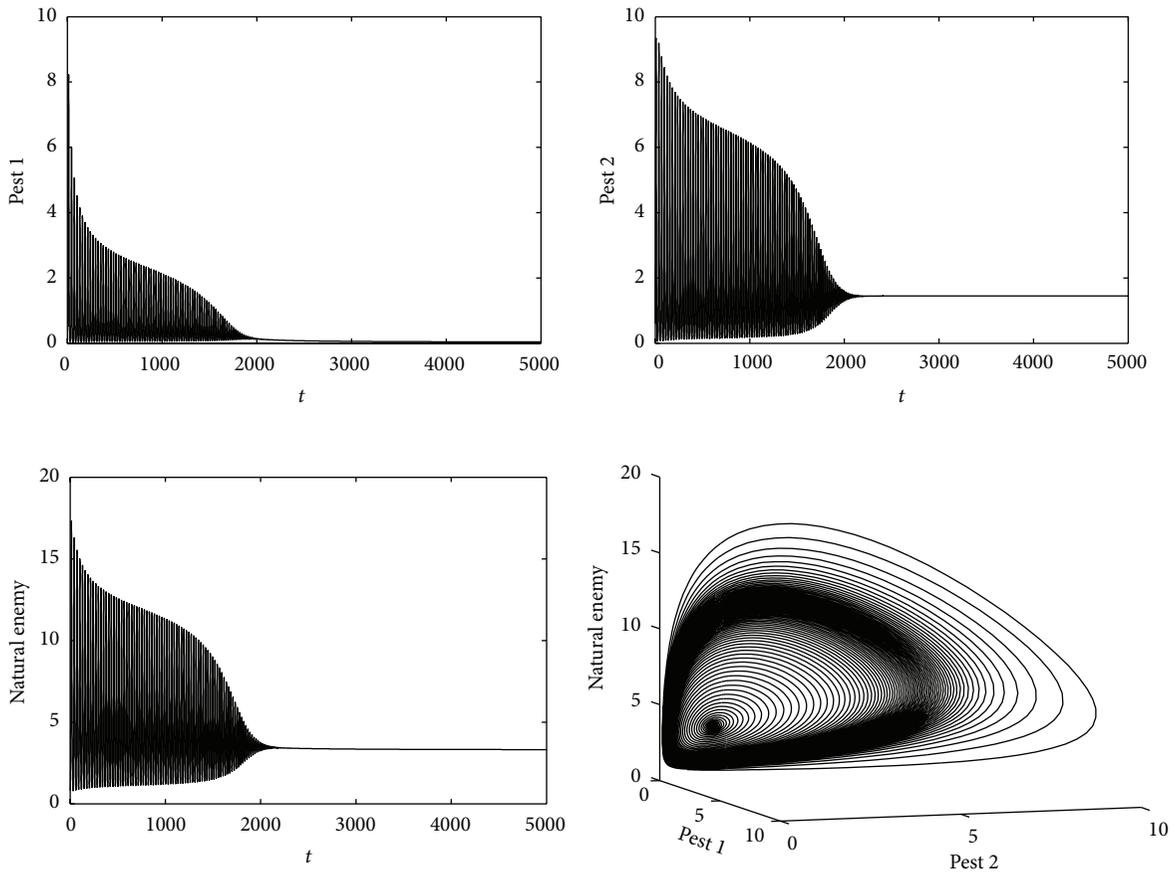

FIGURE 2: Time series and phase portrait of system (1) with $\alpha_1 = 0.6000$ and other parameters the same as Figure 1.

$$y\left(nT^{+}\right) = y\left(nT\right) + \theta,$$

$$t = nT,$$

$$(2)$$

where $nT = \tau_{n0} < \tau_{n1} < \tau_{n2} < \cdots < \tau_{nq} < \tau_{n(q+1)} = (n+1)T$, $n, q \in Z^{+}$; $k = 0, 1, 2, \ldots, q$; $\tau_{n(k+1)} - \tau_{nk} = T/(q+1)$; $0 \le \delta_{k}^{i} < 1$ ($i = 1, 2, 3$) represents an effective kill rate to pest x_{1}, pest x_{2}, and natural enemy y at time $t = \tau_{nk}$, respectively. θ represents natural enemy number of additional release at $t = nT$; T is a release period; τ_{nk} is the kth spraying moment in the nth release period. Other parameters are the same as those in system (1).

Case 2. We suppose that the natural enemies are released several times in a spraying period. The release number of natural enemies (θ_{k}) and an effective kill rate to natural enemies (μ_{k}) are different in different impulsive moments in a spraying period. That is, we consider the following model:

$$\frac{dx_{1}}{dt} = a_{1}x_{1}\left(1 - \frac{x_{1}}{K_{1}} - \frac{c_{1}x_{2}}{K_{1}}\right)$$

$$- \frac{\alpha_{1}x_{1}y}{r + b_{1}x_{1} + b_{2}x_{2} + cy},$$

$$\frac{dx_{2}}{dt} = a_{2}x_{2}\left(1 - \frac{x_{2}}{K_{2}} - \frac{c_{2}x_{1}}{K_{2}}\right)$$

$$- \frac{\alpha_{2}x_{2}y}{r + b_{1}x_{1} + b_{2}x_{2} + cy},$$

$$\frac{dy}{dt} = \frac{\omega_{1}\alpha_{1}x_{1}y}{r + b_{1}x_{1} + b_{2}x_{2} + ey}$$

$$+ \frac{\omega_{2}\alpha_{2}x_{2}y}{r + b_{1}x_{1} + b_{2}x_{2} + ey} - dy,$$

$$t \ne \lambda_{nk}, \quad t \ne nT_{1},$$

$$x_{1}\left(\lambda_{nk}^{+}\right) = 0,$$

$$x_{2}\left(\lambda_{nk}^{+}\right) = 0,$$

$$y\left(\lambda_{nk}^{+}\right) = \left(1 - \mu_{k}\right)y\left(\lambda_{nk}\right) + \theta_{k},$$

$$t = \lambda_{nk},$$

$$x_{1}\left(nT_{1}^{+}\right) = \left(1 - \delta_{1}^{1}\right)x_{1}\left(nT_{1}\right),$$

$$x_{2}\left(nT_{1}^{+}\right) = \left(1 - \delta_{1}^{2}\right)x_{2}\left(nT_{1}\right),$$

$$y\left(nT_{1}^{+}\right) = \left(1 - \delta_{1}^{3}\right)y\left(nT_{1}\right),$$

$$t = nT_{1},$$

$$(3)$$

where $nT_{1} = \lambda_{n0} < \lambda_{n1} < \lambda_{n2} < \cdots < \lambda_{np} < \lambda_{n(p+1)} = (n+1)T_{1}$, $n, p \in Z^{+}$; $k = 0, 1, 2, \ldots, p$; $\lambda_{n(k+1)} - \lambda_{nk} = T_{1}/(p+1)$; $0 \le \delta_{1}^{i} < 1$ ($i = 1, 2, 3$) represents an effective kill rate to pest x_{1}, pest x_{2} and natural enemy y at time $t = \lambda_{nk}$, respectively. μ_{k} represents reduced proportion of natural enemies owing to the delay effect of pesticides and eating the deleterious pests;

θ_{k} represents natural enemy number of additional release at $t = \lambda_{nk}$; T_{1} is a spraying period; λ_{nk} is the kth releasing moment in the spraying nth period. Other parameters are the same as those in system (1).

Furthermore, some essential notations, definitions and lemmas are given as follows.

Let $R_{+} = [0, \infty)$ and $R_{+}^{3} = \{\mathbf{x} = (x_{1}(t), x_{2}(t), y(t) \in R^{3} : x_{1}(t), x_{2}(t), y(t) \ge 0)\}$. Denote $\mathbf{f} = (f_{1}, f_{2}, f_{3})$ the map defined by the right-hand side of the first, second, and third equations of system (2). The solution of system (2), denoted by $\mathbf{x} : R_{+} \to R_{+}^{3}$, is piecewise continuous, and it is continuous on $((n-1)T, \tau_{n1}], (\tau_{n1}, \tau_{n2}], \ldots, (\tau_{nq}, nT]$. Note that $\mathbf{x}((\tau_{nk}^{+})) = \lim_{t \to \tau_{nk}^{+}} x(t)$ and $x(nT^{+}) = \lim_{t \to nT^{+}} x(t)$ exist. The existence and uniqueness of solutions of system (2) are guaranteed by the smoothness of $\mathbf{f}$ (see [22]).

Lemma 1. *Assume that $\mathbf{x}(t)$ is a solution of system (2) with $\mathbf{x}(0^{+}) \ge 0$, then $\mathbf{x}(t) \ge 0$ for $t \ge 0$.*

Lemma 2 (see Lakshmikantham et al. [22]). *Consider the following impulsive differential inequalities:*

$$m'\left(t\right) \le \left(\ge\right) p\left(t\right) m\left(t\right) + q\left(t\right), \quad t \ne t_{k},$$

$$m\left(t_{k}^{+}\right) \le \left(\ge\right) d_{k} m\left(t_{k}\right) + b_{k}, \quad t = t_{k}, \ k \in N,$$

$$(4)$$

where $p(t), q(t) \in C(R_{+}, R)$, $d_{k} \ge 0$, and b_{k} are constants. Assume that:

(A_{0}) *the sequence $\{t_{k}\}$ satisfies $0 \le t_{0} < t_{1} < t_{2} < \cdots$, with $\lim_{t \to \infty} t_{k} = \infty$;*

(A_{1}) *$m(t) \in PC'(R_{+}, R)$ and $m(t)$ is left continuous at t_{k}, $k = 1, 2, \ldots$;*

(A_{2}) *for $k = 1, 2, \ldots, t \ge t_{0}$.*

Then,

$$m\left(t\right) \le \left(\ge\right) m\left(t_{0}\right) \prod_{t_{0} < t_{k} < t} d_{k} \exp\left(\int_{t_{0}}^{t} p\left(s\right) ds\right)$$

$$+ \sum_{t_{0} < t_{k} < t}\left(\prod_{t_{k} < t_{j} < t} d_{j} \exp\left(\int_{t_{k}}^{t} p\left(s\right) ds\right)\right) b_{k}$$

$$+ \int_{t_{0}}^{t} \prod_{s < t_{k} < t} d_{k} \exp\left(\int_{s}^{t} p\left(\sigma\right) d\sigma\right) q\left(s\right) ds, \quad t \ge t_{0}.$$

$$(5)$$

Remark 3. The previous definitions and lemmas may similarly be applied in system (3).

3. Dynamical Analysis of Case 1 and Its Biological Implications

For Case 1, the basic properties of the following subsystem:

$$\frac{dy}{dt} = -dy(t), \quad t \neq nT, \ t \neq \tau_{nk},$$

$$y(t^+) = \left(1 - \delta_k^3\right) y(t), \quad t = \tau_{nk}, \ k = 1, 2, \ldots, q,$$

$$y(t^+) = y(t) + \theta, \quad t = nT,$$

$$y(0^+) = y_0$$

(6)

play a key role in analyzing the pest control.

It is shown in the Appendix that there exists a globally stable periodic solution $y_T(t)$ for the subsystem (6). Therefore, the complete expression for the pest-eradication periodic solution of system (6) over the $(n-1)$th time interval $(n-1)T \leq t \leq nT$ is given by $(0, 0, y_T(t))$. Furthermore, if the following threshold condition:

$$\lambda_i = \prod_{k=1}^{q} \left(1 - \delta_k^i\right) \exp\left(a_i T + \frac{\alpha_i B_1}{cd}\right) < 1 \quad (i = 1, 2)$$

(7)

is satisfied, then the pest-eradication periodic solution $(0, 0, y_T(t))$ is globally asymptotically stable, where $Y^* = \theta / [1 - (1 - \delta_1^3) \cdots (1 - \delta_q^3) e^{-dT}]$ and

$$B_1 = \ln \left(\left(r + cY^* e^{-dT/(q+1)}\right) \left(r + cY^* \left(1 - \delta_1^3\right) e^{-2dT/(q+1)}\right) \right.$$

$$\cdots \left(r + cY^* \prod_{k=1}^{q} \left(1 - \delta_k^3\right) e^{-dT}\right)$$

$$\times \left(\left(r + cY^*\right) \left(r + cY^* \left(1 - \delta_1^3\right) e^{-dT/(q+1)}\right) \right.$$

$$\left. \left. \cdots \left(r + cY^* \prod_{k=1}^{q} \left(1 - \delta_k^3\right) e^{-dqT/(q+1)}\right) \right)^{-1} \right).$$

(8)

Denote $1 - \delta_k^1 = \delta_1$, $1 - \delta_k^2 = \delta_2$, and $1 - \delta_k^3 = \delta_3$, where $k = 1, 2, \ldots, q$. What we want to address in the following is how control tactics including residual rates δ_1, δ_2, and δ_3 release constant θ, timing of pesticide application q, and timing of release period T affect the threshold condition λ_i. Firstly, in the following section, we take Figure 2 as an example to control pests by the control strategy of Case 1. It implies that only the pest x_2 needs be eradicated. We firstly use the traditional control method, spraying insecticides, in order to know well the impact of insecticides on pests and natural enemy species. In general, pesticides tend to be harmful to most natural enemies [24], which may be associated with the acute poisoning. It is significant to understand the acute poisoning of insecticides to natural enemies for the research of IPM strategy.

We only apply insecticides but we do not release natural enemies as shown in Figure 3. The simulation results indicate that pesticide applications (number of pesticide applications $q = 1$ in a period shown in Figure 3(a)) do not lead to the extinction of pest x_2, and on the contrary, they can result in the recurrence of pest x_1, and with the increase of the number q, the quantity of both pest x_1 and pest x_2 increases (see Figure 3(b)). Only when the dosages of pesticides are increased enough can the pests become extinct, but natural enemies also become extinct at the same time (see Figure 3(c)). This shows that the extinction of pests needs plenty of pesticides. Nevertheless, pesticide abuse can bring about environmental contamination, which can also result in human exposure through consumption of residues of pesticides in food and, possibly, drinking water. While developed countries have systems already in place to register pesticides and control their trade and usage, this is not always the case elsewhere, especially in China. Moreover, the pesticides have a serious impact on the natural enemies (Figures 3(a)–3(c)), and the repeated use of the same pesticides can result in one or more population pest outburst (Figures 3(a)-3(b)). The previous results show that pest control of multispecies is much more complicated than single pests [7–9].

By the previous analysis, the additional release of natural enemies is an indispensable part for pest control. Without loss of generality, we assume that natural enemies have food preference phenomenon with pest x_1; that is, $\alpha_1 > \alpha_2$, the intrinsic growth rate $a_1 \leq a_2$, and the residual ratio of pests $\delta_1 \leq \delta_2$ (here the reason is explained in the following section) after spraying; thus, according to Theorem A.2, $\lambda_1 < \lambda_2$ and the condition of pest extinction only needs $\lambda_2 < 1$. In Figure 4, we fix the other parameters of λ_2 and let the residual ratio δ_3 vary. The simulation results indicate that if the pesticide poisons the natural enemies with a relatively low residual ratio δ_3 (e.g., $\delta_3 = 0.9$), the threshold value λ_2 is a monotonically increasing function with respect to the number of pesticide applications q (Figure 4(a)). This further explains that if the pesticide has a severe impact on the natural enemies, repeated use of the same pesticides can result in pest resurgence. If the residual ratio δ_3 is slightly increased from 0.9 to 0.92, Figure 4(b) shows that the threshold value λ_2 is not monotonic with respect to the number of pesticide applications q. So in this case we must carefully select the number of pesticide applications (one to three events in this case). If the pesticides do not kill the natural enemies so much, Figures 4(c) and 4(d) clarify that the threshold value λ_2 is a monotonically decreasing function with respect to the number of pesticide applications q. In Figures 5(a)–5(d), similarly, we fix the other parameters of λ_2 and let the release period T vary. The simulation results indicate that the small change of release period T can lead to the change of the number of pesticide applications q. All these simulations show that the releasing period, the number of times of spraying pesticides within this period, and the residual ratio of natural enemies are crucial to eradicate pest x_1 and pest x_2. Figure 6 shows the relationship between the controllable parameters and the threshold condition λ_2. All simulation results demonstrate that λ_2 seems to be quite sensitive to small changes in residual ratio δ_1 and δ_3, release constant θ,

FIGURE 3: The impact of the number and dosage of application insecticides on pests and natural enemy with parameters of (a) $\delta_1 = 0.9$, $\delta_2 = 0.85$, $\delta_3 = 0.85$, $\theta = 0$, $T = 6$, and $q = 1$, (b) $\delta_1 = 0.95$, $\delta_2 = 0.85$, $\delta_3 = 0.85$, $\theta = 0$, $T = 6$, and $q = 6$, (c) $\delta_1 = 0.58$, $\delta_2 = 0.62$, $\delta_3 = 0.85$, $\theta = 0$, $T = 6$, and $q = 6$, and other parameters the same as Figure 1.

and release period T. By the simulation results, we obtain that the optimum time and frequency of pesticide application; the protection of natural enemies and pesticides choice are the key factors to pest outburst or eradication. The results may provide a theoretical basis for agricultural practitioners to guide them to spray pesticides and release natural enemies more efficiently.

4. Dynamical Analysis of Case 2 and Its Biological Implications

In this section, similar to the study method of system (2), we will give the dynamic property of system (3).

According to Floquet theory [16], if the following threshold condition:

$$\lambda_i = \delta_i \exp\left(a_i T_1 + \frac{\alpha_i B_2}{cd}\right) < 1 \quad (i = 1, 2) \qquad (9)$$

is satisfied, then the pest-eradication periodic solution $(0, 0, y_{T_1}(t))$ of system (3) is globally asymptotically stable, where

$$B_2 = \ln\left(\left(r + cY_1^* e^{-dT_1/(p+1)}\right)\right.$$

$$\times \left(r + c\left(Y_1^* \eta_1 e^{-2dT_1/(p+1)} + \theta_1 e^{-dT_1/(p+1)}\right)\right)$$

$$\cdots \left(r + c\left(Y_1^* \prod_{k=1}^{p} \eta_k e^{-dT_1} + \theta_1 \prod_{k=2}^{p} \eta_k e^{-dqT/(q+1)}\right)\right.$$

$$\left. + \cdots + \theta_p e^{-dT_1/(p+1)}\right)\right)$$

$$\times \left(\left(r + cY_1^*\right)\left(r + c\left(Y_1^* \eta_1 e^{-dT_1/(p+1)} + \theta_1\right)\right)\right.$$

$$\cdots \left(r + c\left(Y_1^* \prod_{k=1}^{p} \eta_k e^{-pdT_1/(p+1)}\right.\right.$$

$$+ \theta_1 \prod_{k=2}^{p} \eta_k e^{(-d(p-1)T)/(p+1)}$$

$$\left.\left.\left.\left. + \cdots + \theta_p\right)\right)\right)^{-1}\right),$$

$$Y_1^* = \frac{S}{1 - (1 - \mu_1)(1 - \mu_2)\cdots(1 - \mu_p)\delta_3 e^{-dT_1}},$$

$$\eta_k = 1 - \mu_k,$$

$$\delta_1 = 1 - \delta_1^1, \qquad \delta_2 = 1 - \delta_1^2, \qquad \delta_3 = 1 - \delta_1^3.$$

$$\qquad (10)$$

For Case 2, there are p times releasing natural enemy during spraying period T_1. Denote $\eta_k = \mu$, $\theta_k = \theta$, where $k = 1, 2, \ldots, p$. Since the release of natural enemies in this case is more frequent than spraying pesticides, the side effects of pesticides on the natural enemy population are largely reduced. Moreover, the threshold condition λ_2 can be strongly affected by the additional release of natural enemies.
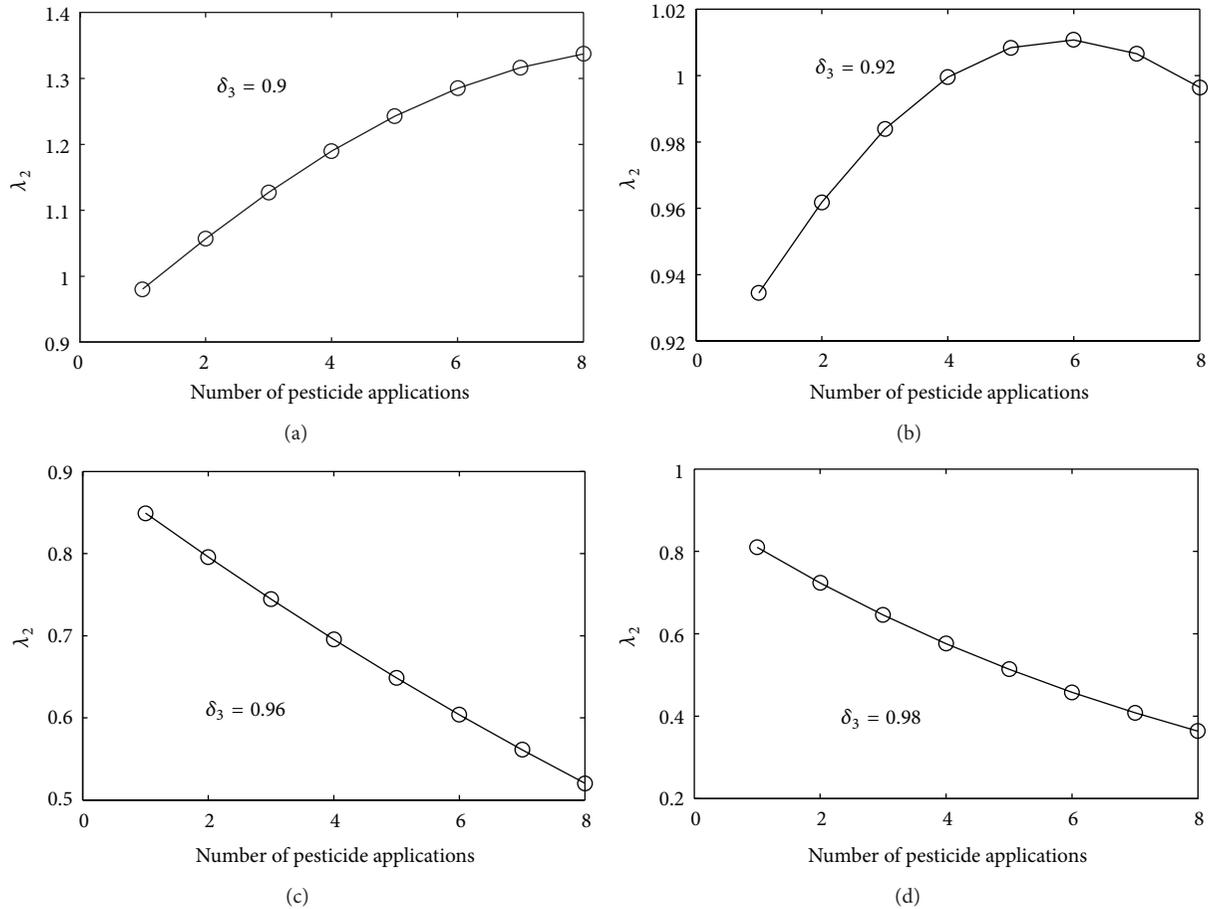
(a)

(b)

(c)

(d)

FIGURE 4: The effects of number of pesticide applications and the residual ratio of natural enemy on the threshold level λ_2. The parameter values are as follows: $\delta_1 = 0.85$, $\delta_2 = 0.85$, $T = 10.7$, $\theta = 6.3$, $d = 0.25$, $r = 1.2$, $c = 0.3$, $a_1 = 0.75$, $a_2 = 0.8$, and $\alpha_2 = 0.7$.

In Figure 7, we fix all the other parameters and choose a different releasing constant θ and different release times p. The simulation results indicate that slight perturbation of the release constant θ can rapidly reduce the threshold value λ_2 (Figure 7), while increasing the number of natural enemy releases as well. This shows that repeated releases of a small number of natural enemies in key time of the season can effectively control the pest outburst. In practice, an example of *Liriomyza sativae* and *Trialeurodes vaporariorum* occurs in heliogreenhouse. The parasitic rate of parasitic wasps which is their natural enemy can amount to over fifty percent without drugs. Thence, periodic releases of the parasitic wasps have been used to control the *Liriomyza sativae* and *Trialeurodes vaporariorum* in Anshan city in Liaoning Province, China, where greenhouse agriculture is developing rapidly.

Remark 4. In this paper, we suppose that the natural enemies can first select which are their favorite prey between pest x_1 and pest x_2. It means that the favorite natural enemies may be a profitable pest to them. Thence, as seen in [19], the profitable pest is classified as palatable and the other as unpalatable. In the following section, by numerical simulation, we will explain under what condition the natural enemies can prey on pest x_1 or pest x_2.

5. Hybrid Impulsive Model with Economic Threshold

As the previous simulation indicates, pesticides may seriously influence the survival of natural enemies. They may impact natural enemies indirectly by killing or contaminating their hosts or prey. It is essential to avoid pesticide abuse when biological control is feasible, as shown in systems (2) and (3). Probably the best method for reducing the side effects of pesticides on natural enemies is to apply pesticides only when the sum of density of two pest populations reaches the economic threshold (ET), since a small number of insect pests may have compensation effect on crops [25]. Thence, we formulate the model as follows:

$$\frac{dx_1}{dt} = a_1 x_1 \left(1 - \frac{x_1}{K_1} - \frac{c_1 x_2}{K_1} \right)$$
$$- \frac{\alpha_1 x_1 y}{r + x_1 + b x_2 + c y},$$
$$\frac{dx_2}{dt} = a_2 x_2 \left(1 - \frac{x_2}{K_2} - \frac{c_2 x_1}{K_2} \right)$$
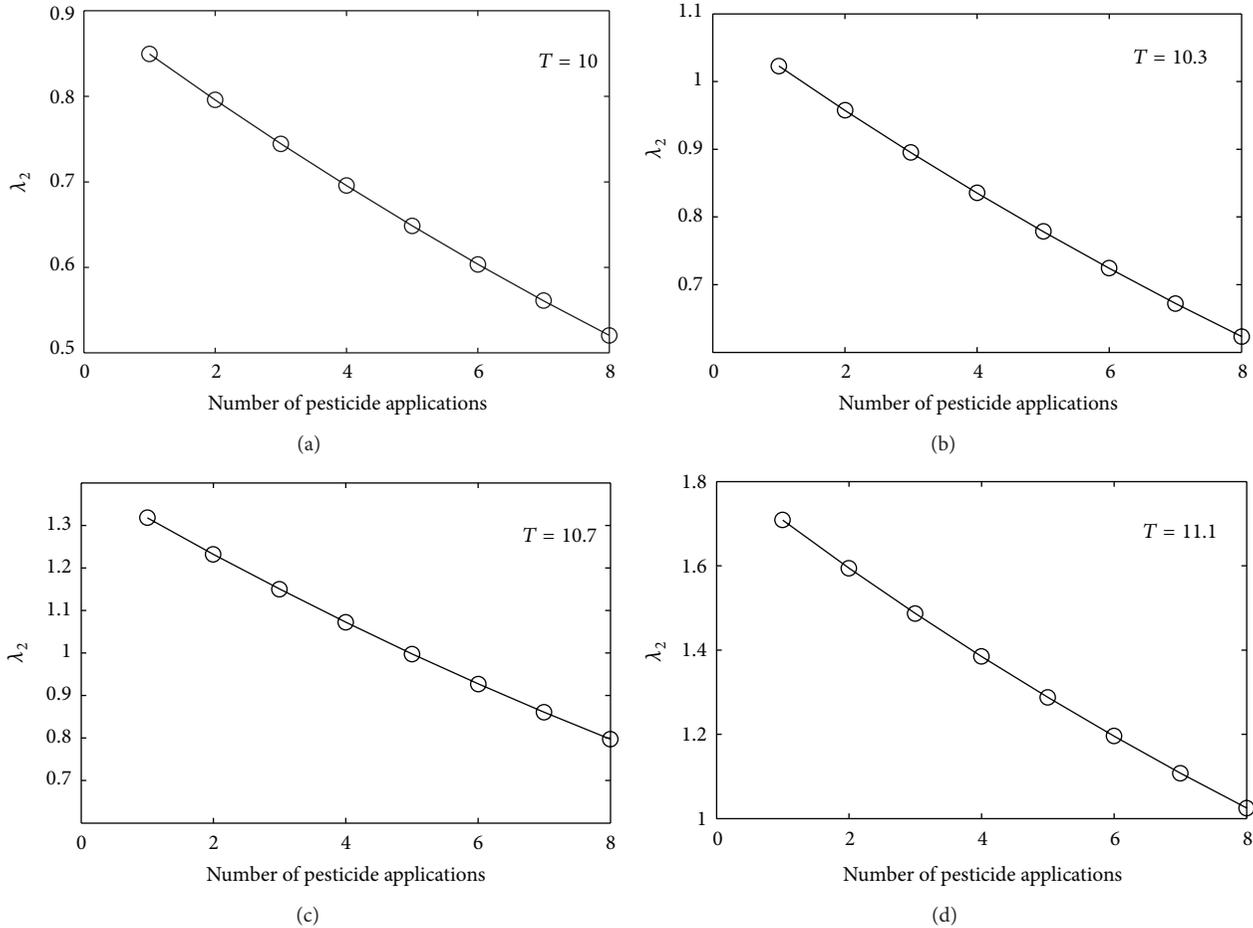
(a)

(b)

(c)

(d)

FIGURE 5: The effects of number of pesticide applications and release period T on the threshold level λ_2 with the parameter value $\delta_3 = 0.93$ and other parameters the same as Figure 4.

$$-\frac{\alpha_2 x_2 y}{r + x_1 + bx_2 + cy},$$

$$\frac{dy}{dt} = \frac{\omega_1 \alpha_1 x_1 y}{r + x_1 + bx_2 + cy}$$

$$+ \frac{\omega_2 \alpha_2 x_2 y}{r + x_1 + bx_2 + cy} - dy,$$

$$x_1 + x_2 < \text{ET}, \quad t \neq nT,$$

$$x_1\left(\tau_{nk}^+\right) = \delta_1 x_1\left(\tau_{nk}\right),$$

$$x_2\left(\tau_{nk}^+\right) = \delta_2 x_2\left(\tau_{nk}\right),$$

$$y\left(\tau_{nk}^+\right) = \delta_3 y\left(\tau_{nk}\right),$$

$$x_1 + x_2 = \text{ET},$$

$$x_1\left(nT^+\right) = 0,$$

$$x_2\left(nT^+\right) = 0,$$

$$y\left(nT^+\right) = y\left(nT\right) + \theta,$$

$$t = nT,$$

(11)

where δ_1, δ_2, and δ_3 are the same as in Section 5 and ET is the economic threshold. T is the releasing period of natural enemies.

The previously mentioned facts show that the effect among the intrinsic growth rate of pests, a_1 and a_2, the predation capacity of natural enemies (or functional response parameter), α_1 and α_2, residual ratio δ_1, δ_2, and δ_3, releasing quantity θ, and other factors (such as ET) may determine dynamic behavior of pests and natural enemy species. How do these key parameters affect the control strategies? In particular, what we want to achieve is to study how the ET, α_1 or α_2, and controllable parameters (such as T) affect the control strategies.

For a fixed ET, by simulation, we obtain the result that the successful control strategies are affected by the predation capacity of natural enemy to different types of pests. To show this, we vary the key parameter α_1 while the other parameters are fixed as those in Figure 8. In Figure 8(a), for the predation capacity of natural enemy $\alpha_1 = 0.74$, the simulation result indicates that the sum of density of the two pests population never reaches the given ET, which implies that $\alpha_1 \geq 0.74$ is free from spraying. If we set $\alpha_1 = 0.737$, Figure 8(b) indicates that the system is free from chemical control after spraying pesticides. If we set $\alpha_1 = 0.736$ or $\alpha_1 = 0.732$
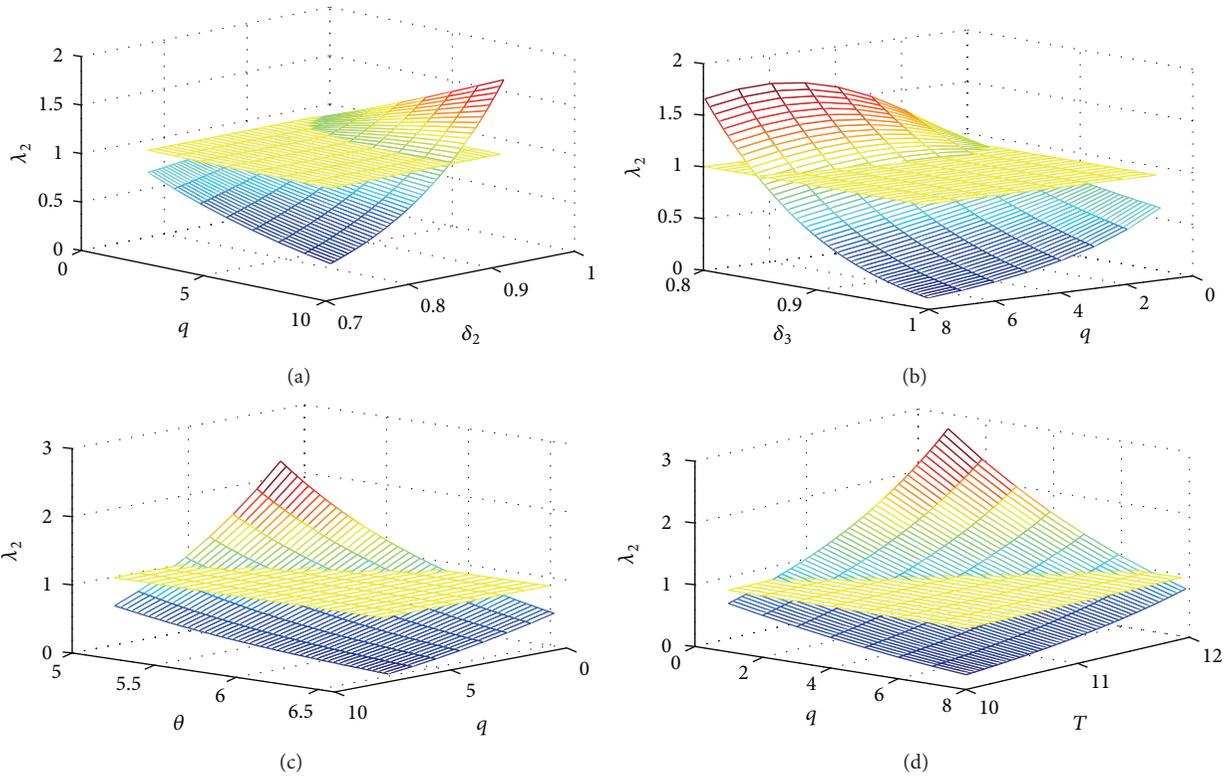
FIGURE 6: The effects of times of spraying pesticides and parameter sets on the threshold level λ_2. The parameter values are as follows: $a_1 = 0.8$, $d = 0.25$, $r = 1.2$, $c = 0.3$, and $\alpha_2 = 0.7$. ((a), (b)) $\delta_2 = 0.75$, $\delta_3 = 0.94$, $\sigma = 6.3$, and $T = 10$; ((c), (d)) $\delta_2 = 0.7$, $\delta_3 = 0.91$, $\sigma = 6.3$, and $T = 10$.
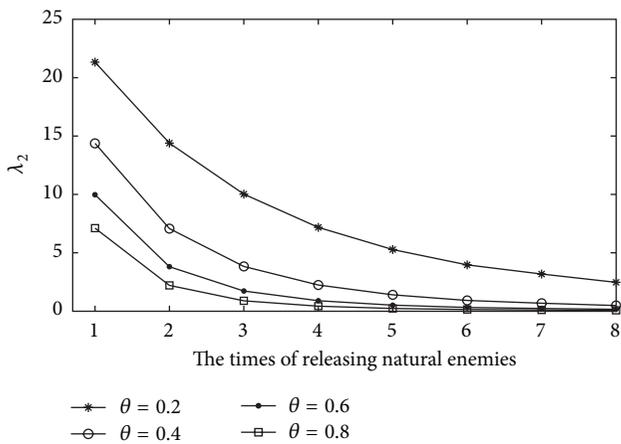

FIGURE 7: The effects of times of releasing natural enemies and releasing constant θ on the threshold level λ_2. The parameter values are as follows: $\delta_1 = 0.92$, $\delta_2 = 0.93$, $\delta_3 = 0.92$, $\mu = 0.99$, $T_1 = 5$, $d = 0.25$, $r = 1.2$, $c = 0.3$, $a_1 = 0.8$, and $\alpha_2 = 0.7$.

or $\alpha_1 = 0.73$, Figures 8(c)–8(e) indicate that the system is free from chemical control after three, four, or five pesticide applications. If we further reduce the predation capacity of natural enemy and set $\alpha_1 = 0.71$, the pest outbreak frequency is sharply increased, as shown in Figure 8(f). As mentioned in Remark 4, in Figure 8, we will expound the interdependent

relationship among the natural enemy, pest x_1, and pest x_2. In the beginning, the density of pest x_2 is larger, and it is regarded as palatable for the natural enemy, which implies that $\alpha_2 \geq \alpha_1$, as shown in Figure 8(f), the pests can break out. However, with the release of natural enemies, an expanded population of natural enemies causes the reduction of pest 2, but when pest x_2 falls below a certain critical value x_2^* [19], the natural enemy begins to eat not only pest x_2 but also pest x_1, which largens α_1 and makes pests gradually no longer break out as shown in Figures 8(e)–8(a). Meanwhile, it causes an immediate recovery of pest x_2, and when pest x_2 raises above the certain critical value x_2^*, the natural enemy begins to eat palatable pest x_2 again, which forms a cycle (it is also called switching between pest x_1 and pest x_2 by natural enemy [26]). The previous analysis illustrates that if the natural enemy and the two pests meet the above relationship, we only need to control pest x_2 falling below the certain critical value without supervising pest x_1.

Finally, we will introduce the definition of pest outbreak duration (period) and analyse the relationship between pest outbreak period and the controllable parameters. We denote the time points at which the solution reaches ET as t_n ($n = 1, 2, \ldots$). If $\mathrm{mod}(t_n, T) = 0$, a chemical control is applied at t_n, and after that a biological control is also applied at the same time. If $\mathrm{mod}(t_n, T) \neq 0$, only a chemical control programme is applied. Further, denote
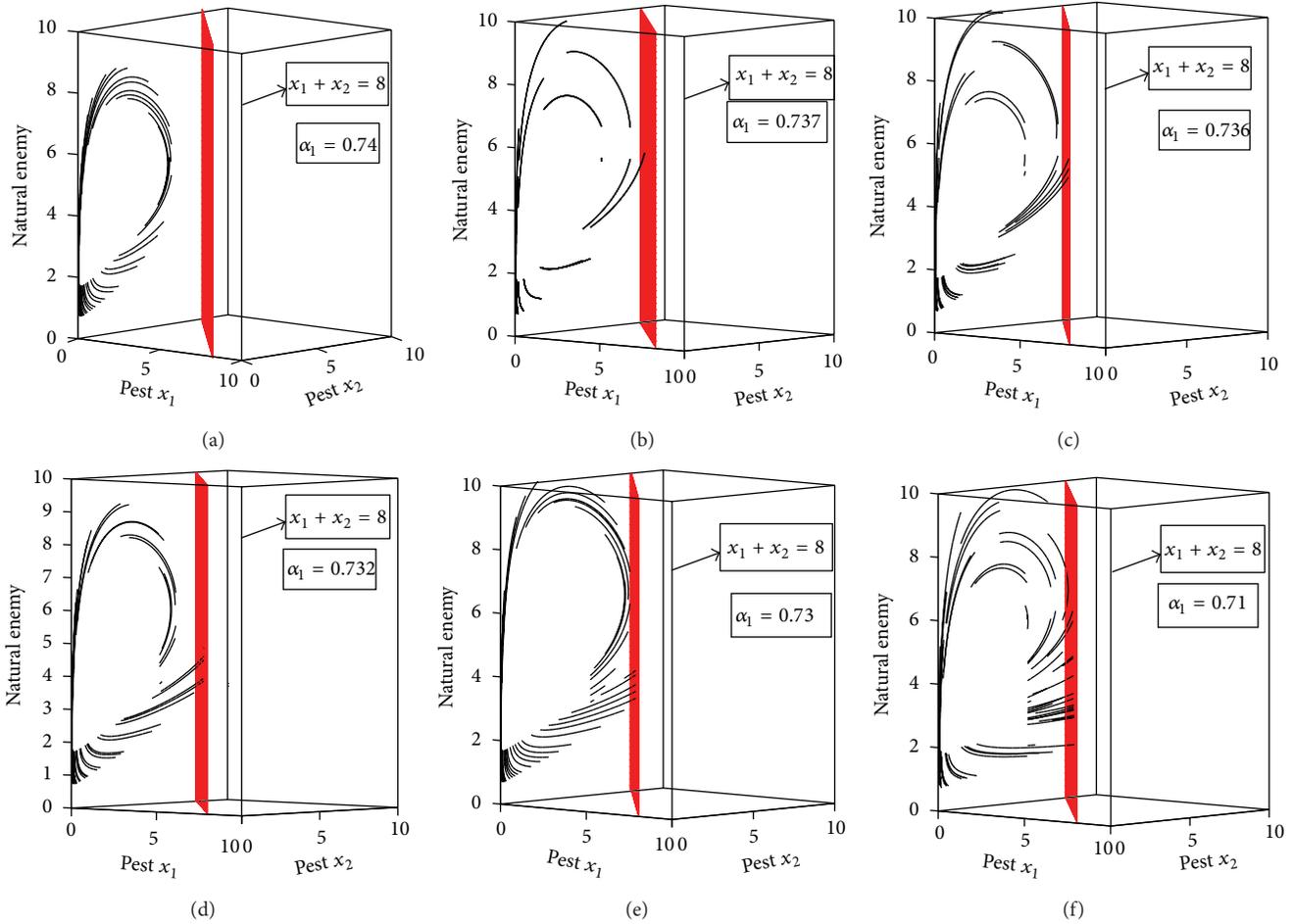
$$T_n = t_n - t_{n-1} \tag{12}$$

(a)   (b)   (c)

(d)   (e)   (f)

FIGURE 8: The effects of ET and α_1 of model (11) on the control strategies. The initial values are $x_1 = 1.3$; $x_2 = 1.5$; and $y = 5$, and parameters are fixed as follows: $\delta_1 = 0.65$, $\delta_2 = 0.7$, $\delta_3 = 0.95$, $\sigma = 1$, ET $= 8$, $T = 4$, $a_1 = 0.75$, $a_2 = 0.85$, $\alpha_2 = 0.85$, $c_1 = 0.2$, $K_1 = 20$, $b = 0.25$, $c = 0.3$, $A_2 = 0.85$, $K_2 = 30$, $c_2 = 0.65$, $W_1 = 0.45$, $W_2 = 0.75$, $d = 0.25$, and $r = 1.2$.

with $t_0 = 0$ as pests-outbreak duration (or period), where n may be finite or infinite which depends on the solutions of the models. The relationship among T, ET, δ_1, δ_2, δ_3, or θ and mean outbreak period of pests can be calculated from model (11) and formula (12) numerically (Figure 9). Mean pest outbreaks period is an average over several pest outbreak (here outbreaks indicate that the sum of densities of the pest x_1 and pest x_2 reaches the given ET). Model (11) predicts that the pests do not break out if the natural enemies are released more transitorily ($T \leq 2$, Figure 9(d)) and the mean outbreak period is decreasing as the release period T or residual ratios of the pests δ_1 and δ_2 increase (Figures 9(a), 9(b), and 9(d)). Conversely, model (11) predicts that with the increase of ET or residual ratios of the natural enemy δ_3, the mean outbreak period becomes longer (Figures 9(c) and 9(e)). In Figure 9(f), let the release period T and the other parameters fixed, and let the release constant θ vary. This indicates that when the release period T is smaller (here $T = 4$), with the increase of release quantity, the pests will not break out (here $\theta \geq 1.5$). And, more remarkably, the mean outbreak period can suddenly jump from a small

value to a larger value at some critical points of δ_3, T, and θ, which implies that the protection of natural enemies, the selection of releasing time, and the quantity may be crucial in prolonging the pest outbreak period. Moreover, the different δ_2 or δ_3 or different values of the release constant θ may have the same mean outbreak period (Figures 9(b), 9(c) and 9(f)). For system (11), the relationship between pest outbreak period and other parameters such as a_1, a_2, α_1, and α_2 can be researched similarly.

6. Discussion

The agricultural pests management plays a decisive role on the survival of people all over the world especially that the impacts of extreme climate change are severer for pest control. For example, the armyworm which is the typical pest threatening corn growth in fall has been widely seen in North China Plain and Northeast China producing regions in August 2012. The leaves of corn stalks in portions of the above regions have been eaten up, cutting corn harvest
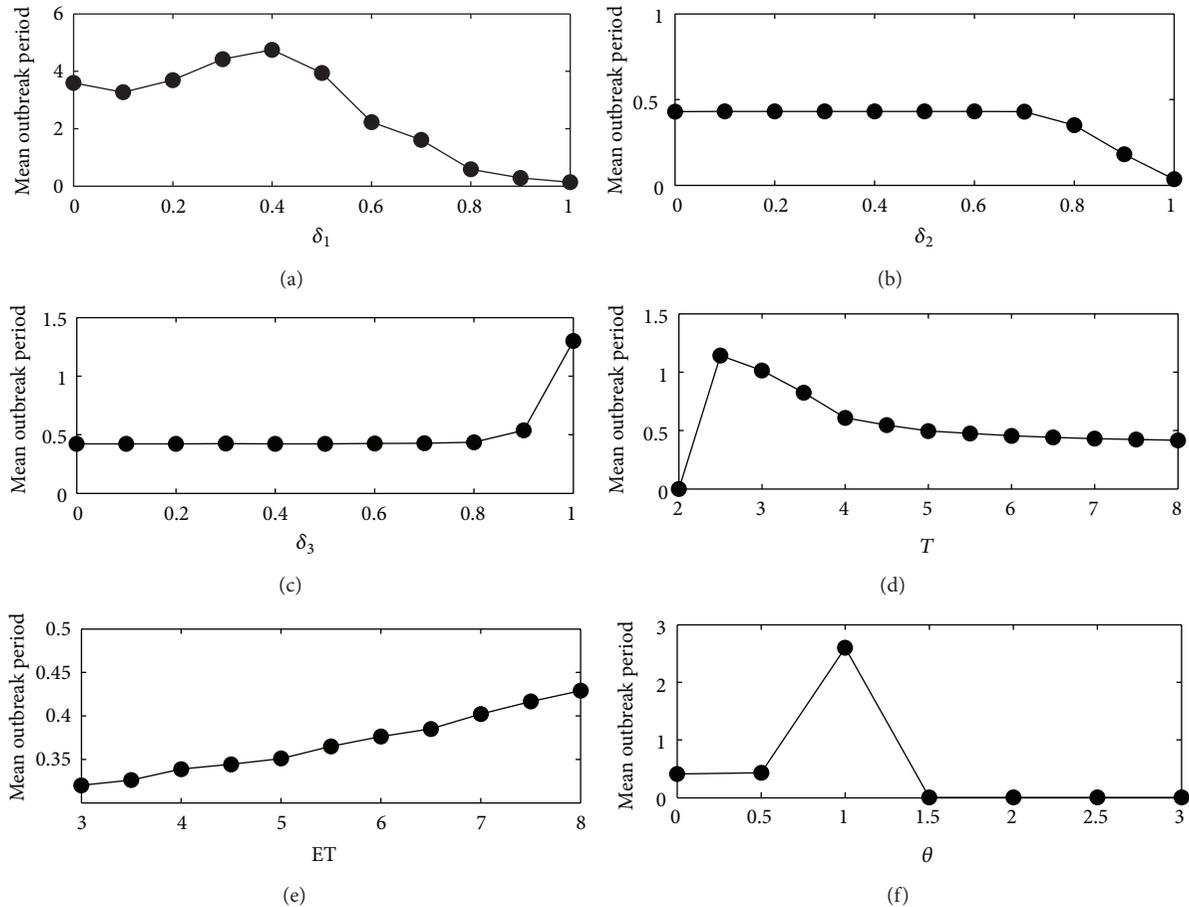
FIGURE 9: The relationship between mean outbreak period and the parameters δ_1, δ_2, δ_3, T, ET, and θ of model (11). The other parameters are the same as Figure 8.

prospect. The pests, common pests, became the most serious threat to the production of corn this summer in the country's major grain-producing regions. This is mainly due to frequent cyclonic activities since mid-July provided favorable conditions to the migration of the armyworms, and then heavy rainfall forced them to stay in the north and northeastern parts of the country. The pest outbreak occurred at the same time as severe droughts in the United States, where the driest conditions in more than half a century have battered corn and soybean crops, causing an upsurge in global grain prices [27, 28].

Thence, it is particularly significant to explore an effective control strategy. In this paper, based on the IPM strategies, we give three different control strategies, which improve traditional IPM control strategies. For system (2), in a releasing natural enemies period, we spray pesticides several times. By the theoretical derivation to system (2), the critical value of pests eradication is figured out. When only using the traditional control method, insecticides, by the numerical simulation, we obtain that the pest x_1 and pest x_2 may break out with the increase of spray times, which is different from the control of a single pest. To better understand how the controllable parameters (here T, q, δ_1, δ_2, and δ_3) impact the pest control, by the numerical simulation, we give

the relationship between the critical value of pests eradication and the controllable parameters. All these results express that the selection of spray times and the protection of the natural enemies are of vital importance for pests eradication. From system (3), we know the real embodiment of the significance of protecting natural enemies. By the analysis of systems (2) and (3), in summary, when insecticides are used excessively, the pests are killed and the natural enemies of the pests are wiped out. In the absence of natural enemies, the surviving population of insect pests multiply rapidly and reach epidemic proportions. Indiscriminate use of pesticides also leads to the development of resistance in pests. This occurs as a result of killing the susceptible genotypes and selecting the more resistant genotypes at every pesticide application event. After several years of using the same pesticide, there would come a time when that particular pesticide will have no effect on the pests because they have developed resistance to the pesticide. Considering the factors, we formulate model (11), which is to apply pesticides only when necessary. By model (11), we also obtain some important conclusions.

Most real communities are more complex than the community analysed here. Therefore, in the future, the factors on the pests of more species and natural enemies, resistance to the pesticide of pests and so on should be considered in the

model to depict the dynamic behavior between populations much more accurately.

Appendix

In any time interval $((n - 1)T, nT]$, we investigate the dynamical behavior of model (6). In fact, integrating the first equation of model (6) from $(n - 1)T$ to τ_{n_1} yields

$$y(t) = y\left((n - 1)T^+\right) e^{-d(t-(n-1)T)}, \quad t \in \left((n - 1)T, \tau_{n_1}\right]. \tag{A.1}$$

At time τ_{n_1}, one pesticide application occurs and

$$y\left(\tau_{n_1}^+\right) = \left(1 - \delta_1^3\right) y\left((n - 1)T^+\right) e^{-dT/(q+1)}. \tag{A.2}$$

Again, integrating the first equation of model (6) from τ_{n_1} to τ_{n_2} yields

$$y(t) = y\left(\tau_{n_1}^+\right) e^{-d(t-\tau_{n_1})}, \quad t \in \left(\tau_{n_1}, \tau_{n_2}\right]. \tag{A.3}$$

At time τ_{n_2}, a single pesticide application occurs and

$$\begin{aligned} y\left(\tau_{n_2}^+\right) &= \left(1 - \delta_2^3\right) y\left(\tau_{n_1}^+\right) e^{-dT/(q+1)} \\ &= \left(1 - \delta_1^3\right)\left(1 - \delta_2^3\right) y\left((n - 1)T^+\right) e^{-2dT/(q+1)}. \end{aligned} \tag{A.4}$$

By induction, we can see that

$$\begin{aligned} y(t) &= \left(1 - \delta_1^3\right)\left(1 - \delta_2^3\right)\cdots\left(1 - \delta_{q-1}^3\right) e^{(-d(q-1)T)/(q+1)} \\ &\quad \times y\left((n - 1)T^+\right) e^{-d(t-\tau_{n(q-1)})}, \quad t \in \left(\tau_{n(q-1)}, \tau_{nq}\right]. \end{aligned} \tag{A.5}$$

At time τ_{nq}, the last time pesticide is applied in the nth period and

$$y\left(\tau_{nq}^+\right) = \left(1 - \delta_1^3\right)\cdots\left(1 - \delta_q^3\right) y\left((n - 1)T^+\right) e^{-dqT/(q+1)}. \tag{A.6}$$

Finally, integrating the first equation of model (6) from τ_{nq} to nT yields

$$\begin{aligned} y(t) &= y\left(\tau_{nq}^+\right) e^{-d(t-\tau_{nq})} = \left(1 - \delta_1^3\right)\cdots\left(1 - \delta_q^3\right) \\ &\quad \times y\left((n - 1)T^+\right) e^{-dqT/(q+1)} e^{-d(t-\tau_{nq})}, \quad t \in \left(\tau_{nq}, nT\right]. \end{aligned} \tag{A.7}$$

At time nT, release of natural enemies occurs once and

$$y\left(nT^+\right) = \left(1 - \delta_1^3\right)\cdots\left(1 - \delta_q^3\right) y\left((n - 1)T^+\right) e^{-dT} + \theta. \tag{A.8}$$

Denote $Y_n = y(nT^+)$, then we have the following difference equation:

$$Y_{n+1} = \left(1 - \delta_1^3\right)\cdots\left(1 - \delta_q^3\right) Y_n e^{-dT} + \theta, \tag{A.9}$$

which has a unique steady state

$$Y^* = \frac{\theta}{1 - \left(1 - \delta_1^3\right)\cdots\left(1 - \delta_q^3\right) e^{-dT}}. \tag{A.10}$$

Let $F = (1 - \delta_1^3)\cdots(1 - \delta_q^3)Ye^{-dT} + \theta$, since $|dF/dY| = 1 - (1 - \delta_1^3)\cdots(1 - \delta_q^3)e^{-dT} < 1$. Therefore, Y^* is a globally asymptotically stable equilibrium of model (A.8), then system (6) has a globally stable T periodic solution $y_T(t)$, which can be calculated as follows:

$$y_T(t)$$
$$= \begin{cases} Y^* e^{-d(t-(n-1)T)}, \quad t \in \left((n - 1)T, \tau_{n1}\right], \\ Y^*\left(1 - \delta_1^3\right) e^{-dT/(q+1)} e^{-d(t-\tau_{n1})}, \\ \qquad\qquad\qquad t \in \left(\tau_{n1}, \tau_{n2}\right], \\ \vdots \\ Y^*\left(1 - \delta_{q-1}^3\right)\cdots\left(1 - \delta_1^3\right) e^{(-d(q-1)T)/(q+1)} e^{-d(t-\tau_{nq-1})}, \\ \qquad\qquad\qquad t \in \left(\tau_{nq-1}, \tau_{nq}\right], \\ Y^*\left(1 - \delta_q^3\right)\cdots\left(1 - \delta_1^3\right) e^{-dqT/(q+1)} e^{-d(t-\tau_{nq})}, \\ \qquad\qquad\qquad t \in \left(\tau_{nq}, nT\right]. \end{cases} \tag{A.11}$$

We have the following.

Lemma A.1. *System (6) has a positive periodic solution $y_T(t)$ and for every solution $y(t)$ of system (6) one has $y(t) \to y_T(t)$ as $t \to \infty$.*

Furthermore, we can obtain the complete expression for the prey-free periodic solution of system (2), $(0, 0, y_T(t))$, for $t \in ((n - 1)T, nT]$. Now, we give the conditions which assure the globally asymptotical stability of the pest-eradication $(0, 0, y_T(t))$.

Theorem A.2. *Let $(x_1(t), x_2(t), y(t))$ be any solution of (2); then, $(0, 0, y_T(t))$ is globally asymptotically stable provided*

$$\ln \frac{1}{\prod_{k=1}^q \left(1 - \delta_k^i\right)} > a_i T + \frac{\alpha_i B_1}{cd}, \quad i = 1, 2, \tag{A.12}$$

where

$$\begin{aligned} B_1 = \ln\Bigg(&\left(r + cY^* e^{-dT/(q+1)}\right)\left(r + cY^*\left(1 - \delta_1^3\right) e^{-2dT/(q+1)}\right) \\ &\cdots\left(r + cY^* \prod_{k=1}^q \left(1 - \delta_k^3\right) e^{-dT}\right) \\ &\times \Bigg(\left(r + cY^*\right)\left(r + cY^*\left(1 - \delta_1^3\right) e^{-dT/(q+1)}\right) \\ &\cdots\left(r + cY^* \prod_{k=1}^q \left(1 - \delta_k^3\right) e^{-dqT/(q+1)}\right) \Bigg)^{-1} \Bigg). \end{aligned} \tag{A.13}$$

Proof. The local stability of periodic solution $(0, 0, y_T(t))$ may be determined by considering the behavior of small amplitude perturbations of the solution. Defining $x_1(t) = U(t)$, $x_2(t) = V(t)$, and $y(t) = y_T(t) + W(t)$, it may be written

$$\begin{pmatrix} U(t) \\ V(t) \\ W(t) \end{pmatrix} = \phi(t) \begin{pmatrix} U(0) \\ V(0) \\ W(0) \end{pmatrix}, \tag{A.14}$$

where ϕ satisfies

$$\frac{d\phi}{dt} = \begin{pmatrix} a_1 - \dfrac{\alpha_1 y_T(t)}{r + cy_T(t)} & 0 & 0 \\ 0 & a_2 - \dfrac{\alpha_2 y_T(t)}{r + cy_T(t)} & 0 \\ 0 & 0 & -d \end{pmatrix} \phi(t), \tag{A.15}$$

and $\phi(0) = I$, the identity matrix. Hence, the fundamental solution matrix is

$$\phi(t)$$

$$= \begin{pmatrix} e^{\int_0^t (a_1 - (\alpha_1 y_T(s)/(r + cy_T(s)))) ds} & 0 & 0 \\ 0 & e^{\int_0^t (a_2 - (\alpha_2 y_T(s)/(r + cy_T(s)))) ds} & 0 \\ 0 & 0 & e^{-dt} \end{pmatrix}. \tag{A.16}$$

The impulsive conditions of system (2) become

$$\begin{pmatrix} U(\tau_{nk}^+) \\ V(\tau_{nk}^+) \\ W(\tau_{nk}^+) \end{pmatrix} = \begin{pmatrix} 1 - \delta_k^1 & 0 & 0 \\ 0 & 1 - \delta_k^2 & 0 \\ 0 & 0 & 1 - \delta_k^3 \end{pmatrix} \begin{pmatrix} U(\tau_{nk}) \\ V(\tau_{nk}) \\ W(\tau_{nk}) \end{pmatrix}, \tag{A.17}$$

where $k = 1, 2, \ldots, q$, and

$$\begin{pmatrix} U(nT^+) \\ V(nT^+) \\ W(nT^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} U(nT) \\ V(nT) \\ W(nT) \end{pmatrix}. \tag{A.18}$$

The stability of the periodic solution $(0, 0, y_T(t))$ is determined by the eigenvalues of the matrix

$$M = \prod_{k=1}^q \begin{pmatrix} 1 - \delta_k^1 & 0 & 0 \\ 0 & 1 - \delta_k^2 & 0 \\ 0 & 0 & 1 - \delta_k^3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \phi(T), \tag{A.19}$$

which are

$$\lambda_1 = \prod_{k=1}^q \left(1 - \delta_k^1\right) e^{\int_0^T (a_1 - (\alpha_1 y_T(t)/(r + cy_T(t)))) dt},$$

$$\lambda_2 = \prod_{k=1}^q \left(1 - \delta_k^2\right) e^{\int_0^T (a_2 - (\alpha_2 y_T(t)/(r + cy_T(t)))) dt}, \tag{A.20}$$

$$\lambda_3 = \prod_{k=1}^q \left(1 - \delta_k^3\right) e^{-dT} < 1.$$

According to Floquet theory [16], $(0, 0, y_T(t))$ is locally stable if $|\lambda_1|, |\lambda_2| < 1$.

Next, integrating the $y_T(t)/(r + cy_T(t))$ from 0 to T yields

$$\int_0^T \frac{y_T(t)}{r + cy_T(t)} dt$$

$$= \int_0^{\tau_{11}} \frac{Y^* e^{-dt}}{r + cY^* e^{-dt}} dt$$

$$+ \int_{\tau_{11}^+}^{\tau_{12}} \frac{Y^* \left(1 - \delta_1^3\right) e^{-dT/(q+1)} e^{-d(t - \tau_{11}^+)}}{r + cY^* \left(1 - \delta_1^3\right) e^{-dT/(q+1)} e^{-d(t - \tau_{11}^+)}} dt + \cdots$$

$$+ \int_{\tau_{1q-1}^+}^{\tau_{1q}} \frac{Y^* \prod_{k=1}^{q-1} \left(1 - \delta_k^3\right) e^{(-d(q-1)T)/(q+1)} e^{-d(t - \tau_{1q-1}^+)}}{r + cY^* \prod_{k=1}^{q-1} \left(1 - \delta_k^3\right) e^{(-d(q-1)T)/(q+1)} e^{-d(t - \tau_{1q-1}^+)}}$$

$$\times dt + \int_{\tau_{1q}}^T \frac{Y^* \prod_{k=1}^q \left(1 - \delta_k^3\right) e^{-dqT/(q+1)} e^{-d(t - \tau_{1q}^+)}}{r + cY^* \prod_{k=1}^q \left(1 - \delta_k^3\right) e^{-dqT/(q+1)} e^{-d(t - \tau_{1q}^+)}} dt$$

$$= \frac{-1}{cd} \ln \frac{r + cY^* e^{-dT/(q+1)}}{r + cY^*}$$

$$+ \frac{-1}{cd} \ln \frac{r + cY^* \left(1 - \delta_1^3\right) e^{-2dT/(q+1)}}{r + cY^* \left(1 - \delta_1^3\right) e^{-dT/(q+1)}}$$

$$+ \frac{-1}{cd} \ln \frac{r + cY^* \prod_{k=1}^{q-1} \left(1 - \delta_k^3\right) e^{-dqT/(q+1)}}{r + cY^* \prod_{k=1}^{q-1} \left(1 - \delta_k^3\right) e^{(-d(q-1)T)/(q+1)}}$$

$$+ \frac{-1}{cd} \ln \frac{r + cY^* \prod_{k=1}^q \left(1 - \delta_k^3\right) e^{-dT}}{r + cY^* \prod_{k=1}^q \left(1 - \delta_k^3\right) e^{-dqT/(q+1)}}. \tag{A.21}$$

Thus, we can obtain

$$\lambda_1$$

$$= \prod_{k=1}^q \left(1 - \delta_k^1\right)$$

$$\times \exp\left(a_1 T + \frac{\alpha_1}{cd}\right.$$

$$\times \ln\left(\left(r + cY^* e^{-dT/(q+1)}\right)\right.$$

$$\times \left(r + cY^* \left(1 - \delta_1^3\right) e^{-2dT/(q+1)}\right)$$

$$\cdots \left(r + cY^* \prod_{k=1}^q \left(1 - \delta_k^3\right) e^{-dT}\right)$$

$$\times \left((r + cY^*)\right.$$

$$\times \left(r + cY^* \left(1 - \delta_1^3\right) e^{-dT/(q+1)}\right)$$

$$\cdots \left(r + cY^* \prod_{k=1}^{q} \left(1 - \delta_k^3\right) e^{-dqT/(q+1)}\right)\right)^{-1}\right)\right),$$

(A.22)

$$\lambda_2$$

$$= \prod_{k=1}^{q} \left(1 - \delta_k^2\right)$$

$$\times \exp\left(a_2 T + \frac{\alpha_2}{cd}\right.$$

$$\times \ln\left(\left(r + cY^* e^{-dT/(q+1)}\right)\right.$$

$$\times \left(r + cY^* \left(1 - \delta_1^3\right) e^{-2dT/(q+1)}\right)$$

$$\cdots \left(r + cY^* \prod_{k=1}^{q} \left(1 - \delta_k^3\right) e^{-dT}\right)$$

$$\times \left(\left(r + cY^*\right)\right.$$

$$\times \left(r + cY^* \left(1 - \delta_1^3\right) e^{-dT/(q+1)}\right)$$

$$\cdots \left(r + cY^* \prod_{k=1}^{q} \left(1 - \delta_k^3\right) e^{-dqT/(q+1)}\right)\right)^{-1}\right)\right).$$

(A.23)

Clearly, the condition (A.12) of Theorem A.2 may be obtained, if we set $|\lambda_1|, |\lambda_2| < 1$. This completes the proof of local stability of periodic solution $(0, 0, y_T(t))$.

We now need to prove the global attractiveness. Choose an $\varepsilon > 0$ such that

$$\xi_i \triangleq \prod_{k=1}^{q} \left(1 - \delta_k^i\right) e^{\int_0^T (a_i - (\alpha_i(y_T(t) - \varepsilon)/(r + c(y_T(t) - \varepsilon)))) dt}$$

(A.24)

$$< 1 \quad (i = 1, 2).$$

Note that $dy(t)/dt \geq -dy(t)$; from Lemma A.1 and comparison theorem of impulsive equation, we have

$$y(t) > y_T(t) - \varepsilon$$

(A.25)

for all sufficiently large t. For simplification, we assume that the inequality (A.25) holds for all $t \geq 0$. Consider the following impulse differential inequalities:

$$\frac{dx_i}{dt} \leq \left(a_i - \frac{\alpha_i(y_T(t) - \varepsilon)}{r + c(y_T(t) - \varepsilon)}\right) x_i(t), \quad t \neq \tau_{nk},$$

$$x_i(\tau_{nk}^+) = \left(1 - \delta_k^i\right) x_1(\tau_{nk}), \quad t = \tau_{nk},$$

(A.26)

where $k = 1, 2, \ldots, q$ and $i = 1, 2$. By using Lemma 2, we have

$$x_i(t) \leq x_i((n-1)T)$$

$$\times \prod_{(n-1)T < \tau_{nk} < t} \left(1 - \delta_k^i\right) e^{\int_{(n-1)T}^{t} e^{a_i - (\alpha_i(y_T(s) - \varepsilon)/(r + c(y_T(s) - \varepsilon)))} ds};$$

(A.27)

then,

$$x_i(nT) \leq x_i((n-1)T)$$

$$\times \prod_{(n-1)T < \tau_{nk} < nT} \left(1 - \delta_k^i\right) e^{\int_{(n-1)T}^{nT} e^{a_i - (\alpha_i y_T(s)/(r + cy_T(s)))} ds}$$

$$= x_i((n-1)T)\, \xi_i.$$

(A.28)

Hence, $x_i(nT) \leq x_i(0^+) \xi_i^n$ and $x_i(nT) \to 0$ as $n \to \infty$. Therefore, $x_i(t) \to 0$ as $t \to \infty$, since $0 < x_i(t) < x_i((n-1)T) \prod_{k=1}^{q} (1 - \delta_k^i) e^{a_i T}$ for $(n-1)T < t < nT$.

Next, we prove that $y(t) \to y_T(t)$ as $t \to \infty$. For $\varepsilon > 0$ sufficiently small, there exists a T' such that $0 < x_1(t) < \varepsilon$ and $0 < x_2(t) < \varepsilon$, $t \geq T'$. Without any loss of generality, we assume that $0 < x_1(t) < \varepsilon$ and $0 < x_2(t) < \varepsilon$ for all $t \geq 0$. Then, from the system (2) we obtain

$$-dy(t) \leq \frac{dy(t)}{dt} \leq (-d + \gamma\varepsilon) y(t), \quad \gamma = \frac{\omega_1 \alpha_1}{r} + \frac{\omega_2 \alpha_2}{r}.$$

(A.29)

From Lemmas 2 and A.1, we have $u(t) \leq y(t) \leq v(t)$ and $u(t) \to y_T(t), v(t) \to y_T(t)$ as $t \to \infty$, where $u(t)$ and $v(t)$ are solutions of the equations

$$\frac{du(t)}{dt} = -du(t), \quad t \neq nT, \ t \neq \tau_{nk},$$

$$u(t^+) = \left(1 - \delta_k^3\right) u(t), \quad t = \tau_{nk}, \ k = 1, 2, \ldots, q,$$

$$u(t^+) = u(t) + \theta, \quad t = nT,$$

$$u(0^+) = y_0,$$

$$\frac{dv(t)}{dt} = (-d + \gamma\varepsilon) v(t), \quad t \neq nT, \ t \neq \tau_{nk},$$

$$v(t^+) = \left(1 - \delta_k^3\right) v(t), \quad t = \tau_{nk}, \ k = 1, 2, \ldots, q,$$

$$v(t^+) = v(t) + \theta, \quad t = nT,$$

$$v(0^+) = y_0,$$

(A.30)

respectively.

We also have

$$v_T(t) = \begin{cases} Y^* e^{(-d+\gamma\varepsilon)(t-(n-1)T)}, & t \in ((n-1)T, \tau_{n1}], \\ Y^* \left(1-\delta_1^3\right) e^{(-d+\gamma\varepsilon)T/(q+1)} e^{(-d+\gamma\varepsilon)(t-\tau_{n1})}, \\ \qquad\qquad t \in (\tau_{n1}, \tau_{n2}], \\ \vdots \\ Y^* \left(1-\delta_{q-1}^3\right)\cdots\left(1-\delta_1^3\right) e^{((-d+\gamma\varepsilon)(q-1)T)/(q+1)} \\ \qquad \times e^{(-d+\gamma\varepsilon)(t-\tau_{nq-1})}, \quad t \in \left(\tau_{n(q-1)}, \tau_{nq}\right], \\ Y^* \left(1-\delta_q^3\right)\cdots\left(1-\delta_1^3\right) e^{((-d+\gamma\varepsilon)qT)/(q+1)} \\ \qquad \times e^{(-d+\gamma\varepsilon)(t-\tau_{nq})}, \quad t \in \left(\tau_{nq}, nT\right]. \end{cases}$$

(A.31)

Then, for any ε_1 there exists a T_1 such that

$$u_T(t) - \varepsilon_1 < y(t) < v_T(t) + \varepsilon_1, \quad t > T_1. \tag{A.32}$$

Let $\varepsilon \to 0$, we have

$$y_T(t) - \varepsilon < y(t) < y_T(t) + \varepsilon \tag{A.33}$$

for t large enough, which implies that $y(t) \to y_T(t)$ as $t \to \infty$. This completes the proof. $\square$

Acknowledgments

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