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Research Article

A Weighted Multiobjective Optimization Method for Mixed-Model Assembly Line Problem

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Mixed-model assembly line (MMAL) is a type of assembly line where several distinct models of a product are assembled. MMAL is applied in many industrial environments today because of its greater variety in demand. This paper considers the objective of minimizing the work overload (i.e., the line balancing problem) and station-to-station product flows. Generally, transportation time between stations are ignored in the literature. In this paper, Multiobjective Mixed-Integer Programming (MOMIP) model is presented to optimize these two criteria simultaneously. Also, this MOMIP model incorporates a practical constraint that allows to add parallel stations to assembly line to decrease higher station time. In the last section, MOMIP is applied to optimize the cycle time and transportation time simultaneously in mixed-model assembly line of a real consumer electronics firm in Turkey, and computational results are presented.

1. Introduction

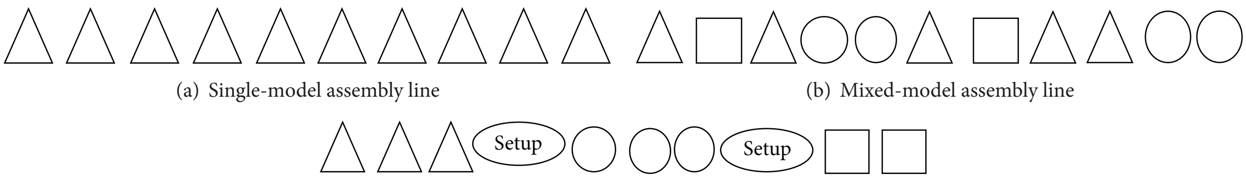
Mixed-model assembly lines are one of the important parts of mass production systems and generally useful in small product variety and high-volume industries such as automotive, electronics, and machinery. An assembly line comprises several successive workstations where a set of parts for one or more product types are assembled [1]. The assembly line balancing problem (ALBP) has had significant industrial importance since Henry Ford's introduction of the assembly line. ALBP is the assignment of product tasks to different stations with considering precedence relations between tasks. In spite of the high cost of settings and operating an assembly line, manufacturers often simultaneously produce one model with different features or several models on a single line [2]. Assembly lines can be classified as "single model," "multimodel," and "mixed model" with respect to the number of different products assembled on an assembly line. We can see difference between single-model, mixed-model, and multimodel assembly lines in Figure 1.

In *single-model assembly lines*, only one model product is produced in the same line. The assembly lines where products of one similar model are assembled with batches are called *multimodel assembly lines*. *Mixed-model assembly lines* are the lines on which in the simultaneous production products of more than one model are assembled [3]. In order to meet the customers' demand of individual and different models of product, mixed-model assembly lines are used. Thus, there can be produced various models but small quantity of products without setup operations on the same line. This is required to respond to the demand of market quickly and adjust to the change of market environment in time. Mixed-model assembly lines have a strong impact to improve the goal of Just in Time (JIT) and balanced production [4].

Mixed-model assembly has had a big importance because of its diversity. Although other goals of mixed model assembly lines such as low costs, high productivity, and standardization are in contrast with diversity, the success of a company is related to its ability to deal with complex products and process designs [5].

There are two types of mixed-model assembly line problems (MMALPs), which are referred as dual problems in the literature.

(i) MMALBP-I minimises the number of workstations for a given cycle time.



(a) Single-model assembly line

(b) Mixed-model assembly line

(c) Multimodel assembly line

Figure 1: Difference between single-model, mixed-model, and multi-model assembly lines.

(ii) MMALBP-II minimises the cycle time for a given number of workstations.

In type I problems, the cycle time (the time elapsed between two consecutive products at the end of the assembly line) and the production rate have to be prespecified. Thus, it is more frequently used in the design of a new assembly line for which the demand can be easily forecasted. Type II problems deal with the maximisation of the production rate of an existing assembly line. For instance, the MMALBP-II is applied when changes in the assembly process or in the product range require the line to be redesigned [6].

Material movement is an important factor in the production line. Although it is an important factor in nonvalue-added cost function, it increases the cost of production. So, in a well-designed system, work and storage areas should be near to their point of use. In an assembly line material handling system, transportation of materials and storage devices require huge capital investment. However, transportation does not add value to the final product.

Transportation time is usually added to the processing time or negligible in assembly line systems. However, minimization of assembly parts from a machine to another decreases the completion time and cost of products. But, minimization of part unit movements can cause workloads imbalance. This leads to unequal total assembly times assigned to stations and longer intermediate queues and bottlenecks in the system as a result. In order to have the best assembly system capabilities, balancing of the line and transfer times between stations should be taken into account simultaneously. Routings of part units are completely defined by the sequence of machines they have to visit.

Although there is a vast literature on the mixed-model assembly line balancing and sequencing problem, transportation times between stations consist of process routings of products and balancing problem generally is not considered in an integrated fashion. In this paper, we propose a mixed-integer programming model to tackle this problem. Also, we assume that there can be parallel machine in some stations, and we add this as a criterion to mathematical model.

With this point, this paper includes two main objectives.

(1) The workload balancing assigns tasks to stations in order to equalize station workloads.

(2) The total amount of transfers of components from a station to another is minimized. In other words, we look for balancing routings of part units to minimize transportation times that cause the increase of completion time of the product.

To reach these objectives, some constraints are added to the mathematical model. If it is required, we add parallel machine to minimize cycle times of stations. MOMIP model also includes a finite workspace constraint that is considered with line balance. Finite working space might be subject to technical restrictions and space requirements of assigned station. The limited workspace capacity restricts the number of tasks which are assigned to each station.

In Section 2, there is a review of literature on MMALP. Then in Sections 3 and 4, MOMIP model is proposed with results of experiments and real-world application in consumer electronics in Turkey. In Section 5, conclusions are presented.

2. Review of the Existing Literature in MMAL

The mixed-model assembly line deals with solving two primary problems in a production line. The first problem is the design and balancing of the production line, whereas the second problem is the determination of the production sequence for different models [7]. The line balancing comprises the assignment of tasks to stations and determination of the work content and model type per station. However, the production sequence is the model mix arranged with regard to minimum overloads on the assembly line. Balancing and sequencing problems are known as an NP-hard class of combinatorial optimization problems in mixed model assembly line literature.

Several researchers have studied MMAL balancing problems. Thomopoulos [8] and Macaskill [9] are the initial researchers in solving this problem. Macaskill [9] and Chakravarty and Shtub [10] have also studied the line balancing for traditional mixed model straight lines (MMSLs). Also, [11] presents a balancing methodology for mixed-model lines with deterministic task times. The objective is minimizing the total cost of stations (essentially the regular time labor cost) and work overload. Erlebacher and Singh [12] proposes a method to allocate a fixed total processing time variance among multiple stations and to minimize the total expected work overload. Zhang and Gen [13] present random key-based representation method with adapting Genetic Algorithm to assign the suitable task to the suitable station and the allocation of the proper worker to the proper station. The objective is to minimizate the variation of workload and the total cost under the constraint of precedence relationships.

The mixed model assembly line sequencing is investigated in [14] for the first time. Various objectives are reported in the literature in determining the optimal sequence for a mixed model assembly line. The common objectives are minimizing the overall line length [15–19], minimizing the risk of stopping a conveyor [20], minimizing the total utility work [21, 22], and keeping a constant rate of part usage [23–27]. Also, Bard et al. [15], Scholl [28], and Yano and Bolat [29] present several procedures for different versions of the mixed-model-sequencing problems [30].

However, Celano et al. [31] investigate the sequencing of MMAL assuming that the parts usage smoothing is the goal of the sequence selection. Mirzapour Al-E-Hashem et al. [7] present a sequencing problem with a bypass subline with the goals of leveling the part usage rates and reducing line stoppages. To solve this problem, a novel hybrid algorithm incorporating genetic algorithm and event-based procedure is developed to solve the problem.

However, there are a couple of papers dealing with simultaneous assembly line balancing and sequencing [32–37]. Hu et al. [38] also analyze the balancing of mixed model assembly lines and design a new algorithm based on the process exchange according to its attributes and characters. The algorithm could be utilized to make further optimization of mixed model assembly lines on the basis of the best production sequence.

Merengo et al. [39] develop a new balancing and production sequencing method for manual mixed-model assembly lines. Minimization of the number of stations is provided by the balancing method, and a uniform part usage is obtained by the sequencing method. Kim et al. [40] present a new method using a coevolutionary algorithm that can simultaneously solve balancing and sequencing problems in mixed-model assembly lines. Karabatı and Sayın [41] propose the MMALBP with the objective of minimizing the total cycle time by combining the cyclic sequencing information. They propose a mathematical model and an alternative heuristic approach to minimize the maximum subcycle time [42]. Fattahi and Salehi [43] consider a mixed-integer programming model with a variable rate launching interval between products on the assembly line, to minimize the idle and utility time cost, with optimization sequencing and launching interval for each sequence.

Although transportation of units is very important in assembly systems, there are few studies that take into account that part units' movements in assembly lines. Transportation time is usually added to the processing times or negligible in assembly systems. However, minimization of assembly parts movements from a machine to another decreases the completion time of products.

In our study, we propose a mixed-integer programming that can simultaneously deal with both balancing and sequencing problems in MMAL, and identical parallel machines are allowed at each stage of the serial system. With this approach, we obtain the best task sequences for the models by taking into consideration the minimization of transportation of models between stations; we also solve the balancing problem in the line at the same time. The mathematical model intends for balancing of workloads and routing of unit parts.

3. Model Description

This paper introduces a mathematical model for MMAL, and it can be categorized as MOMIP model. This mathematical model is integrated to real-world application. Our mathematical model is based on Sawik's integer programming models about mixed model assembly line [44, 45]. The problem objectives are the determination of allocation of assembly tasks among the stations, where they have parallel machines and selection of assembly sequences and assembly routes for a set of products simultaneously to balance station workloads and to minimize total transportation time in assembly line.

We give weights to objectives to solve this multiobjective problem. This problem is solved using ILOG OPL optimization software [46]. The proposed model is used for a mixed model assembly line problem from real world in a Turkish consumer electronics firm. Due to the NP-hard nature of the problem, the size of our model would be large to obtain optimal solutions for problems of realistic sizes. Also, the model suggested in this paper presents a significant improvement relative to the models in the literature.

The assumptions of the proposed model are as follows.

(1) Each assembly task must be assigned to at least one station (alternative assignments are allowed).

(2) There are parallel machines in some stations.

(3) Total space required for the tasks assigned to each station must not exceed the station's finitework space available.

(4) Each product must be routed to the stations subject to precedence relations defined by its assembly plan.

(5) Revisiting of stations is not allowed.

(6) Each station can perform at most one task at any given time.

(7) Transfer times between stations are not negligible.

Notation

Indices

i: assembly station $i \in I$, $I = \{1, \ldots, m\}$

j: assembly task $j \in J = \{1, \ldots, n\}$

p: parallel machine at station i $\{h = 1, \ldots, m_i\}$

k: product, $k \in K = \{1, \ldots, v\}$

s: assembly sequence, $s \in S = \{1, \ldots, w\}$.

Input Parameters

a_{ij}: working space of station i for task j

b_i: working space for station I

m_i: number of parallel machines in station I

p_{jk}: process time for task j of model k

q_{il}: transportation time from station i to station l

I_j: the set of stations capable of performing task j

J_k: the set of tasks required for product k

R_s: the set of immediate predecessor-successor pairs of tasks (j, r) for assembly sequence $s \in S$ such that task j must be performed immediately before task r

S_k: the set of assembly sequences available for product k

T_s: the set of tasks in assembly sequence s.

The following decision variables are introduced to model the loading and routing problem:

α: the weight factor $(0 < \alpha < 1)$,

M: a big number.

Decision Variables

$u_s = 1$, if assembly sequence $s \in S$ is selected; otherwise 0;

$x_{ij} = 1$, if task j is assigned to station $i \in I_j$; otherwise $x_{ij} = 0$;

$z_{sjih} = 1$, if task j is assigned to parallel machine h in station $i \in I_j$; otherwise $z_{sihj} = 0$;

$y_{iljs} = 1$, if product in s sequence is transferred from station i after the completion of task j to station l to perform next task; otherwise $y_{iljs} = 0$;

$P_{\max}$ is the maximum station workload (cycle time),

Q_{sum} represents the weighted sum of total assembly and transportation time.

We can state the problem formally as follows:

$$\text{Min}\left(\alpha P_{\max} + (1-\alpha) Q_{\text{sum}}\right) \quad (1)$$

subject to

$$\sum_{i \in I_j} \sum_{l \geq i} y_{iljs} = u_s; \quad s \in S,\ j \in T_s, \quad (2)$$

$$\sum_{l \leq i} y_{lijs} - \sum_{l \geq i} y_{ilrs} = 0; \quad i \in I_r,\ (j, r) \in R_s,\ s \in S, \quad (3)$$

$$\sum_{k \in K} \sum_{s \in Sk} \sum_{j \in T_s} p_{jk} z_{sjih} \leq P_{\max}; \quad i \in I,\ h \in H, \quad (4)$$

$$\sum_{i \in I} y_{iljs} \leq \sum_{h=1}^{m_i} z_{sjih}; \quad i \in I, j \in J, s \in S, \quad (5)$$

$$\sum_{s \in S} \sum_{h=1}^{m_i} z_{sjhi} \leq m_i * \sum_{i \in I} \sum_{s \in S} y_{iljs}; \quad i \in I, j \in J, \quad (6)$$

$$\sum_{s \in S} \sum_{j \in T_s} \sum_{i \in I} \sum_{l \in I} q_{il} y_{iljs} = Q_{\text{sum}}, \quad (7)$$

$$\sum_{i \in I_j} x_{ij} \geq 1; \quad j \in J, \quad (8)$$

$$\sum_{j \in J} a_{ij} x_{ij} \leq m_i b_i; \quad i \in I, \quad (9)$$

$$\sum_{j \in J} a_{ij} z_{sjih} \leq b_i; \quad i \in I,\ h \in H,\ s \in S, \quad (10)$$

$$\sum_{s \in S} \sum_{i \in I} \sum_{h > m_i} \sum_{j \in J} z_{sjih} = 0, \quad (11)$$

$$z[s][j][i][h] \leq u[s]; \quad i \in I,\ j \in J,\ h \in H,\ s \in S, \quad (12)$$

$$y_{iljs} \leq x_{ij}; \quad i \in I_j,\ l \geq i,\ l \in l_r,\ (j, r) \in R_s,\ s \in S, \quad (13)$$

$$y_{iljs} \leq x_{lr}; \quad i \in I_j,\ l \geq i,\ l \in l_r,\ (j, r) \in R_s,\ s \in S, \quad (14)$$

$$y_{iljs} \leq u_s; \quad i \in I_j,\ l \geq i,\ l \in l_r,\ (j, r) \in R_s,\ s \in S, \quad (15)$$

$$\sum_{s \in S_k} u_s = 1; \quad k \in K, \quad (16)$$

$$\sum_{i \in I} \sum_{l < i} \sum_{s \in S} \sum_{j \in T_s} y_{iljs} = 0, \quad (17)$$

$$\sum_{s \in S} \sum_{j \in J} \sum_{i \in I} \sum_{i \notin I_J} y_{iljs} = 0, \quad (18)$$

$$\sum_{j_2 \in J} \sum_{h_2 \neq h} z_{sj_2ih_2} \leq M\left(1 - z_{sjih}\right); \quad s \in S,\ j \in J,\ i \in I,\ h \in H. \quad (19)$$

The objective function is the minimization of the weighted maximum workload $P_{\max}$ and sum of transportation time Q_{sum}. Equation (2) shows for each product and assembly sequence selected that all of its required tasks are allocated among the stations. Equation (3) is the flow of tasks for each station, for selected assembly sequence, and for successively performed tasks. Equations (4) and (7) ensure the workload of the bottleneck station with parallel machines and the total transportation time, respectively. Equations (5) and (6) define the tasks that are assigned to at least one machine and not more than all "m_i" parallel machines of such a station "i" when the product moves from station "i" to station "l" to perform task "j". Equation (8) ensures that each task is assigned to at least one station, and by this, it admits alternative assembly routes for products. Equation (9) is the station capacity constraint. Equation (10) shows the total flexibility capacity of all parallel machines at related station. Equation (11) represents the capacity constraint of the number of parallel machines in station "i". Equation (12) shows that if the sequence "s" is not selected, all variables of related sequence are made zero. Equation (12) shows that if we do not select any sequence, we make all of variables in this sequence zero. Equations (13), (14), and (15) ensure that each product successively visits stations where the required tasks may be assembled subject to precedence relations defined by the assembly sequence selected. Equation (16) ensures that only one assembly sequence is selected for each product. Equation (17) eliminates upstream flow of products in a unidirectional flow system. Equation (18) eliminates assignment of tasks and products to inappropriate stations. Equation (19) ensures that all tasks, which are in the same "s" sequence, are assigned to the same station and the same parallel machine and that also the tasks of the same product models are assigned to the same station and the same parallel machine.

Multiobjective integer programming problems may be thought as an extension of the classical single objective integer programming problem. Scalarizing functions used in subproblems are very important in solving multi-objective optimization problems [47].

Different scalarization methods have been presented in the literature, some of which include the weighted sum method, ε-constraints method, hierarchical approach, weighted metrics methods, and goal attainment method [48].

In this paper, there are two objective functions to be optimized with multi-objective integer programming technique in mixed model assembly line problem.

We use the most convenient scalarization method with related objective functions of the problem. In this sense, the weighted sum (WS) method is used to solve multi-objective integer programming problem. With solution procedure, we would produce the most preferred nondominated solutions. The weighted sum (WS) method involves a linear or convex combination of the objectives $f_i(x)$, $i = 1, \ldots, p$. Each objective $f_i(x)$ is multiplied by a normalized weight factor w_i and the product added to give the scalar objective $\phi(x, w_i)$ as follows:

$$\varphi(x, w) = \sum_{i=1}^{p} \alpha_i f_i(x), \quad (20)$$

where p is the number of the objectives, $\sum_{i=1}^{p} \alpha_i = 1$, and $\alpha_i > 0$, $i = 1, \ldots, p$.

Its drawbacks are also well known and discussed as follows.

(i) It misses solution points on the nonconvex part of the pareto surface.

(ii) Its diversity cannot be controlled; therefore even the distribution of weights does not translate to uniform the distribution of the solution points.

(iii) The distribution of solution points is highly dependent on the relative scaling of the objective [48].

In this paper, there are two objective functions. We minimize the weighted sum of the objectives to obtain only one solution. The weight factor $\alpha \in [0, 1]$ is used for interactive solution between two objectives. Therefore, decision makers modify the weighting coefficients after each iteration, so he/she would produce the most preferred nondominated solution.

In the following section, firstly we perform experimental test problems to validate this mixed integer model, then we apply the model to real-world mixed model assembly line problem, and computational results are summarized.

4. Experiments and Computational Results

In this section, the experiments are used to test the performance of the mathematical model. Then, their analysis and the application to the real-world consumer electronics firm problem are described. For the experiments, the number of stations and the number of tasks are changed with the weight factor in each iteration. The problem includes optimization of balancing and transportation times of mixed model assembly line. To solve the underlying multi-objective mixed integer programming model, we use ILOG OPL optimization software. The experimental results are seen in Table 1.

Experimental results show that $\alpha = 0$ and $\alpha = 1$ produce weakly nondominated solutions for the objectives. For $\alpha = 0$, transportation time is minimized and dominated whereas for $\alpha = 1$ cycle time is minimized and dominated. Other weights produce strongly nondominated solutions. As a result, while weight of α increases, cycle time decreases and transportation time increases.

In addition to these test instances, we have solved a real-world problem from a consumer electronics plant in Turkey. The firm we have obtained is a household appliances's manufacturer. It principally engages in production, marketing, and after sales service of durable goods, allied components, and consumer electronics. The assembly system of the firm is made up of 18 identical stations which produce three different LCD model TV types, and this number is constant. There are 73 tasks to produce three different model types. Some of these tasks are common. Common tasks can have different task times in each model. Thus, we assume that there is one machine in each station. According to the cycle time we have obtained, we placed one more parallel machine to the station where its cycle time is high. The objectives are the minimization of the weighted maximum workload $P_{\max}$ and the sum of transportation time Q_{sum}. The performance of the proposed algorithm is analyzed through variance between stations and balance loss of the system. In our system, each model has its own set of precedence relationships, but there is a subset of tasks common to all models. To solve this mixed model assembly line problem, the precedence diagrams for all the models are combined, and then we use adjusted processing time to transform the mixed model line balancing problem into a single model line balancing problem and determine average task processing times for the tasks that are required by more than one model [49–52]. The average processing time of task j, t_j, can be calculated by

$$t_j = \sum_k f_k p_{jk}, \quad (21)$$

where p_{jk} equals the processing time of task j in model k, and f_k is the relative frequency or the demand for each model. The demand for each model is assumed to be equal because of the daily production rates, so process time for each task is divided based on demand ratio. Task times of each model are shown in Table 2.

The other variables of the proposed model are as follows.

The transportation times q_{il} from station i to station l are 1 unit time. The set of tasks required for product k (JK) and alternative assembly sequences (plans) (TS) are shown in Tables 3 and 4. Each product modelhas two alternative assembly plans which include task sequences. Working space of station i for task j is 2 units and working space for each stations is 10 units.

The balancing procedure operates as shown in Figure 2.

To determine the weights of objectives in multi-objective integer programming model, we use the preferences of the

Table 1: Experimental results.

Number of product models/Number of tasks	$\alpha = 0$ $P_{\max}$-Q_{sum}	$\alpha = 0.2$ $P_{\max}$-Q_{sum}	$\alpha = 0.4$ $P_{\max}$-Q_{sum}	$\alpha = 0.6$ $P_{\max}$-Q_{sum}	$\alpha = 0.8$ $P_{\max}$-Q_{sum}	$\alpha = 1$ $P_{\max}$-Q_{sum}
3/5	10-8	8-8	6-8	6-8	6-10	6-10
3/10	58-4	25-6	20-8	20-8	18-12	18-12
4/10	46-6	41-6	27-10	20-16	20-16	20-16
3/20	38-6	19-10	19-10	19-10	19-10	19-12
4/20	42-8	41-8	28-12	28-12	27-14	27-16
3/30	58-6	27-8	27-8	25-10	24-12	24-12
4/30	61-8	41-8	41-8	32-16	32-16	32-18

Table 2: Adjusted task times.

Number of task	Model 1	Model 2	Model 3	Number of task	Model 1	Model 2	Model 3	Number of task	Model 1	Model 2	Model 3
1	0,11300	0,11600	0,11600	26	0,10267	0,10267	0,10267	51	—	0,05533	0,02967
2	0,02100	0,02100	0,02100	27	0,01700	—	—	52	—	0,09200	0,09200
3	0,09967	0,09500	0,09600	28	0,08533	0,08933	0,08933	53	—	0,05467	0,01733
4	0,10633	—	—	29	0,05133	0,04800	0,04800	54	—	0,01967	0,01567
5	0,04800	0,04833	0,04833	30	0,15933	0,14567	0,14567	55	—	0,04533	—
6	0,01533	—	0,01533	31	0,11367	—	0,10700	56	—	0,09767	—
7	0,04333	—	—	32	0,03500	—	—	57	—	0,04267	—
8	0,09167	—	—	33	0,12967	—	0,12967	58	—	0,03500	—
9	0,06200	—	—	34	0,04367	—	—	59	—	0,05433	0,06533
10	0,07467	—	—	35	0,02500	—	—	60	—	—	0,07767
11	0,04300	—	—	36	0,07933	—	—	61	—	—	0,03200
12	0,06400	—	—	37	0,06067	0,06067	0,06067	62	—	—	0,03900
13	0,04300	—	—	38	0,07733	0,07733	0,07733	63	—	—	0,08167
14	0,03533	0,03533	0,03533	39	0,02900	—	—	64	—	—	0,04167
15	0,07767	—	—	40	0,02133	—	—	65	—	—	0,02233
16	0,11800	—	—	41	—	0,01833	—	66	—	—	0,04267
17	0,04300	—	—	42	—	0,07067	0,07067	67	—	—	0,05333
18	0,11933	0,11933	0,11933	43	—	0,06733	—	68	—	—	0,07767
19	0,02500	0,02967	0,02967	44	—	0,06533	—	69	—	—	0,01900
20	0,01733	0,01733	0,01733	45	—	0,04433	0,04433	70	—	—	0,02633
21	0,02000	0,01733	0,01733	46	—	0,06567	—	71	—	—	0,01967
22	0,01733	0,01733	0,02133	47	—	0,34000	—	72	—	—	0,07767
23	0,03600	0,03900	0,03200	48	—	0,08700	0,08567	73	—	—	0,07767
24	0,03933	0,02767	0,04067	49	—	0,08067	—				
25	0,08100	—	—	50	—	0,14567	—				

Table 3: Sequencing of tasks for LCD TV product models.

Model	JK
1	1 2 3 4 5 6 7 8 9 10 11 12 13 14 39 15 16 17 18 40 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38
2	1 41 42 2 3 43 44 5 45 46 47 48 49 50 14 51 52 19 53 18 54 22 21 20 23 26 55 24 28 29 56 30 57 58 59 37 38
3	1 42 2 3 60 61 62 5 6 45 63 64 48 65 66 67 51 68 14 52 69 53 70 18 54 22 19 21 20 23 71 26 24 28 29 31 33 30 72 73 59 37 38

decision makers. To determine the important weights of each objective, we select six participants from the production planning department. We ask them about two objectives of the problem in a meeting. Peers being aware of the optimization objectives choose one of the objectives as their preference. The weights assigned to each objective function are f_g. P_{f_g} represents the number of peers that prefer f_i to the other optimization objectives. n is the number of participants in the meeting. The important weights are determined using the number of participants in the meeting. One has

$$\alpha f_i = \frac{p_{f_g}}{n}. \tag{22}$$

Thus, we obtain 0.8 value for the first objective ($P_{\max}$) weight and 0.2 value weight for the second objective (Q_{sum}). Using these weights, we obtain the following results. Also to minimize the cycle time, we assign parallel machines to stations where cycle time is high. Decision of parallel machine

Table 4: Assembly plans for LCD TV product models.

TS	JK
1	1 2 3 4 5 6 7 8 9 10 11 12 13 14 39 15 16 17 18 40 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38
2	1 3 2 4 5 7 6 8 9 10 11 12 14 13 39 15 17 16 18 40 19 20 21 22 23 24 26 25 27 28 29 30 32 31 33 34 35 36 37 38
3	1 41 42 2 3 43 44 5 45 46 47 48 49 50 14 51 52 19 53 18 54 22 21 20 23 26 55 24 28 29 56 30 57 58 59 37 38
4	1 42 41 2 3 44 43 5 45 46 47 48 49 50 14 51 52 53 19 18 54 22 21 20 23 55 26 24 28 29 56 30 57 58 59 37 38
5	1 42 2 3 60 61 62 5 6 45 63 64 48 65 66 67 51 68 14 52 69 53 70 18 54 22 19 21 20 23 71 26 24 28 29 31 33 30 72 73 59 37 38
6	1 2 42 3 60 62 61 5 6 45 63 64 48 65 66 67 51 68 14 52 69 53 70 18 54 19 22 21 20 71 23 26 28 24 29 31 33 30 72 73 59 37 38

Table 5: Sequences after solving mathematical model.

TS	JK
1	1 2 3 4 5 6 7 8 9 10 11 12 13 14 39 15 16 17 18 40 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38
3	1 41 42 2 3 43 44 5 45 46 47 48 49 50 14 51 52 19 53 18 54 22 21 20 23 26 55 24 28 29 56 30 57 58 59 37 38
6	1 2 42 3 60 62 61 5 6 45 63 64 48 65 66 67 51 68 14 52 69 53 70 18 54 19 22 21 20 71 23 26 28 24 29 31 33 30 72 73 59 37 38

Table 6: Assignment of task to stations.

Number of station	Assigned task
1	1, 2, 4, 42
2	2
3	2, 3, 43, 44
4	4, 5, 60, 61, 62, 73
5	6, 7, 8, 9, 45, 46, 47, 48, 63, 64
6	10, 11, 48, 49, 50, 65, 66, 67
7	12, 13, 14, 51, 68
8	52, 53, 69
9	15, 16, 17, 39, 70
10	18, 19, 21, 22, 40, 53, 54
11	20, 21, 23, 26, 71
12	22, 23, 24, 25, 26
13	24, 27, 28, 55
14	29, 30, 56, 57, 58
15	30, 31, 33, 72
16	30, 31, 32, 33, 34
17	35, 36, 37, 59
18	38

depends on the firm's preference because of the high costs of machines. But when we place one more machine to stations $1, 5, 6$, and 10, the cycle time decreases from 0.931 to 0.481. Also, according to the results, transportation time is 51 time unit. After solving this optimization problem in ILOG OPL

Table 7: Stations times (P_{ih}) with parallel machines handled by mixed integer programming model.

Station workload	Machine	P_{ih} (time)	Station	Machine	P_{ih} (time)
1	1	0.3206	9	1	0.294
1	2	0.205	10	1	0.4063
2	1	0.021	10	2	0.2033
3	1	0.4443	11	1	0.2436
4	1	0.4773	12	1	0.418
5	1	0.481	13	1	0.3946
5	2	0.45	14	1	0.4683
6	1	0.1176	15	1	0.460
6	2	0.4316	16	1	0.4813
7	1	0.3756	17	1	0.406
8	1	0.2203	18	1	0.2319

programming $S = 1, 3, 6$ is obtained for sequences of model 1, model 2, and model 3, respectively. Selected sequences after solving mathematical model are shown in Table 5.

Task assignment to stations is shown in Table 6.

After the assignment of tasks to stations with parallel machines, station time (P_{ih}) is handled by mixed integer programming model as shown in Table 7.

After solving integer programming model, performance of the mathematical model is evaluated with balancing loss and variance of station workloads. Balancing loss is a measure of the line inefficiency which results from idle time due to imperfect allocation of work among stations. In this study, we measure system performance with well-balanced assembly line, when the idle time in each work station is at minimum level and the balancing loss is minimized. The balancing loss is shown with E_{line} and calculated by

$$E_{\text{line}} = \frac{n * P_{\max} - \sum_{i=1}^{N} t_i}{n * P_{\max}}. \tag{23}$$

The total work, t_i, content is the aggregate of all the work elements to be done on the line, n is the number of stations, and $P_{\max}$ is the cycle time. One has

$$\begin{gathered} E_{\text{line}} = \frac{n * P_{\max} - \sum_{i=1}^{N} t_i}{n * P_{\max}}, \\ E_{\text{line}} = 18 * 0.481 - \frac{7.55}{18 * 0.481} = 0.12. \end{gathered} \tag{24}$$

As we can observe from the results, decision of parallel machine depends on the firm's preference because of the high cost of machines. But, when we place one more machine to stations $1, 5, 6$, and 10, cycle time decreases from 0.931 to 0.481. In addition, production rate increases from 426 units to 813 units daily. According to the results, transportation time is 51 time unit. After solving this optimization problem in ILOG OPL programming $S = 1, 3, 6$ is obtained for sequences of model 1, model 2, and model 3, respectively. The balancing loss of the system works out of 12%, and the

Figure 2: General flow chart of algorithm.

variance between station times is computed as 0.15 which can be acceptable for MMALBP, because mixed-model assembly occurs when more than one model of the same general product are intermixed on one assembly line. This line design makes the balancing problem more complex than others and causes big balance loss in these systems.

Results show that in terms of total assembly time, processing assembly of different LCD TV models in the same line is more advantageous than processing in different assembly lines. With this study, the workloads at stations are balanced, the production rate is increased, the work flow between stations is improved, and the utilization of machines and labor is increased. Also our research concentrates on testing the method with large-scale instance from real world, eventually solving mixed integer programming models.

5. Conclusion

Mixed model assembly lines are widely used to produce different models as per customer's demands. The problem of mixed-model assembly line balancing is more complex than that of the single-model assembly line, because there are different tasks in different models on mixed assembly line and the processing times for common tasks may be different. In this paper, the MOMIP of a mixed model assembly line problem is formulated.

The goals of this study are to minimize station work loads and to minimize transportation time between stations. In order to achieve these goals, assembly works are assigned to stations optimally, and optimal assembly order and route is determined. The WS method is used to solve multi-objective integer programming problem. Also, to minimize the cycle time in related stations, parallelization is added as a criterion to mathematical model. An other criterion is related to the working space due to transportation times between stations. In the literature, generally, time spent between stations for transporting goods is ignored. In this study, we aim to fill this gap and solve a real-world problem.

In the first step, mixed model assembly line problem is converted to single model assembly line problem. For this, precedence diagrams for each single model are combined in combined precedence diagrams, and adjusted tasks times are computed for each task. The experiments are solved to test the performance of the mathematical model. Then, the proposed model is adapted to the real-world consumer electronics firm problem.

Computational experiments show that this optimization method is capable of obtaining good results both in terms of solution quality and in terms of execution times. The main contribution of this research is the definition of a new problem with practical relevance, and the objective function models trade off between cycle time and transportation time related to selecting route for each product model. Also, the parallelization and station length are added to the mathematical model as a constraint with balancing and sequencing, so transportation of product models is taken into account to be different from other papers in assembly sytems.

After solving mixed model assembly line problem in consumer electronics firm, line efficiency is measured with balancing loss and variance between station workloads.

Processing assembly of different LCD TV models in the same line is more advantageous than processing in different assembly lines in terms of total assembly time. In this way, the firm can meet the demands of different model products at the same time. This means that customer satisfaction and sales increase. Also with this study many improvements are obtained; for example, workloads at stations are balanced, the production rate is increased, the work flow between stations is improved, and the utilization of machines and labor increased. Also our research concentrates on testing the method with large-scale instance from real world, eventually solving mixed integer programming models.

References

[1] Y. Kara, C. Özgüven, N. Yalçin, and C. Chang, "Multi-objective approaches to balance mixed-model assembly lines for model mixes having precedence conflicts and duplicable common tasks," *International Journal of Advanced Manufacturing Technology*, vol. 52, pp. 725–737, 2011.

[2] I. Weida and X. Tianyuan, "Strategic robust mixed model assembly line balancing based on scenario planning," *Tsinghua Science and Technology*, vol. 16, no. 3, pp. 308–314, 2011.

[3] H. Güden, *An Adaptive Simulated Annealing Method For Assembly Line Balancing and A Case Study [In Partial Fulfillment of the Requirements for The Degree of Master of Science In Industrial Engineering]*, Graduate School of Natural And Applied Sciences of Middle East Technical University, 2006.

[4] L.-K. Hu, F.-Y. Shao, and G.-X. He, "Balancing analysis and algorithm design for mixed model assembly lines," in *Proceedings of the 18th International Conference on Industrial Engineering and Engineering Management (IE & EM '11)*, pp. 67–71, September 2011.

[5] F. Hellman, B. Lindahl, and J. Malmberg, *Mixed-Model Assembly Line at Volvo Construction Equipment Requirements for Mixed-Model Assembly Line at Volvo Construction Equipment and a Case Study at the Arvika Plant [Master of Science Thesis in Production Engineering]*, Department of Technology Management and Economics, Division of Logistics and Transportation, Chalmers University of Technology, 2011.

[6] A. S. Simaria and P. M. Vilarinho, "A genetic algorithm based approach to the mixed-model assembly line balancing problem of type II," *Computers and Industrial Engineering*, vol. 47, no. 4, pp. 391–407, 2004.

[7] S. M. J. Mirzapour Al-E-Hashem, M. B. Aryanezhad, and A. Jabbarzadeh, "A new approach to solve a mixed-model assembly line with a bypass subline sequencing problem," *International Journal of Advanced Manufacturing Technology*, vol. 52, no. 9–12, pp. 1053–1066, 2011.

[8] N. T. Thomopoulos, "Mixed model line balancing with smoothed station assignments," Management Science, JSTOR, 1970.

[9] J. L. C. Macaskill, "Production-line balances for mixed-model lines," *Management Science*, vol. 19, no. 4, pp. 423–434, 1972.

[10] A. K. Chakravarty and A. Shtub, "Balancing mixed model lines with in-process inventories," *Management Science*, vol. 31, no. 9, pp. 1161–1174, 1985.

[11] L. Y. Hsu, *Design of an Assembly Line with Stochastic Task Times [Master of Sciencethesis]*, Sloan School of Management, Massachusetts Institute of Technology, Cambridge, Mass, USA, 1992.

[12] S. J. Erlebacher and M. R. Singh, "Optimal variance structures and performance improvement of synchronous assembly lines," *Operations Research*, vol. 47, no. 4, pp. 601–618, 1999.

[13] W. Zhang and M. Gen, "An efficient multiobjective genetic algorithm for mixed-model assembly line balancing problem considering demand ratio-based cycle time," *Journal of Intelligent Manufacturing*, vol. 22, no. 3, pp. 367–378, 2011.

[14] M. D. Kilbridge and L. Wester, "The line model-mix sequencing problem," in *Proceedings of the 3rd International Conference on Operations Research*, p. 247, English Universities Press, Paris, France, 1963.

[15] J. F. Bard, E. Dar-El, and A. Shtub, "An Analytic frame work for sequencing mixed model assembly lines," *International Journal of Production Research*, vol. 30, no. 1, pp. 35–48, 1992.

[16] J. F. Bard, A. Shtub, and S. B. Joshi, "Sequencing mixed-model assembly lines to level parts usage and minimize line length," *International Journal of Production Research*, vol. 32, pp. 2431–2454, 1994.

[17] E. M. Dar-El, "Mixed-model assembly line sequencing problems," *Omega*, vol. 6, no. 4, pp. 313–323, 1978.

[18] E. M. Dar-El and R. F. Cother, "Assembly line sequencing for model mix," *International Journal of Production Research*, vol. 13, no. 5, pp. 463–477, 1975.

[19] M. D. Kilbridge and L. Wester, "The assembly line model-mix sequencing problem," in *Proceedings of the 3rd International Conference on Operations Research*, pp. 247–260, Paris, France, 1963.

[20] K. Okamura and H. Yamashina, "A heuristic algorithms for the assembly line model-mix sequencing problem to minimize the risk of stopping the conveyor," *International Journal of Production Research*, vol. 17, no. 3, pp. 233–247, 1979.

[21] C. A. Yano and R. Rachamadugu, "Sequencing to minimize work overload in assembly lines with product options," *Management Science*, vol. 37, pp. 572–586, 1991.

[22] J. F. Bard, A. Shtub, and S. B. Joshi, "Sequencing mixed-model assembly lines to level parts usage and minimize line length," *International Journal of Production Research*, vol. 32, pp. 2431–2454, 1994.

[23] E. A. Duplaga, C. K. Hahn, and D. Hur, "Mixed-model assembly line sequencing at Hyundai Motor Company," *Production and Inventory Management Journal*, vol. 37, no. 3, pp. 20–25, 1996.

[24] P. R. McMullen, "An ant colony optimization approach to addressing a JIT sequencing problem with multiple objectives," *Artificial Intelligence in Engineering*, vol. 15, no. 3, pp. 309–317, 2001.

[25] J. Miltenburg, "Level schedules for mixed-model assembly lines in just-intime production systems," *Management Science*, vol. 35, no. 2, pp. 192–207, 1989.

[26] Y. Monden, *Toyota Production System: Practical Approach To Production Management*, Industrial Engineering and Management Press, Atlanta, Ga, USA, 1983.

[27] P. R. McMullen and G. V. Frazier, "A simulated annealing approach to mixed-model sequencing with multiple objectives on a just-in-time line," *IIE Transactions*, vol. 32, no. 8, pp. 679–686, 2000.

[28] A. Scholl, *Balancing and Sequencing of Assembly Lines*, Physica, Heidelberg, Germany, 2nd edition, 1999.

[29] C. A. Yano and A. Bolat, "Survey, development and application of algorithms for sequencing paced assembly lines," *Journal of Intelligent Manufacturing*, vol. 2, pp. 172–198, 1989.

[30] C. Copaceanu, "Mixed-model assembly line balancing problem: variants and solving techniques," in *Proceedings of ICMI*, vol. 45, pp. 337–360, Bacau, Romania, September 2006, Stud. Cercet. Stiint. , Ser.Mat., vol. 16, supplement, 2006.

[31] G. Celano, A. Costa, and S. Fichera, "A comparative analysis of sequencing heuristics for solving the Toyota Goal Chasing problem," *Robotics and Computer-Integrated Manufacturing*, vol. 20, no. 6, pp. 573–581, 2004.

[32] P. R. McMullen and G. V. Frazier, "Using simulated annealing to solve a multiobjective assembly line balancing problem with parallel workstations," *International Journal of Production Research*, vol. 36, no. 10, pp. 2717–2741, 1998.

[33] Y. K. Kim, J. Y. Kim, and Y. Kim, "A coevolutionary algorithm for balancing and sequencing in mixed model assembly lines," *Applied Intelligence*, vol. 13, no. 3, pp. 247–258, 2000.

[34] Y. K. Kim, S. J. Kim, and J. Y. Kim, "Balancing and sequencing mixed-model U-lines with a co-evolutionary algorithm," *Production Planning and Control*, vol. 11, no. 8, pp. 754–764, 2000.

[35] J. Miltenburg, "Balancing and scheduling mixed-model U-shaped production lines," *International Journal of Flexible Manufacturing Systems*, vol. 14, no. 2, pp. 123–155, 2002.

[36] T. Sawik, "Monolithic vs. hierarchical balancing and scheduling of a flexible assembly line," *European Journal of Operational Research*, vol. 143, no. 1, pp. 115–124, 2002.

[37] S. Bock, O. Rosenberg, and T. van Brackel, "Controlling mixed-model assembly lines in real-time by using distributed systems," *European Journal of Operational Research*, vol. 168, no. 3, pp. 880–904, 2006.

[38] L.-K. Hu, F.-Y. Shao, and G.-X. He, "Balancing analysis and algorithm design for mixed model assembly lines," in *Proceedings of the 18th International Conference on Industrial Engineering and Engineering Management (IE & EM '11)*, pp. 67–71, chn, September 2011.

[39] C. Merengo, F. Nava, and A. Pozzetti, "Balancing and sequencing manual mixed-model assembly lines," *International Journal of Production Research*, vol. 37, no. 12, pp. 2835–2860, 1999.

[40] Y. K. Kim, J. Y. Kim, and Y. Kim, "A coevolutionary algorithm for balancing and sequencing in mixed model assembly lines," *Applied Intelligence*, vol. 13, no. 3, pp. 247–258, 2000.

[41] S. Karabatı and S. Sayın, "Assembly line balancing in a mixed-model sequencing environment with synchronous transfers," *European Journal of Operational Research*, vol. 149, no. 2, pp. 417–429, 2003.

[42] B. Yagmahan, "Mixed-model assembly line balancing using a multi-objective ant colony optimization approach," *Expert Systems with Applications*, vol. 38, no. 10, pp. 12453–12461, 2011.

[43] P. Fattahi and M. Salehi, "Sequencing the mixed-model assembly line to minimize the total utility and idle costs with variable launching interval," *International Journal of Advanced Manufacturing Technology*, vol. 45, no. 9-10, pp. 987–998, 2009.

[44] T. Sawik, "Flexible assembly line balancing with alternate assembly plans and duplicate task assignments," in *Proceedings of the 6th International Conference on Emerging Technologies and Factory Automation (ETFA '97)*, pp. 171–176, September 1997.

[45] T. Sawik, "Monolithic vs. hierarchical balancing and scheduling of a flexible assembly line," *European Journal of Operational Research*, vol. 143, no. 1, pp. 115–124, 2002.

[46] T. K. Ralphs, M. J. Saltzman, and M. M. Wiecek, "An improved algorithm for solving biobjective integer programs," *Annals of Operations Research*, vol. 147, pp. 43–70, 2006.

[47] ILOG OPL, http://www-01.ibm.com/software/integration/optimization/cplex-optimization-studio/.

[48] A. M. Jubril, "A nonlinear weights selection in weighted sum for convex multiobjective optimization," *Facta Universitatis*, vol. 27, no. 3, pp. 357–372, 2012.

[49] A. L. Arcus, "COMSOAL a computer method of sequencing operations for assembly lines," *The International Journal of Production Research*, vol. 4, no. 4, pp. 259–277, 1966.

[50] R. V. Johnson, "A branch and bound algorithm for assembly line balancing problems with formulation irregularities," *Management Science*, vol. 29, no. 11, pp. 1309–1324, 1983.

[51] G. H. Milas, "Assembly line balancing: let's remove the mystery," *Industrial Engineering*, vol. 22, no. 5, pp. 31–36, 1990.

[52] P. Vrat and A. Virani, "A cost model for optimal mix of balanced stochastic assembly line and the modular assembly system for a customer oriented production system," *International Journal of Production Research*, vol. 14, no. 4, pp. 445–463, 1976.