

## Research Article

# Passivity Analysis and Passive Control for T-S Fuzzy Systems with Leakage Delay and Mixed Time-Varying Delays

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The passivity and passification for Takagi-Sugeno (T-S) fuzzy systems with leakage delay and both discrete and distributed time-varying delays are investigated. By employing the Lyapunov functional method and using the matrix inequality techniques, several delay-dependent criteria to ensure the passivity and passification of the considered T-S fuzzy systems are established in terms of linear matrix inequalities (LMIs) that can be easily checked by using the standard numerical software. The obtained results generalize some previous results. Two examples are given to show the effectiveness of the proposed criteria.

## 1. Introduction

The Takagi-Sugeno (T-S) fuzzy system, initially proposed and studied by Takagi and Sugeno [1], has attracted increasing interest due to the fact that it provides a general framework to represent a nonlinear plant by using a set of local linear models which are smoothly connected through nonlinear fuzzy membership functions [2]. In practice, time delays often occur in many dynamic systems such as chemical processes, metallurgical processes, biological systems, and neural networks [3]. The existence of time delays is usually a source of instability and poor performance [4]. Therefore, the study of stability with consideration of time delays becomes extremely important [5]. Recently, the stability and stabilization of T-S fuzzy systems with delays have been extensively studied; for example, see [3–13] and references therein.

On the other hand, the passivity theory is another effective tool to the stability analysis of the system [14]. The main idea of passivity theory is that the passive properties of the system can keep the system internal stability [15]. For these reasons, the passivity and passification problems have been an active area of research recently. The passification problem, which is also called the passive control problem, is formulated as the problem of finding a suitable controller

such that the resulting closed-loop system is passive. Recently, some authors have studied the passivity of some systems and obtained sufficient conditions for checking the passivity of the systems that include linear systems with delays [14–16], delayed neural networks [17, 18], networked control systems [19], nonlinear discrete-time systems with direct input-output link [20], and T-S fuzzy systems [21–25]. In [21], the stability of fuzzy control loops is proven with the unique condition that the controlled plant can be made passive by zero shifting. For linear time-invariant plants, this approach leads to frequency response conditions similar to the previous results in the literature, but which are more general and can include robust stability considerations. In [23], the passivity and feedback passification of T-S fuzzy systems with time delays are considered. Both delay-independent and delay-dependent results are presented, and the theoretical results are given in terms of LMIs. In [24], discrete-time T-S fuzzy systems with delays were considered, and several sufficient conditions for checking passivity and passification were obtained. In [25], the contiguous-time T-S fuzzy systems with time-varying delays were investigated, and several criteria to ensure the passivity and feedback passification were given. In [22], the passivity and feedback passification of T-S fuzzy systems with both discrete and distributed time-varying

delays were investigated without assuming the differentiability of the time-varying delays. By employing appropriate Lyapunov-Krasovskii functionals, several delay-dependent criteria for the passivity of the considered T-S fuzzy systems were established in terms of LMIs.

Recently, Gopalsamy initially investigated the bidirectional associative memory (BAM) neural networks with constant delays in the leakage terms and derived several sufficient conditions for the existence of a unique equilibrium as well as its asymptotic and exponential stability [26]. Inspired by this work, authors considered the T-S fuzzy systems with constant leakage delay and investigated their stability problem [7]. As pointed out in [7], T-S fuzzy systems with leakage delay are a class of important T-S fuzzy systems: time delay in the leakage term also has great impact on the dynamics of T-S fuzzy systems since time delay in the stabilizing negative feedback term has a tendency to destabilize a system. To the best of the authors knowledge, there is no results on the problem of passivity for T-S fuzzy systems with leakage delay. Therefore, there is a need to further extend the passivity results reported in [22].

Motivated by the above discussions, the objective of this paper is to study the passivity and feedback passification of T-S fuzzy systems with leakage delay and mixed time-varying delays by employing new Lyapunov-Krasovskii functionals and using matrix inequality techniques. The obtained sufficient conditions do not require the differentiability of time-varying delays and are expressed in terms of linear matrix inequalities, which can be checked numerically using the effective LMI toolbox in MATLAB. Two examples are given to show the effectiveness and less conservatism of the proposed criteria.

*Notations.* The notations are quite standard. Throughout this paper,  $\mathbb{R}^n$  and  $\mathbb{R}^{n \times m}$  denote, respectively, the  $n$ -dimensional Euclidean space and the set of all  $n \times m$  real matrices.  $\|\cdot\|$  refers to the Euclidean vector norm.  $A^T$  represents the transpose of matrix  $A$ , and the asterisk “\*” in a matrix is used to represent the term which is induced by symmetry.  $I$  is the identity matrix with compatible dimension.  $X > Y$  means that  $X$  and  $Y$  are symmetric matrices and that  $X - Y$  is positive definite. Matrices, if not explicitly specified, are assumed to have compatible dimensions.

## 2. Model Description and Preliminaries

Consider a continuous time T-S fuzzy system with discrete and distributed time-varying delays as well as leakage delay, and the  $i$ th rule of the model is of the following form.

*Plant Rule  $i$ .* If  $z_1(t)$  is  $M_{i1}$  and...and  $z_p(t)$  is  $M_{ip}$ , then

$$\begin{aligned} \dot{x}(t) = & A_i x(t - \delta) + B_i x(t - \tau(t)) \\ & + W_i \int_{t-d(t)}^t x(s) ds + U_i w(t), \end{aligned}$$

$$\begin{aligned} y(t) = & C_i x(t - \delta) + D_i x(t - \tau(t)) \\ & + H_i \int_{t-d(t)}^t x(s) ds + V_i w(t), \\ x(s) = & \phi(s), \quad s \in [-\rho, 0], \end{aligned} \quad (1)$$

where  $t \geq 0$ ,  $i = 1, 2, \dots, r$ , and  $r$  is the number of If-then rules;  $z_1(t), z_2(t), \dots, z_p(t)$  are the premise variables; each  $M_{ij}$  ( $j = 1, 2, \dots, p$ ) is a fuzzy set;  $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in \mathbb{R}^n$  is the state vector of the system at time  $t$ ;  $w(t) = (w_1(t), w_2(t), \dots, w_q(t))^T \in \mathbb{R}^q$  is the square integrable exogenous input;  $y(t) = (y_1(t), y_2(t), \dots, y_q(t))^T \in \mathbb{R}^q$  is the output vector of the system;  $\delta$ ,  $\tau(t)$ , and  $d(t)$  denote the leakage delay, the discrete time-varying delay, and the distributed time-varying delay, respectively, and satisfy  $0 \leq \tau(t) \leq \tau$ ,  $0 \leq d(t) \leq d$ , where  $\tau$  and  $d$  are constants;  $\phi(s)$  is bounded and continuously differentiable on  $[-\rho, 0]$ , where  $\rho = \max\{\delta, \tau, d\}$ ;  $A_i, B_i, W_i, U_i, C_i, D_i, H_i$ , and  $V_i$  are some given constant matrices with appropriate dimensions.

Let  $\mu_i(t)$  be the normalized membership function of the inferred fuzzy set  $\gamma_i(t)$ ; that is,

$$\mu_i(t) = \frac{\gamma_i(t)}{\sum_{i=1}^r \gamma_i(t)}, \quad (2)$$

where  $\gamma_i(t) = \prod_{j=1}^p M_{ij}(z_j(t))$  with  $M_{ij}(z_j(t))$  being the grade of membership function of  $z_j(t)$  in  $M_{ij}(t)$ . It is assumed that  $\gamma_i(t) \geq 0$  ( $i = 1, 2, \dots, r$ ) and  $\sum_{i=1}^r \gamma_i(t) > 0$  for all  $t$ . Thus,  $\mu_i(t) \geq 0$  and  $\sum_{i=1}^r \mu_i(t) = 1$  for all  $t$ . And the T-S fuzzy model (1) can be defuzzied as

$$\begin{aligned} \dot{x}(t) = & \sum_{i=1}^r \mu_i(t) \left[ A_i x(t - \delta) + B_i x(t - \tau(t)) \right. \\ & \left. + W_i \int_{t-d(t)}^t x(s) ds + U_i w(t) \right], \\ y(t) = & \sum_{i=1}^r \mu_i(t) \left[ C_i x(t - \delta) + D_i x(t - \tau(t)) \right. \\ & \left. + H_i \int_{t-d(t)}^t x(s) ds + V_i w(t) \right], \\ x(s) = & \phi(s), \quad s \in [-\rho, 0]. \end{aligned} \quad (3)$$

In the literature, there are different definitions of passivity. In this paper, we adopt the following widely accepted definition of passivity, which can be found in [22].

*Definition 1.* System (1) is called passive if there exists a scalar  $\gamma > 0$  such that

$$2 \int_0^{t_p} y^T(s) w(s) ds \geq -\gamma \int_0^{t_p} w^T(s) w(s) ds \quad (4)$$

for all  $t_p \geq 0$  and for the solution of (1) with  $\phi(\cdot) \equiv 0$ .

To prove our results, the following lemma that can be found in [27] is necessary.

**Lemma 2** (see [27]). *For any constant matrix  $W \in \mathbb{R}^{m \times m}$ ,  $W > 0$ , scalar  $0 < h(t) < h$ , vector function  $\omega(\cdot) : [0, h] \rightarrow \mathbb{R}^m$  such that the integrations concerned are well defined; then,*

$$\left( \int_0^{h(t)} \omega(s) ds \right)^T W \left( \int_0^{h(t)} \omega(s) ds \right) \leq h(t) \int_0^{h(t)} \omega^T(s) W \omega(s) ds. \tag{5}$$

### 3. Main Results

**Theorem 3.** *Model (1) is passive in the sense of Definition 1 if there exist a scalar  $\gamma > 0$ , three symmetric positive definite matrices  $P_1, P_2$ , and  $P_3$ , and eleven matrices  $Q_1, Q_2, X_{11}, X_{12}, X_{22}, Y_{ij}$  ( $i, j = 1, 2, 3, i \leq j$ ) such that the following LMIs hold for  $i = 1, 2, \dots, r$ :*

$$X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} > 0, \tag{6}$$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ * & Y_{22} & Y_{23} \\ * & * & Y_{33} \end{bmatrix} > 0, \tag{7}$$

$$\Pi_i = \begin{bmatrix} \Pi_{11} & X_{11} - Q_1 & -X_{12} + Q_1 A_i & \Pi_{14,i} & X_{22} & Q_1 W_i & Q_1 U_i \\ * & \Pi_{22} & Q_2 A_i & Q_2 B_i & X_{12} & Q_2 W_i & Q_2 U_i \\ * & * & -P_1 & 0 & -X_{22} & 0 & -C_i^T \\ * & * & * & \Pi_{44} & 0 & 0 & -D_i^T \\ * & * & * & * & -P_2 & 0 & 0 \\ * & * & * & * & * & -P_3 & -H_i^T \\ * & * & * & * & * & * & \Pi_{77,i} \end{bmatrix} < 0, \tag{8}$$

where  $\Pi_{11} = P_1 + \delta^2 P_2 + d^2 P_3 + X_{12} + X_{12}^T + \tau Y_{11} + Y_{13} + Y_{13}^T$ ,  $\Pi_{14,i} = \tau Y_{12} - Y_{13} + Y_{23}^T + Q_1 B_i$ ,  $\Pi_{22} = \tau Y_{33} - Q_2 - Q_2^T$ ,  $\Pi_{44} = \tau Y_{22} - Y_{23} - Y_{23}^T$ , and  $\Pi_{77,i} = -V_i - V_i^T - \gamma I$ .

*Proof.* From condition (7), we know that  $Y_{33} > 0$ . Consider the following Lyapunov-Krasovskii functional as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{9}$$

where

$$\begin{aligned} V_1(t) &= \int_{t-\delta}^t x^T(s) P_1 x(s) ds + \delta \int_{-\delta}^0 \int_{t+\xi}^t x^T(s) P_2 x(s) ds d\xi \\ &\quad + d \int_{-d}^0 \int_{t+\xi}^t x^T(s) P_3 x(s) ds d\xi, \\ V_2(t) &= \begin{bmatrix} x(t) \\ \int_{t-\delta}^t x(s) ds \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \int_{t-\delta}^t x(s) ds \end{bmatrix}, \\ V_3(t) &= \int_{t-\tau}^t \int_{\xi}^t \dot{x}^T(s) Y_{33} \dot{x}(s) ds d\xi, \\ V_4(t) &= \int_0^t \int_{\xi-\tau(\xi)}^{\xi} u^T(\xi, s) Y u(\xi, s) ds d\xi, \end{aligned} \tag{10}$$

and  $u(\xi, s) = (x^T(\xi), x^T(\xi - \tau(\xi)), \dot{x}^T(s))^T$ .

Calculating the time derivative of  $V(t)$  and using Lemma 2, we obtain that

$$\begin{aligned} \dot{V}_1(t) &= x^T(t) (P_1 + \delta^2 P_2 + d^2 P_3) x(t) \\ &\quad - x^T(t - \delta) P_1 x(t - \delta) \\ &\quad - \delta \int_{t-\delta}^t x^T(s) P_2 x(s) ds - d \int_{t-d}^t x^T(s) P_3 x(s) ds \\ &\leq x^T(t) (P_1 + \delta^2 P_2 + d^2 P_3) x(t) \\ &\quad - x^T(t - \delta) P_1 x(t - \delta) \\ &\quad - \left( \int_{t-\delta}^t x(s) ds \right)^T P_2 \left( \int_{t-\delta}^t x(s) ds \right) \\ &\quad - \left( \int_{t-d(t)}^t x(s) ds \right)^T P_3 \left( \int_{t-d(t)}^t x(s) ds \right), \\ \dot{V}_2(t) &= 2 \begin{bmatrix} x(t) \\ \int_{t-\delta}^t x(s) ds \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ x(t) - x(t - \delta) \end{bmatrix} \\ &= x^T(t) (X_{12} + X_{12}^T) x(t) + 2x^T(t) X_{11} \dot{x}(t) \\ &\quad - 2x^T(t) X_{12} x(t - \delta) + 2x^T(t) X_{22} \int_{t-\delta}^t x(s) ds \\ &\quad + 2\dot{x}^T(t) X_{12} \int_{t-\delta}^t x(s) ds \\ &\quad - 2x^T(t - \delta) X_{22} \int_{t-\delta}^t x(s) ds, \\ \dot{V}_3(t) &= \tau \dot{x}^T(t) Y_{33} \dot{x}(t) - \int_{t-\tau}^t \dot{x}^T(s) Y_{33} \dot{x}(s) ds, \\ \dot{V}_4(t) &= \int_{t-\tau(t)}^t (x^T(t), x^T(t - \tau(t)), \dot{x}^T(s)) Y \\ &\quad \times (x^T(t), x^T(t - \tau(t)), \dot{x}^T(s))^T ds \\ &= \tau(t) \begin{bmatrix} x(t) \\ x(t - \tau(t)) \end{bmatrix}^T \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau(t)) \end{bmatrix} \\ &\quad + 2x^T(t) Y_{13} x(t) - 2x^T(t) Y_{13} x(t - \tau(t)) \\ &\quad + 2x^T(t - \tau(t)) Y_{23} x(t) \\ &\quad - 2x^T(t - \tau(t)) Y_{23} x(t - \tau(t)) \\ &\quad + \int_{t-\tau(t)}^t \dot{x}^T(s) Y_{33} \dot{x}(s) ds \\ &\leq x^T(t) (\tau Y_{11} + Y_{13} + Y_{13}^T) x(t) \\ &\quad + 2x^T(t) (\tau Y_{12} - Y_{13} + Y_{23}^T) x(t - \tau(t)) \end{aligned}$$

$$\begin{aligned}
& + x^T(t - \tau(t)) (\tau Y_{22} - Y_{23} - Y_{23}^T) x(t - \tau(t)) \\
& + \int_{t-\tau}^t \dot{x}^T(s) Y_{33} \dot{x}(s) ds.
\end{aligned} \tag{11}$$

It follows from (11) that

$$\begin{aligned}
\dot{V}(t) \leq & x^T(t) (P_1 + \delta^2 P_2 + d^2 P_3 + X_{12} + X_{12}^T \\
& + \tau Y_{11} + Y_{13} + Y_{13}^T) x(t) \\
& + 2x^T(t) X_{11} \dot{x}(t) - 2x^T(t) X_{12} x(t - \delta) \\
& + 2x^T(t) (\tau Y_{12} - Y_{13} + Y_{23}^T) x(t - \tau(t)) \\
& + 2x^T(t) X_{22} \int_{t-\delta}^t x(s) ds + \tau \dot{x}^T(t) Y_{33} \dot{x}(t) \\
& + 2\dot{x}^T(t) X_{12} \int_{t-\delta}^t x(s) ds \\
& + x^T(t - \tau(t)) (\tau Y_{22} - Y_{23} - Y_{23}^T) x(t - \tau(t)) \\
& - x^T(t - \delta) P_1 x(t - \delta) \\
& - 2x^T(t - \delta) X_{22} \int_{t-\delta}^t x(s) ds \\
& - \left( \int_{t-\delta}^t x(s) ds \right)^T P_2 \left( \int_{t-\delta}^t x(s) ds \right) \\
& - \left( \int_{t-d(t)}^t x(s) ds \right)^T P_3 \left( \int_{t-d(t)}^t x(s) ds \right).
\end{aligned} \tag{12}$$

From the first equation of (3), we have

$$\begin{aligned}
0 = & 2(x^T(t) Q_1 + \dot{x}^T(t) Q_2) \\
& \times \sum_{i=1}^r \mu_i(t) \left[ -\dot{x}(t) + A_i x(t - \delta) + B_i x(t - \tau(t)) \right. \\
& \quad \left. + W_i \int_{t-d(t)}^t x(s) ds + U_i w(t) \right] \\
= & \sum_{i=1}^r \mu_i(t) \left( -2x^T(t) Q_1 \dot{x}(t) + 2x^T(t) Q_1 A_i x(t - \delta) \right. \\
& + 2x^T(t) Q_1 B_i x(t - \tau(t)) \\
& + 2x^T(t) Q_1 W_i \int_{t-d(t)}^t x(s) ds \\
& \left. + 2x^T(t) Q_1 U_i w(t) \right)
\end{aligned}$$

$$\begin{aligned}
& - 2\dot{x}^T(t) Q_2 \dot{x}(t) + 2\dot{x}^T(t) Q_2 A_i x(t - \delta) \\
& + 2\dot{x}^T(t) Q_2 B_i x(t - \tau(t)) \\
& + 2\dot{x}^T(t) Q_2 W_i \int_{t-d(t)}^t x(s) ds \\
& + 2\dot{x}^T(t) Q_2 U_i w(t) \Big).
\end{aligned} \tag{13}$$

It follows from (12) and (13) that

$$\begin{aligned}
\dot{V}(t) - 2w^T(t) y(t) - \gamma w^T(t) w(t) \\
\leq \sum_{i=1}^r \mu_i(t) \left[ x^T(t) (P_1 + \delta^2 P_2 + d^2 P_3 + X_{12} \right. \\
\quad \left. + X_{12}^T + \tau Y_{11} + Y_{13} + Y_{13}^T) x(t) \right. \\
+ 2x^T(t) (X_{11} - Q_1) \dot{x}(t) \\
+ 2x^T(t) (-X_{12} + Q_1 A_i) x(t - \delta) \\
+ 2x^T(t) (\tau Y_{12} - Y_{13} + Y_{23}^T + Q_1 B_i) \\
\quad \times x(t - \tau(t)) \\
+ 2x^T(t) X_{22} \int_{t-\delta}^t x(s) ds \\
+ 2x^T(t) Q_1 W_i \int_{t-d(t)}^t x(s) ds \\
+ 2x^T(t) Q_1 U_i w(t) \\
+ \dot{x}^T(t) (\tau Y_{33} - Q_2 - Q_2^T) \dot{x}(t) \\
+ 2\dot{x}^T(t) Q_2 A_i x(t - \delta) \\
+ 2\dot{x}^T(t) Q_2 B_i x(t - \tau(t)) \\
+ 2\dot{x}^T(t) X_{12} \int_{t-\delta}^t x(s) ds \\
+ 2\dot{x}^T(t) Q_2 W_i \int_{t-d(t)}^t x(s) ds \\
+ 2\dot{x}^T(t) Q_2 U_i w(t) \\
- x^T(t - \delta) P_1 x(t - \delta) \\
- 2x^T(t - \delta) X_{22} \int_{t-\delta}^t x(s) ds \\
- 2x^T(t - \delta) C_i^T w(t) \\
+ x^T(t - \tau(t)) (\tau Y_{22} - Y_{23} - Y_{23}^T) \\
\quad \times x(t - \tau(t)) - 2x^T(t - \tau(t)) D_i^T w(t)
\end{aligned}$$

$$\begin{aligned}
 & - \left( \int_{t-\delta}^t x(s) ds \right)^T P_2 \left( \int_{t-\delta}^t x(s) ds \right) \\
 & - \left( \int_{t-d(t)}^t x(s) ds \right)^T P_3 \left( \int_{t-d(t)}^t x(s) ds \right) \\
 & - 2 \left( \int_{t-d(t)}^t x(s) ds \right)^T H_i^T w(t) \\
 & + w^T(t) \left( -V_i - V_i^T - \gamma I \right) w(t) \Big] \\
 & = \sum_{i=1}^r \mu_i(t) z^T(t) \Pi_i z(t), \tag{14}
 \end{aligned}$$

where  $z(t) = (x^T(t), \dot{x}^T(t), x^T(t-\delta), x^T(t-\tau(t)), \int_{t-\delta}^t x^T(s) ds, \int_{t-d(t)}^t x^T(s) ds, w^T(t))^T$ . Thus, one can derive from (8) and (14) that

$$\dot{V}(t) - 2w^T(t) y(t) - \gamma w^T(t) w(t) \leq 0. \tag{15}$$

By integrating (15) with respect to  $t$  from 0 to  $t_p$ , we obtain

$$\begin{aligned}
 & 2 \int_0^{t_p} w^T(s) y(s) ds \geq V(x(t_p)) \\
 & - V(x(0)) - \gamma \int_0^{t_p} w^T(s) w(s) ds. \tag{16}
 \end{aligned}$$

From the definition of  $V(x(t))$ , we have  $V(x(t_p)) \geq 0$  and  $V(x(0)) = 0$  when  $\phi(\cdot) \equiv 0$ . Thus,

$$2 \int_0^{t_p} w^T(s) y(s) ds \geq -\gamma \int_0^{t_p} w^T(s) w(s) ds \tag{17}$$

holds for all  $t_p \geq 0$ . The proof is completed.  $\square$

Next, we consider the passification problem; that is, a state feedback controller is to be designed to make the closed-loop fuzzy system passive. Extending on system (1), we consider the following T-S fuzzy system with control input.

*Plant Rule i.* If  $z_1(t)$  is  $M_{i1}$  and ... and  $z_p(t)$  is  $M_{ip}$ , then

$$\begin{aligned}
 \dot{x}(t) & = A_i x(t - \delta) + B_i x(t - \tau(t)) \\
 & + W_i \int_{t-d(t)}^t x(s) ds + U_i w(t) + R_i u(t), \\
 y(t) & = C_i x(t - \delta) + D_i x(t - \tau(t)) \\
 & + H_i \int_{t-d(t)}^t x(s) ds + V_i w(t), \\
 x(s) & = \phi(s), \quad s \in [-\rho, 0],
 \end{aligned} \tag{18}$$

where  $u(t) \in R^l$  is the control input,  $R_i$  is a constant matrix with appropriate dimension.

*Controller Rule i.* If  $z_1(t)$  is  $M_{i1}$  and ... and  $z_p(t)$  is  $M_{ip}$ , then

$$u(t) = K_i x(t), \quad i = 1, 2, \dots, r. \tag{19}$$

And the overall state feedback controller is presented by

$$u(t) = \sum_{j=1}^r \mu_j(t) K_j x(t), \tag{20}$$

where  $\mu_j(t)$  is defined as before. The closed-loop fuzzy system can be represented as

$$\begin{aligned}
 \dot{x}(t) & = \sum_{i=1}^r \sum_{j=1}^r \mu_i(t) \mu_j(t) \\
 & \times \left[ R_i K_j x(t) + A_i x(t - \delta) + B_i x(t - \tau(t)) \right. \\
 & \quad \left. + W_i \int_{t-\sigma(t)}^t x(s) ds + U_i w(t) \right], \\
 y(t) & = \sum_{i=1}^r \mu_i(t) \\
 & \times \left[ C_i x(t) + D_i x(t - \tau(t)) \right. \\
 & \quad \left. + H_i \int_{t-\sigma(t)}^t x(s) ds + V_i w(t) \right], \\
 x(s) & = \phi(s), \quad s \in [-\rho, 0].
 \end{aligned} \tag{21}$$

The following theorem establishes the main result of the state feedback passification.

**Theorem 4.** *The closed-loop fuzzy system (21) is passive in the sense of Definition 1 if there exist a scalar  $\gamma > 0$ , four symmetric positive definite matrices  $E_1, E_2, E_3$ , and  $S$ , and matrices  $F_{11}, F_{12}, F_{22}, G_{ij}$  ( $i, j = 1, 2, 3, i \leq j$ ), and  $Z_j$  ( $j = 1, 2, \dots, r$ ) such that the following LMIs hold for  $i, j = 1, 2, \dots, r$ :*

$$X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} > 0, \tag{22}$$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ * & Y_{22} & Y_{23} \\ * & * & Y_{33} \end{bmatrix} > 0, \tag{23}$$

$$\Omega_{ij} = \begin{bmatrix} \Omega_{11,ij} & \Omega_{12,ij} & -F_{12} + A_i S & \Omega_{14,i} & F_{22} & W_i S & U_i \\ * & \Omega_{22} & A_i S & B_i S & F_{12} & W_i S & U_i \\ * & * & -E_1 & * & -F_{22} & 0 & -S C_i^T \\ * & * & * & \Omega_{44} & 0 & 0 & -S D_i^T \\ * & * & * & * & -E_2 & 0 & 0 \\ * & * & * & * & * & -E_3 & -S H_i^T \\ * & * & * & * & * & * & \Omega_{77,i} \end{bmatrix} < 0, \tag{24}$$

where  $\Omega_{11,ij} = E_1 + \delta^2 E_2 + d^2 E_3 + F_{12} + F_{12}^T + \tau G_{11} + G_{13} + G_{13}^T + R_i Z_j + Z_j^T R_i^T$ ,  $\Omega_{12,ij} = F_{11} - S + Z_j^T R_i^T$ ,  $\Omega_{14,i} = \tau G_{12} - G_{13} + G_{23}^T + B_i S$ ,  $\Omega_{22} = \tau G_{33} - 2S$ ,  $\Omega_{44} = \tau G_{22} - G_{23} - G_{23}^T$ , and  $\Omega_{77,i} = -V_i - V_i^T - \gamma I$ .

Moreover, the state feedback gains can be constructed as

$$K_j = Z_j S^{-1}, \quad j = 1, 2, \dots, r. \tag{25}$$

*Proof.* Structure Lyapunov-Krasovskii functional (9), where  $P_1 = S^{-1}E_1S^{-1}$ ,  $P_2 = S^{-1}E_2S^{-1}$ ,  $P_3 = S^{-1}E_3S^{-1}$ ,  $X_{ij} = S^{-1}F_{ij}S^{-1}$  ( $i = 1, 2, i \leq j$ ), and  $Y_{ij} = S^{-1}G_{ij}S^{-1}$  ( $i = 1, 2, 3, i \leq j$ ).

From the first equation of (21), we have

$$\begin{aligned}
0 &= 2 \left( x^T(t) + \dot{x}^T(t) \right) S^{-1} \\
&\quad \times \sum_{i=1}^r \sum_{j=1}^r \mu_i(t) \mu_j(t) \\
&\quad \times \left[ -\dot{x}(t) + R_i K_j x(t) + A_i x(t - \delta) \right. \\
&\quad \quad \left. + B_i x(t - \tau(t)) + W_i \int_{t-\sigma(t)}^t x(s) ds + U_i w(t) \right] \\
&= \sum_{i=1}^r \sum_{j=1}^r \mu_i(t) \mu_j(t) \\
&\quad \times \left( 2x^T(t) S^{-1} R_i K_j x(t) \right. \\
&\quad \quad + x^T(t) \left( -2S^{-1} + 2K_j^T R_i^T S^{-1} \right) \dot{x}(t) \\
&\quad \quad + 2x^T(t) S^{-1} A_i x(t - \delta) \\
&\quad \quad + 2x^T(t) S^{-1} B_i x(t - \tau(t)) \\
&\quad \quad + 2x^T(t) S^{-1} W_i \int_{t-d(t)}^t x(s) ds \\
&\quad \quad + 2x^T(t) S^{-1} U_i w(t) - 2\dot{x}^T(t) S^{-1} \dot{x}(t) \\
&\quad \quad + 2\dot{x}^T(t) S^{-1} A_i x(t - \delta) \\
&\quad \quad + 2\dot{x}^T(t) S^{-1} B_i x(t - \tau(t)) \\
&\quad \quad + 2\dot{x}^T(t) S^{-1} W_i \int_{t-d(t)}^t x(s) ds \\
&\quad \quad \left. + 2\dot{x}^T(t) S^{-1} U_i w(t) \right). \tag{26}
\end{aligned}$$

It follows from (12) and (26) that

$$\begin{aligned}
&\dot{V}(t) - 2w^T(t) y(t) - \gamma w^T(t) w(t) \\
&\leq \sum_{i=1}^r \sum_{j=1}^r \mu_i(t) \mu_j(t) \\
&\quad \times \left[ x^T(t) \left( P_1 + \delta^2 P_2 + d^2 P_3 + X_{12} + X_{12}^T \right. \right. \\
&\quad \quad \left. \left. + \tau Y_{11} + Y_{13} + Y_{13}^T + S^{-1} R_i K_j \right. \right. \\
&\quad \quad \left. \left. + K_j^T R_i^T S^{-1} \right) x(t) \right.
\end{aligned}$$

$$\begin{aligned}
&\quad \left. + 2x^T(t) \left( X_{11} - S^{-1} + K_j^T R_i^T S^{-1} \right) \dot{x}(t) \right. \\
&\quad \left. + 2x^T(t) \left( -X_{12} + S^{-1} A_i \right) x(t - \delta) \right. \\
&\quad \left. + 2x^T(t) \left( \tau Y_{12} - Y_{13} + Y_{23}^T + S^{-1} B_i \right) \right. \\
&\quad \left. \times x(t - \tau(t)) \right. \\
&\quad \left. + 2x^T(t) X_{22} \int_{t-\delta}^t x(s) ds \right. \\
&\quad \left. + 2x^T(t) S^{-1} W_i \int_{t-d(t)}^t x(s) ds \right. \\
&\quad \left. + 2x^T(t) S^{-1} U_i w(t) \right. \\
&\quad \left. + \dot{x}^T(t) \left( \tau Y_{33} - 2S^{-1} \right) \dot{x}(t) \right. \\
&\quad \left. + 2\dot{x}^T(t) S^{-1} A_i x(t - \delta) \right. \\
&\quad \left. + 2\dot{x}^T(t) S^{-1} B_i x(t - \tau(t)) \right. \\
&\quad \left. + 2\dot{x}^T(t) X_{12} \int_{t-\delta}^t x(s) ds \right. \\
&\quad \left. + 2\dot{x}^T(t) S^{-1} W_i \int_{t-d(t)}^t x(s) ds \right. \\
&\quad \left. + 2\dot{x}^T(t) S^{-1} U_i w(t) \right. \\
&\quad \left. - x^T(t - \delta) P_1 x(t - \delta) \right. \\
&\quad \left. - 2x^T(t - \delta) X_{22} \int_{t-\delta}^t x(s) ds \right. \\
&\quad \left. - 2x^T(t - \delta) C_i^T w(t) \right. \\
&\quad \left. + x^T(t - \tau(t)) \left( \tau Y_{22} - Y_{23} - Y_{23}^T \right) \right. \\
&\quad \left. \times x(t - \tau(t)) \right. \\
&\quad \left. - 2x^T(t - \tau(t)) D_i^T w(t) \right. \\
&\quad \left. - \left( \int_{t-\delta}^t x(s) ds \right)^T P_2 \left( \int_{t-\delta}^t x(s) ds \right) \right. \\
&\quad \left. - \left( \int_{t-d(t)}^t x(s) ds \right)^T P_3 \left( \int_{t-d(t)}^t x(s) ds \right) \right. \\
&\quad \left. - 2 \left( \int_{t-d(t)}^t x(s) ds \right)^T H_i^T w(t) \right. \\
&\quad \left. + w^T(t) \left( -V_i - V_i^T - \gamma I \right) w(t) \right] \\
&= \sum_{i=1}^r \sum_{j=1}^r \mu_i(t) \mu_j(t) z^T(t) \Xi_{ij} z(t), \tag{27}
\end{aligned}$$

where  $z(t) = (x^T(t), \dot{x}^T(t), x^T(t-\delta), x^T(t-\tau(t)), \int_{t-\delta}^t x^T(s)ds, \int_{t-d(t)}^t x^T(s)ds, w^T(t))^T$ , and

$$\Xi_{ij} = \begin{bmatrix} \Xi_{11,ij} & \Xi_{12,ij} & -S^{-1}F_{12}S^{-1} + S^{-1}A_i & \Xi_{14,i} & S^{-1}F_{22}S^{-1} & S^{-1}W_i & S^{-1}U_i \\ * & \Xi_{22} & S^{-1}A_i & S^{-1}B_i & S^{-1}F_{12}S^{-1} & S^{-1}W_i & S^{-1}U_i \\ * & * & -S^{-1}E_1S^{-1} & 0 & -S^{-1}F_{22}S^{-1} & 0 & -C_i^T \\ * & * & * & \Xi_{44} & 0 & 0 & -D_i^T \\ * & * & * & * & -S^{-1}E_2S^{-1} & 0 & 0 \\ * & * & * & * & * & -S^{-1}E_3S^{-1} & -H_i^T \\ * & * & * & * & * & * & \Xi_{77,i} \end{bmatrix} \quad (28)$$

with  $\Xi_{11,ij} = S^{-1}(E_1 + \delta^2E_2 + d^2E_3 + F_{12} + F_{12}^T + \tau G_{11} + G_{13} + G_{13}^T)S^{-1} + S^{-1}R_iK_j + K_j^TR_i^TS^{-1}$ ,  $\Xi_{12,ij} = S^{-1}F_{11}S^{-1} - S^{-1} + K_j^TR_i^TS^{-1}$ ,  $\Xi_{14,i} = S^{-1}(\tau G_{12} - G_{13} + G_{23}^T + B_iS)S^{-1}$ ,  $\Xi_{22} = \tau S^{-1}G_{33}S^{-1} - 2S^{-1}$ ,  $\Xi_{44} = S^{-1}(\tau G_{22} - G_{23} - G_{23}^T)S^{-1}$ ,  $\Xi_{77,i} = -V_i - V_i^T - \gamma I$ .

Pre- and postmultiply  $\Xi_{ij}$  by matrix  $\text{diag}(S, S, S, S, S, S, I)$ , we get that

$$\Pi_{ij} = \begin{bmatrix} \Pi_{11,ij} & \Pi_{12,ij} & -F_{12} + A_iS & \Pi_{14,i} & F_{22} & W_iS & U_i \\ * & \Pi_{22} & A_iS & B_iS & F_{12} & W_iS & U_i \\ * & * & -E_1 & 0 & -F_{22} & 0 & -SC_i^T \\ * & * & * & \Pi_{44} & 0 & 0 & -SD_i^T \\ * & * & * & * & -E_2 & 0 & 0 \\ * & * & * & * & * & -E_3 & -SH_i^T \\ * & * & * & * & * & * & \Pi_{77,i} \end{bmatrix} \quad (29)$$

with  $\Pi_{11,ij} = E_1 + \delta^2E_2 + d^2E_3 + F_{12} + F_{12}^T + \tau G_{11} + G_{13} + G_{13}^T + R_iK_jS + SK_j^TR_i^T$ ,  $\Pi_{12,ij} = F_{11} - S + SK_j^TR_i^T$ ,  $\Pi_{14,i} = \tau G_{12} - G_{13} + G_{23}^T + B_iS$ ,  $\Pi_{22} = \tau G_{33} - 2S$ ,  $\Pi_{44} = \tau G_{22} - G_{23} - G_{23}^T$ , and  $\Pi_{77,i} = -V_i - V_i^T - \gamma I$ .

Obviously,  $\Xi_{ij} < 0$  and  $\Pi_{ij} < 0$  are equivalent. And we get from condition (25) that  $\Pi_{ij} = \Omega_{ij}$ .

It follows from condition (24) and inequality (27) that

$$\dot{V}(t) - 2y^T(x(t))w(t) - \gamma w^T(t)w(t) \leq 0, \quad (30)$$

which means

$$2 \int_0^{t_p} y^T(x(s))u(s)ds \geq -\gamma \int_0^{t_p} u^T(s)u(s)ds. \quad (31)$$

From Definition 1, we know that the stochastic T-S fuzzy system (1) is passive in the sense of expectation. The proof is completed.  $\square$

### 4. Numerical Examples

To verify the effectiveness of the theoretical results of this paper, consider the following two examples.

*Example 1.* Consider a T-S fuzzy system (1) with  $r = 2$ , where  $\delta = 0.2$ ,  $\tau(t) = 0.5|\cos t|$ , and  $d(t) = 0.1|\cos(2t)|$ ,

$$A_1 = \begin{bmatrix} 0.8 & -0.2 \\ 0 & 0.6 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2 & -0.9 \\ -0.1 & 0.2 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -1.6 & 0.4 \\ 1.2 & 0.3 \end{bmatrix}, \quad U_1 = \begin{bmatrix} 0.1 & -0.2 \\ 0.1 & -0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & -0.1 \\ 0.2 & 0.1 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.2 & -0.1 \\ -0.2 & -0.1 \end{bmatrix}, \quad V_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0 & -0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.7 & -0.6 \\ 1.1 & -0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.1 & -0.4 \\ -0.3 & -0.2 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 0.2 & 0.9 \\ -0.1 & 0.4 \end{bmatrix}, \quad U_2 = \begin{bmatrix} -0.6 & -0.2 \\ 0.5 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.1 & 0.5 \\ 0.2 & -0.4 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.6 & -0.1 \\ -0.2 & 0.1 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} -0.1 & 0.3 \\ -0.7 & -0.6 \end{bmatrix}, \quad V_2 = \begin{bmatrix} -0.1 & -0.2 \\ 0.8 & -0.3 \end{bmatrix}. \quad (32)$$

It can be verified that  $\tau = 0.5$ ,  $d = 0.1$ . By using the MATLAB LMI Control Toolbox, a solution to the LMIs in (6)–(8) is found as follows:

$$P_1 = 10^{-9} \begin{bmatrix} 0.1318 & 0.0174 \\ 0.0174 & 0.1795 \end{bmatrix},$$

$$P_2 = 10^{-8} \begin{bmatrix} 0.1126 & -0.0105 \\ -0.0105 & 0.1813 \end{bmatrix},$$

$$P_3 = 10^{-8} \begin{bmatrix} 0.3237 & -0.0297 \\ -0.0297 & 0.4519 \end{bmatrix},$$

$$Q_1 = 10^{-10} \begin{bmatrix} 0.0548 & 0.0458 \\ 0.7636 & 0.5797 \end{bmatrix},$$

$$Q_2 = 10^{-9} \begin{bmatrix} 0.1094 & 0.0750 \\ 0.0967 & 0.1859 \end{bmatrix},$$

$$X_{11} = 10^{-10} \begin{bmatrix} 0.3230 & 0.3149 \\ 0.3149 & 0.6926 \end{bmatrix},$$

$$X_{12} = 10^{-9} \begin{bmatrix} -0.0972 & 0.0056 \\ -0.0134 & -0.1477 \end{bmatrix},$$

$$X_{22} = 10^{-9} \begin{bmatrix} 0.1644 & -0.0150 \\ -0.0150 & 0.2929 \end{bmatrix},$$

$$Y_{11} = 10^{-8} \begin{bmatrix} 0.1105 & 0.0483 \\ 0.0483 & 0.1743 \end{bmatrix},$$

$$Y_{12} = 10^{-8} \begin{bmatrix} -0.1175 & -0.0473 \\ -0.0510 & -0.1812 \end{bmatrix},$$

$$Y_{13} = 10^{-9} \begin{bmatrix} -0.4500 & -0.1940 \\ -0.1940 & -0.6933 \end{bmatrix},$$

$$\begin{aligned}
Y_{22} &= 10^{-8} \begin{bmatrix} 0.1157 & 0.0497 \\ 0.0497 & 0.1815 \end{bmatrix}, & Z_1 &= 10^3 \begin{bmatrix} -0.1547 & 0.1479 \\ -2.0308 & 2.9733 \end{bmatrix}, \\
Y_{23} &= 10^{-9} \begin{bmatrix} 0.4500 & 0.1940 \\ 0.1940 & 0.6933 \end{bmatrix}, & Z_2 &= 10^3 \begin{bmatrix} -0.1547 & 0.1479 \\ -2.0308 & 2.9733 \end{bmatrix}, \\
Y_{33} &= 10^{-9} \begin{bmatrix} 0.1618 & 0.0970 \\ 0.0970 & 0.2835 \end{bmatrix}, & \gamma &= 405.7666. \\
\gamma &= 6.9261 \times 10^9. & & 
\end{aligned} \tag{33}$$

According to Theorem 3, the considered model (1) is passive in the sense of Definition 1.

*Example 2.* We use the data of Example 1 in addition to

$$R_1 = \begin{bmatrix} 0.3 & 0.1 \\ 0.7 & -0.2 \end{bmatrix}, \quad R_2 = \begin{bmatrix} -0.3 & 0.1 \\ -0.4 & -0.1 \end{bmatrix}. \tag{34}$$

By using the MATLAB LMI Control Toolbox, a solution to the LMIs in (22)–(24) is found as follows:

$$\begin{aligned}
E_1 &= \begin{bmatrix} 176.3972 & -120.7648 \\ -120.7648 & 318.6006 \end{bmatrix}, \\
E_2 &= \begin{bmatrix} 330.6365 & 13.3938 \\ 13.3938 & 485.5898 \end{bmatrix}, \\
E_3 &= \begin{bmatrix} 534.8304 & -124.7740 \\ -124.7740 & 945.8976 \end{bmatrix}, \\
S &= \begin{bmatrix} 49.8112 & 1.0593 \\ 1.0593 & 115.3239 \end{bmatrix}, \\
F_{11} &= \begin{bmatrix} 236.7653 & -230.2321 \\ -230.2321 & 436.0261 \end{bmatrix}, \\
F_{12} &= \begin{bmatrix} -44.4016 & -52.5502 \\ -14.9764 & -39.8274 \end{bmatrix}, \\
F_{22} &= \begin{bmatrix} 54.2464 & 7.9191 \\ 7.9191 & 96.0467 \end{bmatrix}, \\
G_{11} &= \begin{bmatrix} 226.1583 & -23.8308 \\ -23.8308 & 253.3605 \end{bmatrix}, \\
G_{12} &= \begin{bmatrix} -92.2107 & -144.1195 \\ -27.1469 & -174.7165 \end{bmatrix}, \\
G_{13} &= \begin{bmatrix} -46.2910 & -140.9230 \\ -15.2074 & -83.4028 \end{bmatrix}, \\
G_{22} &= \begin{bmatrix} 115.7710 & -1.7962 \\ -1.7962 & 362.7980 \end{bmatrix}, \\
G_{23} &= \begin{bmatrix} 52.3927 & -9.2251 \\ 5.1656 & 258.7547 \end{bmatrix}, \\
G_{33} &= \begin{bmatrix} 29.6087 & 5.0408 \\ 5.0408 & 240.9688 \end{bmatrix},
\end{aligned}$$

Subsequently, we can obtain from (25) that

$$\begin{aligned}
K_1 &= \begin{bmatrix} -3.1335 & 1.3116 \\ -41.3262 & 26.1620 \end{bmatrix}, \\
K_2 &= \begin{bmatrix} -3.1335 & 1.3116 \\ -41.3262 & 26.1620 \end{bmatrix}.
\end{aligned} \tag{36}$$

Thus, a fuzzy controller (20) with feedback gains  $K_1$  and  $K_2$  can be constructed to make the closed-loop T-S fuzzy system (21) passive.

## 5. Conclusions

In this paper, the passivity and passification for T-S fuzzy systems with both discrete and distributed time-varying delays have been investigated without assuming the differentiability of the time-varying delays. By utilizing the Lyapunov functional method and the matrix inequality techniques, several delay-dependent criteria to ensure the passivity of the considered T-S fuzzy systems have been established in terms of linear matrix inequalities (LMIs) that can be easily checked by using the standard numerical software. Two examples have been provided to demonstrate the effectiveness of the proposed criteria since the feasible solutions to the given LMIs criteria in this paper have been found.

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