Research Article

Delay-Dependent Synchronization for Complex Dynamical Networks with Interval Time-Varying and Switched Coupling Delays

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We investigate the local exponential synchronization for complex dynamical networks with interval time-varying delays in the dynamical nodes and the switched coupling term simultaneously. The constraint on the derivative of the time-varying delay is not required which allows the time delay to be a fast time-varying function. By using common unitary matrix for different subnetworks, the problem of synchronization is transformed into the stability analysis of some linear switched delay systems. Then, when subnetworks are synchronizable and nonsynchronizable, a delay-dependent sufficient condition is derived and formulated in the form of linear matrix inequalities (LMIs) by average dwell time approach and piecewise Lyapunov-Krasovskii functionals which are constructed based on the descriptor model of the system and the method of decomposition. The new stability condition is less conservative and is more general than some existing results. A numerical example is also given to illustrate the effectiveness of the proposed method.

1. Introduction

Complex dynamical network, as an interesting subject, has been thoroughly investigated for decades. These networks show very complicated behavior and can be used to model and explain many complex systems in nature such as computer networks [1], the World Wide Web [2], food webs [3], cellular and metabolic networks [4], social networks [5], and electrical power grids [6]. In general, a complex network is a large set of interconnected nodes, in which a node is a fundamental unit with specific contents. As an implicit assumption, these networks are described by the mathematical term *graph*. In such graphs, each vertex represents an individual element in the system, while edges represent the relationship between them. Two nodes are joined by an edge if and only if they interact.

In the last decade, the synchronization of complex dynamical networks has attracted much attention by several researchers in this field [7–12]. Synchronization of complex dynamical networks is one of the most important dynamical mechanisms for creating order in complex dynamical networks. Wang and Chen introduced a uniform dynamical network model and also investigated its synchronization [10, 11, 13]. They have shown that the synchronizability of a scale-free dynamical network is robust against random removal of nodes and yet is fragile to specific removal of the most highly connected nodes [11]. Li and Chen [7] considered the synchronization of complex dynamical network models with coupling delays (continuous and discrete-time) and derived delay-independent and delay-dependent synchronization conditions. By utilizing Lyapunov functional method, Yue and Li [12] studied the synchronization of continuous

and discrete complex dynamical networks with interval time-varying delays in the dynamical nodes and the coupling term simultaneously in which delay-dependent synchronization conditions are derived in the form of linear matrix inequalities (LMIs). In [8], Li et al. studied the synchronization problem of general complex dynamical networks with time-varying delays in the network couplings and in the dynamical nodes, respectively. However, the time-varying delays are required to be differentiable.

It is well known that the existence of time delay in a system may cause instability and oscillations. Examples of time-delay systems are chemical engineering systems, biological modeling, electrical networks, physical networks, and many others, [14–20]. The stability criteria for systems with time delays can be classified into two categories: delay-independent and delay-dependent. Delay-independent criteria do not employ any information on the size of the delay, while delay-dependent criteria make use of such information at different levels. Delay-dependent stability conditions are generally less conservative than the delay-independent ones especially when the delay is small [20]. Recently, delay-dependent stability for interval time-varying delay was investigated in [12, 15–17, 19]. Interval time-varying delay is a time delay that varies in an interval in which the lower bound is not restricted to be 0. Jiang and Han [17] considered the problem of robust $H\infty$ control for uncertain linear systems with interval time-varying delay based on the Lyapunov functional approach in which restriction on the differentiability of the interval time-varying delay was removed. Shao [19] presented a new delay-dependent stability criterion for linear systems with interval time-varying delay without introducing any free-weighting matrices. In order to reduce further the conservatism introduced by descriptor model transformation and bounding techniques, a free-weighting matrices method is proposed in [15, 21–24]. An advantage of this transformation is to transform the original system to an equivalent descriptor representation without introducing additional dynamics in the sense defined in [21]. In [22], based on the descriptor model transformation and decomposition technique of coefficient matrix, some new robust stability criteria are derived. In [12], the synchronization problem has been investigated for continuous/discrete complex dynamical networks with interval time-varying delays. Based on piecewise analysis method and the Lyapunov functional method, some new delay-dependent synchronization criteria are derived in the form of LMIs by introducing free-weighting matrices. It will be pointed out later that some existing results require more free-weighting matrix variables than our result.

Switched systems are special class of hybrid systems which consist of a family of continuous-time or discrete-time subsystems and a switching law that orchestrates the switching between them. Such systems have drawn considerable attention in control and computer communities in the last decade [25–31]. In [32], some stability properties of switched linear systems consisting of both stable and unstable subsystems have been derived by using an average dwell time approach and piecewise Lyapunov functions. It was shown that when the average dwell time is sufficiently large

and the total activation time of the unstable subsystems is relatively small compared with that of stable subsystems, then global exponential stability is guaranteed. Liu and Zhao [9] investigated synchronization of complex delayed networks with switching topology via switched system stability theory. In [9], when all subnetworks are synchronizable, a delay-dependent synchronization condition is given in terms of LMIs which guarantees the solvability of the synchronization problem under an average dwell time scheme. Liu et al. [33] studied the local and global exponential synchronization of a complex dynamical network with switching topology and time-varying coupling delays. However, the time-varying delays in the dynamical nodes were not considered.

In this paper, we will investigate the local exponential synchronization of complex dynamical networks with interval time-varying delays in both dynamical nodes and in switched coupling terms simultaneously. The restriction on the differentiability of interval time-varying delays is removed, which means that a fast time-varying delay is allowed. We use a common unitary matrix for different subnetworks to transform the synchronization problem of the network into the stability analysis of $N - 1$ pieces of linear switched systems. Then by using the average dwell time approach, the descriptor model transformation, and the decomposition technique of coefficient matrix, a new class of piecewise Lyapunov-Krasovskii functionals are constructed in order to get improved delay-dependent synchronization criteria which are derived in the form of LMIs. Finally, numerical examples are given to demonstrate that the derived conditions are less conservative than some existing results given in the literature.

The rest of the paper is organized as follows. In Section 2, a complex dynamical network model with interval time-varying and switched coupling delays as well as some useful lemmas is given. In Section 3, common unitary matrix and descriptor system transformation strategy are introduced to reformulate the network model. Then delay-dependent synchronization criteria are presented based on this novel model. Numerical examples that illustrated the obtained results are given in Section 4. The paper ends with conclusions in Section 5 and cited references.

2. Network Model and Mathematic Preliminaries

The following notation will be used in this paper: R^n denotes the n-dimensional Euclidean space; $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalue of a real symmetric matrix, respectively; $\| \cdot \|$ denotes the Euclidean vector norm of x; A^T denotes the transpose of the vector/matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\| \cdot \|_{cl} = \sup_{-h_M \leq \theta \leq 0} \{\|x(t + \theta)\|, \|\dot{x}(t + \theta)\|\}$. The notation diag{…} stands for a block-diagonal matrix.

Consider a complex dynamical switched network consisting of N identical coupled nodes, with each node being an n-dimensional dynamical system as

$$\dot{x}_i(t) = f(x_i(t), x_i(t - h(t)))$$

$$+ c_{\sigma(t)} \sum_{j=1}^{N} g_{ij}^{\sigma(t)} A_{\sigma(t)} x_j(t - h(t)), \qquad (1)$$

$$x(t) = \phi(t), \quad t \in [t_0 - h_M, t_0], \ h_M \geq 0,$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ is the state vector of ith node, for all $i \in \{1, 2, \ldots, N\}$ are the state of node ith; $f(\cdot) \in R^n$ is the continuously differentiable function; $\sigma(t): [0, +\infty) \rightarrow M = \{1, 2, \ldots, m\}$ is piecewise constant and left continuous and is called a switching signal; the constant $c_{\sigma(t)} > 0$ is the coupling strength; $A_{\sigma(t)} = (a_{ij})_{n \times n}^{\sigma(t)} \in R^{n \times n}$ is a constant inner-coupling matrix, if some pairs (i, j), $1 \leq i, j \leq n$, with $a_{ij}^{\sigma(t)} \neq 0$ which means two coupled nodes are linked through their ith and jth state variables, otherwise $a_{ij}^{\sigma(t)} = 0$; $G_{\sigma(t)} = (g_{ij}^{\sigma(t)})_{N \times N} \in R^{N \times N}$ is the outer-coupling matrix of the network, in which $g_{ij}^{\sigma(t)}$ is defined as follows: if there is a connection between node i and node j $(j \neq i)$, then $g_{ij}^{\sigma(t)} = g_{ji}^{\sigma(t)} = 1$; otherwise, $g_{ij}^{\sigma(t)} = g_{ji}^{\sigma(t)} = 0$ $(j \neq i)$, and the diagonal elements of matrix G are defined by

$$g_{ii}^{\sigma(t)} = -\sum_{j=1, i \neq j}^{N} g_{ij}^{\sigma(t)} = -\sum_{j=1, i \neq j}^{N} g_{ji}^{\sigma(t)}, \quad i = 1, 2, \ldots, N. \quad (2)$$

It is assumed that switched network (1) is connected in the sense that there are no isolated clusters; that is namely, $G_{\sigma(t)}$ is an irreducible matrix. The time delay $h(t)$ is a time-varying continuous function satisfying

$$0 \leq h_m \leq h(t) \leq h_M. \qquad (3)$$

The initial condition function $\phi(t)$ is a continuous vector-valued function for $t \in [t_0 - h_M, t_0]$.

Corresponding to the switching signal $\sigma(t)$, we have the switching sequence $\Sigma = \{x_{t_0}; (i_0, t_0), (i_1, t_1), \ldots, (i_k, t_k), \ldots, | \ i_k \in M\}$, and when $t \in [t_k, t_{k+1})$, the i_k-th subnetwork is activated. Assume that there are no jumps in the state at the switching instants, and every network in (1) is connected; that is, $k \in M$, G_k is irreducible.

Remark 1. For fixed $\sigma(t)$, the system model (1) turns into the delayed complex dynamical network proposed by Yue and Li [12] as

$$\dot{x}_i(t) = f(x_i(t), x_i(t - h(t))) + c \sum_{j=1}^{N} g_{ij} A x_j(t - h(t)),$$

$$i = 1, 2, \ldots, N. \qquad (4)$$

Therefore, (1) is a general complex networks model, with (4) as a special case.

Definition 2 (see [26]). For any $T_2 > T_1 \geq 0$, let $N_{\sigma(t)}(T_1, T_2)$ denote the number of switching of $\sigma(t)$ over (T_1, T_2). If $N_{\sigma(t)}(T_1, T_2) \leq N_0 + ((T_2 - T_1)/T_a)$ holds for $T_a > 0$, $N_0 \geq 0$, then T_a is called average dwell time and N_0 is called a chatter bound.

Definition 3 (see [12]). The delayed dynamical switched network (1) is said to achieve asymptotic synchronization if

$$x_1(t) = x_2(t) = \cdots = s(t) \quad \text{as } t \longrightarrow \infty, \qquad (5)$$

where $s(t)$ is a solution of an isolated node satisfying

$$\dot{s}(t) = f(s(t), s(t - h(t))). \qquad (6)$$

Definition 4 (see [34]). If the matrix $A \in M_{n \times n}$ is similar to a diagonal matrix, then A is said to be diagonalizable.

In this paper, we assume that $s(t)$ is an orbitally stable solution of the above system. Clearly, the stability of the synchronized states (5) of network (1) is determined by the dynamics of the isolated node, the coupling strength $c_{\sigma(t)}$, the inner-coupling matrix $A_{\sigma(t)}$, the outer-coupling matrix $G_{\sigma(t)}$, and the fast time-varying delays $h(t)$.

The following lemmas will be used in the proof of the main result.

Lemma 5 (see [34]). *Let G be a family of diagonalizable matrices. Then G is a commuting family (under multiplication) if and only if it is a simultaneously diagonalizable family.*

Lemma 6 (see [14]). *For any constant symmetric matrix $M \in R^{n \times n}$, $M = M^T > 0$, $0 \leq h_m \leq h(t) \leq h_M$, $t \geq 0$, and any differentiable vector function $x(t) \in R^n$, we have*

(a) $\left[\int_{t-h_m}^{t} \dot{x}(s) \, ds \right]^T M \left[\int_{t-h_m}^{t} \dot{x}(s) \, ds \right]$

$$\leq h_m \int_{t-h_m}^{t} \dot{x}^T(s) M \dot{x}(s) \, ds,$$

(b) $\left[\int_{t-h(t)}^{t-h_m} \dot{x}(s) \, ds \right]^T M \left[\int_{t-h(t)}^{t-h_m} \dot{x}(s) \, ds \right] \qquad (7)$

$$\leq (h(t) - h_m) \int_{t-h(t)}^{t-h_m} \dot{x}^T(s) M \dot{x}(s) \, ds$$

$$\leq (h_M - h_m) \int_{t-h(t)}^{t-h_m} \dot{x}^T(s) M \dot{x}(s) \, ds.$$

3. Synchronization of Switched Network

In this section, we will obtain some delay-dependent synchronization criteria for general complex dynamical network with interval time-varying delay and switched coupling delays (1) by strict LMI approaches. Let $\widetilde{A}_i = J(t)$, $\widetilde{B}_{i\sigma(t)} = J_h(t - h(t)) + c_{\sigma(t)} \lambda_{i\sigma(t)} A_{\sigma(t)}$, $J(t) = f'(s(t))$, and $s(t - h(t))) \in R^{n \times n}$ is the Jacobian of $f(x(t), x(t - h(t)))$ at $s(t)$ with the derivative of $f(x(t), x(t - h(t)))$ with respect to $x(t)$.

$J_h(t - h(t)) = f'(s(t), s(t - h(t))) \in R^{n \times n}$ is the Jacobian of $f(x(t), x(t - h(t)))$ at $s(t - h(t))$ with the derivative of $f(x(t), x(t - h(t)))$ with respect to $x(t - h(t))$.

Lemma 7. *Consider the switched coupling delays dynamical network described by* (1). *Let* $0 = \lambda_{1\sigma(t)} > \lambda_{2\sigma(t)} \geq \lambda_{3\sigma(t)} \geq \cdots \geq \lambda_{N\sigma(t)}$ *be the eigenvalues of the outer-coupling matrix* $G_{\sigma(t)}$. *If the n-dimensional linear switched system*

$$\dot{z}_i(t) = \widetilde{A}_i z_i(t) + \widetilde{B}_{i\sigma(t)} z_i(t - h(t)), \quad i = 2, 3, \ldots, N \quad (8)$$

is asymptotically (or exponentially) stable, then the synchronized state (5) *is asymptotically (or exponentially) stable.*

Proof. To investigate the stability of the synchronized states (5), set

$$e_i(t) = x_i(t) - s(t), \quad i = 1, 2, \ldots, N. \quad (9)$$

Substituting (9) into (1), for $1 \leq i \leq N$, we have

$$\begin{aligned} \dot{e}_i(t) &= \dot{x}_i(t) - \dot{s}(t) \\ &= f(x_i(t), x_i(t - h(t))) - f(s(t), s(t - h(t))) \\ &\quad + c_{\sigma(t)} \sum_{j=1}^{N} g_{ij}^{\sigma(t)} A_{\sigma(t)} e_j(t - h(t)). \end{aligned} \quad (10)$$

Since $f(\cdot)$ is continuous differentiable, it is easy to show that the origin of the nonlinear system (1) is an asymptotically stable equilibrium point if it is an asymptotically stable equilibrium point of the following linear switched time-varying delays systems:

$$\begin{aligned} \dot{e}_i(t) &= J(t) e_i(t) + J_h(t) e_i(t - h(t)) \\ &\quad + c_{\sigma(t)} \sum_{j=1}^{N} g_{ij}^{\sigma(t)} A_{\sigma(t)} e_j(t - h(t)) \\ &= J(t) e_i(t) + J_h(t) e_i(t - h(t)) \\ &\quad + c_{\sigma(t)} A_{\sigma(t)} \left(e_1(t - h(t)), \ldots, e_N(t - h(t)) \right) \\ &\quad \times \left(g_{i1}^{\sigma(t)}, \ldots, g_{iN}^{\sigma(t)} \right)^T, \end{aligned} \quad (11)$$

where $J(t) = f'(s(t), s(t - h(t))) \in R^{n \times n}$ is the Jacobian of $f(x(t), x(t - h(t)))$ at $s(t)$ and $J_h(t - h(t)) = f'(s(t), s(t - h(t))) \in R^{n \times n}$ is the Jacobian of $f(x(t), x(t-h(t)))$ at $s(t-h(t))$.

Letting $e(t) = (e_1(t), \ldots, e_N(t)) \in R^{n \times N}$ and $e(t - h(t)) = (e_1(t - h(t)), \ldots, e_N(t - h(t))) \in R^{n \times N}$, we have

$$\begin{aligned} \dot{e}(t) &= J(t) e(t) + J_h(t - h(t)) e(t - h(t)) \\ &\quad + c_{\sigma(t)} A_{\sigma(t)} e(t - h(t)) G_{\sigma(t)}^T. \end{aligned} \quad (12)$$

Obviously, G_k is diagonalizable for fixed $\sigma(t) = k \in M$. If G_k, G_l, for all k, and $l \in M$ commute pairwise;, that is, $G_k G_l = G_l G_k$, then based on Lemma 5, one can get a common unitary matrix $U \in R^{N \times N}$ with $u_i \in R^N$ such that

$$U^T G_k U = \Gamma_k, \quad (13)$$

where $U^T U = I$, $\Gamma_k = \text{diag}\{\lambda_{1k}, \ldots, \lambda_{Nk}\}$, $\lambda_{ik}, i = 1, 2, \ldots, N$ are eigenvalues of G_k. In addition, with (2) and the irreducible feature of G_k, we may choose $u_1 = (1/\sqrt{N})(1, 1, \ldots, 1)^T$ such that $\lambda_{1k} = 0$, for all $k \in M$.

From (12), by using the nonsingular transform $e(t)U = z(t) = (z_1(t), \ldots, z_N(t)) \in R^{N \times N}$, we have the following matrix equation:

$$\begin{aligned} \dot{z}(t) &= J(t) z(t) + J_h(t - h(t)) z(t - h(t)) \\ &\quad + c_{\sigma(t)} A_{\sigma(t)} z(t - h(t)) \Gamma_{\sigma(t)}, \end{aligned} \quad (14)$$

namely,

$$\dot{z}_i(t) = \widetilde{A}_i z_i(t) + \widetilde{B}_{i\sigma(t)} z_i(t - h(t)), \quad i = 1, 2, \ldots, N, \quad (15)$$

where $\widetilde{A}_i = J(t)$ and $\widetilde{B}_{i\sigma(t)} = J_h(t - h(t)) + c_{\sigma(t)} \lambda_{i\sigma(t)} A_{\sigma(t)}$. Thus, we have transformed the stability problem of the synchronized state (5) to the stability problem of the N pieces of n-dimensional linear switched time-varying delays differential equations (15). Note that $\lambda_{1k} = 0$ corresponding to the synchronization of the system states (5), where the state $s(t)$ is an orbitally stable solution of the isolated node as assumed above in (5). If the following $N - 1$ pieces of n-dimensional linear switched time-varying delays systems

$$\dot{z}_i(t) = \widetilde{A}_i z_i(t) + \widetilde{B}_{i\sigma(t)} z_i(t - h(t)), \quad i = 2, 3, \ldots, N, \quad (16)$$

are asymptotically (or exponentially) stable, then $e(t)$ will tend to the origin asymptotically (or exponentially), which is equivalent to the fact that the synchronized states (5) are asymptotically (or exponentially) stable. This completes the proof. □

Next, we consider the nonswitched system

$$\dot{z}_i(t) = \widetilde{A}_i z_i(t) + \widetilde{B}_i z_i(t - h(t)), \quad i = 2, 3, \ldots, N. \quad (17)$$

We rewrite system (17) in the following equivalent descriptor system form:

$$\dot{z}_i(t) = y_i(t) \quad (18)$$

$$y_i(t) = \widetilde{A}_i z_i(t) + \widetilde{B}_i z_i(t - h(t)), \quad i = 2, 3, \ldots, N. \quad (19)$$

To derive delay-dependent stability conditions, which include the information of the time-varying delay $h(t)$, one usually uses the fact

$$\begin{aligned} z(t - h(t)) &= z(t) - \int_{t-h(t)}^{t} \dot{z}(s) \, ds \\ &= z(t) - \int_{t-h(t)}^{t} y(s) \, ds, \end{aligned} \quad (20)$$

to transform the original system to a system with distributed delays. In order to improve the bound of time-varying delay $h(t)$, let us decompose the matrix $\widetilde{B}_i = \widetilde{B}_{i1} + \widetilde{B}_{i2}$, where $\widetilde{B}_{i1}$ are $\widetilde{B}_{i2}$ are appropriate constant matrices to be determined later. Then the original system (17) can be represented in the

form of a descriptor system with time-varying and distributed delays as

$$\dot{z}_i(t) = y_i(t)$$

$$0 = -y_i(t) + \left(\widetilde{A}_i + \widetilde{B}_{i1}\right) z_i(t) + \widetilde{B}_{i2} z_i(t - h(t))$$

$$- \widetilde{B}_{i1} \int_{t-h(t)}^{t} y_i(s)\, ds, \quad i = 2, 3, \ldots, N. \tag{21}$$

Let $\delta = h_M - h_m$. Let $V_i(t)$ be defined by

$$V_i(t) = V_{i1}(t) + V_{i2}(t) + V_{i3}(t) + V_{i4}(t) + V_{i5}(t) + V_{i6}(t), \tag{22}$$

where

$$V_{i1}(t) = \xi_i^T(t) EP_i \xi_i(t) = z_i^T(t) P_{i1} z_i(t),$$

$$V_{i2}(t) = \int_{t-h_m}^{t} z_i^T(s) Q_i e^{\alpha(s-t)} z_i(s)\, ds,$$

$$V_{i3}(t) = \int_{t-h_M}^{t} z_i^T(s) R_i e^{\alpha(s-t)} z_i(s)\, ds,$$

$$V_{i4}(t) = h_m \int_{-h_m}^{0} \int_{t+\theta}^{t} \dot{z}_i^T(s) U_i e^{\alpha(s-t)} \dot{z}_i(s)\, ds\, d\theta,$$

$$V_{i5}(t) = h_M \int_{-h_M}^{0} \int_{t+\theta}^{t} y_i^T(s) S_i e^{\alpha(s-t)} y_i(s)\, ds\, d\theta, \tag{23}$$

$$V_{i6}(t) = \delta \int_{-h_M}^{-h_m} \int_{t+\theta}^{t} \dot{z}_i^T(s) W_i e^{\alpha(s-t)} \dot{z}_i(s)\, ds\, d\theta,$$

$$\xi_i(t) = \left[z_i^T(t)\ \ y_i^T(t)\ \ \int_{t-h(t)}^{t} y_i^T(s)\, ds \right]^T,$$

$$E = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T, \qquad P_i = \begin{bmatrix} P_{i1} & 0 & 0 \\ P_{i2} & P_{i3} & P_{i4} \end{bmatrix}.$$

For given $\alpha > 0$, the following lemma provides an estimation of $V_i(t)$.

Lemma 8. *Assume that the time-varying delay $h(t)$ satisfies $0 \le h_m \le h(t) \le h_M$ and let $\alpha > 0$. If there exist symmetric positive definite matrices $P_{i1} > 0$, $Q_i > 0$, $R_i > 0$, $S_i > 0$, $U_i > 0$, and $W_i > 0$ and appropriately dimensioned matrices P_{i2}, P_{i3}, and P_{i4} for $i = 2, 3, \ldots, N$, such that the following LMIs hold:*

$$\Sigma_{i1} = \Sigma_i - \begin{bmatrix} 0 & 0 & I & -I & 0 & 0 \end{bmatrix}^T$$
$$\times e^{-\alpha\delta} W_i \begin{bmatrix} 0 & 0 & I & -I & 0 & 0 \end{bmatrix} < 0, \tag{24}$$

$$\Sigma_{i2} = \Sigma_i - \begin{bmatrix} 0 & 0 & 0 & I & -I & 0 \end{bmatrix}^T$$
$$\times e^{-\alpha\delta} W_i \begin{bmatrix} 0 & 0 & 0 & I & -I & 0 \end{bmatrix} < 0, \tag{25}$$

$i = 2, \ldots, N$, *where*

$$\Sigma_i = \begin{bmatrix} \Sigma_{i11} & \Sigma_{i12} & \Sigma_{i13} & \Sigma_{i14} & 0 & \Sigma_{i16} \\ * & \Sigma_{i22} & 0 & \Sigma_{i24} & 0 & \Sigma_{i26} \\ * & * & \Sigma_{i33} & \Sigma_{i34} & 0 & 0 \\ * & * & * & \Sigma_{i44} & \Sigma_{i45} & \Sigma_{i46} \\ * & * & * & * & \Sigma_{i55} & 0 \\ * & * & * & * & * & \Sigma_{i66} \end{bmatrix},$$

$$\Sigma_{i11} = P_{i2}^T \left(\widetilde{A}_i + \widetilde{B}_{i1}\right) + \left(\widetilde{A}_i + \widetilde{B}_{i1}\right)^T P_{i2} + Q_i$$
$$+ R_i - e^{-\alpha h_m} U_i + \alpha P_{i1},$$

$$\Sigma_{i12} = P_{i1}^T - P_{i2}^T + \left(\widetilde{A}_i + \widetilde{B}_{i1}\right)^T P_{i3},$$

$$\Sigma_{i13} = e^{-\alpha h_M} U_i,$$

$$\Sigma_{i14} = P_{i2}^T \widetilde{B}_{i2},$$

$$\Sigma_{i16} = -P_{i2}^T \widetilde{B}_{i1} - \left(\widetilde{A}_i + \widetilde{B}_{i1}\right)^T P_{i4}, \tag{26}$$

$$\Sigma_{i22} = h_m^2 U_i + h_M^2 S_i + \delta^2 W_i - P_{i3}^T - P_{i3},$$

$$\Sigma_{i24} = P_{i3}^T \widetilde{B}_{i2},$$

$$\Sigma_{i26} = -P_{i3}^T \widetilde{B}_{i1} - P_{i4},$$

$$\Sigma_{i33} = -e^{-\alpha h_m} Q_i - e^{-\alpha h_m} U_i - W_i,$$

$$\Sigma_{i34} = e^{-\alpha\delta} W_i,$$

$$\Sigma_{i44} = -2e^{-\alpha\delta} W_i,$$

$$\Sigma_{i45} = e^{-\alpha\delta} W_i,$$

$$\Sigma_{i46} = \widetilde{B}_{i2} P_{i4}^T,$$

$$\Sigma_{i55} = -e^{-\alpha h_M} R_i - e^{-\alpha\delta} W_i,$$

$$\Sigma_{i66} = -e^{-\alpha h_M} S_i - P_{i4}^T \widetilde{B}_{i1} - \widetilde{B}_{i1}^T P_{i4}.$$

Then, along the trajectory of system (17), *we have*

$$V_i(t) \le e^{-\alpha(t-t_0)} V_i(t_0), \quad t \ge t_0 \ge 0. \tag{27}$$

Proof. By taking the derivative of Lyapunov-Krasovskii functional candidate (22) along the trajectory of the system (21), we obtain

$$\dot{V}_i(t) = \dot{V}_{i1}(t) + \dot{V}_{i2}(t) + \dot{V}_{i3}(t) + \dot{V}_{i4}(t) + \dot{V}_{i5}(t) + \dot{V}_{i6}(t), \tag{28}$$

where

On the other hand, we have

$$\dot{V}_{i1}(t) = 2z_i^T(t) P_{i1} \dot{z}_i(t)$$

$$= 2\xi_i^T(t) P_i^T \begin{bmatrix} \dot{z}_i(t) \\ 0 \end{bmatrix},$$

$$\dot{V}_{i2}(t) = -\alpha V_{i2}(t) + z_i^T(t) Q_i z_i(t)$$

$$- z_i^T(t - h_m) Q_i e^{-\alpha h_m} z_i(t - h_m),$$

$$\dot{V}_{i3}(t) = -\alpha V_{i3}(t) + z_i^T(t) R_i z_i(t)$$

$$- z_i^T(t - h_M) R_i e^{-\alpha h_M} z_i(t - h_M),$$

$$\dot{V}_{i4}(t) \le -\alpha V_{i4}(t) + y_i^T(t) h_m^2 U_i y_i(t) \qquad (29)$$

$$- h_m e^{-\alpha h_m} \int_{t-h_m}^t \dot{z}_i^T(s) U_i \dot{z}_i(s)\, ds,$$

$$\dot{V}_{i5}(t) \le -\alpha V_{i5}(t) + y_i^T(t) h_M^2 S_i y_i(t)$$

$$- h_M e^{-\alpha h_M} \int_{t-h_M}^t y_i^T(s) S_i y_i(s)\, ds,$$

$$\dot{V}_{i6}(t) \le -\alpha V_{i6}(t) + y_i^T(t) \delta^2 W_i y_i(t)$$

$$- \delta e^{-\alpha \delta} \int_{t-h_M}^{t-h_m} \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds.$$

Using Lemma 6, we obtain

$$- h_m \int_{t-h_m}^t \dot{z}_i^T(s) U_i \dot{z}_i(s)\, ds$$

$$\le - \left[\int_{t-h_m}^t \dot{z}_i^T(s)\, ds \right]^T U_i \left[\int_{t-h_m}^t \dot{z}_i^T(s)\, ds \right]$$

$$= -[z_i(t) - z_i(t - h_m)]^T U_i [z_i(t) - z_i(t - h_m)] \qquad (30)$$

$$= -z_i^T(t) U_i z_i(t) + 2z_i^T(t) U_i z_i(t - h_m)$$

$$- z_i^T(t - h_m) U_i z_i(t - h_m),$$

$$- h_M \int_{t-h_M}^t y_i^T(s) S_i y_i(s)\, ds$$

$$\le -h(t) \int_{t-h(t)}^t y_i^T(s) S_i y_i(s)\, ds \qquad (31)$$

$$\le - \left[\int_{t-h(t)}^t y_i^T(s)\, ds \right]^T S_i \left[\int_{t-h(t)}^t y_i^T(s)\, ds \right].$$

$$- \delta \int_{t-h_M}^{t-h_m} \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds$$

$$= -\delta \int_{t-h(t)}^{t-h_m} \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds$$

$$- \delta \int_{t-h_M}^{t-h(t)} \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds$$

$$= - (h_M - h(t)) \int_{t-h(t)}^{t-h_m} \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds \qquad (32)$$

$$- (h(t) - h_m) \int_{t-h(t)}^{t-h_m} \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds$$

$$- (h_M - h(t)) \int_{t-h_M}^{t-h(t)} \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds$$

$$- (h(t) - h_m) \int_{t-h_M}^{t-h(t)} \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds.$$

Let $\gamma = (h(t) - h_m)/\delta$. Then, we obtain

$$- (h(t) - h_m) \int_{t-h_M}^{t-h(t)} \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds$$

$$= -\gamma \int_{t-h_M}^{t-h(t)} \delta \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds$$

$$\le -\gamma \int_{t-h_M}^{t-h(t)} (h_M - h(t)) \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds,$$

$$\qquad (33)$$

$$- (h_M - h(t)) \int_{t-h(t)}^{t-h_m} \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds$$

$$= - (1 - \gamma) \int_{t-h(t)}^{t-h_m} \delta \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds$$

$$\le - (1 - \gamma) \int_{t-h(t)}^{t-h_m} (h(t) - h_m) \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds.$$

From Lemma 6, we have

$$- \delta \int_{t-h_M}^{t-h_m} \dot{z}_i^T(s) W_i \dot{z}_i(s)\, ds$$

$$\le -[z_i(t - h_m) - z_i(t - h(t))]^T$$

$$\times W_i [z_i(t - h_m) - z_i(t - h(t))]$$

$$- [z_i(t - h(t)) - z_i(t - h_M)]^T$$

$$\times W_i \left[z_i \left(t - h \left(t \right) \right) - z_i \left(t - h_M \right) \right]$$

$$- \gamma \left[z_i \left(t - h \left(t \right) \right) - z_i \left(t - h_M \right) \right]^T$$

$$\times W_i \left[z_i \left(t - h \left(t \right) \right) - z_i \left(t - h_M \right) \right]$$

$$- \left(1 - \gamma \right) \left[z_i \left(t - h_m \right) - z_i \left(t - h \left(t \right) \right) \right]^T$$

$$\times W_i \left[z_i \left(t - h_m \right) - z_i \left(t - h \left(t \right) \right) \right]. \tag{34}$$

Therefore, from (21) and (30)–(34), it follows that

$$\dot{V}_i \left(t \right) + \alpha V_i \left(t \right) \le \xi_1^T \left(t \right) \Sigma_i \xi_1 \left(t \right)$$

$$- \gamma \left[z_i \left(t - h \left(t \right) \right) - z_i \left(t - h_M \right) \right]^T$$

$$\times e^{-\alpha \delta} W_i \left[z_i \left(t - h \left(t \right) \right) - z_i \left(t - h_M \right) \right]$$

$$- \left(1 - \gamma \right) \left[z_i \left(t - h_m \right) - z_i \left(t - h \left(t \right) \right) \right]^T$$

$$\times e^{-\alpha \delta} W_i \left[z_i \left(t - h_m \right) - z_i \left(t - h \left(t \right) \right) \right]$$

$$= \xi_1^T \left(t \right) \left[\gamma \Sigma_{i1} + \left(1 - \gamma \right) \Sigma_{i2} \right] \xi_1 \left(t \right), \tag{35}$$

where Σ_{i1}, Σ_{i2}, and Σ_i are defined in (24), (25), and (26), respectively, and

$$\xi_1 \left(t \right) = \left[\; z_i^T \left(t \right) \;\; y_i^T \left(t \right) \;\; z_i^T \left(t - h_m \right) \;\; z_i^T \left(t - h \left(t \right) \right) \right.$$

$$\left. z_i^T \left(t - h_M \right) \; \int_{t-h(t)}^{t} y_i^T \left(s \right) ds \; \right]^T . \tag{36}$$

Since $0 \le \gamma \le 1$, $\gamma \Sigma_{i1} + (1 - \gamma) \Sigma_{i2}$ is a convex combination of Σ_{i1} and Σ_{i2}. Therefore, $\gamma \Sigma_{i1} + (1 - \gamma) \Sigma_{i2} < 0$ is equivalent to $\Sigma_{i1} < 0$ and $\Sigma_{i2} < 0$. Thus, it follows from (24), (25), and (35) that

$$\dot{V}_i \left(t \right) + \alpha V_i \left(t \right) \le 0, \quad t \ge t_0 \ge 0. \tag{37}$$

By integrating this inequality from t_0 to t, we obtain (27). $\square$

Remark 9. If $\alpha = 0$ in (24) and (25), then it follows from Lemma 8 that the network system (17) is asymptotically stable.

Next, we define $V(t)$ as follows:

$$V_i \left(t \right) = V_{i1} \left(t \right) + V_{i2} \left(t \right) + V_{i3} \left(t \right) + V_{i4} \left(t \right) + V_{i5} \left(t \right) + V_{i6} \left(t \right), \tag{38}$$

where

$$V_{i1} \left(t \right) = \xi_i^T \left(t \right) E P_i \xi_i \left(t \right) = z_i^T \left(t \right) P_{i1} z_i \left(t \right),$$

$$V_{i2} \left(t \right) = \int_{t-h_m}^{t} z_i^T \left(s \right) Q_i e^{\beta \left(t - s \right)} z_i \left(s \right) ds,$$

$$V_{i3} \left(t \right) = \int_{t-h_M}^{t} z_i^T \left(s \right) R_i e^{\beta \left(t - s \right)} z_i \left(s \right) ds,$$

$$V_{i4} \left(t \right) = h_m \int_{-h_m}^{0} \int_{t+\theta}^{t} \dot{z}_i^T \left(s \right) U_i e^{\beta \left(t - s \right)} \dot{z}_i \left(s \right) ds \, d\theta,$$

$$V_{i5} \left(t \right) = h_M \int_{-h_M}^{0} \int_{t+\theta}^{t} y_i^T \left(s \right) S_i e^{\beta \left(t - s \right)} y_i \left(s \right) ds \, d\theta, \tag{39}$$

$$V_{i6} \left(t \right) = \delta \int_{-h_M}^{-h_m} \int_{t+\theta}^{t} \dot{z}_i^T \left(s \right) W_i e^{\beta \left(t - s \right)} \dot{z}_i \left(s \right) ds \, d\theta,$$

$$\xi_i \left(t \right) = \left[\; z_i^T \left(t \right) \;\; y_i^T \left(t \right) \; \int_{t-h(t)}^{t} y_i^T \left(s \right) ds \; \right]^T ,$$

$$E = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T , \qquad P_i = \begin{bmatrix} P_{i1} & 0 & 0 \\ P_{i2} & P_{i3} & P_{i4} \end{bmatrix} .$$

For given $\alpha > 0$, the following lemma provides an estimation of $V_i(t)$.

Lemma 10. *Assume that the time-varying delay $h(t)$ satisfies $0 \le h_m \le h(t) \le h_M$ and let $\beta > 0$. If there exist symmetric positive definite matrices $P_{i1} > 0$, $Q_i > 0$, $R_i > 0$, $S_i > 0$, $U_i > 0$, and $W_i > 0$ and appropriately dimensioned matrices P_{i2}, P_{i3}, and P_{i4} for $i = 2, 3, \ldots, N$, such that the following symmetric LMIs hold:*

$$\Sigma_{i1} = \Sigma_i - \begin{bmatrix} 0 & 0 & I & -I & 0 & 0 \end{bmatrix}^T$$

$$\times W_i \begin{bmatrix} 0 & 0 & I & -I & 0 & 0 \end{bmatrix} < 0, \tag{40}$$

$$\Sigma_{i2} = \Sigma_i - \begin{bmatrix} 0 & 0 & 0 & I & -I & 0 \end{bmatrix}^T$$

$$\times W_i \begin{bmatrix} 0 & 0 & 0 & I & -I & 0 \end{bmatrix} < 0,$$

$i = 2, \ldots, N$, *where*

$$\Sigma_i = \begin{bmatrix} \Sigma_{i11} & \Sigma_{i12} & \Sigma_{i13} & \Sigma_{i14} & 0 & \Sigma_{i16} \\ * & \Sigma_{i22} & 0 & \Sigma_{i24} & 0 & \Sigma_{i26} \\ * & * & \Sigma_{i33} & \Sigma_{i34} & 0 & 0 \\ * & * & * & \Sigma_{i44} & \Sigma_{i45} & \Sigma_{i46} \\ * & * & * & * & \Sigma_{i55} & 0 \\ * & * & * & * & * & \Sigma_{i66} \end{bmatrix},$$

$$\Sigma_{i11} = P_{i2}^T \left(\widetilde{A}_i + \widetilde{B}_{i1} \right) + \left(\widetilde{A}_i + \widetilde{B}_{i1} \right)^T P_{i2}$$
$$\qquad + Q_i + R_i - U_i - \beta P_{i1},$$

$$\Sigma_{i12} = P_{i1}^T - P_{i2}^T + \left(\widetilde{A}_i + \widetilde{B}_{i1} \right)^T P_{i3},$$

$$\Sigma_{i13} = U_i,$$

$$\Sigma_{i14} = P_{i2}^T \widetilde{B}_{i2},$$

$$\Sigma_{i16} = - P_{i2}^T \widetilde{B}_{i1} - \left(\widetilde{A}_i + \widetilde{B}_{i1} \right)^T P_{i4}, \qquad (41)$$

$$\Sigma_{i22} = h_m^2 U_i + h_M^2 S_i + \delta^2 W_i - P_{i3}^T - P_{i3},$$

$$\Sigma_{i24} = P_{i3}^T \widetilde{B}_{i2},$$

$$\Sigma_{i26} = - P_{i3}^T \widetilde{B}_{i1} - P_{i4},$$

$$\Sigma_{i33} = - Q_i - U_i - W_i,$$

$$\Sigma_{i34} = W_i,$$

$$\Sigma_{i44} = - 2W_i,$$

$$\Sigma_{i45} = W_i,$$

$$\Sigma_{i46} = \widetilde{B}_{i2} P_{i4}^T,$$

$$\Sigma_{i55} = - R_i - W_i,$$

$$\Sigma_{i66} = - S_i - P_{i4}^T \widetilde{B}_{i1} - \widetilde{B}_{i1}^T P_{i4}.$$

Then, along the trajectory of system (17), *we have*

$$V_i(t) \le e^{\beta(t-t_0)} V_i(t_0), \quad t \ge t_0 \ge 0. \qquad (42)$$

Proof. With the same argument as in Lemma 8, by taking the derivative of Lyapunov-Krasovskii functional candidate (38) along the trajectory of the system (21), we obtain

$$\dot{V}_i(t) - \beta V_i(t) \le 0, \quad t \ge t_0 \ge 0. \qquad (43)$$

Integrating this inequality from t_0 to t, we get (42). □

We are now ready to derive some new sufficient conditions for exponential synchronization of the switched network

$$\dot{z}_i(t) = \widetilde{A}_i z_i(t) + \widetilde{B}_{i\sigma(t)} z_i(t - h(t)), \quad i = 2, 3, \ldots, N. \qquad (44)$$

We rewrite system (44) in the following equivalent descriptor system form:

$$\dot{z}_i(t) = y_i(t),$$

$$0 = - y_i(t) + \left(\widetilde{A}_i + \widetilde{B}_{i\sigma(t)1} \right) z_i(t)$$
$$\qquad + \widetilde{B}_{i\sigma(t)2} z_i(t - h(t)) - \widetilde{B}_{i\sigma(t)1} \int_{t-h(t)}^{t} y_i(s) \, ds. \qquad (45)$$

We consider the case when synchronizable and non-synchronizable subnetworks coexist. Without loss of generality, we suppose that each subnetwork $k \in S_s = \{1, 2, \ldots, r\}$ is synchronizable, where $1 \le r < m$, and each subnetwork $l \in S_n = \{r + 1, r + 2, \ldots, m\}$ is non-synchronizable.

For each subnetwork of the system (45) which satisfies Lemma 8, the Lyapunov-Krasovskii functional candidate can be chosen as

$$V_{ik}(t) = z_i^T(t) E P_{ik1} z_i(t) + \int_{t-h_m}^{t} z_i^T(s) Q_{ik} e^{\alpha(s-t)} z_i(s) \, ds$$
$$\qquad + \int_{t-h_M}^{t} z_i^T(s) R_{ik} e^{\alpha(s-t)} z_i(s) \, ds$$
$$\qquad + h_m \int_{-h_m}^{0} \int_{t+\theta}^{t} \dot{z}_i^T(s) U_{ik} e^{\alpha(s-t)} \dot{z}_i(s) \, ds \, d\theta$$
$$\qquad + h_M \int_{-h_M}^{0} \int_{t+\theta}^{t} y_i^T(s) S_{ik} e^{\alpha(s-t)} y_i(s) \, ds \, d\theta$$
$$\qquad + \delta \int_{-h_M}^{-h_m} \int_{t+\theta}^{t} \dot{z}_i^T(s) W_{ik} e^{\alpha(s-t)} \dot{z}_i(s) \, ds \, d\theta, \qquad (46)$$

where $i = 2, 3, \ldots, N$, $k \in S_s$, P_{ik1}, Q_{ik}, R_{ik}, U_{ik}, S_{ik}, and W_{ik} are positive definite matrices and

$$\xi_i(t) = \left[z_i^T(t) \; y_i^T(t) \; \int_{t-h(t)}^{t} y_i^T(s) \, ds \right]^T, \qquad (47)$$

$$E = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T, \qquad P_{ik} = \begin{bmatrix} P_{ik1} & 0 & 0 \\ P_{ik2} & P_{ik3} & P_{ik4} \end{bmatrix}.$$

Similarly, for each subnetwork of the system (45) which satisfies Lemma 10, the Lyapunov-Krasovskii functional candidate can be chosen as

$$
V_{il}(t) = z_i^T(t) E P_{il1} z_i(t) + \int_{t-h_m}^{t} z_i^T(s) Q_{il} e^{\beta(t-s)} z_i(s)\, ds
$$

$$
+ \int_{t-h_M}^{t} z_i^T(s) R_{il} e^{\beta(t-s)} z_i(s)\, ds
$$

$$
+ h_m \int_{-h_m}^{0} \int_{t+\theta}^{t} \dot{z}_i^T(s) U_{il} e^{\beta(t-s)} \dot{z}_i(s)\, ds\, d\theta
$$

$$
+ h_M \int_{-h_M}^{0} \int_{t+\theta}^{t} y_i^T(s) S_{il} e^{\beta(t-s)} y_i(s)\, ds\, d\theta
$$

$$
+ \delta \int_{-h_M}^{-h_m} \int_{t+\theta}^{t} \dot{z}_i^T(s) W_{il} e^{\beta(t-s)} \dot{z}_i(s)\, ds\, d\theta,
$$

$$(48)$$

where $i = 2, 3, \ldots, N$, $l \in S_n$, P_{il1}, Q_{il}, R_{il}, U_{il}, S_{il}, and W_{il} are positive definite matrices.

Consider the following piecewise Lyapunov-Krasovskii functional candidate:

$$
V_i(t) = V_{i\sigma(t)}, \quad i = 2, 3, \ldots, N, \ \sigma(t) \in M. \quad (49)
$$

From Lemmas 8 and 10, the following properties of the Lyapunov-Krasovskii functional candidate (49) are obtained.

(i) There exist $a, b > 0$ such that

$$
a\|z_i(t)\|^2 \le V_{ik,il}(t) \le b\|z_i(t_0)\|_{cl}^2,
$$

$$
i = 2, 3, \ldots, N, \ k, l \in M. \quad (50)
$$

(ii) There exists a constant $\mu \ge 1$ such that

$$
V_{ik}(t) \le \mu V_{il}(t), \quad i = 2, 3, \ldots, N, \ k, l \in M. \quad (51)
$$

(iii) The Lyapunov functional candidate (49) satisfies

$$
V_i(t) \le \begin{cases} e^{-\alpha(t-t_0)} V_{ik}(t_0), & \text{if } \sigma(t) = k \in S_s, \\ e^{\beta(t-t_0)} V_{il}(t_0), & \text{if } \sigma(t) = l \in S_n. \end{cases} \quad (52)
$$

Now, for any piecewise constant switching signal $\sigma(t)$ and any $0 \le t_0 < t$, we let $T^-(t_0, t)$ ($T^+(t_0, t)$ resp.) denote the total activation time of the synchronizable subnetworks (the ones of non-synchronizable subnetworks resp.) during (t_0, t). Then, we choose a scalar $\alpha^* \in (0, \alpha)$ arbitrarily to propose the following switching law.

(S1): Determine the switching signal $\sigma(t)$ so that

$$
\inf_{t \ge t_0} \frac{T^-(t_0, t)}{T^+(t_0, t)} \ge \frac{\beta + \alpha^*}{\alpha - \alpha^*} \quad (53)
$$

holds on time interval (t_0, t). Meanwhile, we choose $\alpha^* < \alpha$ as the average dwell time scheme: for any $t > t_0$,

$$
N_\sigma(t_0, t) \le N_0 + \frac{t - t_0}{T_a}, \quad T_a \ge T_a^* = \frac{\ln \mu}{\alpha^*}. \quad (54)
$$

Theorem 11. *For a given constant $\alpha > 0$, $\beta > 0$ and time-varying delay satisfying (3), suppose that the subnetwork ($1 \le k \le r$) of switched network (1) satisfies the conditions of Lemma 8, and the others satisfy the conditions of Lemma 10. Assume that there exists $\mu \ge 1$ such that*

$$
P_{ip1} \le \mu P_{iq1}, \quad Q_{ip} \le \mu Q_{iq}, \quad R_{ip} \le \mu R_{iq},
$$

$$
U_{ip} \le \mu U_{iq}, \quad S_{ip} \le \mu S_{iq}, \quad W_{ip} \le \mu W_{iq}, \quad (55)
$$

$$
\forall p, q \in M.
$$

Then the synchronous manifold (5) of network (1) is locally exponentially stable for switching signal satisfying (53) and (54), and the state decay estimate is given by

$$
\|z_i(t)\| \le \sqrt{\frac{c_0 b_i}{a_i}} e^{-\lambda(t-t_0)} \|z_i(t_0)\|_{cl}, \quad i = 2, 3, \ldots, N, \quad (56)
$$

where

$$
c_0 = e^{N_0 \ln \mu}, \quad \lambda = \frac{1}{2}\left(\alpha^* - \frac{\ln \mu}{T_a}\right),
$$

$$
a_i = \min_{\forall k \in M} (\lambda_{\min}(P_{ik1})),
$$

$$
b_i = \max_{\forall k \in M} (\lambda_{\max}(P_{ik1})) + h_m \max_{\forall k \in M} (\lambda_{\max}(Q_{ik}))
$$

$$
+ h_M \max_{\forall k \in M} (\lambda_{\max}(R_{ik})) + \frac{h_m^2}{2} \max_{\forall k \in M} (\lambda_{\max}(U_{ik}))
$$

$$
+ \frac{h_M^2}{2} \max_{\forall k \in M} (\lambda_{\max}(S_{ik})) + \frac{\delta^2}{2} \max_{\forall k \in M} (\lambda_{\max}(W_{ik})).
$$

$$(57)$$

Proof. Suppose that $t \in [t_k, t_{k+1})$. For piecewise Lyapunov-Krasovskii functional candidate (49), along trajectory of network system (45), we have

$$
V_i(t) \le \begin{cases} e^{-\alpha(t-t_k)} V_{ik}(t_k), & \text{if } \sigma(t) = k \in S_s, \\ e^{\beta(t-t_k)} V_{il}(t_k), & \text{if } \sigma(t) = l \in S_n. \end{cases} \quad (58)
$$

Since $V_{i\sigma(t_k)}(t_k) \le \mu V_{i\sigma(t_k^-)}(t_k^-)$ is true from (51) at the switching point t_k, where $t_k^- = \lim_{t \to t_k} t$ and from $k = N_\sigma(t_0, t) \le N_0 + ((t - t_0)/T_a)$, we obtain

$$
V_i(t) \le e^{\beta T^+(t_k, t) - \alpha T^-(t_k, t)} V_{i\sigma(t_k)}(t_k)
$$

$$
\le e^{\beta T^+(t_k, t) - \alpha T^-(t_k, t)} \mu V_{i\sigma(t_k^-)}(t_k^-)
$$

$$
\le e^{\beta T^+(t_k, t) - \alpha T^-(t_k, t)} \mu e^{\beta T^+(t_{k-1}, t_k) - \alpha T^-(t_{k-1}, t_k)}
$$

$$
\times V_{i\sigma(t_{k-1})}(t_{k-1})
$$

$$
\le \mu e^{\beta T^+(t_{k-1}, t) - \alpha T^-(t_{k-1}, t)} V_{i\sigma(t_{k-1})}(t_{k-1})
$$

$$
\vdots
$$

$$
\le \mu^k e^{\beta T^+(t_0, t) - \alpha T^-(t_0, t)} V_{i\sigma(t_0)}(t_0)
$$

$$
\le e^{\beta T^+(t_0, t) - \alpha T^-(t_0, t) + (N_0 + ((t-t_0)/T_a)) \ln \mu} V_{i\sigma(t_0)}(t_0).
$$

$$(59)$$

TABLE 1: Comparison of the maximum value h_M for the asymptotic stability of system (65) by different methods.

	c	0.3	0.4	0.5	0.6
	Li et al. 2008 [8]	0.960	0.710	0.562	0.464
$h_m = 0$	Yue and Li 2010 [12]	1.345	0.950	0.731	0.587
	Ours	1.8894	1.3923	1.0983	0.9049
$h_m = 0.1$	Yue and Li 2010 [12]	1.354	0.951	0.731	0.587
	Ours	1.9158	1.4171	1.1219	0.9276
$h_m = 0.5$	Yue and Li 2010 [12]	1.389	0.967	0.740	0.605
	Ours	2.1331	1.6165	1.3127	1.1145

Under the switching law (S1) for any t_0, t, we have

$$\beta T^+(t_0, t) - \alpha T^-(t_0, t) \leq -\alpha^* \left(T^+(t_0, t) + T^-(t_0, t)\right)$$
$$= -\alpha^*(t - t_0).$$
(60)

Thus,

$$V_i(t) \leq e^{N_0 \ln \mu} e^{-(\alpha^* - ((\ln \mu)/T_a))(t-t_0)} V_{i\sigma(t_0)}(t_0)$$
$$\leq c_0 e^{-2\lambda(t-t_0)} V_{i\sigma(t_0)}(t_0),$$
(61)

where $c_0 = e^{N_0 \ln \mu}$, $\lambda = (1/2)(\alpha^* - ((\ln \mu)/T_a))$. According to (49), we have

$$a_i \|z_i(t)\|^2 \leq V_i(t), \qquad V_{i\sigma(t_0)}(t_0) \leq b_i \|z_i(t_0)\|^2_{\text{cl}}. \quad (62)$$

Combining (61) and (62) leads to

$$\|z_i(t)\|^2 \leq \frac{1}{a_i} V_i(t) \leq \frac{b_i}{a_i} c_0 e^{-2\lambda(t-t_0)} \|z_i(t_0)\|^2_{\text{cl}}. \quad (63)$$

Therefore,

$$\|z_i(t)\| \leq \sqrt{\frac{c_0 b_i}{a_i}} e^{-\lambda(t-t_0)} \|z_i(t_0)\|_{\text{cl}}. \quad (64)$$

which means that synchronous solution to (1) is locally exponentially stable. The proof is completed. □

4. Numerical Examples

In this section, we give examples to show the effectiveness of theoretical results obtained in Lemma 8 and Theorem 11.

Example 12. Consider a nonswitched network model with 5 nodes, where each node is a three-dimensional stable linear system described by

$$\dot{x}_{i1}(t) = -x_{i1}(t)$$
$$\dot{x}_{i2}(t) = -2x_{i2}(t) \quad (65)$$
$$\dot{x}_{i3}(t) = -3x_{i3}(t),$$

which is asymptotically stable at the equilibrium point $s(t) = 0$, and its Jacobin matrices are $J(t) = \text{diag}\{-1, -2, -3\}$,

$J_h(t - h(t)) = 0$. Assume that the inner-coupling matrix is $A = \text{diag}\{1, 1, 1\}$, and the outer-coupling matrix is given by the following irreducible symmetric matrix satisfying condition (2):

$$G = \begin{bmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -3 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{bmatrix}. \quad (66)$$

The eigenvalues of G are $\lambda_i = \{0, -1.382, -2.382, -3.618, -4.618\}$, $i = 1, 2, \ldots, 5$. Therefore, if the delayed subsystem in (17) is asymptotically stable, then the synchronized state $s(t)$ is asymptotically stable.

In this example, we have $\widetilde{A}_i = \text{diag}\{-1, -2, -3\}$, $\widetilde{B}_i = \text{diag}\{c\lambda_i, c\lambda_i, c\lambda_i\}$, and $i = 2, \ldots, 5$. Let us decompose the matrix $\widetilde{B}_i = \widetilde{B}_{i1} + \widetilde{B}_{i2}$, where

$$\widetilde{B}_{i1} = \begin{bmatrix} 0.39c\lambda_i & 0 & 0 \\ 0 & 0.39c\lambda_i & 0 \\ 0 & 0 & 0.39c\lambda_i \end{bmatrix},$$
$$\widetilde{B}_{i2} = \begin{bmatrix} 0.61c\lambda_i & 0 & 0 \\ 0 & 0.61c\lambda_i & 0 \\ 0 & 0 & 0.61c\lambda_i \end{bmatrix}, \quad (67)$$
$$i = 2, \ldots, 5.$$

By using Lemma 8, we will give the maximum upper bounds h_M of the time-varying delay for different lower bounds h_m and coupling strength c. Applying Lemma 8 with $\alpha = 0$, we obtain the maximum upper bound of delays h_M as shown in Table 1. We see that, Lemma 8 provides a less conservative result than those obtained via the methods of [8, 12]. When $h_m \neq 0$ especially, the result in [8] is not discussed while Lemma 8 in this paper also considers the case $h_m \neq 0$. Note that we use MATLAB LMI Control Toolbox in order to solve the LMI in (24) and (25).

The numerical simulations are carried out using the explicit Runge-Kutta-like method (dde45), interpolation and extrapolation by spline of the third order. Figure 1 shows the synchronization state curves $x_{i1}(t), x_{i2}(t), x_{i3}(t)$,
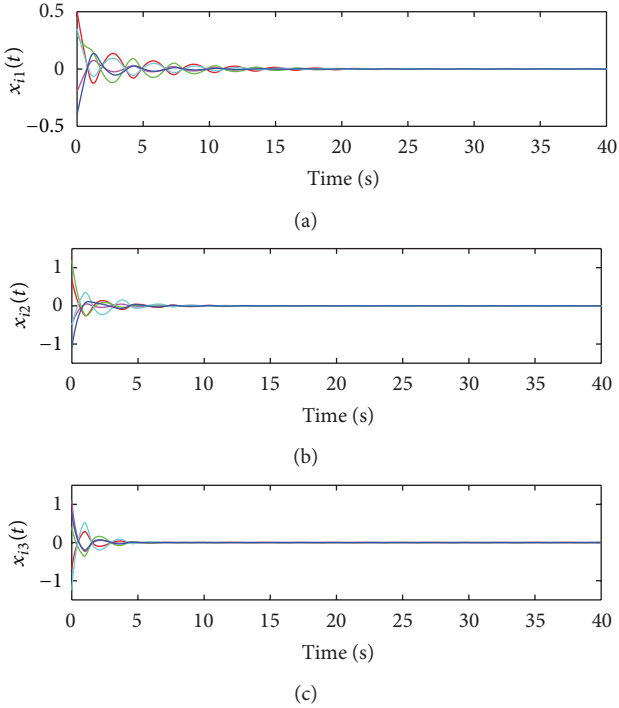
FIGURE 1: Synchronization state curves for the dynamic nodes delayed network (65) with $c = 0.5$ and $h(t) = 0.5 + 0.715|\cos t|$.

$i = 1, 2, \ldots, 5$ with $c_1 = 0.5$, $h(t) = 0.5 + 0.715|\cos t|$ and the initial function

$$\phi_1(t) = [-0.2\cos t, -0.5\cos t, 1.1\cos t],$$

$$\phi_2(t) = [0.5\cos t, 0.7\cos t, -0.7\cos t],$$

$$\phi_3(t) = [0.3\cos t, 1.2\cos t, 0.4\cos t], \tag{68}$$

$$\phi_4(t) = [0.35\cos t, -0.5\cos t, -1.25\cos t],$$

$$\phi_5(t) = [-0.4\cos t, -1.1\cos t, 0.8\cos t].$$

Example 13. Consider a linear switched network model with 5 nodes where each node is described by the following delayed dynamical system:

$$\dot{x}_{i1}(t) = -1x_{i1}(t) - x_{i1}(t - h(t))$$

$$\dot{x}_{i2}(t) = -2x_{i2} + x_{i1}(t - h(t)) - x_{i2}(t - h(t)) \tag{69}$$

$$\dot{x}_{i3}(t) = -1.5x_{i3} - x_{i3}(t - h(t)),$$

which is asymptotically stable at the equilibrium point $s(t) = 0$, $s(t - h(t)) = 0$. Jacobian matrices $J(t)$ and $J_h(t - h(t))$ are

$$J(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1.5 \end{bmatrix}, \qquad J_h(t - h(t)) = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}. \tag{70}$$

Assume that the inner-coupling matrices are $A_1 = \text{diag}\{0.5, 0.5, 0.5\}$, $A_2 = \text{diag}\{-1, -1, -1\}$ and the coupling

strengths are $c_1 = 0.3$ and $c_2 = 0.6$, and the outer-coupling matrices are

$$G_1 = \begin{bmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -3 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{bmatrix},$$

$$\tag{71}$$

$$G_2 = \begin{bmatrix} -4 & 1 & 1 & 1 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -4 & 1 & 1 \\ 1 & 1 & 1 & -4 & 1 \\ 1 & 1 & 1 & 1 & -4 \end{bmatrix}.$$

The eigenvalues of G_1 and G_2 are $\lambda_{i1} = \{0, -1.382, -2.382, -3.618, -4.618\}$, $\lambda_{i2} = \{0, -5, -5, -5, -5\}$, and $i = 1, 2, \ldots, 5$, respectively. Obviously, there exists a unitary matrix U which diagonalizes G_1 and G_2 simultaneously. The subnetwork associated with $\{c_1, A_1, G_1\}$ is synchronizable and the subnetwork associated with $\{c_2, A_2, G_2\}$ is non-synchronizable.

From the above conditions, we obtain matrices

$$\widetilde{A}_i = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1.5 \end{bmatrix},$$

$$\tag{72}$$

$$\widetilde{B}_{ik} = \begin{bmatrix} -1 + c_k a_{11}^k \lambda_{ik} & 0 & 0 \\ 1 & -1 + c_k a_{22}^k \lambda_{ik} & 0 \\ 0 & 0 & -1 + c_k a_{33}^k \lambda_{ik} \end{bmatrix},$$

$i = 2, \ldots, 5$, and $k = 1, 2$. Let us decompose the matrix $\widetilde{B}_{ik} = \widetilde{B}_{ik1} + \widetilde{B}_{ik2}$, where

$$\widetilde{B}_{i11} = 0.93\widetilde{B}_{i1}, \qquad \widetilde{B}_{i12} = 0.07\widetilde{B}_{i1},$$

$$\tag{73}$$

$$\widetilde{B}_{i21} = 0.86\widetilde{B}_{i2}, \qquad \widetilde{B}_{i22} = 0.14\widetilde{B}_{i2}, \quad i = 2, \ldots, 5.$$

Given $h_1 = 0.3$, $h_2 = 0.68$, $\alpha = 1.25$, and $\beta = 0.6$, it is found that LMIs (24), (25), (42), and (42) have feasible solutions. We may choose $\mu = 1.35$. Let $\alpha^* = 0.55 < \alpha$. Then, from the switching law (S1), it is required that $\inf_{t \geq t_0}(T^-(t_0, t)/T^+(t_0, t)) \geq (\beta + \alpha^*)/(\alpha - \alpha^*) = 1.6429$ and the average dwell time is computed as $T_a \geq T_a^* = (\ln \mu)/\alpha^* = 0.5456$.

Let $h(t) = 0.3 + 0.38|\sin t|$ and the initial function

$$\phi_1(t) = [-0.4\cos t, -0.5\cos t, 1.1\cos t],$$

$$\phi_2(t) = [0.5\cos t, 0.7\cos t, -0.7\cos t],$$

$$\phi_3(t) = [0.89\cos t, 1.2\cos t, 0.4\cos t], \tag{74}$$

$$\phi_4(t) = [1.35\cos t, -1.5\cos t, -1.25\cos t],$$

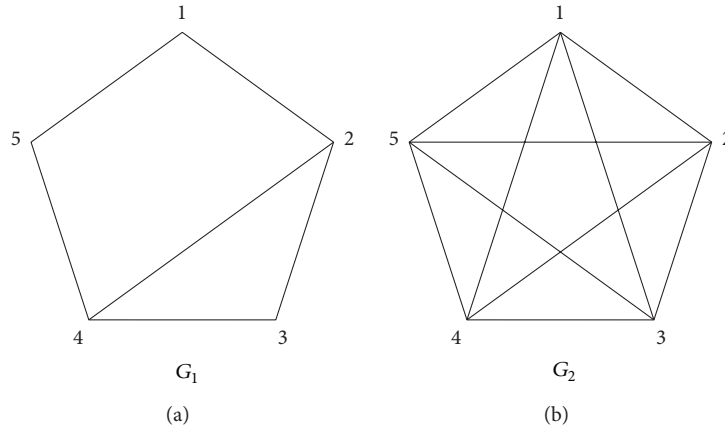$$\phi_5(t) = [-0.8\cos t, -1.1\cos t, 0.8\cos t].$$

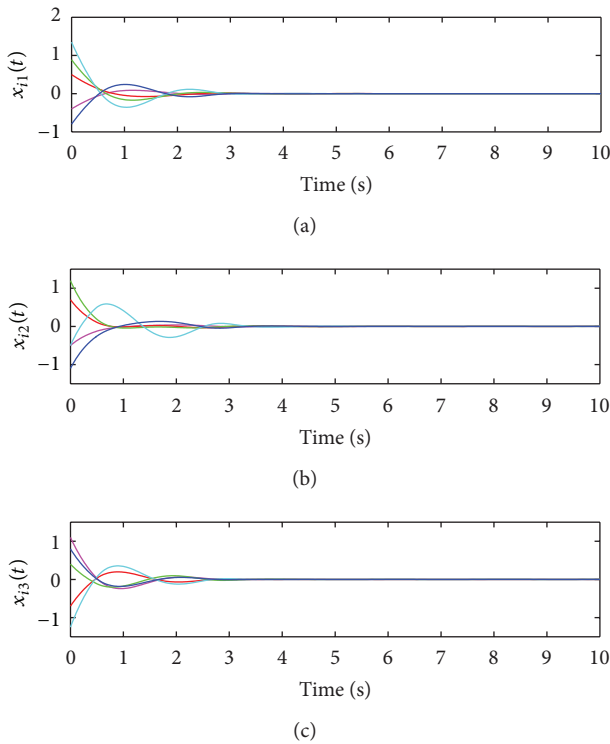FIGURE 2: The structure of complex networks with $N = 5$.

FIGURE 3: Synchronization state curves for the synchronizable subnetwork (69).
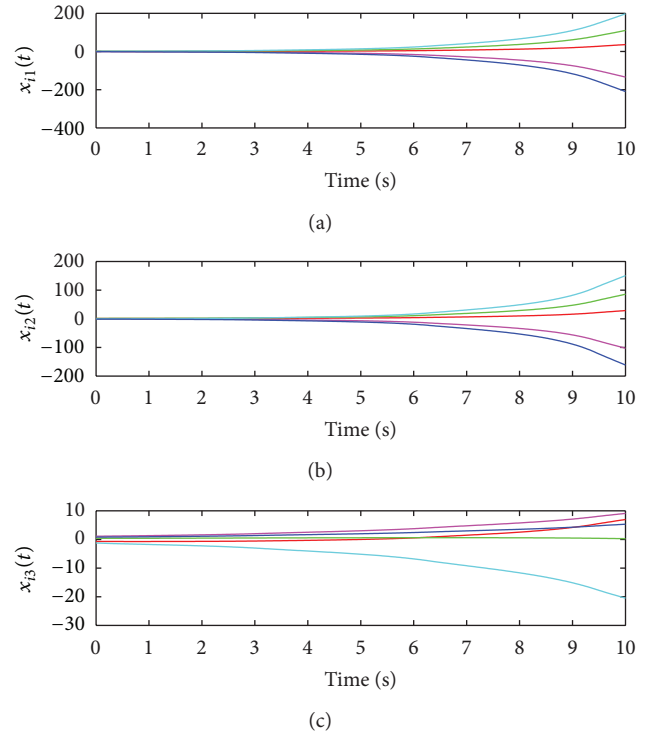

FIGURE 4: Synchronization state curves for the nonsynchronizable subnetwork (69).

Figure 2 shows the structure of complex networks. Figures 3 and 4 show the synchronization state curves of the synchronizable subnetwork and non-synchronizable subnetwork, respectively. Figure 5 shows the synchronization state curves $x_{i1}(t), x_{i2}(t), x_{i3}(t), i = 1, 2, \ldots, 5$ of switched network with $\sigma(t)$ satisfying (S1). Figure 6 shows the switching $\sigma(t)$ of the switched delay network with average dwell time. We see that the synchronization state converges to zero under the above conditions.

Example 14. We consider nonlinear switched network model with 5 nodes in which each node is a Lorenz chaotic system with time-varying delay described by [35] as

$$\dot{x}_{i1}(t) = a\left(x_{i2}(t) - x_{i1}(t)\right),$$

$$\dot{x}_{i2}(t) = c x_{i1} - x_{i2}(t) - x_{i1}(t) x_{i3}(t - h(t)), \qquad (75)$$

$$\dot{x}_{i3}(t) = x_{i1}(t) x_{i2}(t - h(t)) - b x_{i3}(t - h(t)),$$

(a)

(b)

(c)
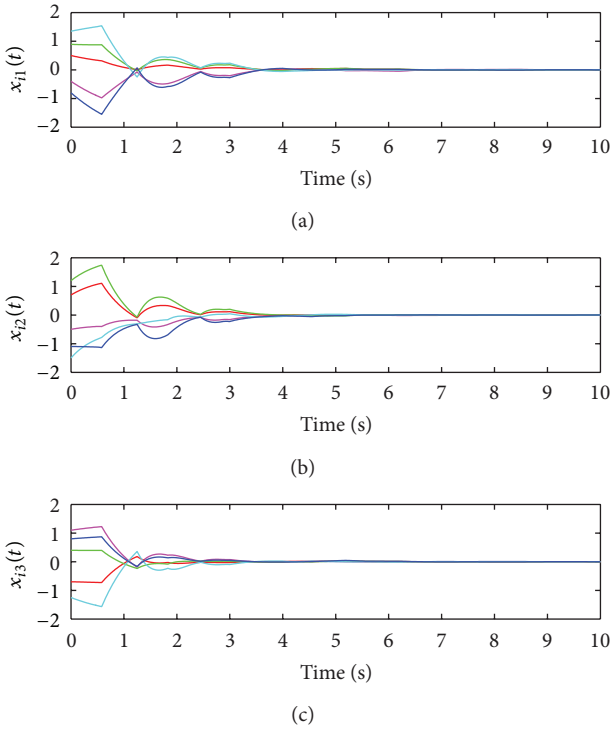
FIGURE 5: Synchronization state curves for the switched delay network (69).
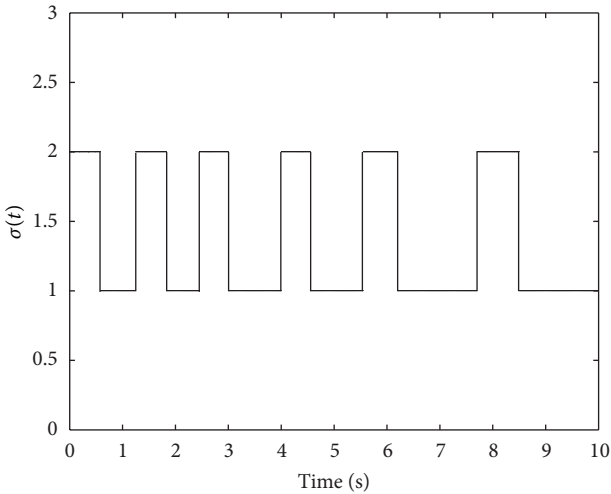

FIGURE 6: The switching signal $\sigma(t)$ of the switched delay network (69) with average dwell time.

where $a = 10$, $b = 1.3$, and $c = -28$. It is asymptotically stable at the equilibrium point $s(t) = 0$, $s(t - h(t)) = 0$ and its Jacobian matrices are

$$J(t) = \begin{bmatrix} -10 & 10 & 0 \\ -28 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad J_h(t - h(t)) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & -1.3 \end{bmatrix}. \tag{76}$$

Assume that the inner-coupling matrices are $A_1 = \mathrm{diag}\{0.4, 0.4, 0.4\}$ and $A_2 = \mathrm{diag}\{-0.2, -0.2, -0.2\}$, the coupling strengths $c_1 = 0.5$, and $c_2 = 0.3$, and the outer-coupling matrices G_1, and G_2 are the same as in Example 13. The subnetwork associated with $\{c_1, A_1, G_1\}$ is synchronizable and the subnetwork associated with $\{c_2, A_2, G_2\}$ is non-synchronizable.

From the above conditions, we obtain

$$\widetilde{A}_i = \begin{bmatrix} -10 & 10 & 0 \\ -28 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{B}_{ik} = \begin{bmatrix} c_k a_{11}^k \lambda_{ik} & 0 & 0 \\ 0 & c_k a_{22}^k \lambda_{ik} & -1 \\ 0 & 0 & -1.3 + c_k a_{33}^k \lambda_{ik} \end{bmatrix}, \tag{77}$$

$i = 2, \ldots, 5$, and $k = 1, 2$. Let us decompose the matrix $\widetilde{B}_{ik} = \widetilde{B}_{ik1} + \widetilde{B}_{ik2}$, where

$$\widetilde{B}_{i11} = 0.8\widetilde{B}_{i1}, \qquad \widetilde{B}_{i12} = 0.2\widetilde{B}_{i1},$$

$$\widetilde{B}_{i21} = 0.86\widetilde{B}_{i2}, \qquad \widetilde{B}_{i22} = 0.14\widetilde{B}_{i2}, \quad i = 2, \ldots, 5. \tag{78}$$

Given $h_1 = 0.2$, $h_2 = 0.55$, $\alpha = 0.9$, and $\beta = 0.3$, it is found that LMIs (24), (25), (42), and (42) have feasible solutions. We may choose $\mu = 1.253$. Let $\alpha^* = 0.45 < \alpha$. Then, from the switching law (S1), it is required that $\inf_{t \geq t_0}(T^-(t_0, t)/T^+(t_0, t)) \geq (\beta + \alpha^*)/(\alpha - \alpha^*) = 1.6667$ and the average dwell time is computed as $T_a \geq T_a^* = (\ln \mu)/\alpha^* = 0.5012$.

Let $h(t) = 0.2 + 0.35|\cos t|$ and the initial function

$$\phi_1(t) = [15 \cos t, 20 \cos t, 5 \cos t],$$

$$\phi_2(t) = [5 \cos t, 15 \cos t, -2 \cos t],$$

$$\phi_3(t) = [10 \cos t, 5 \cos t, 3 \cos t], \tag{79}$$

$$\phi_4(t) = [20 \cos t, 10 \cos t, -1 \cos t],$$

$$\phi_5(t) = [15 \cos t, 20 \cos t, 8 \cos t].$$

Figures 7 and 8 show the synchronization errors between the states of node i and node $i+1$, $e_{ij}(t) = x_{ij} - x_{(i+1)j}$, $i = 1, \ldots, 4$, and $j = 1, \ldots, 3$ of the synchronizable subnetwork and non-synchronizable subnetwork, respectively. Figure 9 shows the synchronization errors between the states of node i and node $i + 1$ of switched network with $\sigma(t)$ satisfying (S1). Figure 10 shows the switching $\sigma(t)$ of the switched delay network with average dwell time. We see that the synchronization state converges to zero under the above conditions.

Remark 15. The advantage of Examples 13 and 14 is the lower bound of the delay $h_m \neq 0$. Moreover, in these examples we still investigate interval time-varying delays in the dynamical nodes and the switched coupling term simultaneously, hence the synchronization conditions derived in [33] cannot be applied to these examples.
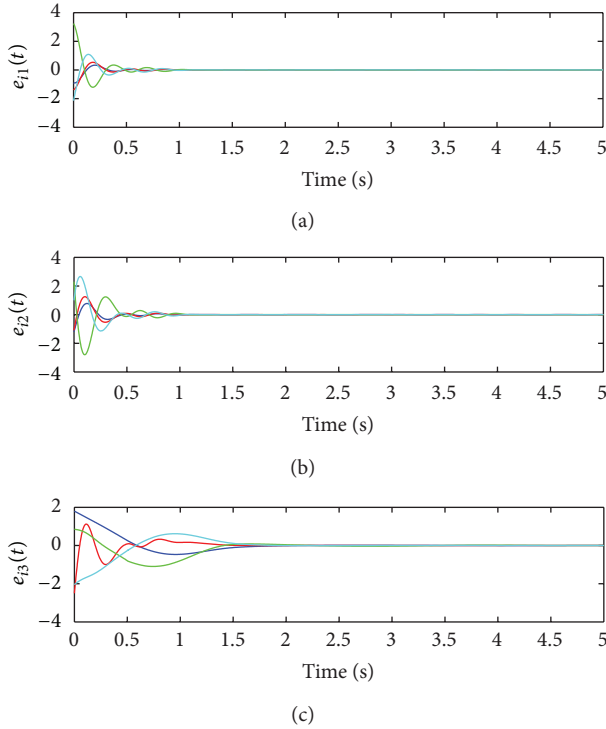
Figure 7: Synchronization errors between the states of node i and node $i + 1$ for the synchronizable subnetwork (75).
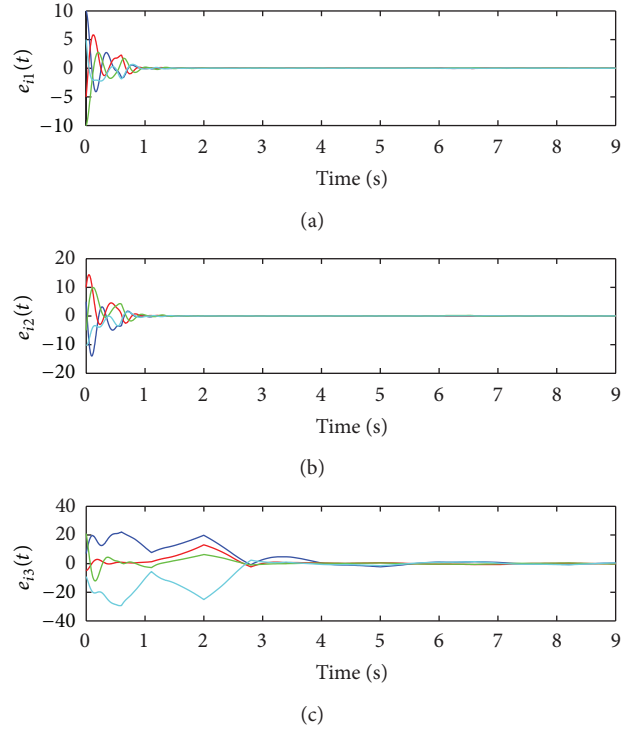

Figure 8: Synchronization errors between the states of node i and node $i + 1$ for the non-synchronizable subnetwork (75).

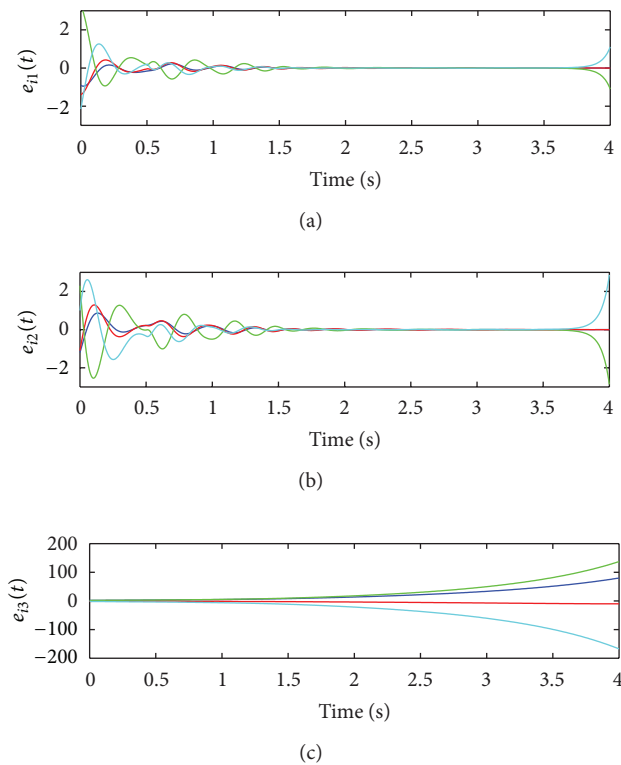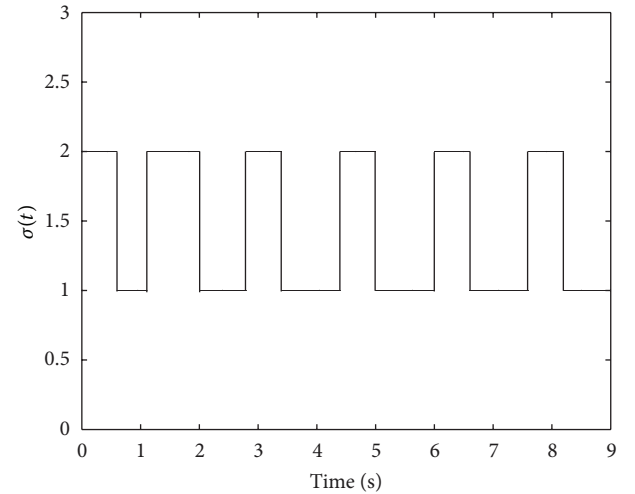Figure 9: Synchronization errors between the states of node i and node $i + 1$ for the switched delay network (75).

Figure 10: The switching signal $\sigma(t)$ of the switched delay network (75) with average dwell time.

5. Conclusion

In this paper, the synchronization problem has been investigated for complex dynamical networks with interval time-varying and switched coupling delays. Both interval time-varying delays in dynamical nodes and interval time-varying delays in switched couplings have been considered. We transformed the synchronization problem of the switched

network into the stability analysis of linear switched systems. By using the average dwell time approach and piecewise Lyapunov-Krasovskii functionals which are constructed based on the descriptor model transformation and decomposition technique of coefficient matrix, new delay-dependent synchronization criteria was derived in terms of linear matrix inequalities. Numerical examples were given to illustrate that the derived criteria are less conservative than some existing results.

Acknowledgments

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