Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2013, Article ID 353757, 8 pages
http://dx.doi.org/10.1155/2013/353757

Research Article

Asymptotic Behavior of Solutions to the Damped Nonlinear Hyperbolic Equation

Yu-Zhu Wang

School of Mathematics and Information Sciences, North China University of Water Resources and Electric Power, Zhengzhou 450011, China

Correspondence should be addressed to Yu-Zhu Wang; yuzhu108@163.com

Received 17 December 2012; Revised 15 May 2013; Accepted 30 May 2013

Academic Editor: Roberto Natalini

We consider the Cauchy problem for the damped nonlinear hyperbolic equation in n-dimensional space. Under small condition on the initial value, the global existence and asymptotic behavior of the solution in the corresponding Sobolev spaces are obtained by the contraction mapping principle.

1. Introduction

We investigate the Cauchy problem for the following damped nonlinear hyperbolic equation:

$$u_{tt} + k_1\Delta^2 u + k_2\Delta^2 u_t = \Delta f(\Delta u) \tag{1}$$

with the initial value

$$t = 0 : u = u_0(x), \qquad u_t = u_1(x). \tag{2}$$

Here $u = u(x,t)$ is the unknown function of $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ and $t > 0$, $k_1 > 0$ and $k_2 > 0$ are constants. The nonlinear term $f(u) = O(u^{1+\theta})$ and θ is a positive integer.

Equation (1) is a model in variational form for the neo-Hookean elastomer rod and describes the motion of a neo-Hookean elastomer rod with internal damping; for more detailed physical background, we refer to [1]. In [1], the authors have studied a general class of abstract evolution equations

$$w_{tt} + A_1 w + A_2 w_t + N^* g(Nw) = f, \tag{3}$$

where A_1, A_2, N, and f satisfy certain assumptions. For quite general conditions on the nonlinear term, global existence, uniqueness, regularity, and continuous dependence on the initial value of a generalized solution to (3) in a bounded domain of $\mathbb{R}^n$ were obtained. Equation (1) fits the abstract framework of [1]. The local well-posedness for the Cauchy problem for (1), (2) in three-dimensional space was obtained by Chen and Da [2]. More precisely, they proved local existence and uniqueness of weak solutions to (1), (2) under the assumption that $u_0 \in H^6(\mathbb{R}^3)$, $u_1 \in H^4(\mathbb{R}^3)$. Local existence and uniqueness of classical solutions to (1), (2) were also established, provided that $u_0 \in H^{12}(\mathbb{R}^3)$, $u_1 \in H^{10}(\mathbb{R}^3)$. Their method is to first establish local-in-time well-posedness of a periodic version of (1), (2) and then construc a solution to (1), (2) as a limit of periodic solutions with divergent periods. This paper also arrived at some sufficient conditions for blow-up of the solution in finite time, and an example was given. Song and Yang [3] studied the existence and nonexistence of global solutions to the Cauchy problem for (1) in one-dimensional space. The boundary value problems for (1) are investigated (see [4, 5]). Equation (1) is a fifth-order wave equation. For more higher order wave equations, we refer to [6–8] and references therein.

The main purpose of this paper is to establish global existence and asymptotic behavior of solutions to (1), (2) by using the contraction mapping principle. Firstly, we consider the decay property of the following linear equation:

$$u_{tt} + k_1\Delta^2 u + k_2\Delta^2 u_t = 0. \tag{4}$$

We obtain the following decay estimate of solutions to (4), (2)

$$\left\|\partial_x^k u(t)\right\|_{L^2} \leq C(1+t)^{-(n/8+k/4)} \times \left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}} + \|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right) \quad \text{for } (k \leq s+4),$$
$$\left\|\partial_x^l u_t(t)\right\|_{L^2} \leq C(1+t)^{-(n/8+l/4+1/2)} \times \left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}} + \|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right) \quad \text{for } (l \leq s). \tag{5}$$

Based on the above estimates, we define a solution space with time weighted norms, and then global existence and asymptotic behavior of solutions to (1), (2) are obtained by using the contraction mapping principle. More precisely, we prove global existence and the following decay estimate of solution to (1), (2):

$$\left\|\partial_x^k u(t)\right\|_{L^2} \leq CE_0(1+t)^{-(n/8+k/4)},$$
$$\left\|\partial_x^l u_t(t)\right\|_{L^2} \leq CE_0(1+t)^{-(n/8+l/4+1/2)} \tag{6}$$

for $k \leq s+4$, $l \leq s$, and $s \geq [n/2]+1$. Here $u_0 \in H^{s+4}(\mathbb{R}^n) \bigcap L^1(\mathbb{R}^n)$, $u_1 \in H^s(\mathbb{R}^n) \bigcap \dot{H}_1^{-2}(\mathbb{R}^n)$, and $E_0 = \|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}} + \|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}$ is assumed to be suitably small. When $n = 3$, our result allows for the initial data $u_0 \in H^6(\mathbb{R}^3)$, $u_1 \in H^2(\mathbb{R}^3)$. But in [2], the authors proved local existence and uniqueness of weak solutions to (1), (2) under the assumption that $u_0 \in H^6(\mathbb{R}^3)$, $u_1 \in H^4(\mathbb{R}^3)$, so our result improves the regularity of the initial condition for the time derivative. This improvement is due to the strong damping term $\Delta^2 u_t$ since the strong damping term $\Delta^2 u_t$ has stronger dissipative effect than the damping u_t. The stronger dissipative effect has been exhibited in the study of the strongly damped wave equation and related problems; see, for instance, [9].

The global existence and asymptotic behavior of solutions to hyperbolic-type equations have been investigated by many authors. We refer to [10–15] for hyperbolic equations, [16–21] for damped wave equation, and [22, 23] for various aspects of dissipation of the plate equation.

We give some notations which are used in this paper. Let $\mathcal{F}[u]$ denote the Fourier transform of u defined by

$$\hat{u}(\xi) = \mathcal{F}[u] = \int_{\mathbb{R}^n} e^{-i\xi\cdot x} u(x)\,dx, \tag{7}$$

and we denote its inverse transform by $\mathcal{F}^{-1}$.

For $1 \leq p \leq \infty$, $L^p = L^p(\mathbb{R}^n)$ denotes the usual Lebesgue space with the norm $\|\cdot\|_{L^p}$. The usual Sobolev space of s is defined by $H_p^s = (I-\Delta)^{-s/2}L^p$ with the norm $\|f\|_{H_p^s} = \|(I-\Delta)^{s/2}f\|_{L^p}$; the homogeneous Sobolev space of s is defined by $\dot{H}_p^s = (-\Delta)^{-s/2}L^p$ with the norm $\|f\|_{\dot{H}_p^s} = \|(-\Delta)^{s/2}f\|_{L^p}$; especially $H^s = H_2^s$, $\dot{H}^s = \dot{H}_2^s$. Moreover, we know that $H_p^s = L^p \bigcap \dot{H}_p^s$ for $s \geq 0$.

Finally, in this paper, we denote every positive constant by the same symbol C or c without confusion. $[\cdot]$ is the Gauss symbol.

The paper is organized as follows. In Section 2 we derive the solution formula of our semilinear problem. We study the decay property of the solution operators appearing in the solution formula in Section 3. Then, in Section 4, we discuss the linear problem and show the decay estimates. Finally, we prove global existence and asymptotic behavior of solutions for the Cauchy problem (1), (2) in Section 5.

2. Solution Formula

The aim of this section is to derive the solution formula for the problem (1), (2). We first investigate (4). Taking the Fourier transform, we have

$$\hat{u}_{tt} + k_2|\xi|^4\hat{u}_t + k_1|\xi|^4\hat{u} = 0. \tag{8}$$

The corresponding initial value is given as

$$t = 0 : \hat{u} = \hat{u}_0(\xi), \qquad \hat{u}_t = \hat{u}_1(\xi). \tag{9}$$

The characteristic equation of (8) is

$$\lambda^2 + k_2|\xi|^4\lambda + k_1|\xi|^4 = 0. \tag{10}$$

Let $\lambda = \lambda_\pm(\xi)$ be the corresponding eigenvalues of (10), and we obtain

$$\lambda_\pm(\xi) = \frac{-k_2|\xi|^4 \pm |\xi|^2\sqrt{k_2^2|\xi|^4 - 4k_1}}{2}. \tag{11}$$

The solution to the problem (8)-(9) is given in the form

$$\hat{u}(\xi,t) = \widehat{G}(\xi,t)\,\hat{u}_1(\xi) + \widehat{H}(\xi,t)\,\hat{u}_0(\xi), \tag{12}$$

where

$$\widehat{G}(\xi,t) = \frac{1}{\lambda_+(\xi) - \lambda_-(\xi)}\left(e^{\lambda_+(\xi)t} - e^{\lambda_-(\xi)t}\right), \tag{13}$$

$$\widehat{H}(\xi,t) = \frac{1}{\lambda_+(\xi) - \lambda_-(\xi)}\left(\lambda_+(\xi)\,e^{\lambda_-(\xi)t} - \lambda_-(\xi)\,e^{\lambda_+(\xi)t}\right). \tag{14}$$

We define $G(x,t)$ and $H(x,t)$ by $G(x,t) = \mathcal{F}^{-1}[\widehat{G}(\xi,t)](x)$ and $H(x,t) = \mathcal{F}^{-1}[\widehat{H}(\xi,t)](x)$, respectively, where $\mathcal{F}^{-1}$ denotes the inverse Fourier transform. Then, applying $\mathcal{F}^{-1}$ to (12), we obtain

$$u(t) = G(t) * u_1 + H(t) * u_0. \tag{15}$$

By the Duhamel principle, we obtain the solution formula to (1), (2)

$$u(t) = G(t) * u_1 + H(t) * u_0 + \int_0^t G(t-\tau) * \Delta f(\Delta u)(\tau)\,d\tau. \tag{16}$$

3. Decay Property

The aim of this section is to establish decay estimates of the solution operators $G(t)$ and $H(t)$ appearing in the solution formula (15).

Lemma 1. *The solution of the problem* (8), (9) *satisfies*

$$\left(|\xi|^4+|\xi|^8\right)|\hat{u}(\xi,t)|^2+|\hat{u}_t(\xi,t)|^2 \le Ce^{-c\omega(\xi)t}\left(\left(|\xi|^4+|\xi|^8\right)|\hat{u}_0(\xi)|^2+|\hat{u}_1(\xi)|^2\right) \tag{17}$$

for $\xi\in\mathbb{R}^n$ *and* $t\ge 0$, *where*

$$\omega(\xi)=\begin{cases}|\xi|^4, & |\xi|\le R_0\\ 1, & |\xi|\ge R_0.\end{cases} \tag{18}$$

Proof. Multiplying (8) by $\bar{\hat{u}}_t$ and taking the real part yield

$$\frac{1}{2}\frac{d}{dt}\left\{|\hat{u}_t|^2+k_2|\xi|^4|\hat{u}|^2\right\}+k_1|\xi|^4|\hat{u}_t|^2=0. \tag{19}$$

Multiplying (8) by $\bar{\hat{u}}$ and taking the real part, we obtain

$$\frac{1}{2}\frac{d}{dt}\left\{k_1|\xi|^4|\hat{u}|^2+2\operatorname{Re}\left(\hat{u}_t\bar{\hat{u}}\right)\right\}+k_2|\xi|^4|\hat{u}|^2-|\hat{u}_t|^2=0. \tag{20}$$

Multiplying both sides of (19) and (20) by 2 and $k_1|\xi|^4$ and summing up the resulting equation yield

$$\frac{d}{dt}E+F=0, \tag{21}$$

where

$$E=|\hat{u}_t|^2+k_2|\xi|^4|\hat{u}|^2+\frac{1}{2}|\xi|^8|\hat{u}|^2+k_1|\xi|^4\operatorname{Re}\left(\hat{u}_t\bar{\hat{u}}\right),\qquad F=k_1k_2|\xi|^8|\hat{u}|^2+k_1|\xi|^4|\hat{u}_t|^2. \tag{22}$$

A simple computation implies that

$$CE_0\le E\le CE_0, \tag{23}$$

where

$$E_0=\left(|\xi|^4+|\xi|^8\right)|\hat{u}|^2+|\hat{u}_t|^2. \tag{24}$$

Note that

$$F\ge\begin{cases}c|\xi|^4\left(\left(|\xi|^4+|\xi|^8\right)|\hat{u}|^2+|\hat{u}_t|^2\right), & |\xi|\le R_0\\ c\left(\left(|\xi|^4+|\xi|^8\right)|\hat{u}|^2+|\hat{u}_t|^2\right), & |\xi|\ge R_0.\end{cases} \tag{25}$$

It follows from (23) that

$$F\ge c\omega(\xi)E. \tag{26}$$

Using (21) and (26), we get

$$\frac{d}{dt}E+c\omega(\xi)E\le 0. \tag{27}$$

Thus

$$E(\xi,t)\le e^{-c\omega(\xi)t}E(\xi,0), \tag{28}$$

which together with (23) proves the desired estimates (17). Then we have completed the proof of the lemma. □

Lemma 2. *Let* $\widehat{G}(\xi,t)$ *and* $\widehat{H}(\xi,t)$ *be the fundamental solution of* (4) *in the Fourier space, which are given in* (13) *and* (14), *respectively. Then one has the estimates*

$$\left(|\xi|^4+|\xi|^8\right)\left|\widehat{G}(\xi,t)\right|^2+\left|\widehat{G}_t(\xi,t)\right|^2\le Ce^{-c\omega(\xi)t}, \tag{29}$$

$$\left(|\xi|^4+|\xi|^8\right)\left|\widehat{H}(\xi,t)\right|^2+\left|\widehat{H}_t(\xi,t)\right|^2\le C\left(|\xi|^4+|\xi|^8\right)e^{-c\omega(\xi)t} \tag{30}$$

for $\xi\in\mathbb{R}^n$ *and* $t\ge 0$, *where*

$$\omega(\xi)=\begin{cases}|\xi|^4, & |\xi|\le R_0\\ 1, & |\xi|\ge R_0.\end{cases} \tag{31}$$

Proof. If $\hat{u}_0(\xi)=0$, from (12), we obtain

$$\hat{u}(\xi,t)=\widehat{G}(\xi,t)\hat{u}_1(\xi),\qquad \hat{u}_t(\xi,t)=\widehat{G}_t(\xi,t)\hat{u}_1(\xi). \tag{32}$$

Substituting the equalities into (17) with $\hat{u}_0(\xi)=0$, we get (29).

In what follows, we consider $\hat{u}_1(\xi)=0$, and it follows from (12) that

$$\hat{u}(\xi,t)=\widehat{H}(\xi,t)\hat{u}_0(\xi),\qquad \hat{u}_t(\xi,t)=\widehat{H}_t(\xi,t)\hat{u}_0(\xi). \tag{33}$$

Substituting the equalities into (17) with $\hat{u}_1(\xi)=0$, we get the desired estimate (30). The lemma is proved. □

Lemma 3. *Let* k *and* j *be nonnegative integer. Then one has*

$$\left\|\partial_x^k G(t)*\phi\right\|_{L^2}\le C(1+t)^{-(n/8+k/4+j/4-1/2)}\times\|\phi\|_{\dot{H}_1^{-j}}+Ce^{-ct}\left\|\partial_x^{(k-4)_+}\phi\right\|_{L^2}, \tag{34}$$

$$\left\|\partial_x^k H(t)*\phi\right\|_{L^2}\le C(1+t)^{-(n/8+k/4+j/4)}\times\|\phi\|_{\dot{H}_1^{-j}}+Ce^{-ct}\left\|\partial_x^{k}\phi\right\|_{L^2}, \tag{35}$$

$$\left\|\partial_x^k G_t(t)*\phi\right\|_{L^2}\le C(1+t)^{-(n/8+k/4+j/4)}\times\|\phi\|_{\dot{H}_1^{-j}}+Ce^{-ct}\left\|\partial_x^{k}\phi\right\|_{L^2}, \tag{36}$$

$$\left\|\partial_x^k H_t(t)*\phi\right\|_{L^2}\le C(1+t)^{-(n/8+k/4+j/4+1/2)}\times\|\phi\|_{\dot{H}_1^{-j}}+Ce^{-ct}\left\|\partial_x^{(k+4)}\phi\right\|_{L^2}, \tag{37}$$

$$\left\|\partial_x^k G(t)*\Delta g\right\|_{L^2}\le C(1+t)^{-(n/8+k/4)}\times\|g\|_{L^1}+Ce^{-ct}\left\|\partial_x^{(k-2)_+}g\right\|_{L^2}, \tag{38}$$

$$\left\|\partial_x^k G_t(t)*\Delta g\right\|_{L^2}\le C(1+t)^{-(n/8+k/4+1/2)}\times\|g\|_{L^1}+Ce^{-ct}\left\|\partial_x^{(k+2)}g\right\|_{L^2}. \tag{39}$$

Here $(k-4)_+=\max\{0,k-4\}$ *in* (34) *and* $(k-2)_+=\max\{0,k-2\}$ *in* (38).

Proof. By the Plancherel theorem and (29), Hausdorff-Young inequality, we obtain

$$\begin{aligned}
&\left\|\partial_x^k G(t) * \phi\right\|_{L^2}^2 \\
&= \int_{|\xi|\le R_0} |\xi|^{2|k|}\left|\widehat{G}(\xi,t)\right|^2\left|\widehat{\phi}(\xi)\right|^2 d\xi \\
&\quad + \int_{|\xi|\ge R_0} |\xi|^{2|k|}\left|\widehat{G}(\xi,t)\right|^2\left|\widehat{\phi}(\xi)\right|^2 d\xi \\
&\le \int_{|\xi|\le R_0} |\xi|^{2|k|-4} e^{-c|\xi|^4 t}\left|\widehat{\phi}(\xi)\right|^2 d\xi \\
&\quad + Ce^{-ct}\int_{|\xi|\ge R_0} |\xi|^{2k}\left(|\xi|^8+|\xi|^4\right)^{-1}\left|\widehat{\phi}(\xi)\right|^2 d\xi \\
&\le \int_{|\xi|\le R_0} |\xi|^{2k-4+2j} e^{-c|\xi|^4 t}|\xi|^{-2j}\left|\widehat{\phi}(\xi)\right|^2 d\xi \\
&\quad + Ce^{-ct}\left\|\partial_x^{(k-4)_+}\phi\right\|_{L^2}^2 \\
&\le C\left\||\xi|^{-j}\widehat{\phi}(\xi)\right\|_{L^\infty}^2 \int_{|\xi|\le R_0} |\xi|^{2k-4+2j} e^{-c|\xi|^4 t} d\xi \\
&\quad + Ce^{-ct}\left\|\partial_x^{(k-4)_+}\phi\right\|_{L^2}^2 \\
&\le C(1+t)^{-(n/4+k/2+j/2-1)}\left\|(-\Delta)^{-j/2}\phi\right\|_{L^1}^2 \\
&\quad + Ce^{-ct}\left\|\partial_x^{(k-4)_+}\phi\right\|_{L^2}^2 .
\end{aligned} \tag{40}$$

Here $(k-4)_+ = \max\{0, k-4\}$ and R_0 is a small positive constant in Lemma 1. Thus (34) follows.

Similarly, using (29) and (30), respectively, we can prove (35)–(37).

In what follows, we prove (38). By the Plancherel theorem, (29), and Hausdorff-Young inequality, we have

$$\begin{aligned}
&\left\|\partial_x^k G(t) * \Delta g\right\|_{L^2}^2 \\
&= \int_{|\xi|\le R_0} |\xi|^{2|k|}\left|\widehat{G}(\xi,t)\right|^2|\xi|^4\left|\widehat{g}(\xi)\right|^2 d\xi \\
&\quad + \int_{|\xi|\ge R_0} |\xi|^{2k}\left|\widehat{G}(\xi,t)\right|^2|\xi|^4\left|\widehat{g}(\xi)\right|^2 d\xi \\
&\le \int_{|\xi|\le R_0} |\xi|^{2k} e^{-c|\xi|^4 t}\left|\widehat{g}(\xi)\right|^2 d\xi \\
&\quad + Ce^{-ct}\int_{|\xi|\ge R_0} |\xi|^{2k}\left(1+|\xi|^4\right)^{-1}\left|\widehat{g}(\xi)\right|^2 d\xi \\
&\le C\left\|\widehat{g}(\xi)\right\|_{L^\infty}^2 \int_{|\xi|\le R_0} |\xi|^{2k} e^{-c|\xi|^4 t} d\xi \\
&\quad + Ce^{-ct}\left\|\partial_x^{(k-2)_+} g\right\|_{L^2}^2 \\
&\le C(1+t)^{-(n/4+k/2)}\|g\|_{L^1}^2 + Ce^{-ct}\left\|\partial_x^{(k-2)_+} g\right\|_{L^2}^2 ,
\end{aligned} \tag{41}$$

where R_0 is a small positive constant in Lemma 1. Thus (38) follows. Similarly, we can prove (39). Thus we have completed the proof of the lemma. □

4. Decay Estimate of Solutions to (4), (2)

Theorem 4. *Assume that* $u_0 \in H^{s+4}(\mathbb{R}^n)\bigcap L^1(\mathbb{R}^n)$, $u_1 \in H^s(\mathbb{R}^n)\bigcap \dot{H}_1^{-2}(\mathbb{R}^n)(s \ge [n/2]+1)$. *Then the classical solution* $u(x,t)$ *to* (4), (2), *which is given by the formula* (15), *satisfies the decay estimate*

$$\begin{aligned}
\left\|\partial_x^k u(t)\right\|_{L^2} &\le C(1+t)^{-(n/8+k/4)} \\
&\quad \times \left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}} + \|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right) \\
&\qquad \text{for } (k \le s+4),
\end{aligned} \tag{42}$$

$$\begin{aligned}
\left\|\partial_x^l u_t(t)\right\|_{L^2} &\le C(1+t)^{-(n/8+l/4+1/2)} \\
&\quad \times \left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}} + \|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right) \\
&\qquad \text{for } (l \le s),
\end{aligned} \tag{43}$$

$$\begin{aligned}
\left\|\partial_x^h u(t)\right\|_{L^\infty} &\le C(1+t)^{-(n/4+h/4)} \\
&\quad \times \left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}} + \|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right) \\
&\qquad \text{for } \left(h \le s - \left[\frac{n}{2}\right] + 3\right).
\end{aligned} \tag{44}$$

Proof. Firstly, we prove (42). Using (34) and (35), for $k \le s+4$, we obtain

$$\begin{aligned}
&\left\|\partial_x^k u(t)\right\|_{L^2} \\
&\le \left\|\partial_x^k G(t) * u_1\right\|_{L^2} + C\left\|\partial_x^k H(t) * u_0\right\|_{L^2} \\
&\le C(1+t)^{-(n/8+k/4)}\left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}}\right) \\
&\quad + Ce^{-ct}\left(\|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right) \\
&\le C(1+t)^{-(n/8+k/4)} \\
&\quad \times \left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}} + \|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right).
\end{aligned} \tag{45}$$

For $l \le s$, it follows from (36) and (37) that

$$\begin{aligned}
&\left\|\partial_x^l u_t(t)\right\|_{L^2} \\
&\le \left\|\partial_x^l G_t(t) * u_1\right\|_{L^2} + C\left\|\partial_x^l H_t(t) * u_0\right\|_{L^2}
\end{aligned}$$

$$\le C(1+t)^{-(n/8+l/4+1/2)}\left(\|u_0\|_{L^1}+\|u_1\|_{\dot H_1^{-2}}\right)$$
$$+Ce^{-ct}\left(\|u_0\|_{H^{s+4}}+\|u_1\|_{H^s}\right)$$
$$\le C(1+t)^{-(n/8+l/4+1/2)}$$
$$\times\left(\|u_0\|_{L^1}+\|u_1\|_{\dot H_1^{-2}}+\|u_0\|_{H^{s+4}}+\|u_1\|_{H^s}\right). \tag{46}$$

Equation (44) follows from (42) and Gagliardo-Nirenberg inequality. The lemma is proved. □

5. Global Existence and Asymptotic Behavior

The purpose of this section is to prove global existence and asymptotic behavior of solutions to the Cauchy problem (1), (2). We need the following lemma, which comes from [24] (see also [25]).

Lemma 5. *Let s and θ be positive integers, $\delta > 0$, $p, q, r \in [1,\infty]$ satisfy $1/r = 1/p + 1/r$, and let $k \in \{0,1,2,\ldots,s\}$. Assume that $F(v)$ is class of C^s and satisfies*

$$\left|\partial_v^l F(v)\right| \le C_{l,\delta}|v|^{\theta+1-l},\quad |v|\le\delta,\ 0\le l\le s,\ l<\theta+1,$$
$$\left|\partial_v^l F(v)\right| \le C_{l,\delta},\quad |v|\le\delta,\ l\le s,\ \theta+1\le l. \tag{47}$$

If $v \in L^p\bigcap W^{k,q}\bigcap L^\infty$ and $\|v\|_{L^\infty}\le\delta$, then for $|\alpha|\le k$ one has

$$\|F(v)\|_{W^{k,r}} \le C_{k,\delta}\|v\|_{W^{k,q}}\|v\|_{L^p}\|v\|_{L^\infty}^{\theta-1},$$
$$\left\|\partial_x^\alpha F(v)\right\|_{L^r} \le C_{k,\delta}\left\|\partial_x^\alpha v\right\|_{L^q}\|v\|_{L^p}\|v\|_{L^\infty}^{\theta-1}. \tag{48}$$

Lemma 6. *Let s and θ be positive integers, let $\delta > 0$, $p, q, r \in [1,\infty]$ satisfy $1/r = 1/p + 1/r$, and let $k \in \{0,1,2,\ldots,s\}$. Let $F(v)$ be a function that satisfies the assumptions of Lemma 5. Moreover, assume that*

$$\left|\partial_v^s F(v_1)-\partial_v^s F(v_2)\right|$$
$$\le C_\delta\left(|v_1|+|v_2|\right)^{\max\{\theta-s,\theta\}}|v_1-v_2|, \tag{49}$$
$$|v_1|\le\delta,\ |v_2|\le\delta.$$

If $v_1, v_2 \in L^p\bigcap W^{k,q}\bigcap L^\infty$ and $\|v_1\|_{L^\infty}\le\delta$, $\|v_2\|_{L^\infty}\le\delta$, then for $|\alpha|\le k$, one has

$$\left\|\partial_x^\alpha\left(F(v_1)-F(v_2)\right)\right\|_{L^r}$$
$$\le C_{k,\delta}\left\{\left(\left\|\partial_x^\alpha v_1\right\|_{L^q}+\left\|\partial_x^\alpha v_2\right\|_{L^q}\right)\|v_1-v_2\|_{L^p}\right.$$
$$\left.+\left(\|v_1\|_{L^p}+\|v_2\|_{L^p}\right)\left\|\partial_x^\alpha(v_1-v_2)\right\|_{L^q}\right\} \tag{50}$$
$$\times\left(\|v_1\|_{L^\infty}+\|v_2\|_{L^\infty}\right)^{\theta-1}.$$

Based on the estimates (42)–(44) *of solutions to the linear problem* (4), (2), *one defines the following solution space:*

$$X=\left\{u\in C\left([0,\infty);H^{s+4}(\mathbb{R}^n)\right)\right.$$
$$\left.\bigcap C^1\left([0,\infty);H^s(\mathbb{R}^n)\right):\|u\|_X<\infty\right\}, \tag{51}$$

where

$$\|u\|_X=\sup_{t\ge0}\left\{\sum_{k\le s+4}(1+t)^{n/8+k/4}\left\|\partial_x^k u(t)\right\|_{L^2}\right.$$
$$\left.+\sum_{l\le s}(1+t)^{n/8+l/4+1/2}\left\|\partial_x^l u_t(t)\right\|_{L^2}\right\}. \tag{52}$$

For $R > 0$, one defines

$$X_R=\left\{u\in X:\|u\|_X\le R\right\}, \tag{53}$$

where R depends on the initial value, which is chosen in the proof of main result.

For $h \le s-[n/2]+3$, using Gagliardo-Nirenberg inequality, one obtains

$$\left\|\partial_x^h u(t)\right\|_{L^\infty}\le C(1+t)^{-(n/4+h/4)}\|u\|_X. \tag{54}$$

Theorem 7. *Assume that $u_0 \in H^{s+4}(\mathbb{R}^n)\bigcap L^1(\mathbb{R}^n)$, $u_1 \in H^s(\mathbb{R}^n)\bigcap \dot H_1^{-2}(\mathbb{R}^n)\left(s\ge\left[\frac{n}{2}\right]+1\right)$, and integer $\theta \ge 1$. $f(u)$ satisfies the assumptions of Lemmas 5 and 6. Put*

$$E_0=\|u_0\|_{L^1}+\|u_1\|_{\dot H_1^{-2}}+\|u_0\|_{H^{s+4}}+\|u_1\|_{H^s}. \tag{55}$$

If E_0 is suitably small, the Cauchy problem (1)-(2) *has a unique global classical solution $u(x,t)$ satisfying*

$$u\in C\left([0,\infty);H^{s+4}(\mathbb{R}^n)\right),$$
$$u_t\in C\left([0,\infty);H^s(\mathbb{R}^n)\right). \tag{56}$$

Moreover, the solution satisfies the decay estimate

$$\left\|\partial_x^k u(t)\right\|_{L^2}\le CE_0(1+t)^{-(n/8+k/4)},$$
$$\left\|\partial_x^l u_t(t)\right\|_{L^2}\le CE_0(1+t)^{-(n/8+l/4+1/2)} \tag{57}$$

for $k \le s+4$ and $l \le s$.

Proof. Define the mapping

$$\mathbb{T}(u)=G(t)*u_1+H(t)*u_0$$
$$+\int_0^t G(t-\tau)*\Delta f(\Delta u(\tau))\,d\tau. \tag{58}$$

Using (34)-(35), (38), Lemma 5, and (54), for $k \le s+4$, we obtain

$$\left\|\partial_x^k\mathbb{T}(u)\right\|_{L^2}$$
$$\le C\left\|\partial_x^k G(t)*u_1\right\|_{L^2}+C\left\|\partial_x^k H(t)*u_0\right\|_{L^2}$$
$$+C\int_0^t\left\|\partial_x^k G(t-\tau)*\Delta f(\Delta u(\tau))\right\|_{L^2}d\tau$$

$$\begin{aligned}
&\le C(1+t)^{-(n/8+k/4)}\|u_1\|_{\dot{H}_1^{-2}} + Ce^{-ct}\left\|\partial_x^{(k-4)_+}u_1\right\|_{L^2} \\
&\quad + C(1+t)^{-(n/8+k/4)}\|u_0\|_{L^1} + Ce^{-ct}\left\|\partial_x^k u_0\right\|_{L^2} \\
&\quad + C\int_0^{t/2}(1+t-\tau)^{-(n/8+k/4)}\|f(\Delta u)\|_{L^1}d\tau \\
&\quad + C\int_{t/2}^{t}(1+t-\tau)^{-n/8}\left\|\partial_x^k f(\Delta u)\right\|_{L^1}d\tau \\
&\quad + C\int_0^{t}e^{-c(t-\tau)}\left\|\partial_x^{(k-2)_+}f(\Delta u)\right\|_{L^2}d\tau \\
&\le C(1+t)^{-(n/8+k/4)}\left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}}\right) \\
&\quad + Ce^{-ct}\left(\|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right) \\
&\quad + C\int_0^{t/2}(1+t-\tau)^{-(n/8+k/4)}\|\Delta u\|_{L^2}^2\|\Delta u\|_{L^\infty}^{\theta-1}d\tau \\
&\quad + C\int_{t/2}^{t}(1+t-\tau)^{-n/8-1/2}\|\Delta u\|_{L^2} \\
&\quad \times\left\|\partial_x^{(k-2)_+}\Delta u\right\|_{L^2}\|\Delta u\|_{L^\infty}^{\theta-1}d\tau \\
&\quad + C\int_0^{t}e^{-c(t-\tau)}\left\|\partial_x^{(k-2)_+}\Delta u\right\|_{L^2}\|\Delta u\|_{L^\infty}^{\theta}d\tau \\
&\le C(1+t)^{-(n/8+k/4)}\left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}}\right) \\
&\quad + Ce^{-ct}\left(\|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right) \\
&\quad + CR^{\theta+1}\int_0^{t/2}(1+t-\tau)^{-(n/8+k/4)}(1+\tau)^{-(n/4+1)} \\
&\quad \times(1+\tau)^{-(n/4+1/2)(\theta-1)}d\tau \\
&\quad + CR^{\theta+1}\int_{t/2}^{t}(1+t-\tau)^{-n/8-1/2}(1+\tau)^{-(n/4+k/4)} \\
&\quad \times(1+\tau)^{-(n/4+1/2)(\theta-1)}d\tau \\
&\quad + CR^{\theta+1}\int_0^{t}e^{-c(t-\tau)}(1+\tau)^{-(n/8+((k-2)_++2)/4)} \\
&\quad \times(1+\tau)^{-(n/4+1/2)\theta}d\tau \\
&\le C(1+t)^{-(n/8+k/4)} \\
&\quad \times\left\{\left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}} + \|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right) + R^{\theta+1}\right\}.
\end{aligned} \tag{59}$$

Thus

$$(1+t)^{n/8+k/4}\left\|\partial_x^k\mathbb{T}(u)\right\|_{L^2} \le CE_0 + CR^{\theta+1}. \tag{60}$$

It follows from (58) that

$$\begin{aligned}
\mathbb{T}(u)_t &= G_t(t)*u_1 + H_t(t)*u_0 \\
&\quad + \int_0^t G_t(t-\tau)*\Delta f(\Delta u(\tau))\,d\tau.
\end{aligned} \tag{61}$$

Using (36)-(37), (39), Lemma 5, and (54), for $l \le s$, we have

$$\begin{aligned}
&\left\|\partial_x^l\mathbb{T}(u)_t\right\|_{L^2} \\
&\quad \le C\left\|\partial_x^l G_t(t)*u_1\right\|_{L^2} + C\left\|\partial_x^l H_t(t)*u_0\right\|_{L^2} \\
&\qquad + C\int_0^t\left\|\partial_x^l G_t(t-\tau)*\Delta f(\Delta u(\tau))\right\|_{L^2}d\tau \\
&\quad \le C(1+t)^{-(n/8+l/4+1/2)}\|u_1\|_{\dot{H}_1^{-2}} + Ce^{-ct}\left\|\partial_x^l u_1\right\|_{L^2} \\
&\qquad + C(1+t)^{-(n/8+l/4+1/2)}\|u_0\|_{L^1} + Ce^{-ct}\left\|\partial^{l+4}u_0\right\|_{L^2} \\
&\qquad + C\int_0^{t/2}(1+t-\tau)^{-(n/8+l/4+1/2)}\|f(\Delta u)\|_{L^1}d\tau \\
&\qquad + C\int_{t/2}^{t}(1+t-\tau)^{-(n/8+1/2)}\left\|\partial_x^l f(\Delta u)\right\|_{L^1}d\tau \\
&\qquad + C\int_0^{t}e^{-c(t-\tau)}\left\|\partial_x^{l+2}f(\Delta u)\right\|_{L^2}d\tau \\
&\quad \le C(1+t)^{-(n/8+l/4+1/2)} \\
&\qquad \times\left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}} + \|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right) \\
&\qquad + C\int_0^{t/2}(1+t-\tau)^{-(n/8+l/4+1/2)}\|\Delta u\|_{L^2}^2\|\Delta u\|_{L^\infty}^{\theta-1}d\tau \\
&\qquad + C\int_{t/2}^{t}(1+t-\tau)^{-(n/8+1/2)}\left\|\partial_x^l\Delta u\right\|_{L^2}^2\|\Delta u\|_{L^\infty}^{\theta-1}d\tau \\
&\qquad + C\int_0^{t}e^{-c(t-\tau)}\left\|\partial_x^{l+4}u\right\|_{L^2}\|\Delta u\|_{L^\infty}^{\theta}d\tau \\
&\quad \le C(1+t)^{-(n/8+l/4+1/2)} \\
&\qquad \times\left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}} + \|u_0\|_{H^{s+4}} + \|u_1\|_{H^s}\right) \\
&\qquad + CR^{\theta+1}\int_0^{t/2}(1+t-\tau)^{-(n/8+l/4+1/2)}(1+\tau)^{-(n/4+1)} \\
&\qquad \times(1+\tau)^{-(n/4+1/2)(\theta-1)}d\tau \\
&\qquad + CR^{\theta+1}\int_{t/2}^{t}(1+t-\tau)^{-(n/8+1/2)}(1+\tau)^{-(n/4+l/2+1)} \\
&\qquad \times(1+\tau)^{-(n/4+1/2)(\theta-1)}d\tau \\
&\qquad + CR^{\theta+1}\int_0^{t}e^{-c(t-\tau)}(1+\tau)^{-(n/8+(l+4)/4)}
\end{aligned}$$

$$\times (1+\tau)^{-(n/4+1/2)\theta} d\tau$$
$$\le C(1+t)^{-(n/8+l/4+1/2)}$$
$$\times \left\{ \left(\|u_0\|_{L^1} + \|u_1\|_{\dot{H}_1^{-2}} + \|u_0\|_{H^{s+4}} + \|u_1\|_{H^s} \right) + R^{\theta+1} \right\}. \quad (62)$$

Thus

$$(1+t)^{n/8+l/4+1/2} \left\| \partial_x^l \mathbb{T}(u)_t \right\|_{L^2} \le CE_0 + CR^{\theta+1}. \quad (63)$$

Combining (60), (63) and taking $R = 2CE_0$ and E_0 suitably small yield

$$\|\mathbb{T}(u)\|_X \le 2CE_0. \quad (64)$$

For $\tilde{u}, \overline{u} \in X_R$, by using (58), we have

$$\mathbb{T}(\tilde{u}) - \mathbb{T}(\overline{u}) = \int_0^t G(t-\tau) * \Delta \left[f(\Delta\tilde{u}) - f(\Delta\overline{u}) \right] d\tau. \quad (65)$$

Exploiting (65), (38) Lemma 6, and (54), for $k \le s+4$, we obtain

$$\begin{aligned}
&\left\| \partial_x^k (\mathbb{T}(\tilde{u}) - \mathbb{T}(\overline{u})) \right\|_{L^2} \\
&\quad \le \int_0^t \left\| \partial_x^k G(t-\tau) * \Delta \left[f(\Delta\tilde{u}) - f(\Delta\overline{u}) \right] \right\|_{L^2} d\tau \\
&\quad \le C \int_0^{t/2} (1+t-\tau)^{-(n/8+k/4)} \left\| \left(f(\Delta\tilde{u}) - f(\Delta\overline{u}) \right) \right\|_{L^1} d\tau \\
&\qquad + C \int_{t/2}^t (1+t-\tau)^{-n/8} \left\| \partial_x^k \left(f(\Delta\tilde{u}) - f(\Delta\overline{u}) \right) \right\|_{L^1} d\tau \\
&\qquad + C \int_0^t e^{-c(t-\tau)} \left\| \partial_x^{(k-2)_+} \left(f(\Delta\tilde{u}) - f(\Delta\overline{u}) \right) \right\|_{L^2} d\tau \\
&\quad \le C \int_0^{t/2} (1+t-\tau)^{-(n/8+k/4)} \left(\|\Delta\tilde{u}\|_{L^2} + \|\Delta\overline{u}\|_{L^2} \right) \\
&\qquad \times \|\Delta(\tilde{u} - \overline{u})\|_{L^2} \left(\|\Delta\tilde{u}\|_{L^\infty} + \|\Delta\overline{u}\|_{L^\infty} \right)^{\theta-1} d\tau \\
&\qquad + C \int_{t/2}^t (1+t-\tau)^{-n/8-1/2} \\
&\qquad \times \left\{ \left(\left\| \partial_x^{(k-2)_+} \Delta\tilde{u} \right\|_{L^2} + \left\| \partial_x^{(k-2)_+} \Delta\tilde{u} \right\|_{L^2} \right) \|\Delta(\tilde{u} - \overline{u})\|_{L^2} \right. \\
&\qquad\qquad \left. + \left(\|\Delta\tilde{u}\|_{L^2} + \|\Delta\overline{u}\|_{L^2} \right) \left\| \partial_x^{(k-2)_+} \Delta(\tilde{u} - \overline{u}) \right\|_{L^2} \right\} \\
&\qquad \times \left(\|\Delta\tilde{u}\|_{L^\infty} + \|\Delta\overline{u}\|_{L^\infty} \right)^{\theta-1} d\tau \\
&\qquad + C \int_0^t e^{-c(t-\tau)} \\
&\qquad \times \left\{ \left(\left\| \partial_x^{(k-2)_+} \Delta\tilde{u} \right\|_{L^2} + \left\| \partial_x^{(k-2)_+} \Delta\tilde{u} \right\|_{L^2} \right) \right. \\
&\qquad\qquad \times \|\Delta(\tilde{u} - \overline{u})\|_{L^\infty} + \left(\|\Delta\tilde{u}\|_{L^\infty} + \|\Delta\overline{u}\|_{L^\infty} \right) \\
&\qquad\qquad \left. \times \left\| \partial_x^{(k-2)_+} \Delta(\tilde{u} - \overline{u}) \right\|_{L^2} \right\} \\
&\qquad \times \left(\|\Delta\tilde{u}\|_{L^\infty} + \|\Delta\overline{u}\|_{L^\infty} \right)^{\theta-1} d\tau \\
&\quad \le CR^\theta \|\tilde{u} - \overline{u}\|_X \int_0^{t/2} (1+t-\tau)^{-(n/8+k/4)} \\
&\qquad \times (1+\tau)^{-(n/4+1+(n/4+1/2)(\theta-1))} d\tau \\
&\qquad + CR^\theta \|\tilde{u} - \overline{u}\|_X \int_{t/2}^t (1+t-\tau)^{-n/8-1/2} \\
&\qquad \times (1+\tau)^{-(n/4+k/4+1/2+(n/4+1/2)(\theta-1))} d\tau \\
&\qquad + CR^\theta \|\tilde{u} - \overline{u}\|_X \int_0^t e^{-c(t-\tau)} \\
&\qquad \times (1+\tau)^{-(n/8+((k-2)_+ +2)/4+(n/4+1/2)\theta)} d\tau \\
&\quad \le CR^\theta (1+t)^{-(n/8+k/4)} \|\tilde{u} - \overline{u}\|_X,
\end{aligned} \quad (66)$$

which implies

$$(1+t)^{n/8+k/4} \left\| \partial_x^k (\mathbb{T}(\tilde{u}) - \mathbb{T}(\overline{u})) \right\|_{L^2} \le CR^\theta \|\tilde{u} - \overline{u}\|_X. \quad (67)$$

Similarly for $l \le s$, from (61), (39), and (54), we have

$$\begin{aligned}
&\left\| \partial_x^l (\mathbb{T}(\tilde{u}) - \mathbb{T}(\overline{u}))_t \right\|_{L^2} \\
&\quad \le \int_0^t \left\| \partial_x^l G_t(t-\tau) * \Delta \left[f(\Delta\tilde{u}) - f(\Delta\overline{u}) \right] \right\|_{L^2} d\tau \\
&\quad \le C \int_0^{t/2} (1+t-\tau)^{-(n/8+l/4+1/2)} \left\| \left(f(\Delta\tilde{u}) - f(\Delta\overline{u}) \right) \right\|_{L^1} d\tau \\
&\qquad + C \int_{t/2}^t (1+t-\tau)^{-(n/8+1/2)} \left\| \partial_x^l \left(f(\Delta\tilde{u}) - f(\Delta\overline{u}) \right) \right\|_{L^1} d\tau \\
&\qquad + C \int_0^t e^{-c(t-\tau)} \left\| \partial_x^{l+2} \left(f(\Delta\tilde{u}) - f(\Delta\overline{u}) \right) \right\|_{L^2} d\tau \\
&\quad \le CR^\theta (1+t)^{-(n/8+l/4+1/2)} \|\tilde{u} - \overline{u}\|_X,
\end{aligned} \quad (68)$$

which implies

$$(1+t)^{n/8+l/4+1/2} \left\| \partial_x^l (\mathbb{T}(\tilde{u}) - \mathbb{T}(\overline{u}))_t \right\|_{L^2} \le CR^\theta \|\tilde{u} - \overline{u}\|_X. \quad (69)$$

Noting $R = 2CE_0$, by using (67), (69) and taking E_0 suitably small, yields

$$\|\mathbb{T}(\tilde{u}) - \mathbb{T}(\overline{u})\|_X \leq \frac{1}{2}\|\tilde{u} - \overline{u}\|_X. \tag{70}$$

From (64) and (70), we know that $\mathbb{T}$ is strictly contracting mapping. Consequently, we conclude that there exists a fixed point $u \in X_R$ of the mapping $\mathbb{T}$, which is a classical solution to (1), (2). We have completed the proof of Theorem 7. □

Acknowledgments

This work was supported in part by the NNSF of China (Grant no. 11101144) and Innovation Scientists and Technicians Troop Construction Projects of Henan Province.

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