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Research Article

Multiple-Set Split Feasibility Problems for κ-Strictly Pseudononspreading Mapping in Hilbert Spaces

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The purpose of this paper is to prove some weak and strong convergence theorems for solving the multiple-set split feasibility problems for κ-strictly pseudononspreading mapping in infinite-dimensional Hilbert spaces by using the proposed iterative method. The main results presented in this paper extend and improve the corresponding results of Xu et al. (2006), of Osilike et al. (2011), and of many other authors.

1. Introduction and Preliminaries

Censor and Elfving first introduced the split feasibility problem (SFP) [1] in finite dimensional spaces for modeling inverse problems. The SFP can be used in various disciplines such as medical image reconstruction [2], image restoration, computer tomograph, and radiation therapy treatment planning [3–5]. The multiple-set split feasibility problem (MSSFP) was studied in [4–6].

In the sequel, we always assume that H_1, H_2 are two real Hilbert spaces and denote by "$\rightarrow$" and "$\rightharpoonup$" the strong and weak convergence, respectively.

The so-called *multiple-set split feasibility problem* (MSSFP) is to find $x^* \in C$ such that $Ax^* \in Q$, where $A : H_1 \rightarrow H_2$ is a bounded linear operator, S_i and T_i, $i = 1, 2, \ldots, N$ are the families of mappings, $S_i : H_1 \rightarrow H_1$ and $T_i : H_2 \rightarrow H_2$, $C := \bigcap_{i=1}^{N} F(S_i)$, and $Q := \bigcap_{i=1}^{N} F(T_i)$, where $F(S_i) = \{x \in H_1 : S_i x = x\}$ and $F(T_i) = \{y \in H_2 : T_i y = y\}$ denote the sets of fixed points of S_i and T_i, respectively. In the sequel, we use Γ to denote the set of solutions of the MSSFP; that is,

$$\Gamma = \{x \in C : Ax \in Q\}. \tag{1}$$

Recently, Kohsaka and Takahashi [7, 8] introduced an important class of mappings which is called the class of *nonspreading mappings*.

Definition 1 (see [7, 8]). Let K to be a nonempty closed convex subset of a Hilbert space H. A mapping $T : K \rightarrow K$ is said to be nonspreading, if

$$2\|Tx - Ty\|^2 \leq \|Tx - y\|^2 + \|Ty - x\|^2, \quad \forall x, y \in K. \tag{2}$$

In [9], Iemoto and Takahashi proved that this definition is equivalent to the following.

Definition 2 (see [9]). Let K be a nonempty closed convex subset of a Hilbert space H. A mapping $T : K \rightarrow K$ is said to be nonspreading, if

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + 2\langle x - Tx, y - Ty\rangle, \quad \forall x, y \in K. \tag{3}$$

Browder and Petryshyn [10] proposed the following κ-*strictly pseudononspreading mapping*.

Definition 3 (see [10]). Let H be a real Hilbert space. We say that a mapping $T : D(T) \subset H \to H$ is κ-strictly pseudononspreading if there exists $\kappa \in [0,1)$, such that

$$\begin{aligned}\|Tx - Ty\|^2 \le \|x - y\|^2 + \kappa\|x - Tx - (y - Ty)\|^2 \\ + 2\langle x - Tx, y - Ty\rangle, \quad \forall x, y \in D(T).\end{aligned} \tag{4}$$

Clearly every nonspreading mapping is κ-strictly pseudononspreading.

Osilike and Isiogugu [11] introduced a class of nonspreading type mappings which is more general than that of the mappings studied in [12] in Hilbert spaces and proved some weak and strong convergence theorems in real Hilbert spaces. Recently, the split feasibility problem also was considered in the work by Deepho and Kumam [13, 14] and Sunthrayuth et al. [15], and some weak and strong convergence theorems are proved in real Hilbert spaces.

The purpose of this paper is to study the multiple-set split feasibility problem (MSSFP) for κ-strictly pseudononspreading mappings in the framework of infinite dimensional Hilbert spaces.

In the sequel, we recall some definitions, notations, and conclusions which will be needed in proving our main results.

Definition 4 (see [3]). Let E be a real Banach space. A mapping T with domain $D(T)$ and range $R(T)$ in E is said to be demiclosed at origin if for any sequence $\{x_n\}$ in $D(T)$ which converges weakly to a point $x^* \in D(T)$ and $\|(I - T)x_n\|$ converges strongly to 0, then $Tx^* = x^*$.

Definition 5. A Banach space E is said to have *Opial property* if, for any sequence $\{x_n\}$ with $x_n \rightharpoonup x^*$, we have

$$\liminf_{n\to\infty} \|x_n - x^*\| < \liminf_{n\to\infty} \|x_n - y\| \tag{5}$$

for all $y \in E$ with $y \ne x^*$.

Remark 6. It is well known that each Hilbert space possesses Opial property.

Definition 7. A mapping $S : K \to K$ is said to be semicompact, if, for any bounded sequence $\{x_n\} \subset K$ with $\lim_{n\to\infty}\|x_n - Sx_n\| = 0$, there exists a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $\{x_{n_i}\}$ converges strongly to some point $x^* \in K$.

Lemma 8 (see [11]). *Let H be a real Hilbert space; then the following results hold.*

(i) *For all $x, y \in H$ and for all $t \in [0,1]$,*

$$\begin{aligned}\|tx + (1-t)y\|^2 = t\|x\|^2 + (1-t)\|y\|^2 \\ - t(1-t)\|x - y\|^2.\end{aligned} \tag{6}$$

(ii) $\|x + y\|^2 \le \|x\|^2 + 2\langle y, x + y\rangle$.

(iii) *If $\{x_n\}_{n=1}^{\infty}$ is a sequence in H which converges weakly to $z \in H$, then*

$$\begin{aligned}\limsup_{n\to\infty} \|x_n - y\|^2 = \limsup_{n\to\infty} \|x_n - z\|^2 \\ + \|z - y\|^2, \quad \forall y \in H.\end{aligned} \tag{7}$$

Definition 9. Let K be a nonempty closed convex subset of a real Hilbert space H. The *metric projection* $P_K : H \to K$ is a mapping such that, for each $x \in H$, $P_K x$ is the unique point in K such that $\|x - P_K x\| \le \|x - y\|$, $\forall y \in K$. It is known that, for each $x \in H$,

$$\langle x - P_K x, y - P_K x\rangle \le 0, \quad \forall y \in K. \tag{8}$$

Lemma 10 (see [11]). *Let K be a nonempty closed convex subset of a real Hilbert space H, and let $T : K \to K$ be a κ-strictly pseudononspreading mapping. If $F(T) \ne \emptyset$, then it is closed and convex.*

Lemma 11 (see [11]). *Let K be a nonempty closed convex subset of a real Hilbert space H, and let $T : K \to K$ be a κ-strictly pseudononspreading mapping. Then $(I - T)$ is demiclosed at 0.*

2. Main Results

Theorem 12. *Let H_1, H_2, A, $\{S_i\}$, $\{T_i\}$, C, Q be the same as aforementioned. For each $i = 1, 2, \ldots, N$, let T_i be a κ_i-strictly pseudononspreading mapping and let S_i be a ϱ_i-strictly pseudononspreading mapping. Let $\{x_n\}$ be the sequence generated by*

$$\begin{aligned}x_1 &\in H_1 \text{ chosen arbitrarily}, \\ u_n &= x_n + \gamma A^*\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n, \\ x_{n+1} &= (1 - \alpha_n)u_n + \alpha_n S_{n(\mathrm{mod}\,N)}u_n,\end{aligned} \tag{9}$$

where $\gamma \in (0, (1-\kappa)/\lambda)$ with λ being the spectral of the operator A^*A and $\kappa = \max\{\kappa_1, \kappa_2, \ldots, \kappa_N\} \in (0,1)$, and α_n is a sequence in $(0, 1-\varrho]$ with $\varrho = \max\{\varrho_1, \varrho_2, \ldots, \varrho_N\} \in (0,1)$. If $\Gamma \ne \emptyset$ (where Γ is the set of solutions of the MSSFP defined by (1)), then the sequence $\{x_n\}$ converges weakly to a point $x^* \in \Gamma$.

Proof. The proof is divided into four steps.

(I) We first prove that $\lim_{n\to\infty}\|x_n - p\|$ exists for any $p \in \Gamma$.

Since $p \in \Gamma$, $p \in C := \bigcap_{i=1}^{N} F(S_i)$ and $Ap \in Q := \bigcap_{i=1}^{N} F(T_i)$. It follows from (9) that

$$\begin{aligned}\|x_{n+1} - p\|^2 &= \|u_n - p + \alpha_n\left(S_{n(\mathrm{mod}\,N)}u_n - u_n\right)\|^2 \\ &= \|u_n - p\|^2 + 2\alpha_n\langle u_n - p, S_{n(\mathrm{mod}\,N)}u_n - u_n\rangle \\ &\quad + \alpha_n^2\|u_n - S_{n(\mathrm{mod}\,N)}u_n\|^2.\end{aligned} \tag{10}$$

Because S_i is ϱ_i-strictly pseudononspreading, for each $v \in H_1$, we have

$$\begin{aligned}&\left\|S_{n(\mathrm{mod}\,N)}u_n - S_{n(\mathrm{mod}\,N)}v\right\|^2\\&\quad\le \left\|u_n - v\right\|^2 + \varrho\left\|u_n - S_{n(\mathrm{mod}\,N)}u_n - (v - S_{n(\mathrm{mod}\,N)}v)\right\|^2\\&\qquad + 2\left\langle u_n - S_{n(\mathrm{mod}\,N)}u_n, v - S_{n(\mathrm{mod}\,N)}v\right\rangle.\end{aligned} \tag{11}$$

Taking $v = p$, we have

$$\left\|S_{n(\mathrm{mod}\,N)}u_n - p\right\|^2 \le \left\|u_n - p\right\|^2 + \varrho\left\|u_n - S_{n(\mathrm{mod}\,N)}u_n\right\|^2. \tag{12}$$

This implies that

$$\begin{aligned}&\left\|S_{n(\mathrm{mod}\,N)}u_n - p\right\|^2\\&\quad= \left\|S_{n(\mathrm{mod}\,N)}u_n - u_n + u_n - p\right\|^2\\&\quad= \left\|S_{n(\mathrm{mod}\,N)}u_n - u_n\right\|^2 + 2\left\langle S_{n(\mathrm{mod}\,N)}u_n - u_n, u_n - p\right\rangle\\&\qquad + \left\|u_n - p\right\|^2\\&\quad\le \left\|u_n - p\right\|^2 + \varrho\left\|u_n - S_{n(\mathrm{mod}\,N)}u_n\right\|^2.\end{aligned} \tag{13}$$

Thus it yields that

$$\begin{aligned}&2\alpha_n\left\langle S_{n(\mathrm{mod}\,N)}u_n - u_n, u_n - p\right\rangle\\&\qquad\le \alpha_n(\varrho - 1)\left\|u_n - S_{n(\mathrm{mod}\,N)}u_n\right\|^2.\end{aligned} \tag{14}$$

Substituting (14) into (10) and simplifying, we have

$$\begin{aligned}\left\|x_{n+1} - p\right\|^2 &\le \left\|u_n - p\right\|^2 + \alpha_n(\varrho - 1)\left\|u_n - S_{n(\mathrm{mod}\,N)}u_n\right\|^2\\&\quad + \alpha_n^2\left\|u_n - S_{n(\mathrm{mod}\,N)}u_n\right\|^2\\&= \left\|u_n - p\right\|^2 - \alpha_n(1 - \varrho - \alpha_n)\left\|u_n - S_{n(\mathrm{mod}\,N)}u_n\right\|^2.\end{aligned} \tag{15}$$

On the other hand,

$$\begin{aligned}&\left\|u_n - p\right\|^2\\&\quad= \left\|x_n - p + \gamma A^*(T_{n(\mathrm{mod}\,N)} - I)Ax_n\right\|^2\\&\quad= \left\|x_n - p\right\|^2 + 2\gamma\left\langle x_n - p, A^*\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\rangle\\&\qquad + \gamma^2\left\|A^*(T_{n(\mathrm{mod}\,N)} - I)Ax_n\right\|^2\\&\quad= \left\|x_n - p\right\|^2 + 2\gamma\left\langle x_n - p, A^*\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\rangle\\&\qquad + \gamma^2\left\langle A^*\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n, A^*\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\rangle\\&\quad= \left\|x_n - p\right\|^2 + 2\gamma\left\langle x_n - p, A^*\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\rangle\\&\qquad + \gamma^2\left\langle AA^*\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n, \left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\rangle\\&\quad\le \left\|x_n - p\right\|^2 + 2\gamma\left\langle x_n - p, A^*\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\rangle\\&\qquad + \gamma^2\|A\|^2\left\|\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\|^2.\end{aligned} \tag{16}$$

Since T_i is κ_i-strictly pseudononspreading and noting that $Ap \in \bigcap_{i=1}^{N} F(T_i)$, we have

$$\begin{aligned}&\left\|T_{n(\mathrm{mod}\,N)}Ax_n - Ap\right\|^2\\&\quad= \left\|T_{n(\mathrm{mod}\,N)}Ax_n - Ax_n + Ax_n - Ap\right\|^2\\&\quad= \left\|T_{n(\mathrm{mod}\,N)}Ax_n - Ax_n\right\|^2 + \left\|Ax_n - Ap\right\|^2\\&\qquad + 2\left\langle T_{n(\mathrm{mod}\,N)}Ax_n - Ax_n, Ax_n - Ap\right\rangle\\&\quad\le \left\|Ax_n - Ap\right\|^2 + \kappa\left\|T_{n(\mathrm{mod}\,N)}Ax_n - Ax_n\right\|^2.\end{aligned} \tag{17}$$

This leads to

$$\begin{aligned}&\left\langle T_{n(\mathrm{mod}\,N)}Ax_n - Ax_n, Ax_n - Ap\right\rangle\\&\qquad\le \frac{\kappa - 1}{2}\left\|T_{n(\mathrm{mod}\,N)}Ax_n - Ax_n\right\|^2.\end{aligned} \tag{18}$$

By (18), we have

$$\begin{aligned}&\left\langle T_{n(\mathrm{mod}\,N)}Ax_n - Ax_n, T_{n(\mathrm{mod}\,N)}Ax_n - Ap\right\rangle\\&= \left\langle T_{n(\mathrm{mod}\,N)}Ax_n - Ax_n, T_{n(\mathrm{mod}\,N)}Ax_n + Ax_n - Ax_n - Ap\right\rangle\\&= \left\|\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\|^2 + \left\langle T_{n(\mathrm{mod}\,N)}Ax_n - Ax_n, Ax_n - Ap\right\rangle\\&\le \left\|(T_{n(\mathrm{mod}\,N)} - I)Ax_n\right\|^2 + \frac{\kappa - 1}{2}\left\|(T_{n(\mathrm{mod}\,N)} - I)Ax_n\right\|^2\\&= \frac{\kappa + 1}{2}\left\|(T_{n(\mathrm{mod}\,N)} - I)Ax_n\right\|^2.\end{aligned} \tag{19}$$

It follows from (19) that

$$\begin{aligned}&2\gamma\left\langle x_n - p, A^*\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\rangle\\&\quad= 2\gamma\left\langle A(x_n - p), \left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\rangle\\&\quad= 2\gamma\left\langle A(x_n - p) + \left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right.\\&\qquad\left. - \left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n, \left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\rangle\\&\quad= 2\gamma\left\langle T_{n(\mathrm{mod}\,N)}Ax_n - Ap, \left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\rangle\\&\qquad - 2\gamma\left\|\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\|^2\\&\quad\le [\gamma(1 + \kappa) - 2\gamma]\left\|\left(T_{n(\mathrm{mod}\,N)} - I\right)Ax_n\right\|^2\\&\quad= [\gamma(\kappa - 1)]\left\|(T_{n(\mathrm{mod}\,N)} - I)Ax_n\right\|^2.\end{aligned} \tag{20}$$

By using (15), (16), (19), and (20), we have

$$\begin{aligned}
&\|x_{n+1}-p\|^2 \\
&\le \|x_n-p\|^2+\gamma^2\|A\|^2\|(T_{n(\mathrm{mod}\,N)}-I)Ax_n\|^2 \\
&\quad+[\gamma(\kappa-1)]\|(T_{n(\mathrm{mod}\,N)}-I)Ax_n\|^2 \\
&\quad-\alpha_n(1-\sigma-\alpha_n)\|u_n-S_{n(\mathrm{mod}\,N)}u_n\|^2 \\
&\le \|x_n-p\|^2-\gamma\left(1-\kappa-\gamma\|A\|^2\right)\|(T_{n(\mathrm{mod}\,N)}-I)Ax_n\|^2 \\
&\quad-\alpha_n(1-\varrho-\alpha_n)\|u_n-S_{n(\mathrm{mod}\,N)}u_n\|^2 \\
&\le \|x_n-p\|^2.
\end{aligned} \tag{21}$$

This shows that $\lim_{n\to\infty}\|x_n-p\|$ exists.

(II) We now prove that $\lim_{n\to\infty}\|u_n-p\|$ exists.

In fact, by (21), we have

$$\begin{aligned}
&\left[\gamma\left(1-\kappa-\gamma\|A\|^2\right)\right]\|(T_{n(\mathrm{mod}\,N)}-I)Ax_n\|^2 \\
&\quad+\alpha_n(1-\varrho-\alpha_n)\|u_n-S_{n(\mathrm{mod}\,N)}u_n\|^2 \\
&\le \|x_n-p\|^2-\|x_{n+1}-p\|^2.
\end{aligned} \tag{22}$$

This implies that

$$\lim_{n\to\infty}\|(T_{n(\mathrm{mod}\,N)}-I)Ax_n\|=0, \tag{23}$$

$$\lim_{n\to\infty}\|u_n-S_{n(\mathrm{mod}\,N)}u_n\|=0. \tag{24}$$

By virtue of (16), (23), and (24), it follows that $\lim_{n\to\infty}\|u_n-p\|$ exists and $\lim_{n\to\infty}\|x_n-p\|=\lim_{n\to\infty}\|u_n-p\|$.

(III) Now, we prove that $\lim_{n\to\infty}\|x_{n+1}-x_n\|=0$ and $\lim_{n\to\infty}\|u_{n+1}-u_n\|=0$.

In fact, it follows from (9) that

$$\begin{aligned}
&\|x_{n+1}-x_n\| \\
&=\|(1-\alpha_n)u_n+\alpha_nS_{n(\mathrm{mod}\,N)}u_n-x_n\| \\
&=\|(1-\alpha_n)(x_n+\gamma A^*(T_{n(\mathrm{mod}\,N)}-I)Ax_n) \\
&\qquad+\alpha_nS_{n(\mathrm{mod}\,N)}u_n-x_n\| \\
&=\|(1-\alpha_n)(\gamma A^*(T_{n(\mathrm{mod}\,N)}-I)Ax_n) \\
&\qquad+\alpha_n(S_{n(\mathrm{mod}\,N)}u_n-x_n)\| \\
&=\|(1-\alpha_n)(\gamma A^*(T_{n(\mathrm{mod}\,N)}-I)Ax_n) \\
&\qquad+\alpha_n(S_{n(\mathrm{mod}\,N)}u_n-u_n)+\alpha_n(u_n-x_n)\| \\
&=\|(1-\alpha_n)(\gamma A^*(T_{n(\mathrm{mod}\,N)}-I)Ax_n) \\
&\qquad+\alpha_n(S_{n(\mathrm{mod}\,N)}u_n-u_n) \\
&\qquad+\alpha_n\gamma A^*(T_{n(\mathrm{mod}\,N)}-I)Ax_n\| \\
&=\|\gamma A^*(T_{n(\mathrm{mod}\,N)}-I)Ax_n+\alpha_n(S_{n(\mathrm{mod}\,N)}u_n-u_n)\|.
\end{aligned} \tag{25}$$

This together with (23) and (24) leads to $\lim_{n\to\infty}\|x_{n+1}-x_n\|=0$.

Similarly, it follows from (9), (23), and (25) that

$$\begin{aligned}
&\|u_{n+1}-u_n\| \\
&\quad=\|x_{n+1}+\gamma A^*(T_{n+1(\mathrm{mod}\,N)}-I)Ax_{n+1} \\
&\qquad-[x_n+\gamma A^*(T_{n(\mathrm{mod}\,N)}-I)Ax_n]\| \\
&\quad\le\|x_{n+1}-x_n\|+\|\gamma A^*(T_{n+1(\mathrm{mod}\,N)}-I)Ax_{n+1}\| \\
&\qquad+\|[\gamma A^*(T_{n(\mathrm{mod}\,N)}-I)Ax_n]\| \\
&\quad\longrightarrow 0\quad(\text{as } n\to\infty).
\end{aligned} \tag{26}$$

(IV) Finally, we prove that $x_n\rightharpoonup x^*$ and $u_n\rightharpoonup x^*$, which is a solution of the MSSFP.

In fact, since $\{u_n\}$ is bounded, there exists a subsequence $\{u_{n_i}\}\subset\{u_n\}$ such that $\{u_{n_i}\}\rightharpoonup x^*\in H_1$. Hence, for any positive integer $j=1,2,\ldots,N$, there exists a subsequence $\{n_i(j)\}\subset\{n_i\}$ with $n_i(j)(\mathrm{mod}\,N)=j$ such that $\{u_{n_i(j)}\}\rightharpoonup x^*$. Again, by (24) we know that $\|u_{iN+j}-S_ju_{iN+j}\|\to 0$, as $n\to\infty$; therefore, we have that

$$\left\|u_{n_i(j)}-S_ju_{n_i(j)}\right\|\longrightarrow 0,\quad\text{as } n_{i(j)}\longrightarrow\infty. \tag{27}$$

Since S_j is demiclosed at zero, it follows that $x\in F(S_j)$. By the arbitrariness of $j=1,2,\ldots,N$, we have

$$x^*\in C:=\bigcap_{i=1}^{N}F(S_i). \tag{28}$$

Moreover, from (9) and (24), we have $x_{n_i}=u_{n_i}-\gamma A^*(T_{n_i(\mathrm{mod}\,N)}-I)Ax_{n_i}\rightharpoonup x^*$. Since A is a bounded linear operator, it follows that $Ax_{n_i}\rightharpoonup Ax^*$. For any positive integer $k=1,2,\ldots,N$, there exists a subsequence $\{x_{n_i(k)}\}\subset\{x_{n_i}\}$ with $n_i(k)(\mathrm{mod}\,N)=k$ such that $Ax_{n_i(k)}\rightharpoonup Ax^*$ and $\|Ax_{n_i(k)}-T_kAx_{n_i(k)}\|\to 0$. Since T_k is demiclosed at zero, we have $Ax^*\in F(T_k)$. By the arbitrariness of $k\in\{1,2,\ldots,N\}$, it follows that $Ax^*\in Q:=\bigcap_{k=1}^{N}F(T_k)$. This together with $x^*\in C$ shows that $x^*\in\Gamma$; that is, x^* is a solution to the MSSFP.

Now, we prove that $x_n\rightharpoonup x^*$ and $u_n\rightharpoonup x^*$.

Suppose on the contrary that there exists another subsequence $\{u_{n_j}\}\subset\{u_n\}$ such that $\{u_{n_j}\}\rightharpoonup y^*\in\Gamma$ with $y^*\ne x^*$. Consequently, by virtue of the existence of $\lim_{n\to\infty}\|x_n-p\|$ and the Opial property of Hilbert space, we have

$$\begin{aligned}
\liminf_{n_i\to\infty}\left\|u_{n_i}-x^*\right\| &< \liminf_{n_i\to\infty}\left\|u_{n_i}-y^*\right\| \\
&=\liminf_{n\to\infty}\|u_n-y^*\|=\liminf_{n_j\to\infty}\left\|u_{n_j}-y^*\right\| \\
&<\liminf_{n_j\to\infty}\left\|u_{n_j}-x^*\right\|=\liminf_{n\to\infty}\|u_n-x^*\| \\
&=\liminf_{n_i\to\infty}\left\|u_{n_i}-x^*\right\|.
\end{aligned} \tag{29}$$

This is a contradiction. Therefore $u_n \rightharpoonup x^*$. By (9) and (24), we have

$$x_n = u_n - \gamma A^* \left(T^n_{n(\mathrm{mod}\, N)} - I\right) Ax_n \rightharpoonup x^*. \tag{30}$$

Therefore, the conclusion follows.

This completes the proof of Theorem 12. □

Theorem 13. *Let H_1, H_2, A, $\{S_i\}$, $\{T_i\}$, C, Q be the same as in Theorem 12. For each $i = 1, 2, \ldots, N$, let T_i be a κ_i-strictly pseudononspreading mapping, and S_i be a ϱ_i-strictly pseudononspreading mapping. Let $\{x_n\}$ be the sequence generated by*

$$\begin{aligned} &x_1 \in H_1 \text{ chosen arbitrarily}, \\ &u_n = x_n + \gamma A^* \left(T_{n(\mathrm{mod}\, N)} - I\right) Ax_n, \\ &x_{n+1} = (1 - \alpha_n) u_n + \alpha_n S_{n(\mathrm{mod}\, N)} u_n, \end{aligned} \tag{31}$$

where $\gamma \in (0, (1-\kappa)/\lambda)$ with λ being the spectral of the operator A^*A and $\kappa = \max\{\kappa_1, \kappa_2, \ldots, \kappa_N\} \in (0, 1)$, and $\{\alpha_n\}$ is a sequence in $(0, 1 - \varrho]$ with $\varrho = \max\{\varrho_1, \varrho_2, \ldots, \varrho_N\} \in (0, 1)$. If $\Gamma \neq \emptyset$ and if there exists a positive integer j such that S_j is semicompact, then the sequence $\{x_n\}$ converges strongly to a point $x^* \in \Gamma$.

Proof. Without loss of generality, we can assume that S_1 is semicompact. It follows from (27) that

$$\left\| u_{n_i(1)} - S_1 u_{n_i(1)} \right\| \longrightarrow 0, \quad n_{i(1)} \longrightarrow \infty. \tag{32}$$

Therefore, there exists a subsequence of $\{u_{n_i(1)}\}$ (for the sake of convenience, we still denote it by $\{u_{n_i(1)}\}$), such that $u_{n_i(1)} \rightarrow u^* \in H_1$. Since $u_{n_i(1)} \rightharpoonup x^*$, $x^* = u^*$, and so $u_{n_i(1)} \rightarrow x^* \in \Gamma$. By virtue of the existence of $\lim_{n\to\infty}\|x_n - p\|$, we know that

$$\lim_{n\to\infty} \left\| u_n - x^* \right\| = 0, \qquad \lim_{n\to\infty} \left\| x_n - x^* \right\| = 0. \tag{33}$$

That is, $\{u_n\}$ and $\{x_n\}$ both converge strongly to the point $x^* \in \Gamma$. This completes the proof of Theorem 13. □

Conflict of Interests

The authors declare that they have no conflict of interests.

Authors' Contribution

All the authors contributed equally and significantly in writing this paper. All the authors read and approved the final paper.

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