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Research Article

Memory State-Feedback Stabilization for a Class of Time-Delay Systems with a Type of Adaptive Strategy

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Stabilization of a class of systems with time delay is studied using adaptive control. With the help of the "error to error" technique and the separated "descriptor form" technique, the memory state-feedback controller is designed. The adaptive controller designed can guarantee asymptotical stability of the closed-loop system via a suitable Lyapunov-Krasovskii functional. Some sufficient conditions are derived for the stabilization together with the linear matrix inequality (LMI) design approach. Finally, the effectiveness of the proposed control design methodology is demonstrated in numerical simulations.

1. Introduction

Time delay is one of the instability sources for various systems in practice. With the aid of memoryless state-feedback controllers, Choi and Chung [1] extended the Riccati equation approach to uncertain dynamic systems with time-varying delay in both the system state and control. Since that, considerable attention has been devoted to the problem of delay-dependent stability analysis and controller design for time-delay systems; see, for example, Fridman [2, 3], where the functional was based on the "descriptor form," Liu et al. [4] and He et al. [5], where free weighting matrix technique and Leibniz-Newton formula were utilized to reduce the conservativeness, Park and Jeong [6], where delay-upper-bounded state was exploited, and Tian and Zhou [7], where a less conservative conclusion was investigated on neural networks by integral inequality and taking delay upper bound into account. By constructing the whole state-space trajectory solution, De la Sen [8] investigated the stabilization problems for time-delay time-invariant systems and switched dynamic systems with incommensurate point delays. Some robust controllers for uncertain time-delay systems can also be seen in Tsai et al. [9], Han [10], Chen et al. [11], and Zheng et al. [12], where some different kinds of functional were employed to design controllers. Apparently, the memory state-feedback controller is less conservative than the memoryless one [4], as the former can take delayed states into account. However, the memory control results in the mentioned results require precise information of the time delay; that is, the time-delay parameter must be known exactly. If this crucial piece of information is not available, the memory control schemes developed so far cannot be implemented. For these papers mentioned above, only stability analysis can be pursued for systems with unknown time-delay parameter. To the best of the authors' knowledge, few results have been reported to design memory controllers with unknown time-delay parameter. Jiang et al. [13] provided a type of memory controller with adaptation to the unknown delay parameter; however, the estimate value of the unknown delay was limited to be larger than its real value, which results in the adaptive regulation being unavailable in practice. The bound of the unknown delay was not used to construct the adaptive controller, which is an important factor in leading conservatism. In addition, the LMIs seem unsolvable. Specifically, unknown matrixes like P and P^{-1} exist in the same LMI. On the other hand, the aforementioned results have not considered the time-delay effect which is actually very common in input. The problem of memory state-feedback controllers for systems with control input delay and available adaptation to unknown time-delay parameter remains open, which motivates the research in this paper.

The main contributions of this paper can be illustrated in the following two aspects: (1) for the unknown time-delay parameter, a new adaptive strategy design method is proposed, considering both the current estimate value and the bound of the time-delay; (2) this work simultaneously constructs a memory controller for a class of systems, which are delayed in the system state, control input, and system matrix.

This paper is organized as follows: in Section 2, we first formulate the problem of the memory controller and adaptive regulation for a class of time-delay systems of a particularly complex nature, since the unknown delay parameter exists in the system state, control input, and system matrix. A lemma is provided to introduce the novel kind of adaptive idea, which can be described as the error to error adaptive technique. In Section 3, for the stabilization problem, by using the function based on "descriptor form" and the technique of separation, the memory controller and the adaptive regulation for the unknown delay parameter can be obtained despite the delayed control input. The bound of the unknown delay parameter is considered in the adaptive strategy, which guarantees the less conservatism. The estimate value is not limited to be larger than the real value; what is more is that it is contained in the memory controller, and thus the conservatism can be reduced further. All the necessary matrixes can be obtained by calculating a solvable LMI. In Section 4, a numerical example is presented to demonstrate the effectiveness of the design method. Finally, several formulations are collected in the appendices.

2. Problem Statements

Consider the following class of time-delay systems as follows:

$$\begin{gathered}\dot{x}(t) = Ax(t) + \overline{A}(\tau_1)x(t-\tau_1) + Bu(t-\tau_2),\\ x(t) = \phi(t), \quad \forall t \in [-\max(\tau_1,\tau_2), 0],\end{gathered} \tag{1}$$

where $x(t) \in R^n$ is the state vector and $u(t) \in R^{n_1}$ is the control input vector. A, B are known constant matrices with appropriate dimensions, and $\overline{A}(\tau_1)$ is the matrix depending on τ_1. $\tau_2 \geq 0$ is the time delay in control input with known value. $\tau_1 \geq 0$ is the time delay in system state which is not known exactly, but the upper bound τ_1^* and the lower bound τ_{1*} are available. $\phi(t) \in C[-\tau, 0]$ is a given continuous vector-valued initial function of system (1). Moreover, there exists a positive constant $\overline{h}_1$ such that $0 < \tau_1^* \leq \overline{h}_1 \leq \tau_{1*}$ holds. Generally, the value $\overline{h}_1$ can be chosen as the mean value between τ_1^* and τ_{1*}; that is, $\overline{h}_1 = (\tau_1^* + \tau_{1*})/2$.

Assumption 1. The system matrix $\overline{A}(\tau_1)$ is composed of a constant matrix A_1 and a matrix $A_2(\tau_1)$ which is linear with τ_1; that is,

$$\overline{A}(\tau_1) = A_1 + \tau_1 A_2(\tau_1). \tag{2}$$

Remark 2. The system matrix $\overline{A}(\tau_1)$ is a function of the state delay, which can be seen in the model of measuring intensity for nuclear physics system obtained by [14]. So the problem of stabilization for this type of systems is of some practical significance, and the difficulty in constructing controllers is obvious.

We consider the following feedback controller with memory, with all time delays known:

$$u(t) = K_1x(t) + K_2x(t-\tau_1) + \tau_1K_3x(t-\tau_1). \tag{3}$$

If the time-delay constant τ_1 of system (1) is not known exactly, which has been introduced above, our main result on memory feedback controller with adaptation to delay parameter for system (1) is presented as follows:

$$\begin{aligned}u(t) = {} & K_1x(t) + K_2x\left(t - a_1\hat{\tau}_1(t) - (\hat{\tau}_1(t) - h_1)^2\right)\\ & + \left(a_1\hat{\tau}_1(t) + (\hat{\tau}_1(t) - h_1)^2\right)\\ & \times K_3x\left(t - a_1\hat{\tau}_1(t) - (\hat{\tau}_1(t) - h_1)^2\right),\end{aligned} \tag{4}$$

where $\hat{\tau}_1(t)$ is the estimate value of the unknown delay parameter τ_1, satisfying $\dot{\hat{\tau}}_1(t)[2(\hat{\tau}_1(t) - h_1) + a_1] \leq 0$, for all $t \geq 0$. By using past state information, $(a_1\hat{\tau}_1(t) + (\hat{\tau}_1(t) - h_1)^2)K_3x(t - a_1\hat{\tau}_1(t) - (\hat{\tau}_1(t) - h_1)^2)$ in the controller (4) is designed for $\tau_1A_2(\tau_1)$ in system (1); thus the controller allows for the property of the time-delay system. The constants a_1, h_1 and the matrices K_i $(i = 1, 2, 3)$ wait to be determined.

The objective of this paper is to stabilize the system (1) by using the controller (4), obtaining the adaptation law for $\hat{\tau}_1(t)$, which is everywhere time differentiable, at the same time. In order to prove our results, we introduce the following lemmas.

Lemma 3 (see [15]). *Given matrices X and Y with the appropriate dimensions,*

$$X^TY + Y^TX \leq X^TTX + Y^TT^{-1}Y, \quad \forall T > 0. \tag{5}$$

Lemma 4. *Considering the following gain adaptive law for the estimator*

$$\dot{\hat{\tau}}_1(t) = -\left[2(\hat{\tau}_1(t) - h_1) + a_1\right]m(t), \tag{6}$$

where a_1 and h_1 are positive constants and $m(t) \geq 0$ is a positive derivative function, which will be determined later. If we choose a_1 and h_1 with the following equations:

$$h_1 = \sqrt{\overline{h}_1 + \overline{h}_1^2}, \qquad a_1 = 2\left(\sqrt{\overline{h}_1 + \overline{h}_1^2} - \overline{h}_1\right), \tag{7}$$

then $\hat{\tau}_1(t)$ is bounded, and $a_1\hat{\tau}_1(t) + (\hat{\tau}_1(t) - h_1)^2$ can be bounded with $\overline{h}_1$.

Proof. From (6), it is obvious that $\hat{\tau}_1(t)$ satisfies $\dot{\hat{\tau}}_1(t)[2(\hat{\tau}_1(t) - h_1) + a_1] \leq 0$, for all $t \geq 0$.

Let us prove the boundedness of $\hat{\tau}_1(t)$, for which a Lyapunov function can be constructed as $\overline{V}(\hat{\tau}_1(t)) = \hat{\tau}_1^2(t)/2$. The time derivative of $\overline{V}(\hat{\tau}_1(t))$ along the adaptive strategy of (6) is

$$\dot{\overline{V}}(\hat{\tau}_1(t)) = -\hat{\tau}_1(t)\left[2(\hat{\tau}_1(t) - h_1) + a_1\right]m(t). \tag{8}$$

If $\widehat{\tau}_1(t) > (2h_1 - a_1)/2$, it results in

$$\dot{\overline{V}}(\widehat{\tau}_1(t)) < \widehat{\tau}_1(t)\, m(t) \le 0, \quad (9)$$

which implies the boundedness of $\widehat{\tau}_1(t)$ at $(2h_1 - a_1)/2$. If $\widehat{\tau}_1(t) \le (2h_1 - a_1)/2$, together with (8), we have

$$\dot{\overline{V}}(\widehat{\tau}_1(t)) \ge \widehat{\tau}_1(t)\, m(t) \ge 0. \quad (10)$$

Therefore, $\widehat{\tau}_1(t)$ is increased only when it is less than $(2h_1 - a_1)/2$. Once $\widehat{\tau}_1(t) = (2h_1 - a_1)/2$, $\widehat{\tau}_1(t)$ will be fixed at this value. Then from (7) yields

$$\widehat{\tau}_1(t) = \frac{2h_1 - a_1}{2} = \frac{2\sqrt{\overline{h}_1 + \overline{h}_1^2} - 2\left(\sqrt{\overline{h}_1 + \overline{h}_1^2} - \overline{h}_1\right)}{2} = \overline{h}_1. \quad (11)$$

It can also be obtained that if $\widehat{\tau}_1(t)$ is fixed at $(2h_1 - a_1)/2$, the value of $a_1\widehat{\tau}_1(t) + (\widehat{\tau}_1(t) - h_1)^2$ can also be bounded as follows:

$$\begin{aligned} a_1\widehat{\tau}_1(t) + (\widehat{\tau}_1(t) - h_1)^2 &= a_1\frac{2h_1 - a_1}{2} + \left(\frac{2h_1 - a_1}{2} - h_1\right)^2 \\ &= \frac{4a_1h_1 - a_1^2}{4} \\ &= \frac{a_1(4h_1 - a_1)}{4} \\ &= h_1^2 - \overline{h}_1^2 = \overline{h}_1 + \overline{h}_1^2 - \overline{h}_1^2 = \overline{h}_1. \end{aligned} \quad (12)$$

This completes the proof. □

Remark 5. $\widehat{\tau}_1(t)$ is the estimate value of the unknown delay constant τ_1, satisfying $\dot{\widehat{\tau}}_1(t)[2(\widehat{\tau}_1(t) - h_1) + a_1] \le 0$. Then $\dot{\widehat{\tau}}_1(t)[\widehat{\tau}_1(t) - \overline{h}_1] \le 0$ can be obtained by fixing h_1 and a_1 in Lemma 4. Apparently, the difference between $\widehat{\tau}_1(t)$ and $\overline{h}_1$ determines the variation of $\widehat{\tau}_1(t)$. So the adaptive strategy for $\widehat{\tau}_1(t)$ is based on a novel type of adaptive idea, which can be described as error to error adaptive technique. This adaptive strategy imposes no limitation on the estimate value. It also guarantees $\widehat{\tau}_1(t)$ always in its bound; that is, $0 < \tau_1^* \le \widehat{\tau}_1(t) \le \tau_{1*}$, for $\widehat{\tau}_1(t)$ can be fixed at $\overline{h}_1$.

3. Main Results

Using the controller (4), the closed-loop system (1) can be written as

$$\begin{aligned} \dot{x}(t) = {} & Ax(t) + [A_1 + \tau_1 A_2(\tau_1)]\, x(t - \tau_1) \\ & + BK_1 x(t - \tau_2) \\ & + \left[BK_2 + \left(a_1\widehat{\tau}_1(t - \tau_2) + (\widehat{\tau}_1(t - \tau_2) - h_1)^2\right) BK_3\right] x \\ & \times \left(t - \tau_2 - a_1\widehat{\tau}_1(t - \tau_2) - (\widehat{\tau}_1(t - \tau_2) - h_1)^2\right), \\ x(t) = {} & \phi(t), \quad \forall t \in [-\overline{\tau}, 0], \end{aligned} \quad (13)$$

where $\overline{\tau} = \max(\tau_1^*, \overline{\tau}_1^2 + 2\tau_1^*(\sqrt{\overline{h}_1 + \overline{h}_1^2} - \overline{h}_1) + \tau_2)$. Considering the "descriptor form" in [2, 3], we can rewrite (13) as

$$\begin{aligned} \dot{x}(t) &= y(t) + z(t), \\ y(t) &= Ax(t) + [A_1 + \tau_1 A_2(\tau_1)]\, x(t - \tau_1), \end{aligned} \quad (14)$$

where

$$\begin{aligned} z(t) = {} & BK_1 x(t - \tau_2) \\ & + \left[BK_2 + \left(a_1\widehat{\tau}_1(t - \tau_2) + (\widehat{\tau}_1(t - \tau_2) - h_1)^2\right) BK_3\right] \\ & \times x\left(t - \tau_2 - a_1\widehat{\tau}_1(t - \tau_2) - (\widehat{\tau}_1(t - \tau_2) - h_1)^2\right) \end{aligned} \quad (15a)$$

which yields consequently

$$\begin{aligned} y(t) + z(t) = {} & [A + A_1 + \tau_1 A_2(\tau_1) + BK]\, x(t) \\ & - [A_1 + \tau_1 A_2(\tau_1)] \int_{t-\tau_1}^{t} (y(s) + z(s))\, ds \\ & - BK_1 \int_{t-\tau_2}^{t} (y(\xi) + z(\xi))\, d\xi \\ & - BK_2 \int_{t-\tau_3}^{t} (y(s) + z(s))\, ds \\ & - \tau_4 BK_3 \int_{t-\tau_4}^{t} (y(s) + z(s))\, ds \end{aligned} \quad (16a)$$

with $K = K_1 + K_2 + \tau_3 K_3$, and $\tau_3 = \tau_4 = \tau_2 + a_1\widehat{\tau}_1(t - \tau_2) + (\widehat{\tau}_1(t - \tau_2) - h_1)^2$.

Remark 6. By separating the "descriptor form" $\dot{x}(t)$ into two parts $y(t)$ and $z(t)$, the specific adaptive regulation constructed later can be obtained in spite of the control input delay.

For systems (15a) and (16a), we consider the following Lyapunov-Krasovskii functional as

$$V(x_t) = V_1(x_t) + V_2(x_t) + \frac{l}{2}[2(\widehat{\tau}_1(t - \tau_2) - h_1) + a_1]^2, \quad (17)$$

where $V_1(x_t) = x^T P x = \begin{bmatrix} x^T & (y+z)^T \end{bmatrix} E\overline{P}^T \begin{bmatrix} x \\ y+z \end{bmatrix}$, $E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $\overline{P} = \begin{bmatrix} P & P_1 \\ 0 & P_2 \end{bmatrix}$,

$$\begin{aligned} V_2(x_t) = {} & \sum_{i=1}^{3} \int_{-\tau_i}^{0} \int_{t+\theta}^{t} (y(s) + z(s))^T \\ & \times \overline{A}_i^T Q_i^{-1} \overline{A}_i (y(s) + z(s))\, ds\, d\theta \\ & + \tau_4 \int_{-\tau_4}^{0} \int_{t+\theta}^{t} (y(s) + z(s))^T \\ & \times \overline{A}_4^T Q_4^{-1} \overline{A}_4 (y(s) + z(s))\, ds\, d\theta, \end{aligned} \quad (18)$$

with $\overline{A}_1 = A_1 + \tau_1 A_2(\tau_1)$, $\overline{A}_2 = BK_1$, $\overline{A}_3 = BK_2$, $\overline{A}_4 = BK_3$, $\overline{A}_5 = \tau_4 \overline{A}_4$, $l > 0$, $h_1 > 0$, and $a_1 > 0$ being constants, and matrices $P > 0$, $Q_i > 0$ $(i = 1, 2, 3, 4)$ are waiting to be determined.

Remark 7. It is easy to see that $V_1(x_t) = x^T P x > 0$ and $V_2(x_t) \geq 0$. From (13), (15a), and (16a), it is not difficult to observe that if the norm of $x(t)$ diverges to infinity, then $V_1(x_t)$ will also diverge to infinity. If $\overline{A}_i$ for $i = 1, 2, 3, 4$ are singular, thus $V_2(x_t) = 0$. Otherwise, the norms of $y(t)$ and $z(t)$ are also infinite when the norm of $x(t)$ is unbounded, resulting in $V_2(x_t)$ diverging to infinity. Thus the Lyapunov-Krasovskii functional $V(x_t)$ defined in (17) can be guaranteed to be radially unbounded, that is, a well-posed candidate Lyapunov functional.

Hence, the derivative of $V_1(x_t)$ along the systems (15a) and (16a) is given by

$$\begin{aligned}
\dot{V}_1(x_t) &= 2x^T P (y+z) \\
&= 2\left[x^T \ \ (y+z)^T\right] \begin{bmatrix} P & P_1 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} y+z \\ 0 \end{bmatrix} \\
&= 2\left[x^T \ \ (y+z)^T\right] \begin{bmatrix} P & P_1 \\ 0 & P_2 \end{bmatrix} \\
&\quad \times \left\{ \begin{bmatrix} y+z \\ \overline{A}x - y - z \end{bmatrix} + \begin{bmatrix} 0 \\ \tau_1 A_2(\tau_1) x \end{bmatrix} \right. \\
&\quad - \sum_{i=1}^{3} \begin{bmatrix} 0 \\ \overline{A}_i \end{bmatrix} \int_{t-\tau_i}^{t} (y(s) + z(s))\, ds \\
&\quad \left. - \begin{bmatrix} 0 \\ \overline{A}_5 \end{bmatrix} \int_{t-\tau_4}^{t} (y(s) + z(s))\, ds \right\} \\
&= 2\left[x^T \ \ (y+z)^T\right] \begin{bmatrix} P & P_1 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} y+z \\ \overline{A}x - y - z \end{bmatrix} - \sum_{i=1}^{5} \eta_i,
\end{aligned} \tag{19}$$

where $\overline{A} = A + A_1 + BK$, $\eta_5(t) = -2\left[x^T \ \ (y+z)^T\right] \overline{P} \left[\begin{smallmatrix} 0 \\ I \end{smallmatrix}\right] \tau_1 A_2(\tau_1) x$,

$$\begin{aligned}
\eta_i(t) &= -2 \int_{t-\tau_i}^{t} \left[x^T \ \ (y+z)^T\right] \overline{P} \begin{bmatrix} 0 \\ I \end{bmatrix} \overline{A}_i \\
&\qquad \times (y(s) + z(s))\, ds \quad (i = 1, 2, 3), \\
\eta_4(t) &= -2 \int_{t-\tau_4}^{t} \left[x^T \ \ (y+z)^T\right] \overline{P} \begin{bmatrix} 0 \\ I \end{bmatrix} \overline{A}_5 \\
&\qquad \times (y(s) + z(s))\, ds.
\end{aligned} \tag{20}$$

By means of Lemma 3, we have

$$\begin{aligned}
\eta_i &\leq \tau_i \left[x^T \ \ (y+z)^T\right] \overline{P} \begin{bmatrix} 0 \\ I \end{bmatrix} R_i \begin{bmatrix} 0 & I \end{bmatrix} \overline{P}^T \begin{bmatrix} x \\ y+z \end{bmatrix} \\
&\quad + \int_{t-\tau_i}^{t} (y(s) + z(s))^T \overline{A}_i^T R_i^{-1} \overline{A}_i (y(s) + z(s))\, ds,
\end{aligned} \tag{21}$$

$$\begin{aligned}
\eta_4 &\leq \tau_4^2 \left[x^T \ \ (y+z)^T\right] \overline{P} \begin{bmatrix} 0 \\ I \end{bmatrix} R_4 \begin{bmatrix} 0 & I \end{bmatrix} \overline{P}^T \begin{bmatrix} x \\ y+z \end{bmatrix} \\
&\quad + \tau_4 \int_{t-\tau_4}^{t} (y(s) + z(s))^T \overline{A}_4^T R_4^{-1} \overline{A}_4 (y(s) + z(s))\, ds,
\end{aligned} \tag{22}$$

$$\begin{aligned}
\eta_5 &\leq \tau_1 \left[x^T \ \ (y+z)^T\right] \overline{P} \begin{bmatrix} 0 \\ I \end{bmatrix} Q_5 \begin{bmatrix} 0 & I \end{bmatrix} \overline{P}^T \begin{bmatrix} x \\ y+z \end{bmatrix} \\
&\quad + \tau_1 x^T (A_2(\tau_1))^T Q_5^{-1} A_2(\tau_1) x.
\end{aligned} \tag{23}$$

Note that

$$\begin{aligned}
&\left[x^T \ \ (y+z)^T\right] \overline{P} \begin{bmatrix} 0 \\ I \end{bmatrix} R_i \begin{bmatrix} 0 & I \end{bmatrix} \overline{P}^T \begin{bmatrix} x \\ y+z \end{bmatrix} \\
&\quad = x^T P_1 R_i P_1^T x + 2x^T P_1 R_i P_2^T (y+z) \\
&\qquad + (y+z)^T P_2 R_i P_2^T (y+z) \\
&\quad = x^T P_1 R_i P_1^T x + 2x^T P_1 R_i P_2^T (y+z) \\
&\qquad + 2z^T P_2 R_i P_2^T y + y^T P_2 R_i P_2^T y \\
&\qquad + z^T P_2 R_i P_2^T z, \quad i = 3, 4,
\end{aligned} \tag{24}$$

$$\begin{aligned}
\tau_3 &= \tau_2 + a_1 \hat{\tau}_1 (t - \tau_2) + (\hat{\tau}_1 (t - \tau_2) - h_1)^2 \\
&= \frac{1}{4} \left\{ \left[2(\hat{\tau}_1 (t - \tau_2) - h_1) + a_1\right]^2 - a_1^2 + 4h_1 a_1 \right\} + \tau_2, \\
\tau_4^2 &= \left[\tau_2 + a_1 \hat{\tau}_1 (t - \tau_2) + (\hat{\tau}_1 (t - \tau_2) - h_1)^2\right]^2 \\
&= \tau_2^2 + a_1^2 (\hat{\tau}_1 (t - \tau_2))^2 + (\hat{\tau}_1 (t - \tau_2) - h_1)^4 \\
&\quad + 2\tau_2 a_1 \hat{\tau}_1 (t - \tau_2) + 2\tau_2 (\hat{\tau}_1 (t - \tau_2) - h_1)^2 \\
&\quad + 2a_1 \hat{\tau}_1 (t - \tau_2) (\hat{\tau}_1 (t - \tau_2) - h_1)^2 \\
&= a_1^2 (\hat{\tau}_1 (t - \tau_2))^2 + (\hat{\tau}_1 (t - \tau_2) - h_1)^4 \\
&\quad + 2a_1 \hat{\tau}_1 (t - \tau_2) (\hat{\tau}_1 (t - \tau_2) - h_1)^2 \\
&\quad + \tau_2 \frac{1}{2} \left\{ \left[2(\hat{\tau}_1 (t - \tau_2) - h_1) + a_1\right]^2 - a_1^2 + 4h_1 a_1 \right\}.
\end{aligned} \tag{25}$$

Besides,

$$\begin{aligned}
&\frac{d\left(\int_{-\tau_i}^{0} \int_{t+\theta}^{t} \dot{x}(s)^T \overline{A}_i^T Q_i^{-1} \overline{A}_i \dot{x}(s)\, ds\, d\theta\right)}{dt} \\
&\quad = \dot{\tau}_i(t) \int_{t-\tau_i}^{t} \dot{x}(s)^T \overline{A}_i^T Q_i^{-1} \overline{A}_i \dot{x}(s)\, ds \\
&\qquad + \int_{-\tau_i}^{0} \left[\dot{x}(t)^T \overline{A}_i^T Q_i^{-1} \overline{A}_i \dot{x}(t) \right. \\
&\qquad \left. - \dot{x}(t+\theta)^T \overline{A}_i^T Q_i^{-1} \overline{A}_i \dot{x}(t+\theta)\right] d\theta, \quad i = 1, 2, 3, \\
&\frac{d\left(\int_{-\tau_4}^{0} \int_{t+\theta}^{t} \tau_4(t)\, \dot{x}(s)^T \overline{A}_4^T Q_4^{-1} \overline{A}_4 \dot{x}(s)\, ds\, d\theta\right)}{dt} \\
&\quad = \dot{\tau}_4(t) \int_{t-\tau_4}^{t} \tau_4(t)\, \dot{x}(s)^T \overline{A}_4^T Q_4^{-1} \overline{A}_4 \dot{x}(s)\, ds
\end{aligned}$$

$$+\int_{-\tau_4}^{0}\tau_4(t)\Big[\dot{x}(t)^T\overline{A}_4^T Q_4^{-1}\overline{A}_4\dot{x}(t) - \dot{x}(t+\theta)^T\overline{A}_4^T Q_4^{-1}\overline{A}_4\dot{x}(t+\theta)\Big]d\theta + \int_{-\tau_4}^{0}\int_{t+\theta}^{t}\dot{\tau}_4(t)\,\dot{x}(s)^T\overline{A}_4^T Q_i^{-1}\overline{A}_4\dot{x}(s)\,ds\,d\theta. \quad (26)$$

Since $\dot{\hat{\tau}}_1(t)[2(\hat{\tau}_1(t)-h_1)+a_1]\le 0$, that is, $\dot{\hat{\tau}}_1(t-\tau_2)[2(\hat{\tau}_1(t-\tau_2)-h_1)+a_1]\le 0$, and

$$\begin{aligned}\dot{\tau}_3(t) &= \dot{\tau}_4(t) \\ &= \frac{d\left(\tau_2 + a_1\hat{\tau}_1(t-\tau_2) + (\hat{\tau}_1(t-\tau_2)-h_1)^2\right)}{dt} \\ &= a_1\dot{\hat{\tau}}_1(t-\tau_2) + 2(\hat{\tau}_1(t-\tau_2)-h_1)\dot{\hat{\tau}}_1(t-\tau_2) \\ &= \dot{\hat{\tau}}_1(t-\tau_2)[a_1 + 2(\hat{\tau}_1(t-\tau_2)-h_1)],\end{aligned} \quad (27)$$

we can have

$$\begin{aligned}\dot{V}_2(x_t) \le \sum_{i=1}^{3}\Big[&\tau_i(y(t)+z(t))^T\overline{A}_i^T Q_i^{-1}\overline{A}_i(y(t)+z(t)) \\ &- \int_{t-\tau_i}^{t}(y(s)+z(s))^T\overline{A}_i^T Q_i^{-1}\overline{A}_i \times (y(s)+z(s))\,ds\Big] \\ &+ \tau_4^2(t)\,\dot{x}(t)^T\overline{A}_4^T Q_4^{-1}\overline{A}_4\dot{x}(t) \\ &- \tau_4(t)\int_{-\tau_4}^{0}\dot{x}(t+\theta)^T\overline{A}_4^T Q_4^{-1}\overline{A}_4\dot{x}(t+\theta)\,d\theta.\end{aligned} \quad (28)$$

Let $R_i = Q_i$ $(i = 1,\ldots,4)$, according to (17)–(28), the following inequalities are obvious by means of Schur complement:

$$\begin{aligned}\dot{V}(x_t) \le\; &\tilde{x}(t)^T\Xi_0\tilde{x}(t) + [a_1 + 2(\hat{\tau}_1(t-\tau_2)-h_1)] \\ &\times\Big\{l\dot{\hat{\tau}}_1(t-\tau_2) + \frac{1}{4}[a_1 + 2(\hat{\tau}_1(t-\tau_2)-h_1)] \\ &\times z^T P_2(Q_3 + 2\tau_2 Q_4)P_2^T z\Big\},\end{aligned} \quad (29)$$

where

$$\tilde{x}(t)^T = \begin{bmatrix} x^T & y^T & z^T\end{bmatrix}, \qquad \Xi_0 = \begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33}\end{bmatrix},$$

$$\Xi_{11} = P_1\overline{A} + \overline{A}^T P_1^T + \sum_{i=1}^{5}\tilde{\tau}_i P_1 Q_i P_1^T + \tau_1(A_2(\tau_1))^T Q_5^{-1}A_2(\tau_1),$$

$$\Xi_{12} = P - P_1 + \overline{A}^T P_2^T + \sum_{i=1}^{5}\tilde{\tau}_i P_1 Q_i P_2^T = \Xi_{13},$$

$$\tilde{\tau}_i = \tau_i \quad i = 1,2,3, \quad \tilde{\tau}_4 = \tau_4^2, \quad \tilde{\tau}_5 = \tau_1,$$

$$\Xi_{23} = \sum_{i=1}^{5}\tilde{\tau}_i P_2 Q_i P_2^T - P_2 - P_2^T + \sum_{i=1}^{4}\tilde{\tau}_i\overline{A}_i^T Q_i^{-1}\overline{A}_i = \Xi_{22},$$

$$\begin{aligned}\Xi_{33} = &\sum_{i=1}^{2}\tau_i P_2 Q_i P_2^T + \tau_1 P_2 Q_5 P_2^T - P_2 - P_2^T \\ &+ \sum_{i=1}^{3}\tau_i\overline{A}_i^T Q_i^{-1}\overline{A}_i + \left[\frac{1}{4}\left(4h_1 a_1 - a_1^2\right) + \tau_2\right]P_2, \\ &Q_3 P_2^T + \tilde{\tau}_6 P_2 Q_4 P_2^T + \tilde{\tau}_4\overline{A}_4^T Q_4^{-1}\overline{A}_4, \\ &\tilde{\tau}_6 = \tilde{\tau}_4 - \tau_2\frac{1}{2}[2(\hat{\tau}_1(t-\tau_2)-h_1)+a_1]^2.\end{aligned} \quad (30)$$

Results can be obtained as follows.

Theorem 8. *Consider the time-delay system* (1) *with unknown time-delay parameter* τ_1; *the system* (1) *can be stabilized by the state-feedback controller* (4) *if there exist matrices* U_i $(i = 1,2,3)$ *and positive-definite matrices* X, Q_i $(i = 1,\ldots,4)$ *such that the linear matrix inequalities* (32) *hold, with the parameters* a_1 *and* h_1 *selected as* (7) *in Lemma 4. Moreover, the adaptive strategy about the unknown delay constant* τ_1 *can be obtained from* (31), *and the feedback gains of the controller* (4) *are given by* $K_i = U_i X^{-1}$ $(i = 1,2,3)$.

Proof. Consider the following adaptive control:

$$\begin{aligned}\dot{\hat{\tau}}_1(t-\tau_2) &= -\frac{1}{4l}[a_1 + 2(\hat{\tau}_1(t-\tau_2)-h_1)]z^T P_2 \\ &\quad\times(Q_3 + 2\tau_2 Q_4)P_2^T z. \text{ That is,} \\ \dot{\hat{\tau}}_1(t) &= -\frac{1}{4l}[a_1 + 2(\hat{\tau}_1(t)-h_1)]m(t),\end{aligned} \quad (31)$$

where $m(t) = z(t+\tau_2)^T P_2(Q_3 + 2\tau_2 Q_4)P_2^T z(t+\tau_2)$ satisfying the adaptive strategy as (6) in Lemma 4. Thus by using (29), we have $\dot{V}(x_t) \le \tilde{x}^T(t)\Xi_0\tilde{x}(t)$. So if $S \overset{\text{def}}{=} \Xi_0 < 0$, under the action of the controller (4), the system (1) will be asymptotically stable. The most important work of the memory feedback control problem is how to solve the matrix inequality $S < 0$. Obviously, there exists $S(\tau_1) \le S(\tau_1^*)$, for $\tau_1 \le \tau_1^*$. So $S(\tau_1^*) < 0$ can guarantee $\dot{V}(x_t) < 0$ satisfied, which means that the time-delay system (8) is asymptotically stabilizable by using feedback controller (4). Let $\Xi \overset{\text{def}}{=} S(\tau_1^*)$, and consider the Lyapunov matrix $\overline{P}$ with $E\overline{P}^T = \overline{P}E$. In this case, we can suggest that the P_1 and P_2 in $\overline{P}$ can be substituted as $P_1 = n_1/n_2 P$ and $P_2 = 1/n_2 P$, where n_1 and n_2 are real scalars. In this way, we can solve the above problem; furthermore, by making n_1 and n_2 line search parameters (i.e., plain search), we anticipate that less conservative conditions are given. Now before and after multiplying both sides of $\Xi < 0$ by $\operatorname{diag}(X_1 \;\; X_2 \;\; X_3)$, where $X_1 = P_1^{-1} = n_2/n_1 X$, $X_2 = X_3 = P_2^{-1} = n_2 X$, and $X = P^{-1}$. After substituting

$h_1 = \sqrt{\overline{h}_1 + \overline{h}_1^2}$ and $a_1 = 2(\sqrt{\overline{h}_1 + \overline{h}_1^2} - \overline{h}_1)$ into $\Xi < 0$, the following linear matrix inequalities can be obtained by applying Schur complement.

Consider

$$\widetilde{\Xi} = \begin{bmatrix} \overline{\Xi} & \widetilde{\Xi}_1 & \cdots & \widetilde{\Xi}_5 \\ \widetilde{\Xi}_1^T & \mathrm{M}_1 & & \\ \vdots & & \ddots & \\ \widetilde{\Xi}_5^T & & & \mathrm{M}_5 \end{bmatrix} < 0, \quad (32)$$

where $\widetilde{\Xi}_1^T = [0 \;\; n_2 A_1 X \;\; n_2 A_1 X]$, $\widetilde{\Xi}_{i+1}^T = [0 \;\; n_2 B_2 U_i \;\; n_2 B_2 U_i \;\; 0_1 \;\; \cdots \;\; 0_i]$, $i = 1, 2, 3$, $M_i = -(\tau_i^*)^{-1} Q_i$, $i = 1, 2, 3$, $\tau_2^* = \tau_2$, $M_4 = -(\widetilde{\tau}_4^*)^{-1} Q_4$, $M_5 = -(\tau_1^*)^{-1} Q_5$, $\widetilde{\Xi}_5^T = [n_2/n_1 A_2(\tau_1) X \;\; 0_1 \;\; \cdots \;\; 0_6]$, $U_j = K_j X$, $j = 1, 2$, $\overline{\Xi} = \begin{bmatrix} \overline{\Xi}_{11} & \overline{\Xi}_{12} & \overline{\Xi}_{13} \\ * & \overline{\Xi}_{22} & \overline{\Xi}_{23} \\ * & * & \overline{\Xi}_{33} \end{bmatrix}$, $\overline{\Xi}_{11} = n_2/n_1 (A + A_1) X + n_2/n_1 \sum_{i=1}^{3} B_2 U_i + n_2/n_1 X (A + A_1)^T + n_2/n_1 \sum_{i=1}^{3} (B_2 U_i)^T + \sum_{i=1}^{5} \widetilde{\tau}_i^* Q_i$, $\overline{\Xi}_{12} = n_2/n_1 (n_2 - n_1) X + n_2/n_1 X (A + A_1)^T + n_2/n_1 \sum_{i=1}^{3} (B_2 U_i)^T + \sum_{i=1}^{5} \widetilde{\tau}_i^* Q_i = \overline{\Xi}_{13}$, $\overline{\Xi}_{23} = \sum_{i=1}^{5} \widetilde{\tau}_i^* Q_i - 2 n_2 X = \overline{\Xi}_{22}$, and $\overline{\Xi}_{33} = \sum_{i=1}^{2} \tau_i^* Q_i + \tau_5^* Q_5 - 2 n_2 X + (\overline{h}_1 + \tau_2) Q_3 + \widetilde{\tau}_6^* Q_4$. $\widetilde{\tau}_i = \tau_i$, $i = 1, 2, 3$, $\widetilde{\tau}_5 = \tau_1$, $\widetilde{\tau}_4 = \tau_4^2$, and $\widetilde{\tau}_6 = \widetilde{\tau}_4 - \tau_2 (1/2) [2(\widehat{\tau}_1(t - \tau_2) - h_1) + a_1]^2$.

The linear matrix inequality (32) can be directly solved by LMI toolbox in MATLAB software, and the matrices U_i ($i = 1, 2, 3$) and positive-definite matrices X, Q_3, and Q_4 can also be acquired. Consequently, we have $K_i = U_i X^{-1}$ ($i = 1, 2, 3$). □

Remark 9. For the value of τ_3^*, $\widetilde{\tau}_4^*$, and $\widetilde{\tau}_6^*$ see Appendices. The LMI provided here is solvable, while, as for the results in Jiang et al. [13], unknown matrixes like P and P^{-1} exist in the same LMI, resulting in the LMI unsolvable.

4. Numerical Example

We consider a system with the same structure as (1), and the model matrices are

$$A = \begin{bmatrix} -4 & 14 \\ -15 & -3 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 8 \\ 1 \end{bmatrix}. \quad (33)$$

The known time delay in control input is $\tau_2 = 0.008$. We consider the following uncertainty in the time delay τ_1: $\tau_1 \in [0.2, 0.3]$; that is, the upper bound $\tau_1^* = 0.3$, the lower bound $\tau_{1*} = 0.2$, and $\overline{h}_1$ is selected as $(\tau_1^* + \tau_{1*})/2 = 0.25$. According to Lemma 4, we can select $h_1 = \sqrt{\overline{h}_1 + \overline{h}_1^2} = 0.559$, and $a_1 = 2(\sqrt{\overline{h}_1 + \overline{h}_1^2} - \overline{h}_1) = 0.618$. By applying Theorem 8, the feasible solution can be obtained with $K_1 = [0.5501 \;\; -1.6962]$, $K_2 = 1.0e - 006 * [0.0806 \;\; -0.1576]$, $K_3 = 1.0e - 005 * [0.0575 \;\; -0.1365]$, and $P_2 (Q_3 + Q_4) P_2^T = \begin{bmatrix} 0.0126 & 0.0026 \\ 0.0026 & 0.0006 \end{bmatrix}$. Moreover $\widetilde{\tau}_4^* = \widetilde{\tau}^* = \max\{\widetilde{\tau}(\widehat{\tau}_1(t - \tau_2) = \{\tau_1^*, \tau_{1*}\})\} = 0.0623$ can be obtained from Appendix B. And the global minimum for LMI (32) is $t_{\min} = -2.4706e - 007$, while by, the controller proposed in Jiang et al. [13], the global minimum $t_{\min} = -2.4292e-008$, which means that our result is less conservation. If the initial conditions are chosen as

$$\widehat{\tau}_1(-\overline{\tau}) = 0.2,$$
$$\begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} = \begin{bmatrix} \dfrac{2 \sin 4\pi (t - \overline{\tau})}{\overline{\tau}} \\ -\dfrac{3 \sin 4\pi (t - \overline{\tau})}{\overline{\tau}} \end{bmatrix}, \quad -\overline{\tau} \le t \le 0, \quad (34)$$

where $\overline{\tau} = \max(\tau_1^*, \overline{\tau}_1^2 + \tau_1^* a_1 + \tau_2) = \tau_1^* = 0.3$, and the parameter l is chosen as $l = 0.4$, then the system state under adaptive memory controller is shown in Figure 1. At this time, the estimate value of the unknown time-delay parameter, that is, $\widehat{\tau}_1(t)$, is shown in Figure 2.

Remark 10. In Jiang et al. [13], $\widehat{\tau}_1(t)$ was limited to be larger than the real unknown value τ_1. However, since τ_1 is unknown, it is difficult to satisfy the limitation. So the memory controller with such $\widehat{\tau}_1(t)$ cannot be implemented as it was described. Besides, $\widehat{\tau}_1(t)$ remains decreasing until the system is stabilized. If the memory controller does not perform well, $\widehat{\tau}_1(t)$ will remain decreasing, which deteriorates the function of the controller. In this paper, $\widehat{\tau}_1(t)$ is maintained between the lower bound τ_* and the upper bound τ^*, which is much easier to be implemented. With the error to error adaptive technique, $\widehat{\tau}_1(t)$ will always stay between τ_* and τ^*, so the memory controller with such $\widehat{\tau}_1(t)$ can allow for more information of the system, which reduces the conservativeness.

5. Conclusions

In this paper, the problem of memory feedback controller with adaptation to unknown time delay parameter is addressed. The system investigated is with time delay in system state, control input, and system matrix, and additionally the state time-delay is unknown. By using a novel type of adaptive strategy with the idea of error to error and separated "descriptor form" functional technique, the estimate value of the time-delay constant can always be reflected by the feedback controller. Since more information in the system is presented, the controller proposed in this paper is much less conservative. Moreover, the adaptive strategy about time-delay parameter can achieve that no limitation is imposed on the estimate value, so it is more simple and convenient than the existing adaptive controllers. The sufficient condition for stabilization is presented in the form of LMI. To illustrate efficiency of the proposed technique, a numerical example has been provided.

Appendices

A. The Value of τ_3^*

$\tau_3 = \tau_4 = \tau_2 + a_1 \widehat{\tau}_1(t - \tau_2) + (\widehat{\tau}_1(t - \tau_2) - h_1)^2$ as derivative of $\tau_3(t)$ can be obtained for $\widehat{\tau}_1(t - \tau_2) \in [\tau_{1*}, \tau_1^*]$, and we have

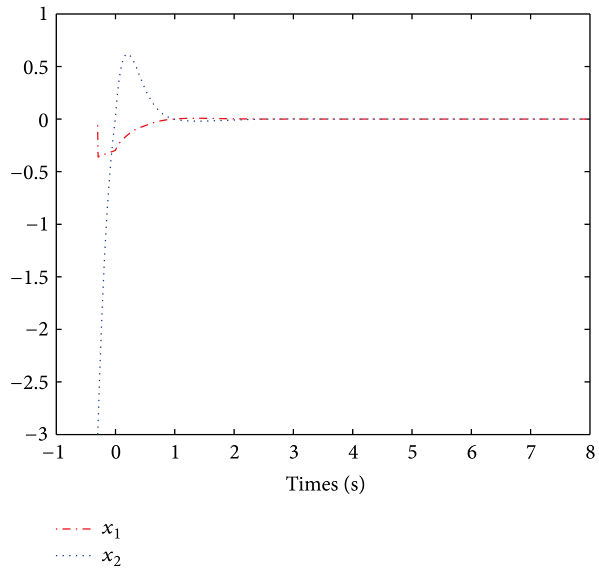

FIGURE 1: The system state under adaptive memory controller.

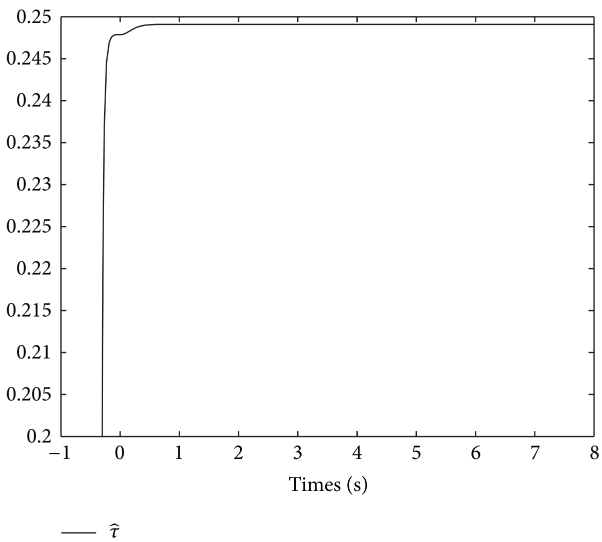

FIGURE 2: The estimate value of the unknown time delay.

$d\tau_3(\hat{\tau}_1(t-\tau_2))/d(\hat{\tau}_1(t-\tau_2)) = 2\hat{\tau}_1(t-\tau_2) - 2h_1 + a_1 = 0$, so $\tau_3(t)$ can achieve extremum when $\hat{\tau}_1(t-\tau_2) = (2h_1 - a_1)/2$. Furthermore, as $\tau_3''((2h_1 - a_1)/2) = 2 > 0$, so $\tau_3(t)$ can achieve minimum when $\hat{\tau}_1(t-\tau_2) = (2h_1 - a_1)/2 = \bar{h}_1$. As a result, the maximum for $\tau_3(t)$, that is, $\tau_3^* = \max\{\tau_3(\hat{\tau}_1(t-\tau_2) = \{\tau_1^*, \tau_{1*}\})\}$.

B. The Value of $\tilde{\tau}_4^*$

Consider

$$\begin{aligned}\tilde{\tau}_4 = \tau_4^2 &= \left[\tau_2 + a_1\hat{\tau}_1(t-\tau_2) + (\hat{\tau}_1(t-\tau_2) - h_1)^2\right]^2 \\ &= \tau_2^2 + a_1^2(\hat{\tau}_1(t-\tau_2))^2 + (\hat{\tau}_1(t-\tau_2) - h_1)^4 \\ &\quad + 2\tau_2 a_1\hat{\tau}_1(t-\tau_2) + 2\tau_2(\hat{\tau}_1(t-\tau_2) - h_1)^2 \\ &\quad + 2a_1\hat{\tau}_1(t-\tau_2)(\hat{\tau}_1(t-\tau_2) - h_1)^2.\end{aligned} \tag{B.1}$$

As it is well known that the value of third power of a variable is difficult to obtain, since

$$\hat{\tau}_1 - h_1 \le \tau_1^* - h_1 = \tau_1^* - \sqrt{\bar{h}_1 + \bar{h}_1^2} < \tau_1^* - \bar{h}_1 \le \bar{\tau}_1, \tag{B.2}$$

then substituting (B.2) into (B.1), we have

$$\begin{aligned}\tau_4^2 \le{}& \tau_2^2 + a_1^2(\hat{\tau}_1(t-\tau_2))^2 + \bar{\tau}_1^4 + 2\tau_2 a_1\hat{\tau}_1(t-\tau_2) \\ &+ 2\tau_2\bar{\tau}_1^2 + 2a_1\hat{\tau}_1(t-\tau_2)(\hat{\tau}_1(t-\tau_2) - h_1)^2 \triangleq \tilde{\tau}.\end{aligned} \tag{B.3}$$

By the similar deduction in Appendix A, we have the following conclusion under two kinds of situations.

(1) If $2\bar{h}_1\sqrt{\bar{h}_1 + \bar{h}_1^2} > 2\bar{h}_1 + \bar{h}_1^2 + 3\tau_2$, we can obtain the maximum of $\tilde{\tau}$ which is

$$\tilde{\tau}^* = \max\{\tilde{\tau}(\hat{\tau}_1(t-\tau_2) = \{\tau_1^*, \tau_{1*}, \tilde{\tau}_2\})\}, \tag{B.4}$$

where $\tilde{\tau}_2 = (4h_1 - a_1 - \sqrt{a_1^2 - 8a_1h_1 + 4h_1^2 - 12\tau_2})/6$.

(2) If $2\bar{h}_1\sqrt{\bar{h}_1 + \bar{h}_1^2} \le 2\bar{h}_1 + \bar{h}_1^2 + 3\tau_2$, we can obtain that

$$\tilde{\tau}^* = \max\{\tilde{\tau}(\hat{\tau}_1(t-\tau_2) = \{\tau_1^*, \tau_{1*}\})\}. \tag{B.5}$$

C. The Value of $\tilde{\tau}_6^*$

From Appendices A and B, we can obtain that $\tilde{\tau}_6^* = \tilde{\tau}_4^* - 2\tau_2(\tau_{3*} - \tau_2 - (1/4)\bar{h}_1)$.

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