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Research Article

Controller Design of Multiinput Multioutput Time-Delay Large-Scale System

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The paper presents a novel feedback linearization controller of nonlinear multiinput multioutput time-delay large-scale systems to obtain both the tracking and almost disturbance decoupling (ADD) performances. The significant contribution of this paper is to build up a control law such that the overall closed-loop system is stable for given initial condition and bounded tracking trajectory with the input-to-state-stability characteristic and almost disturbance decoupling performance. We have simulated the two-inverted-pendulum system coupled by a spring for networked control systems which has been used as a test bed for the study of decentralized control of large-scale systems.

1. Introduction

Recently, robust stabilization of system with time delay has been of a challenging and interesting problem [1]. As we know, in general, the existence of time-delay degrades the control performance and sometimes makes the closed-loop stabilization difficult, especially for nonlinear systems. Appropriate mathematical descriptions incorporating the time delay are the differential-difference equations, that is, differential equations with deviating arguments. Several related reports have shown that differential-difference equations have been widely applied in theory of automatic control, the theory of self-oscillating systems, the study of problems connected with combustion in rocket motion, the problem of long-range planning in economics, a series of biological problems, and in many other areas of science and technology [2, 3]. In the past, there have been a number of interesting developments in stability criteria and controller designs for time-delay control systems but mostly were restricted to linear cases; see, for example, [1, 4–6]. In general, the global stability test of control systems with time delays is not as easy, even in the linear case, as without time delays. It requires some disgusting tasks as solving nonlinear matrix equations [7]. It is clear that the investigation of nonlinear time-delay systems is worthwhile. In this paper, the globally tracking and almost disturbance decoupling problem of a general class of nonlinear time-delay control systems is investigated.

A large-scale system is organized as some interconnected subsystems, such as industrial control system, power systems, computer, biomedical networks, district heating systems [8], and wind farm power control [9]. Due to the inherited complexity of control approach and physical constraints on information communication between subsystems, it is necessary to design for each subsystem a decentralized controller depending on local data, even if to construct an object for the whole large-scale system. Recently networked control systems have attracted many research interests. The significant characteristics of networked control systems are controller decentralization, interconnected diagnostics, convenient maintenance, and low cost [10]. The application of networked control architecture will increase the performances of interconnected applications via reduced maintenance costs [11]. The major limitations in the existing networked control systems research are that (a) most researches are limited to linear cases and (b) the significant applications of networked control systems are for distributed control systems. However, some theoretical results are not valid for distributed control systems and then have little application value [12, 13]. We have proposed a study of two-pendulum system coupled by a spring for networked control systems which has been used as a test bed for the investigation of decentralized control of large-scale systems.

Recently, variable control method has been utilized to investigate nonlinear industrial system. However, nasty chattering characteristics may excite unmodeled high frequency and even force system into instable state for variable control method [14]. The backstepping approach has been an important method for designing controller for nonlinear industrial systems. However, a disadvantage with the backstepping approach is the intricacy which is generated by the complex duplicated differentiations of some nonlinear functions [15]. The output regulation control method [16] is utilized to the industrial system in which the output terminals are derived with an exosystem. However, the output regulation problem should solve the complicated solution of partial-differential equation and the problem of creating the transient tracking errors. The nonlinear H^{∞} control method generally has to find out the solution for the complex Hamilton-Jacobi equation [17]. However, it is unfortunate that we can solve a closed-form solution only for some special industrial control systems [18]. The internal-model-principle method, solving a partial-differential center manifold equation [19], transforms the original tracking problem to output regulation structure. Only for some particular industrial systems and desired trajectories, the asymptotic solutions can be found out [20]. The H^{∞} adaptive fuzzy control approach has been utilized to systematically address some industrial control systems [1]. Its shortcoming is that the troublesome parameter update law makes the method impractical. During the past decade, the feedback linearization control has been the significant research direction for nonlinear systems [21] and has been addressed successfully to investigate many industrial control systems including the shunt hybrid power filter [22], high-power self-commutated voltage-source converter and current-source converter [23], and differential-drive wheeled mobile robots [24].

The almost disturbance decoupling control problem, originally developed for linear and nonlinear control systems by [25, 26], respectively, exploits the research that a controller reduces the effect of the disturbance on the outputs to an arbitrary degree of accuracy, and then many important researches have been proposed for nonlinear industrial control systems [27, 28]. The authors of [26] exploit the fact that the almost disturbance decoupling performance cannot be achieved for the following control system: $\dot{x}_1(t) = \tan^{-1}(x_2) + \theta(t)$, $\dot{x}_2(t) = u$, $y = x_1$, where u, y are the input and output, respectively, and θ is the disturbance term. It is fortunate that this example can be easily solved via the proposed approach in this study. In order to exploit the significant industrial applicability, this study has favorably designed the almost disturbance decoupling controller for a two-inverted-pendulum system.

2. Tracking and Almost Disturbance Decoupling Controller Design

Consider the following nonlinear time-delay large-scale system with disturbance that is organized into N subsystems interconnected by their output terminals. The ith subsystem S_i, $1 \le i \le N$, is shown as

$$\begin{aligned}
\dot{x}_{i1} &= f_{i1}\left(x_{i1}, x_{i2}, \ldots, x_{in_i}\right) + d_{i1}\left(y_i\ \left(t - \tau_{i1}(t)\right)\right) \\
&\quad + \Delta f_{i1}\left(y_1, y_2, \ldots, y_N\right) + \theta_{i1}, \\
\dot{x}_{i2} &= f_{i2}\left(x_{i1}, x_{i2}, \ldots, x_{in_i}\right) + d_{i2}\left(y_i\left(t - \tau_{i2}(t)\right)\right) \\
&\quad + \Delta f_{i2}\left(y_1, y_2, \ldots, y_N\right) + \theta_{i2} \\
&\ \vdots \\
&\ \vdots \\
\dot{x}_{in_i} &= f_{in_i}\left(x_{i1}, x_{i2}, \ldots, x_{in_i}\right) + d_{in_i}\left(y_i\left(t - \tau_{in_i}(t)\right)\right) \\
&\quad + \Delta f_{in_i}\left(y_1, y_2, \ldots, y_N\right) + \theta_{in_i} + g_i u_i, \\
y_i &= x_{i1},
\end{aligned} \tag{1}$$

where $\left[x_{i1}(t)\ \ x_{i2}(t)\ \ \cdots\ \ x_{in_i}(t)\right]^T \in \mathfrak{R}^{n_i}$, $u_{in_i} \in \mathfrak{R}$, and $y_i \in \mathfrak{R}$ are the state vector, the input vector, and the output vector of subsystem S_i, respectively. f_{in_i} and g_{in_i} are smooth nonlinear system functions. Δf_{in_i} represents the nonlinearity in the ith subsystem and the interconnection function between the ith subsystem and other subsystems. θ_{in_i} is a bounded time-varying disturbances vector, and $\tau_{in_i}(t)$ is a time-delay term.

Define

$$\begin{aligned}
N_t &\equiv \sum_{i=1}^{N} n_i, \\
x_1 &\equiv x_{11}, x_2 \equiv x_{12}, \ldots, x_{n_1} \equiv x_{1n_1}, \\
x_{n_1+1} &\equiv x_{21}, x_{n_1+2} \equiv x_{22}, \ldots, x_{n_1+n_2} \equiv x_{2n_2}, \\
&\ \vdots \\
x_{n_1+n_2+\cdots+n_{N-1}+1} &\equiv x_{N1}, x_{n_1+n_2+\cdots+n_{N-1}+2} \\
&\equiv x_{N2}, \ldots, x_{n_1+n_2+\cdots+n_N} \equiv x_{Nn_N}, \\
n_0 &\equiv 0, \\
x_{n_0+n_1+\cdots+n_i+j} &\equiv x_{(i+1)(j)}, \tau_{n_0+n_1+\cdots+n_i+j} \\
&\equiv \tau_{(i+1)(j)}, \Delta f_{n_0+n_1+\cdots+n_i+j} \\
&\equiv \Delta f_{(i+1)(j)}, d_{n_0+n_1+\cdots+n_i+j} \equiv d_{(i+1)(j)}, \\
D_{e(n_0+n_1+\cdots+n_i+j)} &\equiv d_{(i+1)(j)}.
\end{aligned} \tag{2}$$

The nonlinear time-delay large-scale system will be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{N_t} \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_{N_t}) \\ f_2(x_1, x_2, \ldots, x_{N_t}) \\ \vdots \\ f_{N_t}(x_1, x_2, \ldots, x_{N_t}) \end{bmatrix} + \begin{bmatrix} 0_{(n_1-1)\times 1} \\ g_1 u_1 \\ 0_{(n_2-1)\times 1} \\ g_2 u_2 \\ \vdots \\ 0_{(n_N-1)\times 1} \\ g_N u_N \end{bmatrix} + \begin{bmatrix} D_{e1}(\tau_1) \\ D_{e2}(\tau_2) \\ \vdots \\ D_{eN_t}(\tau_{N_t}) \end{bmatrix} + \begin{bmatrix} \Delta f_1(x_1, x_2, \ldots, x_{N_t}) \\ \Delta f_2(x_1, x_2, \ldots, x_{N_t}) \\ \vdots \\ \Delta f_{N_t}(x_1, x_2, \ldots, x_{N_t}) \end{bmatrix} + \sum_{j=1}^{N_t} q_j^* \theta_{jd}, \tag{3a}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \equiv \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_N \end{bmatrix} = \begin{bmatrix} x_1 \\ x_{n_1+1} \\ \vdots \\ x_{n_1+n_2+\cdots+n_N+1} \end{bmatrix}. \tag{3b}$$

That is,

$$\dot{X}(t) = f(X(t)) + gu + D_e + \Delta f + \sum_{j=1}^{N_t} q_j^* \theta_{jd}, \qquad y(t) = h(X(t)), \tag{4}$$

where $X(t) \equiv [x_1(t)\ x_2(t)\ \cdots\ x_{N_t}(t)]^T \in \mathfrak{R}^{N_t}$ is the state vector, $gu \equiv [0_{(n_1-1)\times 1}\ g_1u_1\ 0_{(n_2-1)\times 1}\ g_2u_2\ \cdots\ 0_{(n_N-1)\times 1}\ g_Nu_N]^T \in \mathfrak{R}^{N_t}$ is the input vector, $y \equiv [y_1\ y_2\ \cdots\ y_N]^T \in \mathfrak{R}^N$ is the output vector, $\theta_d \equiv [\theta_{1d}(t)\ \theta_{2d}(t)\ \cdots\ \theta_{N_td}(t)]^T \in \mathfrak{R}^{N_t}$ is a bounded time-varying disturbances vector, and $\Delta f \equiv [\Delta f_1\ \Delta f_2\ \cdots\ \Delta f_{N_t}] \in \mathfrak{R}^{N_t}$. $f \equiv [f_1\ f_2\ \cdots\ f_{N_t}]^T \in \mathfrak{R}^{N_t}$, $D \equiv [D_{e1}\ D_{e2}\ \cdots\ D_{eN_t}]^T \in \mathfrak{R}^{N_t}$, $D_e \equiv \sum_{i=1}^{N_t} q_i^* D_{ei} \in \mathfrak{R}^{N_t}$, and $h \equiv [h_1\ h_2\ \cdots\ h_N]^T \in \mathfrak{R}^N$ are smooth vector fields. The nominal system is then defined as follows:

$$\dot{X}(t) = f(X(t)) + gu, \tag{5a}$$

$$y(t) = h(X(t)). \tag{5b}$$

The nominal system of the form (5a) and (5b) is said to have the vector relative degree $\{r_1, r_2, \ldots, r_N\}$ [29] with the following properties for all $X \in \mathfrak{R}^{N_t}$:

(i)

$$L_{g_j} L_f^k h_i(X) = 0, \quad 1 \le i \le N, \quad 1 \le j \le N,\ k < r_i - 1, \tag{6}$$

where the operator L is the Lie derivative [30] and $r_1 + r_2 + \cdots + r_N = r$;

(ii) the $N \times N$ matrix

$$A_{\text{non}} \equiv \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1(X) & \cdots & L_{g_N} L_f^{r_1-1} h_1(X) \\ L_{g_1} L_f^{r_2-1} h_2(X) & \cdots & L_{g_N} L_f^{r_2-1} h_2(X) \\ \vdots & & \vdots \\ L_{g_1} L_f^{r_N-1} h_N(X) & & L_{g_N} L_f^{r_N-1} h_N(X) \end{bmatrix} \tag{7}$$

is nonsingular.

The desired output trajectory y_d^i, $1 \le i \le N$ and its first r_i derivatives are all uniformly bounded, and

$$\left\| \left[y_d^i, y_d^{i\,(1)}, \ldots, y_d^{i\,(r_i)} \right] \right\| \le B_d^i, \quad 1 \le i \le N, \tag{8}$$

where B_d^i is some positive real constant. Based on the existence of vector relative degree, it has been shown [30] that the function

$$\Omega : \mathfrak{R}^{N_t} \longrightarrow \mathfrak{R}^{N_t} \tag{9}$$

defined as

$$\xi_i \equiv \begin{bmatrix} \xi_1^i \\ \xi_2^i \\ \vdots \\ \xi_{r_i}^i \end{bmatrix} \equiv \begin{bmatrix} \Omega_1^i \\ \Omega_2^i \\ \vdots \\ \Omega_{r_i}^i \end{bmatrix} \equiv \begin{bmatrix} L_f^0 h_i(X) \\ L_f^1 h_i(X) \\ \vdots \\ L_f^{r_i-1} h_i(X) \end{bmatrix}, \quad i = 1, 2, \ldots, N,$$
$$\Omega_k(X(t)) \equiv \eta_k(t), \quad k = r+1, r+2, \ldots, N_t,$$
$$L_{g_j} \Omega_k(X(t)) = 0, \quad k = r+1,\ r+2, \ldots, N_t,\ 1 \le j \le N \tag{10}$$

is a diffeomorphism, if (i) the distribution

$$g_{\text{span}} \equiv \text{span}\{g_1, g_2, \ldots, g_N\} \tag{11}$$

has involutive property; (ii) the vector fields

$$Z_j^k, \quad 1 \le j \le N,\ 1 \le k \le r_j \tag{12}$$

are complete, where

$$Z_j^k \equiv (-1)^{k-1} ad_{\tilde{f}}^{k-1} \tilde{g}_j, \quad 1 \le j \le N,\ 1 \le k \le r_j,$$
$$\tilde{f}(X) \equiv f(X) - g(X) A_{\text{non}}^{-1}(X) s(X),$$
$$s(X) \equiv \left[L_f^{r_1} h_1(X)\ \ L_f^{r_2} h_2(X)\ \cdots\ L_f^{r_N} h_N(X) \right]^T,$$
$$\tilde{g} \equiv [\tilde{g}_1\ \tilde{g}_2\ \cdots\ \tilde{g}_N] \equiv g(X) A_{\text{non}}^{-1}(X), \tag{13}$$
$$ad_p^k q \equiv \left[p\ \ ad_p^{k-1} q \right],$$
$$[p\ \ q] \equiv \frac{\partial q}{\partial X} p(X) - \frac{\partial p}{\partial X} q(X).$$

Denote the trajectory error to be

$$e_j^i \equiv \xi_j^i - y_d^{i(j-1)}, \quad i = 1,2,\ldots,N, \; j = 1,2,\ldots,r_i, \tag{14}$$

$$e^i \equiv \left[e_1^i \;\; e_2^i \;\; \cdots \;\; e_{r_i}^i\right]^T \in \mathfrak{R}^{r_i}, \tag{15}$$

$$\overline{e_j^i} \equiv \kappa^{j-1} e_j^i, \quad i = 1,2,\ldots,N, \; j = 1,2,\ldots,r_i, \tag{16}$$

$$\overline{e^i} \equiv \left[\overline{e_1^i} \;\; \overline{e_2^i} \;\; \cdots \;\; \overline{e_{r_i}^i}(t)\right]^T \in \mathfrak{R}^{r_i}, \tag{17}$$

$$\overline{e} \equiv \left[\overline{e^1} \;\; \overline{e^2} \;\; \ldots \;\; \overline{e^N}\right]^T \in \mathfrak{R}^{r}, \tag{18}$$

$$\xi \equiv \left[\xi_1 \;\; \xi_2 \;\; \cdots \;\; \xi_r\right]^T \in \mathfrak{R}^{r}, \tag{19}$$

$$\eta(t) \equiv \left[\eta_{r+1}(t) \;\; \eta_{r+2}(t) \;\; \cdots \;\; \eta_{N_t}(t)\right]^T \in \mathfrak{R}^{N_t - r}, \tag{20}$$

$$\begin{aligned} q(\xi(t), \eta(t)) &\equiv \left[L_f\phi_{r+1}(t) \;\; L_f\phi_{r+2}(t) \;\; \cdots \;\; L_f\phi_{N_t}(t)\right]^T \\ &\equiv \left[q_{r+1} \;\; q_{r+2} \;\; \cdots \;\; q_{N_t}\right]^T. \end{aligned} \tag{21}$$

Define a phase-variable canonical matrix A_{phase}^i to be

$$A_{\text{phase}}^i \equiv \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_1^i & -a_2^i & -a_3^i & \cdots & -a_{r_i}^i \end{bmatrix}_{r_i \times r_i}, \quad 1 \le i \le N, \tag{22}$$

where $a_1^i, a_2^i, \ldots, a_{r_i}^i$ are any chosen variable such that A_{phase}^i is Hurwitz, and define the vector B^i to be

$$B^i \equiv \left[0 \;\; 0 \;\; \cdots \;\; 0 \;\; 1\right]_{r_i \times 1}^T, \quad 1 \le i \le N. \tag{23}$$

Define the positive definite matrix Q^i to be solution of the following Lyapunov equation:

$$\begin{gathered} \left(A_{\text{phase}}^i\right)^T Q^i + Q^i A_{\text{phase}}^i = -I, \quad 1 \le i \le N, \\ \lambda_{\max}\left(Q^i\right) \equiv \text{the maximum eigenvalue of } Q^i, \quad 1 \le i \le N, \\ \lambda_{\min}\left(Q^i\right) \equiv \text{ the minimum eigenvalue of } Q^i, \quad 1 \le i \le N, \\ \lambda_{\max}^* \equiv \min\left\{\lambda_{\max}\left(Q^1\right), \lambda_{\max}\left(Q^2\right), \ldots, \lambda_{\max}\left(Q^N\right)\right\}, \\ \lambda_{\min}^* \equiv \min\left\{\lambda_{\min}\left(Q^1\right), \lambda_{\min}\left(Q^2\right), \ldots, \lambda_{\min}\left(Q^N\right)\right\}. \end{gathered} \tag{24}$$

Assumption 1. For all $t \ge 0$, $\eta \in \mathfrak{R}^{N_t - r}$, and $\xi \in \mathfrak{R}^r$, there exists a positive constant L_m such that the following inequality holds:

$$\left\|q_{22}(t, \eta, \overline{e}) - q_{22}(t, \eta, 0)\right\| \le L_m(\|\overline{e}\|), \tag{25}$$

where $q_{22}(t, \eta, \overline{e}) \equiv q(\xi, \eta)$.

For the sake of describing precisely the considered problem, denote

$$\begin{gathered} d_{ij} \equiv L_{g_j} L_f^{r_i - 1} h_i(X), \quad 1 \le i \le N, \; 1 \le j \le N, \\ c_i \equiv L_f^{r_i} h_i(X), \quad 1 \le i \le N, \\ \overline{\overline{e^i}} \equiv a_1^i \overline{e_1^i} + a_2^i \overline{e_2^i} + \cdots + a_{r_i}^i \overline{e_{r_i}^i}, \quad 1 \le i \le N. \end{gathered} \tag{26}$$

Definition 2 (see [31]). Consider the nonlinear system $\dot{z} = f(t, z, n)$, where $f : [0, \infty) \times \mathfrak{R}^n \times \mathfrak{R}^n \rightarrow \mathfrak{R}^n$ is piecewise continuous in t and has the locally Lipschitz property in z and n. This nonlinear system is said to has the input-to-state stable property if there exist a class KL function β, a class K function γ, and positive real constants λ_1 and λ_2 such that, for given initial state $z(t_0)$ with $\|z(t_0)\| < \lambda_1$ and any bounded noise $n(t)$ with $\sup_{t \ge t_0}\|n(t)\| < \lambda_2$, the system state has the following property:

$$\|z(t)\| \le \beta\left(\left\|z(t_0)\right\|, t - t_0\right) + \gamma\left(\sup_{t_0 \le \tau \le t} \|n(\tau)\|\right) \tag{27}$$

for all $t \ge t_0 \ge 0$.

Definition 3 (see [32]). The tracking problem with almost disturbance decoupling performance is denoted to be globally solved by the control law u for the overall system, if the control law u has the following characteristics.

(i) The control system is input-to-state stable for disturbance inputs.

(ii) For any given initial value $\overline{x}_{e0} \equiv \left[\overline{e}(t_0) \;\; \eta(t_0)\right]^T$, for any $t \ge t_0$, and for any $t_0 \ge 0$,

$$\begin{aligned} \left|y(t) - y_d(t)\right| \le{}& \beta_{11}\left(\left\|x(t_0)\right\|, t - t_0\right) \\ &+ \frac{1}{\sqrt{\beta_{22}}}\beta_{33}\left(\sup_{t_0 \le \tau \le t} \|\theta(\tau)\|\right), \end{aligned} \tag{28}$$

$$\begin{aligned} &\int_{t_0}^t \left[y(\tau) - y_d(\tau)\right]^2 d\tau \\ &\quad \le \frac{1}{\beta_{44}}\left[\beta_{55}\left(\|\overline{x}_{e0}\|\right) + \int_{t_0}^t \beta_{33}\left(\|\theta(\tau)\|^2\right) d\tau\right], \end{aligned} \tag{29}$$

where β_{22}, β_{44} are positive real constants, β_{33}, β_{55} are class K functions, and β_{11} is a class KL function.

Theorem 4. *Assume that there exists a continuously differentiable function $T : \mathfrak{R}^{N_t - r} \rightarrow \mathfrak{R}^+$ such that the following properties hold for all $\eta \in \mathfrak{R}^{N_t - r}$:*

$$\Delta_1\|\eta\|^2 \le T(\eta) \le \Delta_2\|\eta\|^2, \quad \Delta_1, \Delta_2 > 0, \tag{30a}$$

$$\nabla_t T + \left(\nabla_\eta T\right)^T q_{22}(t, \eta, 0) \le -2\alpha_x\|\eta\|^2, \quad \alpha_x > 0, \tag{30b}$$

$$\left\|\nabla_\eta T\right\| \le \Delta_3\|\eta\|, \quad \Delta_3 > 0. \tag{30c}$$

Then the tracking problem with almost disturbance decoupling performance for time-delay large-scale system is globally solved by the control law:

$$u = A_{non}^{-1}\left\{-u_b + u_v\right\}, \tag{31}$$

$$u_b \equiv \left[L_f^{r_1}h_1 \;\; L_f^{r_2}h_2 \;\; \cdots \;\; L_f^{r_N}h_N\right]^T, \tag{32}$$

$$u_v \equiv \left[v_1 \;\; v_2 \;\; \cdots \;\; v_N\right]^T, \tag{33}$$

$$\begin{aligned} v_i \equiv\; & y_d^{i\,(r_i)} - \kappa^{-r_i} a_1^i \left[L_f^0 h_i(X) - y_d^i\right] \\ & - \kappa^{1-r_i} a_2^i \left[L_f^1 h_i(X) - y_d^{i\,(1)}\right] \\ & - \cdots - \kappa^{-1} a_{r_i}^i \left[L_f^{r_i-1} h_i(X) - y_d^{i\,(r_i-1)}\right], \quad 1 \le i \le N. \end{aligned} \tag{34}$$

Moreover, the influence of disturbances on the L_2 norm of the tracking error can be arbitrarily reduced by adjusting parameter $N_2 > 1$:

$$k_{11} \equiv \frac{k}{2\kappa} - \frac{25k^2}{\kappa^2}\left\|\phi_\xi^1\right\|^2\left\|Q^1\right\|^2 - \cdots - \frac{25k^2}{\kappa^2}\left\|\phi_\xi^N\right\|^2\left\|Q^N\right\|^2 - 25, \tag{35a}$$

$$k_{22} \equiv 22\alpha_x - \frac{1}{100}\Delta_3^2 L_m^2 - 25\Delta_3^2\left\|\phi_\eta\right\|^2, \tag{35b}$$

$$N_2 \equiv \min\left\{k_{11}, k_{22}\right\}, \tag{35c}$$

$$\Omega_\xi^i(\kappa) \equiv \begin{bmatrix} \kappa\frac{\partial}{\partial X}h_i q_1^* & \cdots & \kappa\frac{\partial}{\partial X}h_i q_p^* \\ \vdots & & \vdots \\ \kappa^{r_i}\frac{\partial}{\partial X}L_f^{r_i-1}h_i q_1^* & \cdots & \kappa^{r_i}\frac{\partial}{\partial X}L_f^{r_i-1}h_i q_q^* \end{bmatrix}, \quad 1 \le i \le N, \tag{35d}$$

$$\Omega_\eta(\kappa) \equiv \begin{bmatrix} \frac{\partial}{\partial X}\phi_{r+1} q_1^* & \cdots & \frac{\partial}{\partial X}\phi_{r+1} q_p^* \\ \vdots & & \vdots \\ \frac{\partial}{\partial X}\phi_{N_t} q_1^* & \cdots & \frac{\partial}{\partial X}\phi_{N_t} q_q^* \end{bmatrix}, \tag{35e}$$

where $k(\kappa) : \mathfrak{R}^+ \rightarrow \mathfrak{R}^+$ is a continuous function satisfying

$$\lim_{\varepsilon \to 0} k(\kappa) = 0, \qquad \lim_{\kappa \to 0}\frac{\kappa}{k(\kappa)} = 0. \tag{35f}$$

Proof. Applying the coordinate transformation (9) yields

$$\begin{aligned}
\dot{\xi}_1^1 &= \frac{\partial h_1}{\partial X} f + \sum_{j=1}^{N_t}\frac{\partial h_1}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right) \\
&= \xi_2^1 + \sum_{j=1}^{N_t}\frac{\partial h_1}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right), \\
&\;\;\vdots \\
\dot{\xi}_{r_1-1}^1 &= \frac{\partial L_f^{r_1-2}h_1}{\partial X} f + \sum_{j=1}^{N_t}\frac{\partial L_f^{r_1-2}h_1}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right) \\
&= L_f^{r_1-1}h_1 + \sum_{j=1}^{N_t}\frac{\partial L_f^{r_1-2}h_1}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right), \\
\dot{\xi}_{r_1}^1 &= c_1 + d_{11}u_1 + \cdots + d_{1N}u_N \\
&\quad + \sum_{j=1}^{N_t}\frac{\partial L_f^{r_1-1}h_1}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right), \\
&\;\;\vdots \\
\dot{\xi}_1^N &= L_f^1 h_N + \sum_{j=1}^{N_t}\frac{\partial h_1}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right) \\
&= \xi_2^N + \sum_{j=1}^{N_t}\frac{\partial h_N}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right), \\
&\;\;\vdots \\
\dot{\xi}_{r_N-1}^N &= L_f^{r_N-1}h_N + \sum_{j=1}^{N_t}\frac{\partial L_f^{r_N-2}h_N}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right) \\
&= \xi_{r_N}^N + \sum_{j=1}^{N_t}\frac{\partial L_f^{r_N-2}h_N}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right), \\
\dot{\xi}_{r_N}^N &= c_N + d_{N1}u_1 + \cdots + d_{NN}u_N \\
&\quad + \sum_{j=1}^{N_t}\frac{\partial L_f^{r_N-1}h_N}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right), \\
\dot{\eta}_k(t) &= L_f\Omega_k + \sum_{j=1}^{N_t}\frac{\partial \Omega_k}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right) \\
&= q_k + \sum_{j=1}^{N_t}\frac{\partial \Omega_k}{\partial X} q_j^*\left(\theta_{jd} + D_{ej}\right).
\end{aligned} \tag{36}$$

Since

$$c_i(\xi(t),\eta(t)) \equiv L_f^{r_i} h_i(X(t)), \quad 1 \le i \le N, \tag{37}$$

$$d_{ij} \equiv L_{g_j} L_f^{r_i-1} h_i(X), \quad 1 \le i \le N,\ 1 \le j \le N, \tag{38}$$

$$q_k(\xi(t),\eta(t)) = L_f \Omega_k(X), \quad k = r+1,\ r+2,\dots,N_t. \tag{39}$$

The dynamic equations of systems (3a) and (3b) can be rewritten as follows:

$$\dot{\xi}_i^1(t) = \xi_{i+1}^1(t) + \sum_{j=1}^{N_t} \frac{\partial}{\partial X} L_f^{i-1} h_1 q_j^*(\theta_{jd} + D_{ej}), \quad i = 1,2,\dots, r_1 - 1, \tag{40}$$

$$\begin{aligned}\dot{\xi}_{r_1}^1(t) &= c_1(\xi(t),\eta(t)) + d_{11}(\xi(t),\eta(t))u_1 \\ &+ \cdots + d_{1N}(\xi(t),\eta(t))u_N \\ &+ \sum_{j=1}^{N_t} \frac{\partial}{\partial X} L_f^{r_1-1} h_1 q_j^*(\theta_{jd} + D_{ej}),\end{aligned} \tag{41}$$

$$\vdots$$

$$\dot{\xi}_i^N(t) = \xi_{i+1}^N(t) + \sum_{j=1}^{N_t} \frac{\partial}{\partial X} L_f^{i-1} h_N q_j^*(\theta_{jd} + D_{ej}), \quad i = 1,2,\dots, r_N - 1, \tag{42}$$

$$\begin{aligned}\dot{\xi}_{r_N}^N(t) &= c_N(\xi(t),\eta(t)) + d_{N1}(\xi(t),\eta(t))u_1 \\ &+ \cdots + d_{NN}(\xi(t),\eta(t))u_N \\ &+ \sum_{j=1}^{N_t} \frac{\partial}{\partial X} L_f^{r_N-1} h_N q_j^*(\theta_{jd} + D_{ej}),\end{aligned} \tag{43}$$

$$\dot{\eta}_k(t) = q_k(\xi(t),\eta(t)) + \sum_{j=1}^{N_t} \frac{\partial}{\partial X} \Omega_k(X) q_j^*(\theta_{jd} + D_{ej}), \quad k = r+1,\dots,N_t, \tag{44}$$

$$y_i(t) = \xi_1^i(t), \quad 1 \le i \le N. \tag{45}$$

Applying (14), (34), (37), and (38) yields the controller as

$$u = A_{\text{non}}^{-1}[-u_b + u_v]. \tag{46}$$

The dynamic equations of systems (3a) and (3b) can be rewritten as follows by substituting (46) into (41) and (43):

$$\begin{aligned}
\begin{bmatrix} \dot{\xi}_1^i(t) \\ \dot{\xi}_2^i(t) \\ \vdots \\ \dot{\xi}_{r_i-1}^i(t) \\ \dot{\xi}_{r_i}^i(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ & \vdots & & & & \vdots \\ 0 & 0 & 0 & & \cdots & 1 \\ 0 & 0 & 0 & & \cdots & 0 \end{bmatrix} \begin{bmatrix} \xi_1^i(t) \\ \xi_2^i(t) \\ \vdots \\ \xi_{r_i-1}^i(t) \\ \xi_{r_i}^i(t) \end{bmatrix} \\
&+ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} v_i + \begin{bmatrix} \sum_{j=1}^{N_t} \frac{\partial}{\partial X} h_i q_j^*(\theta_{jd} + D_{ej}) \\ \sum_{j=1}^{N_t} \frac{\partial}{\partial X} L_f^1 h_i q_j^*(\theta_{jd} + D_{ej}) \\ \vdots \\ \sum_{j=1}^{N_t} \frac{\partial}{\partial X} L_f^{r_i-1} h_i q_j^*(\theta_{jd} + D_{ej}) \end{bmatrix}, \\
\begin{bmatrix} \dot{\eta}_{r+1}(t) \\ \dot{\eta}_{r+2}(t) \\ \vdots \\ \dot{\eta}_{N_t-1}(t) \\ \dot{\eta}_{N_t}(t) \end{bmatrix} &= \begin{bmatrix} q_{r+1}(t) \\ q_{r+2}(t) \\ \vdots \\ q_{N_t-1}(t) \\ q_{N_t}(t) \end{bmatrix} + \begin{bmatrix} \sum_{j=1}^{N_t} \frac{\partial}{\partial X} \Omega_{r+1} q_j^*(\theta_{jd} + D_{ej}) \\ \sum_{j=1}^{N_t} \frac{\partial}{\partial X} \Omega_{r+2} q_j^*(\theta_{jd} + D_{ej}) \\ \vdots \\ \sum_{j=1}^{N_t} \frac{\partial}{\partial X} \Omega_{N_t-1} q_j^*(\theta_{jd} + D_{ej}) \\ \sum_{j=1}^{N_t} \frac{\partial}{\partial X} \Omega_{N_t} q_j^*(\theta_{jd} + D_{ej}) \end{bmatrix}, \\
y_i &= [1 \ \ 0 \ \ \cdots \ \ 0 \ \ 0]_{r\times 1} \begin{bmatrix} \xi_1^i(t) \\ \xi_2^i(t) \\ \vdots \\ \xi_{r_i-1}^i(t) \\ \xi_{r_i}^i(t) \end{bmatrix}_{r\times 1} \\
&= \xi_1^i(t), \quad 1 \le i \le N.
\end{aligned} \tag{47}$$

Combining (14), (16), (17), (22), and (34) yields the transformation of (47):

$$\begin{aligned}\dot{\eta}(t) &= q(\xi(t),\eta(t)) + \Omega_\eta(\theta_d + D) \\ &\equiv q_{22}(t,\eta(t),\overline{e}) + \Omega_\eta(\theta_d + D),\end{aligned} \tag{48a}$$

$$\kappa \dot{\overline{e^i}}(t) = A_{\text{phase}}^i \overline{e^i} + \Omega_\xi^i(\theta_d + D), \quad 1 \le i \le N, \tag{48b}$$

$$y_i(t) = \xi_1^i(t), \quad 1 \le i \le N. \tag{49}$$

We consider $L(\bar{e},\eta)$ denoted by a weighted sum of $T(\eta)$ and $W(\bar{e})$, and

$$L(\bar{e},\eta) \equiv T(\eta) + k(\kappa) W(\bar{e}) \equiv T(\eta) + k(\kappa)\left(W^1\left(\overline{e^1}\right) + \cdots + W^N\left(\overline{e^N}\right)\right), \tag{50}$$

where

$$W(\bar{e}) \equiv W^1\left(\overline{e^1}\right) + \cdots + W^N\left(\overline{e^N}\right) \tag{51}$$

as a composite Lyapunov function of the subsystems (48a) and (48b) [33, 34], where $W(\overline{e^i})$ satisfies

$$W^i\left(\overline{e^i}\right) \equiv \frac{1}{2}\overline{e^i}^T Q^i \overline{e^i}. \tag{52}$$

Utilizing (14), (25) and (30a), (30b), and (30c) result in the derivative of L along the trajectories of (48a) and (48b) as

$$\begin{aligned}
\dot{L} &= \left[\nabla_t T + \left(\nabla_\eta T\right)^T \dot{\eta}\right] \\
&\quad + \frac{k}{2}\left[\left(\dot{\overline{e^1}}\right)^T Q^1 \overline{e^1} + \left(\overline{e^1}\right)^T Q^1 \left(\dot{\overline{e^1}}\right) + \cdots + \left(\dot{\overline{e^N}}\right)^T Q^N \overline{e^N} + \left(\overline{e^N}\right)^T Q^N \left(\dot{\overline{e^N}}\right)\right] \\
&\leq \left[\nabla_t T + \left(\nabla_\eta T\right)^T q_{22}\left(t, \eta(t), \bar{e}\right) + \left(\nabla_\eta V\right)^T \Omega_\eta\left(\theta_d + D\right)\right] \\
&\quad - \frac{k}{2\kappa}\left[\left(\overline{e^1}\right)^T \overline{e^1} + \cdots + \left(\overline{e^N}\right)^T \overline{e^N}\right] \\
&\quad + \frac{k}{\kappa}\left[\left\|\left(\theta_d + D\right)\right\| \left\|\Omega_\xi^1\right\| \left\|Q^1\right\| \left\|\overline{e^1}\right\| + \cdots + \left\|\left(\theta_d + D\right)\right\| \left\|\Omega_\xi^N\right\| \left\|Q^N\right\| \left\|\overline{e^N}\right\|\right] \\
&\leq -\|\eta\|^2\left[2\alpha_x - \frac{1}{100}\Delta_3^2 L_m^2 - 25\Delta_3^2\left\|\Omega_\eta\right\|^2\right] \\
&\quad - \|\bar{e}\|^2\left[\frac{k}{2\kappa} - \frac{25k^2}{\kappa^2}\left\|\Omega_\xi^1\right\|^2\left\|Q^1\right\|^2 - \cdots - \frac{25k^2}{\kappa^2}\left\|\Omega_\xi^N\right\|^2\left\|Q^N\right\|^2 - 25\right] \\
&\quad + \frac{N+1}{100}\left\|\left(\theta_d + D\right)\right\|^2.
\end{aligned} \tag{53}$$

That is,

$$\dot{L} \leq -k_{11}\|\bar{e}\|^2 - k_{22}\|\eta\|^2 + \frac{N+1}{100}\left\|\left(\theta_d + D\right)\right\|^2. \tag{54}$$

From (35c), we obtain

$$\dot{L} \leq -N_2\left(\|\bar{e}\|^2 + \|\eta\|^2\right) + \frac{N+1}{100}\left\|\left(\theta_d + D\right)\right\|^2. \tag{55}$$

Define

$$\bar{e} \equiv \begin{bmatrix} \overline{e^1} \\ \overline{e^2} \\ \vdots \\ \overline{e^N} \end{bmatrix} \equiv \begin{bmatrix} \overline{e_1^1} \\ \overline{e_{\text{rem}}^1} \end{bmatrix}, \quad \overline{e_{\text{rem}}^1} \in \mathfrak{R}^{r-1}. \tag{56}$$

Hence

$$\dot{L} \leq -N_2\left(\|\eta\|^2 + \left\|\overline{e_1^1}\right\|^2 + \left\|\overline{e_{\text{rem}}^1}\right\|^2\right) + \frac{N+1}{100}\left\|\left(\theta_d + D\right)\right\|^2. \tag{57}$$

Utilizing (57) easily yields

$$\int_{t_0}^{t}\left(y_1(\tau) - y_d^1(\tau)\right)^2 d\tau \leq \frac{L\left(t_0\right)}{N_2} + \frac{N+1}{100N_2}\int_{t_0}^{t}\left\|\left(\theta_d(\tau) + D\right)\right\|^2 d\tau. \tag{58}$$

Similarly, it is easy to prove that

$$\int_{t_0}^{t}\left(y_i(\tau) - y_d^i(\tau)\right)^2 d\tau \leq \frac{L\left(t_0\right)}{N_2} + \frac{N+1}{100N_2}\int_{t_0}^{t}\left\|\left(\theta_d(\tau) + D\right)\right\|^2 d\tau, \quad 2 \leq i \leq N \tag{59}$$

so that statement (29) is satisfied. From (55), we get

$$\dot{L} \leq -N_2\left(\left\|y_{\text{total}}\right\|^2\right) + \frac{N+1}{100}\left\|\left(\theta_d + D\right)\right\|^2, \tag{60a}$$

where

$$\left\|y_{\text{total}}\right\|^2 \equiv \|\bar{e}\|^2 + \|\eta\|^2. \tag{60b}$$

Utilizing [31, Theorem 5.2] and (60a) implies the input-to-state stable property for the overall system. Furthermore, it is easy to obtain the following inequality:

$$\Delta_{\min}\left(\|\bar{e}\|^2 + \|\eta\|^2\right) \leq L \leq \Delta_{\max}\left(\|\bar{e}\|^2 + \|\eta\|^2\right). \tag{61}$$

That is,

$$\Delta_{\min}\left(\left\|y_{\text{total}}\right\|^2\right) \leq L \leq \Delta_{\max}\left(\left\|y_{\text{total}}\right\|^2\right), \tag{62}$$

where $\Delta_{\min} \equiv \min\{\Delta_1, (k/2)\lambda_{\min}^*\}$ and $\Delta_{\max} \equiv \max\{\Delta_2, (k/2)\lambda_{\max}^*\}$. From (55) and (62), we get

$$\dot{L} \leq -\frac{N_2}{\Delta_{\max}}L + \frac{N+1}{100}\left(\sup_{t_0\leq\tau\leq t}\left\|\left(\theta_d(\tau) + D\right)\right\|\right)^2. \tag{63}$$

Hence,

$$L(t) \leq L\left(t_0\right) e^{(-N_2/\Delta_{\max})(t-t_0)} + \frac{\Delta_{\max}(N+1)}{100N_2}\left(\sup_{t_0\leq\tau\leq t}\left\|\left(\theta_d(\tau) + D\right)\right\|\right)^2, \quad t \geq t_0 \tag{64}$$

which implies

$$\left|y_1(t) - y_d^1(t)\right| \le \sqrt{\frac{2L(t_0)}{k\lambda^*_{\min}}} e^{(-N_2/2\Delta_{\max})(t-t_0)} + \sqrt{\frac{\Delta_{\max}(N+1)}{50k\lambda^*_{\min}N_2}} \left(\sup_{t_0\le\tau\le t} \left\|(\theta_d(\tau) + D)\right\|\right). \tag{65}$$

Similarly, it is easy to prove that

$$\left|y_i(t) - y_d^1(t)\right| \le \sqrt{\frac{2L(t_0)}{k\lambda^*_{\min}}} e^{(-N_2/2\Delta_{\max})(t-t_0)} + \sqrt{\frac{\Delta_{\max}(N+1)}{50k\lambda^*_{\min}N_2}} \times \left(\sup_{t_0\le\tau\le t} \left\|(\theta_d(\tau) + D)\right\|\right), \quad 2 \le i \le N \tag{66}$$

so that statement (28) is proved, and then the tracking problem with almost disturbance decoupling is globally solved. □

If the sum $r_1+r_2+\cdots+r_m$ is equal to the system dimension n, then Theorem 4 will be reduced to the following simplified version with cancelling Assumption 1 and (30a), (30b), and (30c).

Theorem 5. *The tracking problem with almost disturbance decoupling performance for time-delay large-scale system is globally solved by the control law:*

$$\begin{aligned} u &= A_{non}^{-1}\left\{-u_b + u_v\right\}, \\ u_b &\equiv \left[L_f^{r_1}h_1 \;\; L_f^{r_2}h_2 \;\; \cdots \;\; L_f^{r_N}h_N\right]^T, \\ u_v &\equiv \left[v_1 \;\; v_2 \;\; \cdots \;\; v_N\right]^T, \\ v_i &\equiv y_d^{i\,(r_i)} - \kappa^{-r_i}a_1^i\left[L_f^0 h_i(X) - y_d^i\right] \\ &\quad - \kappa^{1-r_i}a_2^i\left[L_f^1 h_i(X) - y_d^{i\,(1)}\right] \\ &\quad - \cdots - \kappa^{-1}a_{r_i}^i\left[L_f^{r_i-1}h_i(X) - y_d^{i\,(r_i-1)}\right], \quad 1 \le i \le N. \end{aligned} \tag{67}$$

Moreover, the influence of disturbances on the L_2 norm of the tracking error can be arbitrarily reduced by adjusting parameter $k_{11} > 1$:

$$k_{11} \equiv \frac{k}{2\kappa} - \frac{25k^2}{\kappa^2}\left\|\phi_\xi^1\right\|^2\left\|Q^1\right\|^2 - \cdots - \frac{25k^2}{\kappa^2}\left\|\phi_\xi^N\right\|^2\left\|Q^N\right\|^2. \tag{68}$$

3. Illustrative Example

Consider the two inverted pendulums coupled by a spring with disturbances as shown in Figure 1. The dynamic equations are given as follows:

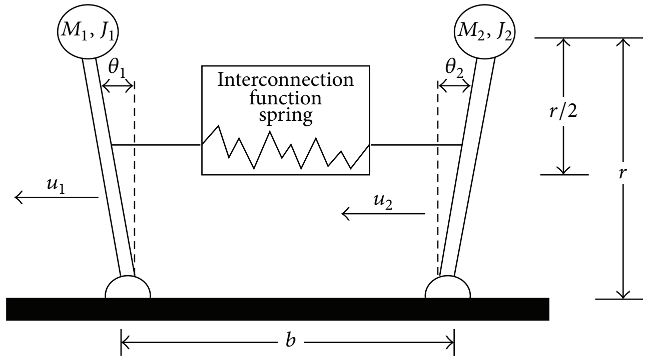

Figure 1: The two-inverted-pendulum system coupled by a spring.

S_1 (inverted pendulum 1):

$$\begin{aligned} \ddot{\theta}_1(t) &= \left(-\frac{kr^2}{4J_1} + \frac{m_1gr}{J_1}\right)\sin(\theta_1(t)) \\ &\quad + \frac{kr(l-b)}{2J_1} + \frac{kr^2}{4J_1}\sin(\theta_2(t)) \\ &\quad + \frac{u_1}{J_1} + \frac{\theta_1(t-\tau_{1,2}(t))}{1+\theta_1^2(t-\tau_{1,2}(t))}, \\ \tau_{1,2}(t) &\equiv 0.4\left(1 + \sin^2(t)\right), \\ y_1 &= \theta_1(t), \end{aligned} \tag{69}$$

S_2 (inverted pendulum 2):

$$\begin{aligned} \ddot{\theta}_2(t) &= \left(-\frac{kr^2}{4J_2} + \frac{m_2gr}{J_2}\right)\sin(\theta_2(t)) \\ &\quad + \frac{kr(l-b)}{2J_2} + \frac{kr^2}{4J_2}\sin\left(\dot{\theta}_1(t)\right) \\ &\quad + \frac{u_2}{J_2} + \frac{\theta_2(t-\tau_{2,2}(t))}{1+\theta_2^2(t-\tau_{2,2}(t))}, \\ \tau_{2,2}(t) &\equiv 0.4\left(1 + \sin^2(t)\right) \\ y_2 &= \theta_2(t), \end{aligned} \tag{70}$$

where u_i is the torque input generated by the actuator for pendulum i ($i = 1, 2$), θ_i is the angular displacement of pendulum i ($i = 1, 2$), m_i is the mass of pendulum i ($i = 1, 2$), J_i is the moment of inertia ($i = 1, 2$), k is the spring constant, r is the pendulum height, l is the length of pendulum, g is the gravitational acceleration constant, and b is the distance between the pendulums. Defining the state variables

$x_1 \equiv \theta_1(t)$, $x_2 \equiv \dot{\theta}_1(t)$, $x_3 \equiv \theta_2(t)$, and $x_4 \equiv \dot{\theta}_2(t)$ yields the following state space models:

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t), \\
\dot{x}_2(t) &= \left(-\frac{kr^2}{4J_1} + \frac{m_1 gr}{J_1}\right)\sin(x_1(t)) \\
&\quad + \frac{kr(l-b)}{2J_1} + \frac{kr^2}{4J_1}\sin(x_3(t)) \\
&\quad + \frac{u_1}{J_1} + \frac{x_1(t-\tau_{1,2}(t))}{1+x_1^2(t-\tau_{1,2}(t))}, \\
\dot{x}_3(t) &= x_4(t), \\
\dot{x}_4(t) &= \left(-\frac{kr^2}{4J_1} + \frac{m_1 gr}{J_1}\right)\sin(x_3(t)) \\
&\quad + \frac{kr(l-b)}{2J_1} + \frac{kr^2}{4J_1}\sin(x_2(t)) \\
&\quad + \frac{u_2}{J_2} + \frac{x_3(t-\tau_{1,2}(t))}{1+x_3^2(t-\tau_{1,2}(t))}, \\
\tau_{1,2}(t) &\equiv \tau_{2,2}(t) \equiv 0.4\left(1+\sin^2(t)\right), \\
y_1 &= x_1(t), \\
y_2 &= x_3(t).
\end{aligned} \tag{71}$$

The following physical parameters are chosen in our simulation: $m_1 = 2\,\text{kg}$, $m_2 = 2.5\,\text{kg}$, $J_1 = 1\,\text{kg/m}^2$, $J_2 = 1\,\text{kg/m}^2$, $k = 10\,\text{N/m}$, $r = 0.1\,\text{m}$, $l = 0.5\,\text{m}$, $g = 9.8\,\text{m/s}^2$, and $b = 0.5\,\text{m}$. Hence the mathematical model can be rewritten as

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t), \\
\dot{x}_2(t) &= 1.935\sin(x_1(t)) + 0.025\sin(x_3(t)) \\
&\quad + u_1 + \frac{x_1\left(t-0.4\left(1+\sin^2(t)\right)\right)}{1+x_1^2\left(t-0.4\left(1+\sin^2(t)\right)\right)}, \\
\dot{x}_3(t) &= x_4(t), \\
\dot{x}_4(t) &= 2.425\sin(x_3(t)) + \frac{kr(l-b)}{2J_1} \\
&\quad + 0.025\sin(x_2(t)) + u_2 \\
&\quad + \frac{x_3\left(t-0.4\left(1+\sin^2(t)\right)\right)}{1+x_3^2\left(t-0.4\left(1+\sin^2(t)\right)\right)}, \\
y_1 &= x_1(t), \\
y_2 &= x_3(t).
\end{aligned} \tag{72}$$

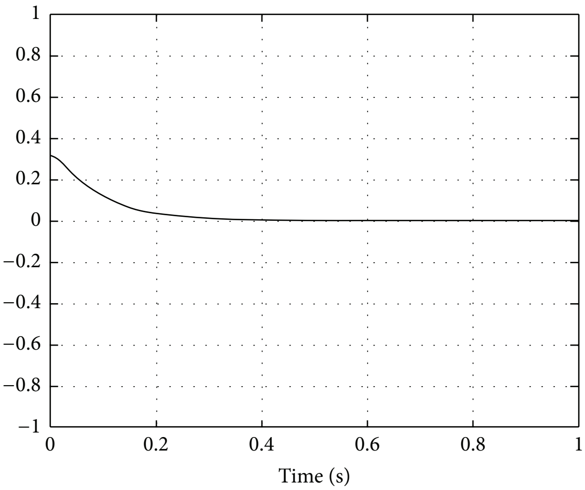

FIGURE 2: The output angular position $x_1(\theta_1)$ of the two inverted pendulums.

Now we will show how to explicitly construct a controller that tracks the desired signals $y_d^1 = y_d^2 = 0$ and attenuates the disturbance's effect on the output terminal to an arbitrary degree of accuracy. Let us arbitrarily choose $a_1^1 = a_2^1 = a_1^2 = a_2^2 = 10$, and the solution of Lyapunov equation is given as $A_{\text{phase}}^1 = A_{\text{phase}}^2 = \left[\begin{smallmatrix} 0 & 1 \\ -10 & -10 \end{smallmatrix}\right]$, $Q^1 = Q^2 = \left[\begin{smallmatrix} 1.05 & 0.05 \\ 0.05 & 0.055 \end{smallmatrix}\right]$, $\lambda_{\max}(Q^1) = \lambda_{\max}(Q^2) = 1.0525$, $\lambda_{\min}(Q^1) = \lambda_{\min}(Q^2) = 0.0525$, $\lambda_{\max}^* = 1.0525$, and $\lambda_{\min}^* = 0.0525$. From (31), we obtain the desired tracking controllers

$$\begin{aligned}
u_1 &= -1000x_1 - 100x_2 - 1.935\sin x_1 - 0.025\sin x_3, \\
u_2 &= -1000x_3 - 100x_4 - 2.425\sin x_3 - 0.025\sin x_2.
\end{aligned} \tag{73}$$

It can be verified that the relative conditions of Theorem 4 are satisfied with $\kappa = 0.1$, $B_d^1 = B_d^2 = 0$, $N_2 = 2.82$, $k = 20\sqrt{\kappa}$, and $k_{11} = 2.82$. Hence the tracking controllers will steer the output tracking errors of the closed-loop system, starting from any initial value to be asymptotically attenuated to zero by virtue of Theorem 4. The complete trajectories of the outputs are depicted in Figures 2 and 3.

4. Conclusion

In this paper, we propose a novel control design which globally solves the almost disturbance decoupling problem for multiinput multioutput nonlinear time delay large-scale system via the fuzzy feedback linearization approach. The investigation of feedback linearization of nonlinear time-delay large-scale control systems by diffeomorphism has been proposed. Moreover, a practical industrial system of two inverted pendulums coupled by a spring demonstrates the applicability of the proposed feedback linearization method. Simulation results exploit the fact that the proposed methodology is successfully utilized to solve the feedback linearization problem and achieve the desired almost disturbance decoupling performance of the overall system.

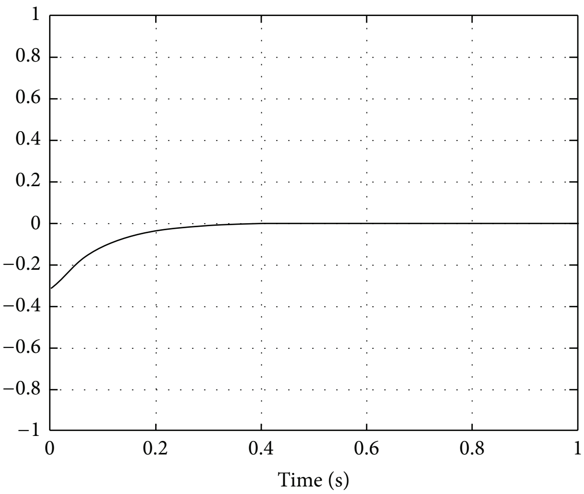

FIGURE 3: The output angular position $x_3(\theta_2)$ of the two inverted pendulums.

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