Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 212753, 10 pages
http://dx.doi.org/10.1155/2013/212753

Research Article

Effects of Time-Varying Impulses on the Synchronization of Delayed Dynamical Networks

Shuiming Cai,[1] Peipei Zhou,[1] and Zengrong Liu[2]

[1] *Faculty of Science, Jiangsu University, Zhenjiang 212013, China*
[2] *Institute of Systems Biology, Shanghai University, Shanghai 200444, China*

Correspondence should be addressed to Zengrong Liu; zrongliu@126.com

Received 4 September 2013; Accepted 29 November 2013

Academic Editor: Allan Peterson

The effects of time-varying impulses on the synchronization of a class of general complex delayed dynamical networks are investigated. Different from the existing works, the impulses discussed here are time-varying, and both synchronizing and desynchronizing impulses are considered in the network model simultaneously. Moreover, the network topology is assumed to be directed and weakly connected with a spanning tree. By using the comparison principle, some simple yet generic globally exponential synchronization criteria are derived. It is shown that besides impulse strengths and impulsive interval, the obtained criteria are also closely related with topology structure of the network. Finally, numerical examples are given to demonstrate the effectiveness of the theoretical results.

1. Introduction

Recently, much efforts have been devoted to complex dynamical networks due to the wide and potential applications in many areas [1–3]. In general, a complex network is a large set of interconnected nodes, in which each node is a fundamental unit with specific dynamics and each edge represents the interactions between them. As a matter of fact, the structure of many real systems in nature including the Internet, food webs, biomolecular networks, and social networks can be described by complex networks [1, 2]. One significant and interesting phenomenon in a complex dynamical network is the synchronization of all its dynamical nodes. In the past few decades, synchronization of complex dynamical networks has been extensively studied from various fields of science and engineering, and a wide variety of synchronization criteria have been presented for various dynamical networks; see [2–22] and the references therein.

From the literature, there exist two common phenomena in many dynamical networks: delay effects and impulsive effects [4, 5, 23–29]. Due to the finite switching speed of amplifiers and finite signal propagation time, time delay is inevitably encountered in many dynamical networks and may cause undesirable dynamical behaviors such as oscillation and instability [4, 5, 23, 24]. On the other hand, the states of many complex systems and realistic networks are often subject to instantaneous perturbations and experience abrupt changes at certain instants, which may be caused by switching phenomenon, frequency change, or other sudden noise; that is, they exhibit impulsive effects [25–29]. Since time delays and impulses can heavily affect the dynamical behaviors of the networks, it is imperative to investigate both effects of time delays and impulses on the synchronization of dynamical networks.

In general, there are two types of impulses in terms of synchronization in dynamical networks. An impulsive sequence is said to be synchronizing if it can enhance the synchronization of dynamical networks. Conversely, an impulsive sequence is said to be desynchronizing if the impulsive effects can suppress the synchronization of dynamical networks. In recent years, many interesting results on the synchronization of complex dynamical networks with impulsive effects have been reported in the literature [6–16]. For instance, Lu et al. [11] established a unified synchronization criterion for impulsive dynamical networks subject to synchronizing impulses or desynchronizing impulses. In addition, Lu et al. [12] also investigated the exponential synchronization of coupled neural networks with impulsive disturbances. Zhang et al. [16]

discussed the synchronization problem of coupled switched neural networks with mode-dependent impulsive effects. Unfortunately, in the most existing literature, it is implicitly assumed that the synchronizing and desynchronizing impulses occur separately. In practice, however, many electronic or biological networks are often subject to instantaneous disturbance and then exposed to time-varying impulsive strengths, and both synchronizing and desynchronizing impulses might exist in realistic networks simultaneously, which was widely overlooked in most of the existing results [30]. To the best of our knowledge, the effects of time-varying impulses on the synchronization of delayed dynamical networks are rarely addressed. Moreover, in much of the literature, time delays in the couplings are considered; however, time delays in the dynamical nodes [5, 9, 17, 18], which are more complex, are still relatively unexplored. In fact, numerous examples can be found in the real world which are characterized by delayed differential equations having time delays in the dynamical nodes [5, 9, 17, 18].

This paper aims at exploring the effects of time-varying impulses on the synchronization of general complex networks with time-varying delays dynamical nodes. Both synchronizing and desynchronizing impulses are considered in the network model simultaneously. The directed and weakly connected topology of networks is focused on. Based on the comparison principle, some simple yet generic globally exponential synchronization criteria are derived. Numerical examples are also provided to illustrate the effectiveness of the theoretical analysis.

2. Model and Preliminaries

Consider a directed complex network consisting of N identical time-varying delays dynamical nodes:

$$\dot{x}_i(t) = f(t, x_i(t), x_i(t-\tau(t))) + c\sum_{j=1}^{N} b_{ij}\Gamma x_j(t), \quad i = 1, \ldots, N, \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^\top \in \mathbb{R}^n$ is the state vector of node i; $f : \mathbb{R}\times\mathbb{R}^n\times\mathbb{R}^n \to \mathbb{R}^n$ is a continuously vector-valued function governing the dynamics of isolated nodes; $\tau(t)$ is a time-varying delay satisfying $0 \le \tau(t) \le \tau$; $c > 0$ is the coupling strength; $\Gamma > 0$ is the inner connecting matrix of nodes; $B = (b_{ij})_{N\times N}$ is the coupling matrix representing the topological structure of the network. Without loss of generality, we assume that the matrix B possesses the following properties: $\sum_{j=1}^{N} b_{ij} = 0$, $b_{ij} \ge 0$, $i \ne j$, and $\operatorname{rank}(B) = N-1$ [9, 18]. It is worth mentioning that the coupling matrix B can be regarded as the Laplacian matrix of a weighted graph with a spanning tree, and B has an eigenvalue 0 with multiplicity 1 [9, 19].

Remark 1. The coupling matrix B represents the topological structure of network (1). In this paper, the matrix B is not restricted to be symmetric or irreducible. A general structure of the network is discussed; that is, the corresponding graph generated by the matrix B can be directed and weakly connected with a spanning tree. Obviously, the network model (1) is a generalization of that discussed in [11, 12].

Due to switching phenomenon, frequency change, or other sudden noise, the states of nodes in many realistic networks are often subject to instantaneous perturbations and experience abrupt changes at certain instants [25–29]. Suppose at time instants t_k, there are "sudden changes" (or "jumps") in the state variable such that

$$\Delta x_i\big|_{t=t_k} \triangleq x_i(t_k^+) - x_i(t_k^-) = D_k x_i(t_k^-), \quad i = 1, 2, \ldots, N, \tag{2}$$

where $\{t_1, t_2, t_3, \ldots\}$ is an impulsive sequence satisfying $t_{k-1} < t_k$ and $\lim_{k\to\infty} t_k = +\infty$, $x_i(t_k^+) = \lim_{t\to t_k^+} x_i(t)$, $x_i(t_k^-) = \lim_{t\to t_k^-} x_i(t)$, and $D_k \in \mathbb{R}^{n\times n}$ is an impulsive gain matrix. For the sake of analytical simplification, we will choose $D_k = d_k I_n$, where $d_k \in \mathbb{R}$ represents the strength of impulses and I_n is an $n\times n$ identity matrix. This simplification does not cause any loss of generality in the sense of synchronization analysis. In fact, when the constant matrix D_k is used to describe the impulsive gain, the matrix product can be used to describe synchronization criteria. Therefore, we can obtain the following impulsive delayed dynamical network:

$$\begin{aligned} \dot{x}_i(t) &= f(t, x_i(t), x_i(t-\tau(t))) + c\sum_{j=1}^{N} b_{ij}\Gamma x_j(t), \\ &\qquad t \ne t_k,\ t \ge t_0, \\ \Delta x_i &= x_i(t_k^+) - x_i(t_k^-) = d_k x_i(t_k^-), \\ &\qquad t = t_k,\ k \in Z^+,\ i = 1, 2, \ldots, N, \end{aligned} \tag{3}$$

where $Z^+ = \{1, 2, \ldots\}$ denotes the set of positive integers. Without loss of generality, we assume that $x_i(t)$ is right continuous at $t = t_k$; that is, $x_i(t_k) = x_i(t_k^+)$. The initial conditions of (3) are given by $x_i(t_0 + s) = \varphi_i(s) \in PC([-\tau, 0], \mathbb{R}^n)$, where $PC([-\tau, 0], \mathbb{R}^n)$ denotes the set of all functions of bounded variation and right-continuous on any compact subinterval of $[-\tau, 0]$. We always assume that (3) has an unique solution with respect to initial conditions.

Assumption 2 (see [18]). For the vector-valued function $f(t, x(t), x(t-\tau(t)))$, suppose that the uniform semi-Lipschitz condition with respect to the time t holds; that is, for any $x(t)$, $y(t) \in \mathbb{R}^n$, there exist positive constants $L_1 > 0$ and $L_2 > 0$ such that

$$\begin{aligned} &(x(t) - y(t))^\top \\ &\quad \times \left(f(t, x(t), x(t-\tau(t))) - f(t, y(t), y(t-\tau(t)))\right) \\ &\quad \le L_1 (x(t) - y(t))^\top (x(t) - y(t)) \\ &\qquad + L_2 (x(t-\tau(t)) - y(t-\tau(t)))^\top \\ &\qquad \times (x(t-\tau(t)) - y(t-\tau(t))). \end{aligned} \tag{4}$$

Remark 3. Assumption 2 gives some requirements for the dynamics of isolated node in network (1). If the function $f(t, x(t), x(t-\tau(t)))$ satisfies the uniform Lipschitz condition [17], that is, $\| f(t, x(t), x(t-\tau(t))) - f(t, y(t), y(t-\tau(t)))\| \le K_1\|x(t)-y(t)\| + K_2\|x(t-\tau(t)) - y(t-\tau(t))\|$, one can choose $L_1 = K_1 + \omega K_2/2$ and $L_2 = K_2/(2\omega)$ to satisfy Assumption 2, where ω is a positive constant. Moreover, it is easy to check that almost all the well-known chaotic systems with delays or without delays, such as the Lorenz system, Rössler system, Chen system, and Chua's circuit as well as delayed Hopfield neural networks and delayed cellular neural networks (see [9, 18], and the references therein) also satisfy Assumption 2.

Definition 4. The impulsive delayed dynamical network (3) is said to be globally exponentially synchronized if there exist constants $\lambda_0 > 0$ and $M_0 > 0$ such that for any initial conditions $\varphi_i(s) \in PC([-\tau, 0], \mathbb{R}^n)$ $(i = 1, 2, \ldots, N)$

$$\left\|x_i(t) - x_j(t)\right\| \le M_0 \exp^{-\lambda_0(t-t_0)}, \quad \forall t \ge t_0. \tag{5}$$

Define $s(t) = (1/N)\sum_{l=1}^{N} x_l(t)$ and error vectors as $e_i(t) = x_i(t) - s(t)$, $i = 1, 2, \ldots, N$, then we have

$$\begin{aligned}
\dot e_i(t) &= \dot x_i(t) - \dot s(t)\\
&= \tilde f(t, e_i(t), e_i(t-\tau(t))) + c\sum_{j=1}^{N} b_{ij}\Gamma e_j(t) + J,\\
&\qquad\qquad t \ne t_k,\\
\Delta e_i(t_k) &= e_i(t_k^+) - e_i(t_k^-)\\
&= x_i(t_k^+) - x_i(t_k^-) - (s(t_k^+) - s(t_k^-))\\
&= x_i(t_k^+) - x_i(t_k^-) - \frac{1}{N}\sum_{l=1}^{N}(x_l(t_k^+) - x_l(t_k^-))\\
&= d_k x_i(t_k^-) - \frac{d_k}{N}\sum_{l=1}^{N} x_l(t_k^-) = d_k e_i(t_k^-), \quad t = t_k,
\end{aligned} \tag{6}$$

where $\tilde f(t, e_i(t), e_i(t-\tau(t))) = f(t, e_i(t) + s(t), e_i(t-\tau(t)) + s(t-\tau(t))) - f(t, s(t), s(t-\tau(t)))$, and $J = f(t, s(t), s(t-\tau(t))) - (1/N)\sum_{l=1}^{N} f(t, x_l(t), x_l(t-\tau(t))) - (c/N)\sum_{l=1}^{N}\sum_{j=1}^{N} b_{lj}\Gamma x_j(t)$.

Thus, the error dynamical system can be written as follows:

$$\begin{aligned}
\dot e_i(t) &= \tilde f(t, e_i(t), e_i(t-\tau(t))) + c\sum_{j=1}^{N} b_{ij}\Gamma e_j(t) + J,\\
&\qquad\qquad t \ne t_k,\ t \ge t_0,\\
e_i(t_k) &= (1 + d_k) e_i(t_k^-), \quad t = t_k,\ k \in Z^+,\ i = 1, 2, \ldots, N.
\end{aligned} \tag{7}$$

Clearly, the globally exponential synchronization of the impulsive delayed dynamical network (3) is achieved if the zero solution of the error dynamical system (7) is globally exponentially stable.

Remark 5. When $|(1 + d_k)| < 1$, the impulses are beneficial for the synchronization of the impulsive delayed dynamical network (3) since the absolute values of the synchronization errors are reduced. Thus, the impulses are synchronizing impulses if $-2 < d_k < 0$. Conversely, when $|(1 + d_k)| > 1$, that is, the impulsive strengths $d_k > 0$ or $d_k < -2$, the impulses are desynchronizing impulses since the absolute values of the synchronization errors are enlarged. In addition, when $|(1 + d_k)| = 1$, the impulses are neither beneficial nor harmful for the synchronization since the absolute values of the synchronization errors are unchanged. This type of impulses are called inactive impulses [11]. We will not consider inactive impulses here because they have no effect on the synchronization dynamics. In this paper, we focus on the synchronization problem of the dynamical network (3) with both synchronizing and desynchronizing impulses, which is different from most existing literature where the synchronizing and desynchronizing impulses are assumed to occur separately.

Lemma 6 (see [29]). *Let $0 \le \tau(t) \le \tau$ and $F(t, x, y) : [t_0, \infty)\times\mathbb{R}\times\mathbb{R} \to \mathbb{R}$ be nondecreasing in y for fixed (t, x), and let $I_k(x) : \mathbb{R} \to \mathbb{R}$ be nondecreasing in x. Suppose that*

$$\begin{aligned}
D^+u(t) &\le F(t, u(t), u(t-\tau(t))), \quad t \ge t_0,\\
u(t_k) &\le I_k(u(t_k^-)), \quad k \in Z^+,\\
D^+v(t) &> F(t, v(t), v(t-\tau(t))), \quad t \ge t_0,\\
v(t_k) &\ge I_k(v(t_k^-)), \quad k \in Z^+.
\end{aligned} \tag{8}$$

If $u(t) \le v(t)$, for $t_0 - \tau \le t \le t_0$, then $u(t) \le v(t)$, $t \ge t_0$.

3. Main Results

In this section, globally exponential synchronization of delayed dynamical networks with time-varying impulses including both synchronizing and desynchronizing impulses simultaneously will be studied. The relationship between the strengths of synchronizing and desynchronizing impulses and the frequency of their occurrence will be established.

Let the matrix $\tilde B$ be defined as $\tilde B \triangleq (B + B^\top) - \Delta$, where $\Delta = \mathrm{diag}(\delta_1, \delta_2, \ldots, \delta_N)$ with $\delta_j = \sum_{k=1}^{N} b_{kj}$. Then the matrix $\tilde B$ is a symmetrical irreducible matrix with zero-sum and nonnegative off-diagonal elements [9, 20]. This implies that zero is an eigenvalue of $\tilde B$ with multiplicity 1, and all the other eigenvalues of $\tilde B$ are strictly negative [9, 19]. The eigenvalues of $\tilde B$ can be ordered as $0 = \tilde\lambda_1 > \tilde\lambda_2 \ge \cdots \ge \tilde\lambda_N$. Since both synchronizing and desynchronizing impulses are taken into the network model simultaneously, without loss of generality, we assume that the strengths of synchronizing and desynchronizing impulses take values from finite sets $\{d_1^{sy}, d_2^{sy}, \ldots, d_{N_1}^{sy}\}$ and $\{d_1^{desy}, d_2^{desy}, \ldots, d_{N_2}^{desy}\}$, respectively, where $-2 < d_i^{sy} < 0$, $d_j^{desy} > 0$ or $d_j^{desy} < -2$ for $i = 1, 2, \ldots, N_1$, $j = 1, 2, \ldots, N_2$. In addition, we assume that $t_{i_k}^{\downarrow}$ and $t_{j_k}^{\uparrow}$ represent the activation time of the synchronizing impulses and that of the desynchronizing impulses, respectively. Let $d_*^{sy} = \max_{1\le i\le N_1} d_i^{sy}$, $d_*^{desy} = \max_{1\le j\le N_2} d_j^{desy}$,

$T^{\downarrow}_{\sup} = \sup\{t^{\downarrow}_{i_k} - t^{\downarrow}_{i_{k-1}}\} < \infty$, and $T^{\uparrow}_{\inf} = \inf\{t^{\uparrow}_{j_k} - t^{\uparrow}_{j_{k-1}}\} > 0$, where $t^{\downarrow}_{i_k}, t^{\uparrow}_{j_k} \in \{t_1, t_2, t_3, \ldots\}$; then we have the following result.

Theorem 7. *Consider the impulsive delayed dynamical network* (3) *with both synchronizing and desynchronizing impulses simultaneously. Under* Assumption 2*, the impulsive delayed dynamical network* (3) *is globally exponentially synchronized if the following condition holds:*

$$\frac{2\ln\left|\left(1+d^{sy}_{*}\right)\right|}{T^{\downarrow}_{\sup}} + \frac{2\ln\left|\left(1+d^{desy}_{*}\right)\right|}{T^{\uparrow}_{\inf}} + 2L_1 + r\lambda(r) + 2d^{*}L_2 < 0, \tag{9}$$

where

$$d^{*} = \left|\frac{\left(1+d^{desy}_{*}\right)}{\left(1+d^{sy}_{*}\right)}\right|^2, \qquad r = c\left(\widetilde{\lambda}_2 + \max_{1\le k\le N}\delta_k\right)$$
$$\text{with } \lambda(r) = \begin{cases}\lambda_{\max}(\Gamma), & \text{if } r>0,\\ 0, & \text{if } r=0,\\ \lambda_{\min}(\Gamma), & \text{if } r<0.\end{cases} \tag{10}$$

Proof. Let $e(t) = (e_1^{\top}(t), e_2^{\top}(t), \ldots, e_N^{\top}(t))^{\top}$, construct a Lyapunov function

$$V(t) = \frac{1}{2}e^{\top}(t)\left(I_N\otimes I_n\right)e(t) = \frac{1}{2}\sum_{i=1}^{N}e_i^{\top}(t)\,e_i(t). \tag{11}$$

Calculating the upper Dini derivative of $V(t)$ along the solution of (3), by using Assumption 2 and note that $\sum_{i=1}^{N} e_i(t) = 0$, we get

$$\begin{aligned}
D^{+}V(t) &= \sum_{i=1}^{N}e_i^{\top}(t) \times \left[\widetilde{f}\left(t, e_i(t), e_i(t-\tau(t))\right) + c\sum_{j=1}^{N}b_{ij}\Gamma e_j(t) + J\right]\\
&= \sum_{i=1}^{N}e_i^{\top}(t)\,\widetilde{f}\left(t, e_i(t), e_i(t-\tau(t))\right) + ce^{\top}(t)(B\otimes\Gamma)e(t)\\
&\le \sum_{i=1}^{N}L_1e_i^{\top}(t)\,e_i(t) + \sum_{i=1}^{N}L_2e_i^{\top}(t-\tau(t))\,e_i(t-\tau(t)) + ce^{\top}(t)(B\otimes\Gamma)e(t)\\
&\le 2L_1V(t) + 2L_2V(t-\tau(t)) + ce^{\top}(t)(B\otimes\Gamma)e(t)\\
&= 2L_1V(t) + 2L_2V(t-\tau(t)) + \frac{c}{2}e^{\top}(t)\left(\left(B+B^{\top}\right)\otimes\Gamma\right)e(t)\\
&= 2L_1V(t) + 2L_2V(t-\tau(t)) + \frac{c}{2}e^{\top}(t)\left(\left(\widetilde{B}+\Delta\right)\otimes\Gamma\right)e(t), \quad t\ne t_k.
\end{aligned} \tag{12}$$

Since $\widetilde{B}$ is a symmetrical matrix, there exists a unitary matrix $U = (u_1, u_2, \ldots, u_N)$ with $UU^{\top} = I_N$ such that $\widetilde{B} = U\,\mathrm{diag}(\widetilde{\lambda}_1, \widetilde{\lambda}_2, \ldots, \widetilde{\lambda}_N)U^{\top}$. Introduce a transformation $Z(t) = (U^{\top}\otimes I_n)e(t)$, where $Z(t) = (z_1^{\top}(t), z_2^{\top}(t), \ldots, z_N^{\top}(t))^{\top}$, $z_k \in \mathbb{R}^n$, then one has

$$Z^{\top}(t)\,Z(t) = \sum_{i=1}^{N}e_i^{\top}(t)\,e_i(t). \tag{13}$$

Notice that $\widetilde{\lambda}_1 = 0$ is an eigenvalue of the matrix $\widetilde{B}$ and its corresponding eigenvector is $u_1 = (1/\sqrt{N}, 1/\sqrt{N}, \ldots, 1/\sqrt{N})^{\top}$, then we have

$$z_1(t) = \left(u_1^{\top}\otimes I_n\right)e(t) = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}e_i(t) = 0. \tag{14}$$

According to (13)-(14) and the property of the Kronecker product of the matrices, we obtain

$$\begin{aligned}
&ce^{\top}(t)\left(\left(\widetilde{B}+\Delta\right)\otimes\Gamma\right)e(t)\\
&\quad= ce^{\top}(t)\left(U\otimes I_n\right) \times \left(\left(\mathrm{diag}\left(\widetilde{\lambda}_1, \widetilde{\lambda}_2, \ldots, \widetilde{\lambda}_N\right) + U^{\top}\Delta U\right)\otimes\Gamma\right) \times \left(U^{\top}\otimes I_n\right)e(t)\\
&\quad\le cZ^{\top}(t) \times \left(\left(\mathrm{diag}\left(\widetilde{\lambda}_1, \widetilde{\lambda}_2, \ldots, \widetilde{\lambda}_N\right) + \lambda_{\max}\left(U^{\top}\Delta U\right)I_N\right) \times \otimes\Gamma\right)Z(t)\\
&\quad\le cZ^{\top}(t) \times \left(\left(\mathrm{diag}\left(\widetilde{\lambda}_1, \widetilde{\lambda}_2, \ldots, \widetilde{\lambda}_N\right) + \max_{1\le k\le N}\delta_k I_N\right)\otimes\Gamma\right)Z(t)\\
&\quad= c\sum_{i=1}^{N}\left(\widetilde{\lambda}_i + \max_{1\le k\le N}\delta_k\right)z_i^{\top}(t)\,\Gamma z_i(t)\\
&\quad\le \sum_{i=1}^{N}c\left(\widetilde{\lambda}_2 + \max_{1\le k\le N}\delta_k\right)z_i^{\top}(t)\,\Gamma z_i(t) \le r\lambda(r) \times \sum_{i=1}^{N}e_i^{\top}(t)\,e_i(t) = 2r\lambda(r)V(t).
\end{aligned} \tag{15}$$

Substituting (15) into (12) gives

$$D^{+}V(t) \le (2L_1 + r\lambda(r))V(t) + 2L_2V(t-\tau(t)), \quad t \ne t_k. \tag{16}$$

When $t = t_k$, we have

$$\begin{aligned} V(t_k) &= \frac{1}{2}\sum_{i=1}^{N} e_i^{\top}(t_k)e_i(t_k) = \frac{1}{2}\sum_{i=1}^{N}(1+d_k)^2 e_i^{\top}(t_k^-)e_i(t_k^-) \\ &= (1+d_k)^2 V(t_k^-). \end{aligned} \tag{17}$$

Denote $p = 2L_1 + r\lambda(r)$ and $q = 2L_2$. For any $\epsilon > 0$, let $\mu_\epsilon(t)$ be unique solution of the following impulsive delayed dynamical system:

$$\begin{aligned} \dot{\mu}_\epsilon(t) &= p\mu_\epsilon(t) + q\mu_\epsilon(t-\tau(t)) + \epsilon, \quad t \ne t_k, \ t \ge t_0, \\ \mu_\epsilon(t_k) &= (1+d_k)^2\mu_\epsilon(t_k^-), \quad t = t_k, \ k \in Z^+, \\ \mu_\epsilon(t) &= \sup_{t_0-\tau\le s\le t_0}\|V(s)\|, \quad t_0-\tau \le t \le t_0. \end{aligned} \tag{18}$$

Let $M_0 = d^*\sup_{t_0-\tau\le s\le t_0}\|V(s)\|$ and $\eta = -(2\ln|(1+d_*^{\mathrm{sy}})|/T_{\sup}^{\downarrow} + 2\ln|(1+d_*^{\mathrm{desy}})|/T_{\inf}^{\uparrow} + p)$. In the following, we will prove that condition (9) implies

$$\mu_\epsilon(t) < M_0\exp^{-\lambda(t-t_0)} + \frac{\epsilon}{(\eta - d^*q)\,d^{*-1}}, \quad t \ge t_0, \tag{19}$$

where $\lambda > 0$ is unique positive solution of the equation

$$\lambda - \eta + d^*q\exp^{\lambda\tau} = 0. \tag{20}$$

Define $H(\lambda) = \lambda - \eta + d^*q\exp^{\lambda\tau}$. From condition (9), one has $H(0) < 0$. Since $H(+\infty) > 0$ and $dH(\lambda)/d\lambda > 0$, (20) must have unique positive solution $\lambda > 0$. By the formula for the variation of parameters [29], it follows from (18) that

$$\begin{aligned} \mu_\epsilon(t) &= W(t,t_0)\mu_\epsilon(t_0) \\ &\quad + \int_{t_0}^{t} W(t,s)\left[q\mu_\epsilon(s-\tau(s)) + \epsilon\right]ds, \quad t \ge t_0, \end{aligned} \tag{21}$$

where $W(t,s), t, s \ge t_0$ is the Cauchy matrix of linear system [29]

$$\begin{aligned} \dot{\psi}(t) &= p\psi(t), \quad t \ne t_k, \ t \ge t_0, \\ \psi(t_k) &= (1+d_k)^2\psi(t_k^-), \quad t = t_k, \ k \in Z^+. \end{aligned} \tag{22}$$

According to the representation of the Cauchy matrix [29], we have

$$W(t,s) = \left(\prod_{s<t_k\le t}(1+d_k)^2\right)\exp^{p(t-s)}, \quad t > s \ge t_0. \tag{23}$$

Suppose that there exist $\kappa_i > 0$ synchronization impulses with the impulsive strength d_i^{sy} and $\kappa_j > 0$ desynchronization impulses with the impulsive strength d_j^{desy} in the interval (s,t), then one can easily get that $(\kappa_i + 1)T_{\sup}^{\downarrow} \ge t - s$ and $(\kappa_j - 1)T_{\inf}^{\uparrow} \le t - s$. Thus, it follows from (23) that

$$\begin{aligned} W(t,s) &= \left(\prod_{s<t_k\le t}(1+d_k)^2\right)\exp^{p(t-s)} \\ &= \left(\prod_{i=1}^{\kappa_i}(1+d_i^{\mathrm{sy}})^2\prod_{j=1}^{\kappa_j}\left(1+d_j^{\mathrm{desy}}\right)^2\right)\exp^{p(t-s)} \\ &\le \left|(1+d_*^{\mathrm{sy}})\right|^{2((t-s)/T_{\sup}^{\downarrow}-1)} \\ &\quad \times \left|\left(1+d_*^{\mathrm{desy}}\right)\right|^{2((t-s)/T_{\inf}^{\uparrow}+1)} \\ &= \left|(1+d_*^{\mathrm{sy}})\right|^{-2}\left|\left(1+d_*^{\mathrm{desy}}\right)\right|^{2} \\ &\quad \times \exp^{(p+2\ln|(1+d_*^{\mathrm{sy}})|/T_{\sup}^{\downarrow}+2\ln|(1+d_*^{\mathrm{desy}})|/T_{\inf}^{\uparrow})(t-s)} \\ &= d^*\exp^{-\eta(t-s)}. \end{aligned} \tag{24}$$

Substituting (24) into (21) gives

$$\begin{aligned} \mu_\epsilon(t) &\le d^*\exp^{-\eta(t-t_0)}\mu_\epsilon(t_0) \\ &\quad + \int_{t_0}^{t} d^*\exp^{-\eta(t-s)}\left[q\mu_\epsilon(s-\tau(s)) + \epsilon\right]ds \\ &= M_0\exp^{-\eta(t-t_0)} \\ &\quad + \int_{t_0}^{t}\exp^{-\eta(t-s)}\left[d^*q\mu_\epsilon(s-\tau(s)) + d^*\epsilon\right]ds, \quad t \ge t_0. \end{aligned} \tag{25}$$

Since $\epsilon, \lambda, \eta - d^*q > 0$, and $d^* > 1$, one has

$$\begin{aligned} \mu_\epsilon(t) &\le d^*\sup_{t_0-\tau\le s\le t_0}\|V(s)\| < M_0\exp^{-\lambda(t-t_0)} \\ &\quad + \frac{\epsilon}{(\eta - d^*q)\,d^{*-1}}, \quad t_0 - \tau \le t \le t_0. \end{aligned} \tag{26}$$

Now, we claim (19) holds; that is,

$$\mu_\epsilon(t) < M_0\exp^{-\lambda(t-t_0)} + \frac{\epsilon}{(\eta - d^*q)\,d^{*-1}}, \quad t \ge t_0. \tag{27}$$

If this is not true, from (26), then there must exists a $t^* > t_0$ such that

$$\mu_\epsilon(t^*) \ge M_0\exp^{-\lambda(t^*-t_0)} + \frac{\epsilon}{(\eta - d^*q)\,d^{*-1}}, \tag{28}$$

$$\mu_\epsilon(t) < M_0\exp^{-\lambda(t-t_0)} + \frac{\epsilon}{(\eta - d^*q)\,d^{*-1}}, \quad t_0 - \tau \le t < t^*. \tag{29}$$

By (20), (25), and (29), we have

$$\begin{aligned}\mu_\epsilon(t^*) &\le M_0 \exp^{-\eta(t^*-t_0)} \\ &\quad + \int_{t_0}^{t^*} \exp^{-\eta(t^*-s)} \left[d^* q \mu_\epsilon(s-\tau(s)) + d^*\epsilon\right] ds \\ &\le \exp^{-\eta(t^*-t_0)} \\ &\quad \times \left\{ M_0 + \int_{t_0}^{t^*} \exp^{\eta(s-t_0)} \right. \\ &\qquad \times \left[d^* q M_0 \exp^{-\lambda(s-\tau(s)-t_0)} \right. \\ &\qquad \left.\left. + \frac{\epsilon d^* q}{(\eta - d^* q) d^{*-1}} + d^*\epsilon \right] ds \right\} \\ &\le \exp^{-\eta(t^*-t_0)} \left\{ M_0 + d^* q M_0 \right. \\ &\qquad \times \exp^{\lambda\tau} \int_{t_0}^{t^*} \exp^{(\eta-\lambda)(s-t_0)} ds \\ &\qquad + \frac{\epsilon\eta}{(\eta - d^* q) d^{*-1}} \\ &\qquad \left. \times \int_{t_0}^{t^*} \exp^{\eta(s-t_0)} ds \right\} \\ &< M_0 \exp^{-\lambda(t^*-t_0)} + \frac{\epsilon}{(\eta - d^* q) d^{*-1}}.\end{aligned} \tag{30}$$

Since $V(t) \le \sup_{t_0-\tau\le s\le t_0} \|V(s)\| = \mu_\epsilon(t)$, for $t_0 - \tau \le t \le t_0$, it follows from (16)–(18) and Lemma 6 that

$$V(t) \le \mu_\epsilon(t) < M_0 \exp^{-\lambda(t-t_0)} + \frac{\epsilon}{(\eta - d^* q) d^{*-1}}, \quad t \ge t_0. \tag{31}$$

Letting $\epsilon \to 0^+$, then we have

$$V(t) \le M_0 \exp^{-\lambda(t-t_0)}, \quad t \ge t_0. \tag{32}$$

This means the zero solution of the error system (7) is globally exponentially stable. The proof of Theorem 7 is thus completed. □

Remark 8. Theorem 7 provides a simple yet generic globally exponential synchronization criterion for directed impulsive delayed dynamical network (3) with time-varying impulses. It should be stressed that here both synchronizing and desynchronizing impulses are considered in the model simultaneously. This is different from previous studies on impulsive dynamical networks in [6–15], where the two kinds of impulses are assumed to occur separately. Thus, the impulses discussed in [6–15] can be viewed as a special case of our proposed time-varying impulses.

Remark 9. It can be seen from Theorem 7 that the derived criterion depends mainly on the synchronization and desynchronizing impulsive strengths d_i^{sy} and d_j^{desy}, the upper and lower bounds of the synchronization and desynchronizing impulsive intervals $T_{\sup}^{\downarrow}$ and $T_{\inf}^{\uparrow}$, and the eigenvalue $\widetilde{\lambda}_2$. Just as stated in [21, 22], the synchronizability of the dynamical network can also be characterized by the second largest eigenvalue $\widetilde{\lambda}_2$ of the specific matrix $\widetilde{B}$. Therefore, the result shows that the network topology also has a great impact on synchronization dynamics of the impulsive delayed dynamical network, which is different from the results presented in [6–8, 14–16].

Remark 10. Condition (9) in Theorem 7 indicates that the sum $((2\ln|(1+d_*^{sy})|/T_{\sup}^{\downarrow})+(2\ln|(1+d_*^{desy})|/T_{\inf}^{\uparrow})) = 2\ln(|(1+d_*^{sy})|^{1/T_{\sup}^{\downarrow}}|(1+d_*^{desy})|^{1/T_{\inf}^{\uparrow}})$ plays an important role in the synchronization criterion. For convenience, let $\widetilde{d} = |(1+d_*^{sy})|^{1/T_{\sup}^{\downarrow}}|(1+d_*^{desy})|^{1/T_{\inf}^{\uparrow}}$; then condition (9) can be rewritten as

$$\ln\widetilde{d} + L_1 + \frac{r\lambda(r)}{2} + d^* L_2 < 0. \tag{33}$$

Since $|(1+d_*^{sy})| < 1$ and $|(1+d_*^{desy})| > 1$, the following two cases will appear.

(i) If $0 < \widetilde{d} < 1$, that is, $|(1+d_*^{desy})|^{1/T_{\inf}^{\uparrow}} < |(1+d_*^{sy})|^{-1/T_{\sup}^{\downarrow}}$, then $\ln\widetilde{d} < 0$. In this case, the restriction condition $L_1 + r\lambda(r)/2 < -L_2$, that is, $-p > q \ge 0$, is not required in inequality (33). According to the Halanay differential inequality on delayed dynamical systems [13, 27] and (16), this implies that the underlying delayed dynamical network (1) without impulses might be asynchronous. Therefore, the result shows that even if desynchronization impulses occur frequently, the underlying delayed dynamical networks without impulses which may be asynchronous itself can also be globally exponentially synchronized if synchronization impulses can prevail over the influence of desynchronization impulsive effects. Such statement will be further verified through numerical examples.

(ii) If $\widetilde{d} \ge 1$, that is, $|(1+d_*^{desy})|^{1/T_{\inf}^{\uparrow}} \ge |(1+d_*^{sy})|^{-1/T_{\sup}^{\downarrow}}$, then $p + d^* q < 0$ since $\ln\widetilde{d} \ge 0$. Note that $d^* > 1$, one has $-p > q > 0$, which means the underlying delayed dynamical network (1) without impulses in fact is globally exponential synchronization itself in this case [13, 27]. Thus, inequality (33) provides a criterion under which globally exponential synchronization of the original delayed dynamical network (1) can be preserved when desynchronization impulses prevail over the influence of synchronization impulsive effects.

In the following, for simplicity, we consider that both the strengths of the synchronization and desynchronization impulses are time-invariant; that is, $d_i^{sy} \equiv d^{sy}$, $d_j^{desy} \equiv d^{desy}$ for $i = 1, 2, \ldots, N_1$, $j = 1, 2, \ldots, N_2$. Then, the following result can be obtained readily from Theorem 7.

Corollary 11. *Consider the impulsive delayed dynamical network* (3) *with time-varying impulses. Under Assumption 2, the impulsive delayed dynamical network* (3) *is globally exponentially synchronized if the following condition holds:*

$$\ln\left(\left|\left(1+d^{sy}\right)\right|^{1/T^{\downarrow}_{\sup}}\left|\left(1+d^{desy}\right)\right|^{1/T^{\uparrow}_{\inf}}\right) + L_1 + \frac{r\lambda(r)}{2} + \overline{d}^{*} L_2 < 0, \tag{34}$$

where $\overline{d}^{*} = (1+d^{desy})^2/(1+d^{sy})^2$ *and the other parameters are given in Theorem 7.*

Furthermore, if we assume the impulsive intervals of the synchronization and desynchronization impulses are equidistant and equal to each other, that is, $t^{\downarrow}_{i_k} - t^{\downarrow}_{i_{k-1}} \equiv \Delta t^{\downarrow}$, $t^{\uparrow}_{j_k} - t^{\uparrow}_{j_{k-1}} \equiv \Delta t^{\uparrow}$, and $\Delta t^{\downarrow} = \Delta t^{\uparrow} = \Delta t$, then Corollary 11 is reduced to the following.

Corollary 12. *Consider the impulsive delayed dynamical network* (3) *with time-varying impulses. Under Assumption 2, the impulsive delayed dynamical network* (3) *is globally exponentially synchronized if the following condition holds:*

$$\frac{\ln\left(\left|\left(1+d^{sy}\right)\left(1+d^{desy}\right)\right|\right)}{\Delta t} + L_1 + \frac{r\lambda(r)}{2} + \overline{d}^{*} L_2 < 0. \tag{35}$$

Remark 13. Corollary 12 shows that if the impulsive intervals of the synchronization and desynchronization impulses are equidistant and equal to each other, when $|(1+d^{sy})(1+d^{desy})| < 1$, that is, when the absolute value of product of $(1+d^{sy})$ and $(1+d^{desy})$ is less than 1, the whole impulsive effects are synchronizing; when the absolute value is equal to 1, the whole impulsive effects are inactive; and when the absolute value is more than 1, the whole impulsive effects are desynchronizing.

4. Numerical Examples

In this section, two numerical examples are given to illustrate the results derived in this work. The delayed Hopfield neural network is chosen as the isolated node of network (1), which can be described by the following [9]:

$$\dot{x}(t) = f(t, x(t), x(t-\tau(t))) = A_1 x(t) + E_1 g_1(x(t)) + F_1 g_2(x(t-\tau(t))), \tag{36}$$

where $x(t) = (x_1(t), x_2(t))^{\top} \in \mathbb{R}^2$, $g_1(x(t)) = g_2(x(t)) = (\tanh(x_1), \tanh(x_2))^{\top}$, $\tau(t) = 1$, and

$$A_1 = \begin{bmatrix} -1.0 & 0 \\ 0 & -1.0 \end{bmatrix}, \qquad E_1 = \begin{bmatrix} 2.0 & -0.1 \\ -5.0 & 3.0 \end{bmatrix}, \qquad F_1 = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{bmatrix}. \tag{37}$$

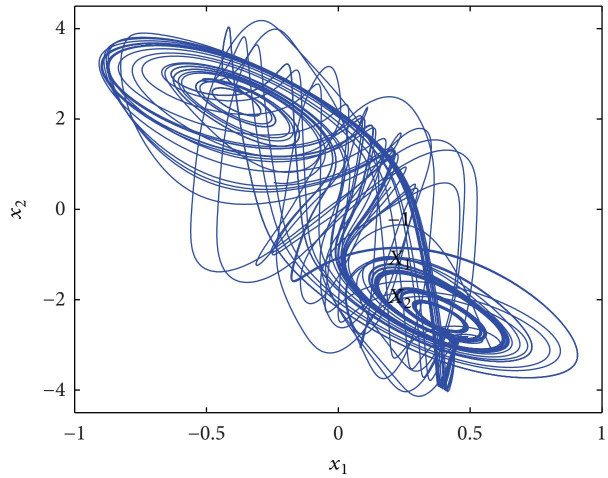

Figure 1: Chaotic attractor of the isolate delayed Hopfield neural network (36) with initial conditions $x_1(0) = 0.2$ and $x_2(0) = 0.5$.

The single delayed Hopfield neural network (36) has a chaotic attractor as shown in Figure 1. By using the inequality $|ab| \le (1/2)a^2 + (1/2)b^2$, it is easy to check that

$$\begin{aligned}
&(x(t)-y(t))^{\top} \\
&\quad\times\left(f(t,x(t),x(t-\tau(t))) - f(t,y(t),y(t-\tau(t)))\right) \\
&\quad\le (x(t)-y(t))^{\top} A_1 (x(t)-y(t)) \\
&\qquad + (4.55+0.8\omega)(x_1(t)-y_1(t))^2 \\
&\qquad + (5.55+1.35\omega)(x_2(t)-y_2(t))^2 \\
&\qquad + \left(\frac{0.85}{\omega}\right)(x_1(t-\tau(t)) - y_1(t-\tau(t)))^2 \\
&\qquad + \left(\frac{1.3}{\omega}\right)(x_2(t-\tau(t)) - y_2(t-\tau(t)))^2 \\
&\quad\le \lambda_{\max}(\widetilde{A}_1)(x(t)-y(t))^{\top}(x(t)-y(t)) \\
&\qquad + \lambda_{\max}(\widetilde{A}_2)(x(t-\tau(t)) - y(t-\tau(t)))^{\top} \\
&\qquad \times (x(t-\tau(t)) - y(t-\tau(t))) \\
&\quad= L_1 (x(t)-y(t))^{\top}(x(t)-y(t)) \\
&\qquad + L_2 (x(t-\tau(t)) - y(t-\tau(t)))^{\top} \\
&\qquad \times (x(t-\tau(t)) - y(t-\tau(t))),
\end{aligned} \tag{38}$$

where $\widetilde{A}_1 = \operatorname{diag}((3.55+0.8\omega), (5.55+1.35\omega))$, $\widetilde{A}_2 = \operatorname{diag}((0.85/\omega), (1.3/\omega))$, $L_1 = \lambda_{\max}(\widetilde{A}_1)$, $L_2 = \lambda_{\max}(\widetilde{A}_2)$, and ω is a positive constant. Thus, the condition of Assumption 2 is satisfied.

For the reason of convenient explanation, we here consider $d^{sy}_i \equiv d^{sy}$, $d^{desy}_j \equiv d^{desy}$, $i = 1, 2, \ldots, N_1$, $j = 1, 2, \ldots, N_2$, $t^{\downarrow}_{i_k} - t^{\downarrow}_{i_{k-1}} = t^{\uparrow}_{j_k} - t^{\uparrow}_{j_{k-1}} = \Delta t$, and the impulsive strength d_k ($k \in Z^{+}$) satisfies

$$d_k = \begin{cases} d^{sy}, & \text{if } \operatorname{mod}(k,2) \ne 0, \\ d^{desy}, & \text{if } \operatorname{mod}(k,2) = 0. \end{cases} \tag{39}$$

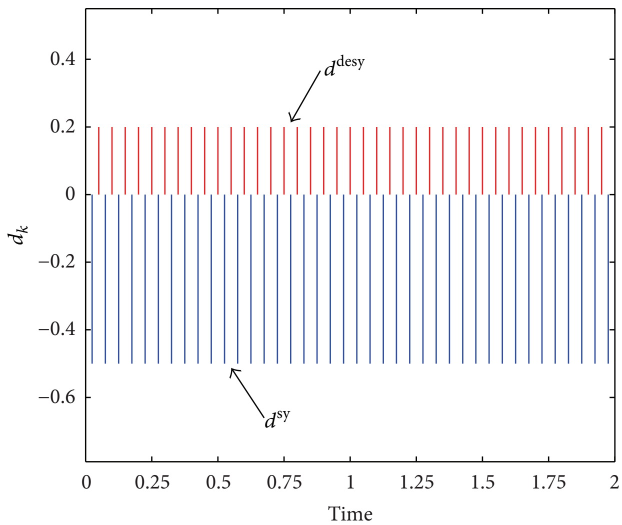

FIGURE 2: Time-varying impulsive sequence with $d^{sy} = -0.5$, $d^{desy} = 0.2$, and $\Delta t = 0.05$ in time interval $[0 \;\; 2]$.

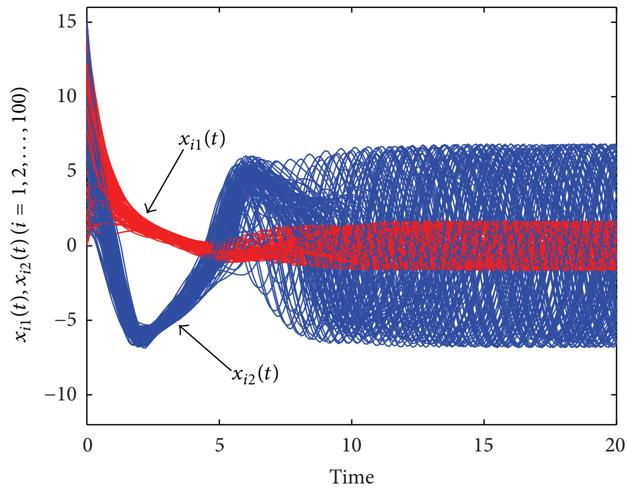

FIGURE 3: Change process of the state variables of the nearest-neighbor unidirectional coupled delayed dynamical network without impulses in time interval $[0 \;\; 20]$.

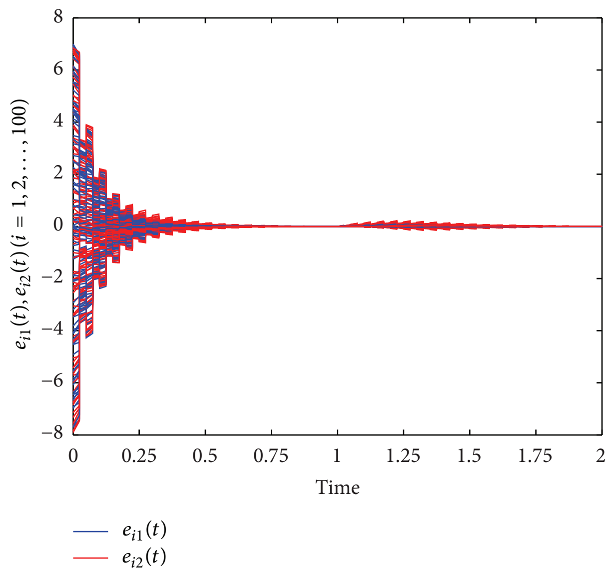

FIGURE 4: Error trajectories of the nearest-neighbor unidirectional coupled delayed dynamical network with the time-varying impulses in time interval $[0 \;\; 2]$.

Example 1. In this example, a nearest-neighbor unidirectional coupled impulsive delayed dynamical network (3) with time-varying impulses is considered. It is well known that the synchronization of the nearest-neighbor coupled dynamical network is difficult to achieve if the number of network nodes is large enough [21]. The coupling matrix B of this network is of the form

$$B = \begin{pmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & 1 \\ 1 & 0 & \cdots & 0 & -1 \end{pmatrix}_{N\times N}. \quad (40)$$

Clearly, the coupling matrix B is a asymmetrical Laplacian matrix of a weighted graph with a spanning tree [9]. In this simulation, choosing $N = 100$, then one has $\tilde{\lambda}_2 = -0.0039$ and $\max_{1\le k\le N}\delta_k = 0$. Let the coupling strength $c = 1$ and the inner coupling matrix $\Gamma = I_2$; then $r\lambda(r) = -0.0039$.

Select $\omega = 2$; one has $L_1 = 8.25$ and $L_2 = 0.65$. Let $d^{sy} = -0.5$ and $d^{desy} = 0.2$, by Corollary 12, then it can be concluded that the nearest-neighbor unidirectional coupled impulsive delayed dynamical network can be globally exponentially synchronized if

$$\Delta t < \frac{-\ln\left(\left|\left(1 + d^{sy}\right)\left(1 + d^{desy}\right)\right|\right)}{L_1 + r\lambda(r)/2 + \overline{d}^{*} L_2} = 0.0525. \quad (41)$$

Let the equidistant impulsive interval be taken as $\Delta t = 0.05$. Figure 2 shows the time-varying impulses sequence. Figure 3 visualizes the change process of the state variables of the nearest-neighbor unidirectional coupled delayed dynamical network without impulses, which clearly indicates desynchronization of the underlying delayed dynamical network without impulses. Error trajectories of the nearest-neighbor unidirectional coupled impulsive delayed dynamical network are plotted in Figure 4. It can be seen that the network achieves quickly synchronization under the time-varying impulses. Since here $|(1 + d^{sy})(1 + d^{desy})| = 0.6 < 1$, the whole impulsive effects are synchronizing. This example verifies the above statement that even if desynchronization impulses occur frequently, an unsynchronized delayed dynamical network (1) can also be globally exponentially synchronized under time-varying impulses if synchronization impulses can prevail over the influence of desynchronization impulsive effects.

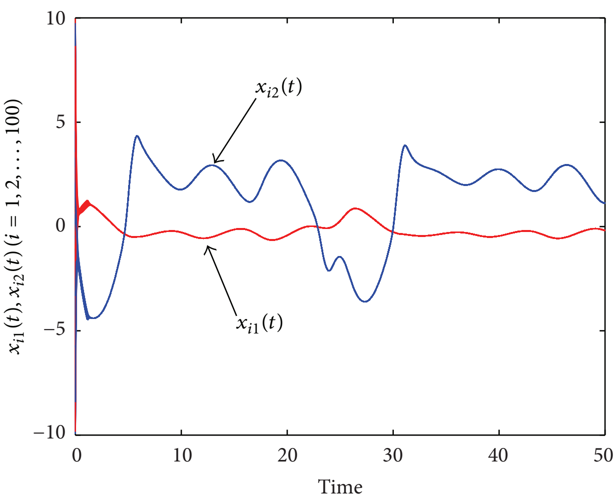

FIGURE 5: Change process of the state variables of the scale-free coupled delayed dynamical network without impulses in time interval $[0\ \ 50]$.

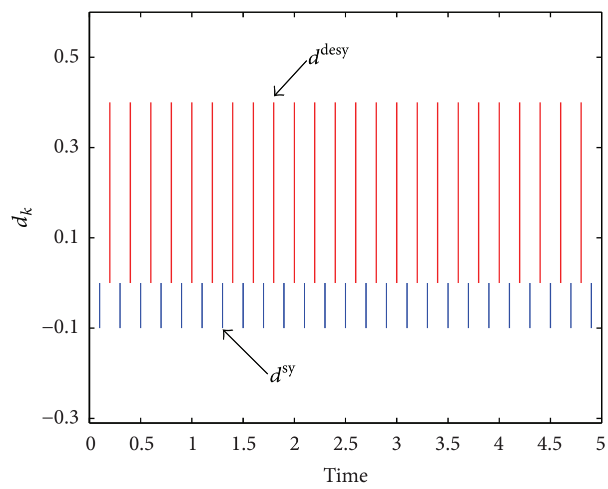

FIGURE 6: Time-varying impulsive sequence with $d^{\text{sy}} = -0.1$, $d^{\text{desy}} = 0.4$, and $\Delta t = 0.2$ in time interval $[0\ \ 5]$.

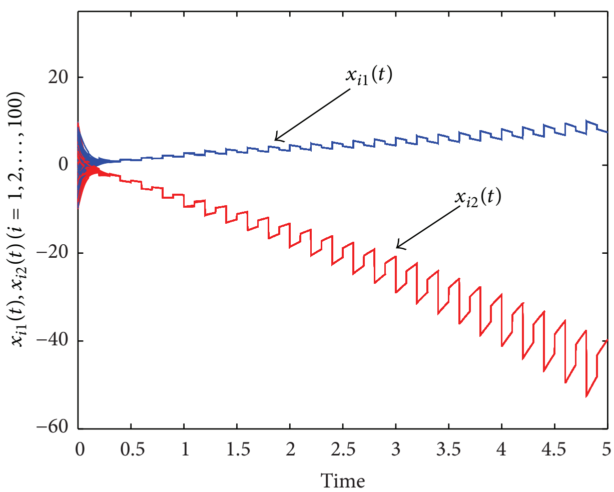

FIGURE 7: Synchronization of the scale-free coupled delayed dynamical network with the time-varying impulses in time interval $[0\ \ 5]$.

Example 2. A BA scale-free [31] coupled impulsive delayed dynamical network (3) is taken as the second example. The parameters of the BA model are given by $m_0 = m = 4$ and $N = 100$. We generate 100 BA scale-free networks randomly, and the network with $\tilde{\lambda}_2 = -4.3532$ is selected for simulation in this example. The remaining 99 scale-free networks can be analyzed similarly [10]. By simple calculation, we get that $\max_{1\le k\le N}\delta_k = 0$. Let the coupling strength $c = 5$ and the inner coupling matrix $\Gamma = I_2$. Select $\omega = 1.5$; one has $L_1 = 7.575$ and $L_2 = 0.8667$. Then, $2L_1 + r\lambda(r) + 2L_2 = -4.8827 < 0$; that is, $-p > q > 0$. Thus, the underlying delayed dynamical network without impulses is globally exponentially synchronized itself in this example, as shown in Figure 5.

Let $d^{\text{sy}} = -0.1$ and $d^{\text{desy}} = 0.4$. By Corollary 12, we can derive the following condition ensuring globally exponential synchronization of the scale-free coupled delayed dynamical network with time-varying impulses will be maintained

$$\Delta t > -\frac{\ln\left(\left|\left(1 + d^{\text{sy}}\right)\left(1 + d^{\text{desy}}\right)\right|\right)}{L_1 + r\lambda(r)/2 + \overline{d}^{*} L_2} = 0.1909. \quad (42)$$

Select $\Delta t = 0.2$; Figure 6 represents the time-varying impulses sequence, and the synchronization process of the scale-free coupled impulsive delayed dynamical network is plotted in Figure 7. In this example, the whole impulsive effects are desynchronizing because $|(1 + d^{\text{sy}})(1 + d^{\text{desy}})| = 1.26 > 1$. We can see that globally exponential synchronization of the underlying delayed dynamical network without impulses is preserved under such time-varying impulses.

5. Conclusion

In this paper, a detailed analysis has been carried out for the synchronization of directed complex networks with time-varying delays dynamical nodes and time-varying impulsive effects. Both synchronizing and desynchronizing impulses were considered in the network model simultaneously. Without assuming symmetry and irreducibility of coupling structure, some globally exponential synchronization criteria for the proposed impulsive delayed dynamical networks with weakly connected topology have been established by using the comparison principle. The obtained results showed that even if desynchronization impulses occur frequently, the underlying delayed dynamical networks without impulses which may be asynchronous itself can also be globally exponentially synchronized if synchronization impulses can prevail over the influence of desynchronization impulsive effects. Two numerical examples and their simulations have been given to verify the effectiveness of the theoretical results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors are grateful to the reviewers and the editor for their valuable comments and suggestions. This work was supported by the National Science Foundation of China (Grant no. 11172158), National Science Foundation of China, Tian Yuan Special Foundation (Grant no. 11326193), the Research Foundation for Advanced Talents of Jiangsu University (Grant no. 13JDG027), and the First-Class Discipline of Universities in Shanghai.

References

[1] S. H. Strogatz, "Exploring complex networks," *Nature*, vol. 410, no. 6825, pp. 268–276, 2001.

[2] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, "Complex networks: structure and dynamics," *Physics Reports*, vol. 424, no. 4-5, pp. 175–308, 2006.

[3] C. W. Wu, *Synchronization in Complex Networks of Nonlinear Dynamical Systems*, World Scientific, Singapore, 2007.

[4] C. Li and G. Chen, "Synchronization in general complex dynamical networks with coupling delays," *Physica A*, vol. 343, no. 1–4, pp. 263–278, 2004.

[5] K. Li, S. Guan, X. Gong, and C.-H. Lai, "Synchronization stability of general complex dynamical networks with time-varying delays," *Physics Letters A*, vol. 372, no. 48, pp. 7133–7139, 2008.

[6] B. Liu, X. Liu, G. Chen, and H. Wang, "Robust impulsive synchronization of uncertain dynamical networks," *IEEE Transactions on Circuits and Systems I*, vol. 52, no. 7, pp. 1431–1441, 2005.

[7] G. Zhang, Z. Liu, and Z. Ma, "Synchronization of complex dynamical networks via impulsive control," *Chaos*, vol. 17, no. 4, Article ID 043126, 9 pages, 2007.

[8] J. Zhou, Q. Wu, and L. Xiang, "Impulsive pinning complex dynamical networks and applications to firing neuronal synchronization," *Nonlinear Dynamics*, vol. 69, no. 3, pp. 1393–1403, 2012.

[9] S. Cai, J. Zhou, L. Xiang, and Z. Liu, "Robust impulsive synchronization of complex delayed dynamical networks," *Physics Letters A*, vol. 372, no. 30, pp. 4990–4995, 2008.

[10] J. Lu, J. Kurths, J. Cao, N. Mahdavi, and C. Huang, "Synchronization control for nonlinear stochastic dynamical networks: pinning impulsive strategy," *IEEE Transactions on Neural Networks*, vol. 23, no. 2, pp. 285–292, 2012.

[11] J. Lu, D. W. C. Ho, and J. Cao, "A unified synchronization criterion for impulsive dynamical networks," *Automatica*, vol. 46, no. 7, pp. 1215–1221, 2010.

[12] J. Lu, D. W. C. Ho, J. Cao, and J. Kurths, "Exponential synchronization of linearly coupled neural networks with impulsive disturbances," *IEEE Transactions on Neural Networks*, vol. 22, no. 2, pp. 329–335, 2011.

[13] J. Zhou, L. Xiang, and Z. Liu, "Synchronization in complex delayed dynamical networks with impulsive effects," *Physica A*, vol. 384, no. 2, pp. 684–692, 2007.

[14] B. Liu and D. J. Hill, "Robust stability of complex impulsive dynamical systems," in *Proceedings of the 46th IEEE Conference on Decision and Control*, p. 103, New Orleans, Lo, USA, 2007.

[15] Y. Yang and J. Cao, "Exponential synchronization of the complex dynamical networks with a coupling delay and impulsive effects," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 3, pp. 1650–1659, 2010.

[16] W. Zhang, Y. Tang, Q. Miao, and W. Du, "Distributed synchronization of coupled neural networks via randomly occurring control," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 24, no. 3, pp. 435–437, 2013.

[17] Q. Zhang, J. Lu, J. Lu, and C. K. Tse, "Adaptive feedback synchronization of a general complex dynamical network with delayed nodes," *IEEE Transactions on Circuits and Systems II*, vol. 55, no. 2, pp. 183–187, 2008.

[18] S. Cai, Q. He, J. Hao, and Z. Liu, "Exponential synchronization of complex networks with nonidentical time-delayed dynamical nodes," *Physics Letters A*, vol. 374, no. 25, pp. 2539–2550, 2010.

[19] T. Chen, X. Liu, and W. Lu, "Pinning complex networks by a single controller," *IEEE Transactions on Circuits and Systems I*, vol. 54, no. 6, pp. 1317–1326, 2007.

[20] Z. Li and J.-J. Lee, "New eigenvalue based approach to synchronization in asymmetrically coupled networks," *Chaos*, vol. 17, no. 4, Article ID 043117, 9 pages, 2007.

[21] C. W. Wu and L. O. Chua, "Synchronization in an array of linearly coupled dynamical systems," *IEEE Transactions on Circuits and Systems I*, vol. 42, no. 8, pp. 430–447, 1995.

[22] X. Li, X. Wang, and G. Chen, "Pinning a complex dynamical network to its equilibrium," *IEEE Transactions on Circuits and Systems I*, vol. 51, no. 10, pp. 2074–2087, 2004.

[23] J. Hale, *Theory of Functional Differential Equations*, vol. 3 of *Applied Mathematical Sciences*, Springer, New York, NY, USA, 2nd edition, 1977.

[24] G. Huang and J. Cao, "Delay-dependent multistability in recurrent neural networks," *Neural Networks*, vol. 23, no. 2, pp. 201–209, 2010.

[25] D. Bainov and P. S. Simeonov, *Systems with Impulsive Effect: Stability, Theory and Applications*, Ellis Horwood, New York, NY, USA, 1989.

[26] T. Yang, *Impulsive Control Theory*, vol. 272 of *Lecture Notes in Control and Information Sciences*, Springer, Berlin, Germany, 2001.

[27] Z. Yang and D. Xu, "Stability analysis of delay neural networks with impulsive effects," *IEEE Transactions on Circuits and Systems II*, vol. 52, no. 8, pp. 517–521, 2005.

[28] Y. Zhang and J. Sun, "Stability of impulsive neural networks with time delays," *Physics Letters A*, vol. 348, no. 1-2, pp. 44–50, 2005.

[29] Z. Yang and D. Xu, "Stability analysis and design of impulsive control systems with time delay," *IEEE Transactions on Automatic Control*, vol. 52, no. 8, pp. 1448–1454, 2007.

[30] W. Zhang, Y. Tang, J. Fang, and X. Wu, "Stability of delayed neural networks with time-varying impulses," *Neural Networks*, vol. 36, pp. 59–63, 2012.

[31] A.-L. Barabási and R. Albert, "Emergence of scaling in random networks," *Science*, vol. 286, no. 5439, pp. 509–512, 1999.