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Research Article

Nontrivial Periodic Solutions of an n-Dimensional Differential System and Its Application

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Two criteria are constructed to guarantee the existence of periodic solutions for a second-order n-dimensional differential system by using continuation theorem. It is noticed that the criteria established are found to be associated with the system's damping coefficient, natural frequency, parametrical excitation, and the coefficient of the nonlinear term. Based on the criteria obtained, we investigate the periodic motions of the simply supported at the four-edge rectangular thin plate system subjected to the parametrical excitation. The effectiveness of the criteria is validated by corresponding numerical simulation. It is found that the existent range of periodic solutions for the thin plate system increases along with the increase of the ratio of the modulus of nonlinear term's coefficient and parametric excitation term, which generalize and improve the corresponding achievements given in the known literature.

1. Introduction

In recent years, thin plates have been widely applied to the fields of automobile, marine, space station, shutter and modern aircraft, and so forth. Therefore, the nonlinear dynamic behavior of the thin plate received very considerable attention within many articles available in the technical and scientific literature. See [1–6], for example, and the references therein.

In the aforementioned works, based on the Schauder second fixed point theorem, Dizaji et al. [2] predicted the existence of periodic solutions of the following governing equations of motion:

$$\ddot{A} + P(t)A + Q(t)A^3 = F(t), \tag{1}$$

which could be derived from the nonlinear simply supported rectangular thin plate system under the influence of a relatively moving mass.

Zhang [5] and Zhang et al. [6] studied the periodic and chaotic motions of the parametrically excited rectangular thin plates on the basis of multiple scales method and continuation theorem, respectively.

As far as we know, there were few researchers who focused on the existence of periodic solutions of the thin plate system subjected to the parametrical excitation with rigorous theoretical proof. In contrast, the existence of periodic solutions is often shown only by numerical simulation. However, it is the rigorously proved theorem that can throw more light than thousands of beautiful pictures on the basic nature of periodicity.

Recently, there are lots of mathematical researchers devoted to the investigation of the periodic solutions for p-Laplacian-like systems, for example, [7–12], which can be reduced to the general second-order systems of ordinary differential equations while $p \equiv 2$. Nevertheless, the results obtained cannot be applied to the general nonlinear equations, for example, see [13–16]. The challenge lies in the growth degree with respect to the nonlinear term which often needs to be less than or equal to $p-1$ for p-Laplacian-like systems. For instance, the one-sided growth condition imposed on the nonlinear terms in [7, 8, 10, 17] was given as follows:

$$\lim_{|u|\to\infty} \frac{|g(t,u)|}{|u|^n} \leqslant r \in [0, +\infty[, \quad \text{uniformly for } t \in [0,T], \quad n \leqslant p-1. \tag{2}$$

In this paper, two criteria are established to guarantee the existence of periodic solutions for a second-order n-dimensional differential system. It is noticed that the existence of nontrivial periodic solutions is found to be influenced by the system's damping coefficient, natural frequency, parametric excitation, and the coefficient of the nonlinear term. Moreover, the parametrical excitation in this paper is not limited to be periodic.

As an application of the criteria obtained, the existence of periodic solutions for the simply supported at the four-edged rectangular thin plate system subjected to parametrical excitation is investigated in Section 4 of this paper. Furthermore, corresponding numerical simulations are carried out to validate the feasibility of the criteria achieved. From the several numerical results, it is noticed that the existent range of periodic solutions for the thin plate system becomes larger with the increase of the ratio of the modulus of nonlinear term's coefficient and parametric excitation term. By means of analytical arguments and numerical simulation runs, it is easy to find that the proposals given in this study are seldom obtained in the known literature, for example, [2, 5, 6].

2. Preliminaries and Notations

Consider a second-order n-dimensional system

$$\ddot{x} + \mu_0 \dot{x} + \left[\omega_0^2 + \alpha(t)\right] x + \boldsymbol{\beta}\left(x_1^3, x_2^3, \ldots, x_n^3\right)^T = e(t), \quad (3)$$

where $x \in \mathbb{R}^n$, $\mu_0 \in \mathbb{R}$, $\omega_0 \in \mathbb{R}^n$, and $\boldsymbol{\beta}$ is an $n \times n$ symmetric matrix of constants. $\alpha, e \in C(\mathbb{R}, \mathbb{R}^n)$ with $\alpha(t+T) \equiv \alpha(t)$, $e(t+T) \equiv e(t)$, and $\int_0^T e(s)\, ds = 0$; T is a positive constant.

Next, we recall an important lemma which will help us to start the corresponding research.

Lemma 1 (see [18]). *Suppose that X and Y are two Banach spaces, and let $L : D(L) \subset X \to Y$ be a Fredholm operator with index zero. Furthermore, $\Omega \subset X$ is an open bounded set and $N : \overline{\Omega} \to Y$ is L-compact in $\overline{\Omega}$. If*

(1) $Lx \neq \epsilon Nx$, $\forall (x, \epsilon) \in (D(L) \cap \partial\Omega) \times]0, 1]$,

(2) $Nx \notin \operatorname{Im} L$, $\forall x \in \operatorname{Ker} L \cap \partial\Omega$,

(3) $\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} \neq 0$, *where* $J : \operatorname{Im} Q \to \operatorname{Ker} L$ *is an isomorphism.*

Then, the equation $Lx = Nx$ has a solution in $D(L) \cap \overline{\Omega}$.

In what follows, for convenience and without loss of generality, some notations are introduced throughout the paper: $|\cdot|$ denotes absolute value and the Euclidean norm on $\mathbb{R}^n$, for $\forall a = (a_1, a_2, \ldots, a_n)^T$ and $|a| = (\sum_{i=1}^n |a_i|^2)^{1/2}$. Also, we set $I = [0, T]$, $C^0 = C^0(I, \mathbb{R}^n)$, $C^1 = C^1(I, \mathbb{R}^n)$ and $C_T = \{u \in C^0 \mid u(0) = u(T)\}$ with the norm $|u|_{\max} = \max_{t \in I} |u(t)|$ and $C_T^1 = \{v \in C^1 \mid v(0) = v(T), \dot{v}(0) = \dot{v}(T)\}$ with the norm $\|v\| = \max\{|v|_{\max}, |\dot{v}|_{\max}\}$. Obviously, C_T and C_T^1 are two Banach spaces. Meanwhile, denote

$$L : D(L) \subset C_T \longrightarrow C_T, \qquad Lx = \ddot{x}, \quad (4)$$

$$N : C_T \longrightarrow C_T, \qquad [Nx](t) = -\mu_0 \dot{x} - \left[\omega_0^2 + \alpha(t)\right] x - \boldsymbol{\beta}\left(x_1^3, x_2^3, \ldots, x_n^3\right)^T + e(t), \quad (5)$$

where $D(L) = \{x \mid x \in C_T^1, \ddot{x} \in C(\mathbb{R}, \mathbb{R}^n)\}$.

It is easily shown that system (3) can be converted into the equivalent abstract equation $Lx = Nx$. Moreover, from the definition of L, we see that $\operatorname{Ker} L = \mathbb{R}^n$, $\operatorname{Im} L = \{y \mid y \in C_T, \int_0^T y(s) ds = 0\}$. Therefore, L is a Fredholm operator with index zero.

Let the projections

$$P : C_T^1 \longrightarrow \operatorname{Ker} L, \qquad [Px](t) = x(0) = x(T),$$
$$Q : C_T \longrightarrow \frac{C_T}{\operatorname{Im} L}, \qquad [Qy](t) = \frac{1}{T} \int_0^T y(s)\, ds, \quad (6)$$

then, $\operatorname{Im} P = \operatorname{Ker} L$, $\operatorname{Ker} Q = \operatorname{Im} L$.

Let L^{-1} represent the inverse of $L|_{D(L) \cap \operatorname{Ker} P}$, $L^{-1} : \operatorname{Im} L \to D(L) \cap \operatorname{Ker} P$; then

$$\left[L^{-1} y\right](t) = \int_0^T G(t, s)\, y(s)\, ds, \quad (7)$$

where

$$G(t, s) = \begin{cases} \dfrac{s(t - T)}{T}, & 0 \leqslant s < t \leqslant T, \\[2mm] \dfrac{t(s - T)}{T}, & 0 \leqslant t \leqslant s \leqslant T. \end{cases} \quad (8)$$

From (5) and (7), it is easily verified that N is L-compact on $\overline{\Omega}$, where Ω is an arbitrary open bounded subset of C_T^1.

3. Main Results

3.1. Theoretical Proof

Theorem 2. *For all $i \in \{1, 2, \ldots, n\}$, assume that the following conditions are satisfied:*

$[C_1]$ $0 \leqslant |\alpha_i|_{\max} |\lambda_i|^{1/3} T / \mu_0 \Delta < 2$, *where λ_i are the eigenvalues of $\boldsymbol{\beta}$ and $\Delta = |\lambda_i|^{1/3} - (\omega_{0i}^2 + |\alpha_i|_{\max})^{1/3}$.*

$[C_2]$ (i) *If $\lambda_i \neq 0$, there is a constant $\gamma > 0$ such that*

$$\lambda_i > -\frac{\omega_{0i}^2 + \alpha_{i,\min}}{\gamma^2}, \quad \lambda_i > 0,$$
$$\lambda_i < -\frac{\omega_{0i}^2 + \alpha_{i,\max}}{\gamma^2}, \quad \lambda_i < 0, \quad (9)$$

where $\alpha_{i,\min} = \min_{t \in I} \alpha_i(t)$ and $\alpha_{i,\max} = \max_{t \in I} \alpha_i(t)$.

(ii) *If* $\lambda_i \equiv 0$, *and* $\omega_{0i}^2 \neq -(1/T)\int_0^T \alpha_i dt$,

then, system (3) *has at least one nontrivial* T*-periodic solution if there exist constants* d_i, *such that*

$$\left|\lambda_i\right|_{\max} < \frac{\omega_{0i}^2 + \alpha_{i,\min}}{d_i^2}. \quad (10)$$

Proof. Let us embed system (3) into one parameter family of the systems as follows:

$$\begin{aligned} &\ddot{x} + \epsilon\mu_0\dot{x} + \epsilon\left[\omega_0^2 + \alpha(t)\right]x + \epsilon\boldsymbol{\beta}\left(x_1^3, x_2^3, \ldots, x_n^3\right)^T \\ &\quad = \epsilon e(t), \quad \epsilon \in \left]0, 1\right]. \end{aligned} \quad (11)$$

Since $\boldsymbol{\beta}$ is the symmetrical matrix, there is an orthogonal matrix U, such that

$$U\boldsymbol{\beta}U^T = \operatorname{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\}. \quad (12)$$

Integrating both sides of (11) from 0 to T gives

$$\begin{aligned} \int_0^T \left|\lambda_i x_i^3\right| dt &\leqslant \int_0^T \left|x_i\left[\omega_{0i}^2 + \alpha_i(t)\right]\right| dt + T|e|_{\max}, \\ &\qquad \forall i \in \{1, 2, \ldots, n\}. \end{aligned} \quad (13)$$

Applying integral mean value theorem, there exists a constant $\xi \in]0, T[$, such that

$$\left|\lambda_i\right|\left|x_i(\xi)\right|^3 \leqslant \frac{1}{T}\int_0^T \left|x_i\left[\omega_{0i}^2 + \alpha_i(t)\right]\right| dt + T|e|_{\max}. \quad (14)$$

Now, we claim that

$$\left|\lambda_i\right|^{1/3}\left|x_i(\xi)\right| \leqslant \left(\omega_{0i}^2 + \left|\alpha_i\right|_{\max}\right)^{1/3}\left|x_i\right|_{\max} + C, \quad (15)$$

where $C = (\omega_{0i}^2 + |\alpha_i|_{\max})^{1/3} + T|e|_{\max} + |\lambda_i|^{1/3}$.

Case 1. If $|x_i(\xi)| \leqslant 1$, then, (15) holds clearly.

Case 2. If $|x_i(\xi)| > 1$, define

$$E_1 = \left\{t : t \in I, \left|x_i\right| > 1\right\}, \qquad E_2 = \left\{t : t \in I, \left|x_i\right| \leqslant 1\right\}. \quad (16)$$

By (14) and simple calculation, we obtain

$$\begin{aligned} \left|\lambda_i\right|^{1/3}\left|x_i(\xi)\right| &\leqslant \left|\frac{1}{T}\int_{E_1} x_i\left[\omega_{0i}^2 + \alpha_i(t)\right] dt\right|^{1/3} \\ &\quad + \left|\frac{1}{T}\int_{E_2} x_i\left[\omega_{0i}^2 + \alpha_i(t)\right] dt\right|^{1/3} \\ &\leqslant \left(\omega_{0i}^2 + \left|\alpha_i\right|_{\max}\right)^{1/3}\left|x_i\right|_{\max} \\ &\quad + \left(\omega_{0i}^2 + \left|\alpha_i\right|_{\max}\right)^{1/3}. \end{aligned} \quad (17)$$

Thus, it can be easily seen that (15) holds.

According to (15), we have

$$\begin{aligned} \left|\lambda_i\right|^{1/3}\left|x_i(t)\right| &\leqslant \left(\omega_{0i}^2 + \left|\alpha_i\right|_{\max}\right)^{1/3}\left|x_i\right|_{\max} \\ &\quad + \left|\lambda_i\right|^{1/3}\int_{t^*}^{t}\left|\dot{x}_i(s)\right| ds + C, \\ &\qquad t \in \left[t^*, t^* + T\right], \end{aligned} \quad (18)$$

$$\begin{aligned} \left|\lambda_i\right|^{1/3}\left|x_i(t)\right| &= \left|\lambda_i\right|^{1/3}\left|x_i(t - T)\right| \\ &\leqslant \left(\omega_{0i}^2 + \left|\alpha_i\right|_{\max}\right)^{1/3}\left|x_i\right|_{\max} \\ &\quad + \left|\lambda_i\right|^{1/3}\int_{t-T}^{t^*}\left|\dot{x}_i(s)\right| ds + C, \\ &\qquad t \in \left[t^*, t^* + T\right]. \end{aligned} \quad (19)$$

Combining inequalities (18) and (19) yields

$$\begin{aligned} \left|\lambda_i\right|^{1/3}\left|x_i\right|_{\max} &\leqslant \left(\omega_{0i}^2 + \left|\alpha_i\right|_{\max}\right)^{1/3}\left|x_i\right|_{\max} \\ &\quad + \frac{\left|\lambda_i\right|^{1/3}}{2}\int_0^T\left|\dot{x}_i(t)\right| dt + C. \end{aligned} \quad (20)$$

Therefore, it follows from condition $[C_1]$ that $\Delta > 0$. Then, we obtain

$$\left|x_i\right|_{\max} \leqslant \frac{\left|\lambda_i\right|^{1/3}}{2\Delta}\int_0^T\left|\dot{x}_i(t)\right| dt + \frac{C}{\Delta}. \quad (21)$$

As $x(t) \in C_T^1$, multiplying both sides of the ith component of (11) by $\dot{x}_i(t)$ and integrating on the interval I lead to

$$\mu_0\int_0^T\left|\dot{x}_i\right|^2 dt + \int_0^T \alpha_i x_i\dot{x}_i\, dt = \int_0^T e_i\dot{x}_i\, dt. \quad (22)$$

Using Hölder's inequality and (22), we have

$$\begin{aligned} \mu_0\int_0^T\left|\dot{x}_i\right|^2 dt &\leqslant \left(\left|\alpha_i\right|_{\max}\left|x_i\right|_{\max} + \left|e_i\right|_{\max}\right)T^{1/2} \\ &\quad \times\left(\int_0^T\left|\dot{x}_i\right|^2 dt\right)^{1/2}. \end{aligned} \quad (23)$$

It is noticed that $\alpha_i(t)$ and $e_i(t)$ are bounded on the interval I. From (21), we obtain

$$\begin{aligned} \mu_0\int_0^T\left|\dot{x}_i\right|^2 dt &\leqslant \frac{\left|\alpha_i\right|_{\max}\left|\lambda_i\right|^{1/3}T}{2\Delta}\int_0^T\left|\dot{x}_i\right|^2 dt \\ &\quad + \left(\frac{\left|\alpha_i\right|_{\max}C}{\Delta} + \left|e_i\right|_{\max}\right)T^{1/2} \\ &\quad \times\left(\int_0^T\left|\dot{x}_i\right|^2 dt\right)^{1/2}. \end{aligned} \quad (24)$$

According to the condition $[C_1]$, it is easily seen that there exists a constant $M > 0$, such that

$$\int_0^T\left|\dot{x}_i\right|^2 dt \leqslant M. \quad (25)$$

Combining (21) and (25), we obtain

$$|x|_{\max} \leqslant \frac{|\lambda_i|^{1/3}\sqrt{nMT}}{2\Delta} + \frac{\sqrt{n}C}{\Delta} \triangleq M_1. \tag{26}$$

For $x(0) = x(T)$, there exists a $t_0 \in]0, T[$, such that $\dot{x}(t_0) = 0$. Then, it follows from (11) that

$$\begin{aligned} |\dot{x}_i| &= \left|\int_{t_0}^{t} \ddot{x}_i(s)\,ds\right| \\ &\leqslant \mu_0 \int_0^T |\dot{x}_i|\,dt + \left(\omega_{0i}^2 + |\alpha_i|_{\max}\right)|x_i|_{\max}T \\ &\quad + |\lambda_i|\,|x_i|_{\max}^3 T + |e_i|_{\max}T. \end{aligned} \tag{27}$$

Therefore, we have

$$\begin{aligned} |\dot{x}|_{\max} &\leqslant \mu_0\sqrt{nMT} + \left(\omega_{0i}^2 + |\alpha|_{\max}\right)\sqrt{n}|x_i|_{\max}T \\ &\quad + \sqrt{n}|\lambda_i|\,|x_i|_{\max}^3 T + \sqrt{n}|e_i|_{\max}T \\ &\leqslant \mu_0\sqrt{nMT} \\ &\quad + \frac{\left(\omega_{0i}^2 + |\alpha|_{\max}\right)\sqrt{n}T}{2\Delta}\left(|\lambda_i|^{1/3}\sqrt{MT} + 2C\right) \\ &\quad + \frac{\sqrt{n}T}{8\Delta^3}\Big[|\lambda_i|(MT)^{3/2} + 6|\lambda_i|^{2/3}CMT \\ &\qquad + 12|\lambda_i|^{1/3}C^2(MT)^{1/2} + 8C^3\Big] \\ &\quad + \sqrt{n}|e_i|_{\max}T \\ &\triangleq M_2. \end{aligned} \tag{28}$$

Let $\Omega := \{x : |x|_{\max} < M_1 + 1, |\dot{x}|_{\max} < M_2 + 1\}$. For $\forall \epsilon \in]0, 1]$, there is not any solutions of (11) on $\partial\Omega$ with for all $x \in \operatorname{Ker} L \cap \partial\Omega$. Based on the condition $[C_2]$, there exist the appropriate constants M_1 and γ, such that the following relationship holds

$$\begin{aligned} &\int_0^T [Nx](t)\,dt \\ &\quad = -\left(\int_0^T x_1\left[\omega_{01}^2 + \alpha_1(t) + \lambda_1 x_1^2\right]dt, \ldots,\right. \\ &\qquad \left.\int_0^T x_n\left[\omega_{0n}^2 + \alpha_n(t) + \lambda_n x_n^2\right]dt\right)^T \\ &\quad \neq (0, 0, \ldots, 0)^T, \quad \text{for } |x_i| > \gamma,\ i = \{1, 2, \ldots, n\}. \end{aligned} \tag{29}$$

Thus, the first two conditions of Lemma 1 are satisfied.

Next, we claim that the third condition of Lemma 1 is also satisfied. To verify this, we define the isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$, $J(x) = x$. For all $\mu^* \in [0, 1]$, $i = 1, 2, \ldots, n$, we denote

$$\begin{aligned} H(x, \mu^*) &= \mu^* x + \frac{1-\mu^*}{T} \\ &\quad \times \left(\operatorname{sgn}(\lambda_1)\int_0^T x_1\left[\omega_{01}^2 + \alpha_1(t) + \lambda_1 x_1^2\right]dt, \ldots,\right. \\ &\quad \left.\operatorname{sgn}(\lambda_n)\int_0^T x_n\left[\omega_{0n}^2 + \alpha_n(t) + \lambda_n x_n^2\right]dt\right)^T, \\ &\qquad \lambda_i \neq 0, \\ H(x, \mu^*) &= \mu^* x + \frac{1-\mu^*}{T} \\ &\quad \times \left(\operatorname{sgn}\left(\int_0^T \left[\omega_{01}^2 + \alpha_1(t)\right]dt\right)\right. \\ &\quad \times \int_0^T x_1\left[\omega_{01}^2 + \alpha_1(t)\right]dt, \ldots, \\ &\quad \operatorname{sgn}\left(\int_0^T \left[\omega_{0n}^2 + \alpha_n(t)\right]dt\right) \\ &\quad \left.\int_0^T x_n\left[\omega_{0n}^2 + \alpha_n(t)\right]dt\right)^T, \\ &\qquad \lambda_i \equiv 0. \end{aligned} \tag{30}$$

By using the condition $[C_2]$ again, the following relationships hold, when: $x \in \partial\Omega \cap \mathbb{R}^n$,

$$\begin{aligned} x_i H(x_i, \mu^*) &= \mu^* x_i^2 + \frac{1-\mu^*}{T}\operatorname{sgn}(\lambda_i) \\ &\quad \times \int_0^T x_i^2\left[\omega_{0i}^2 + \alpha_i(t) + \lambda_i x_i^2\right]dt > 0, \\ &\qquad \lambda_i \neq 0, \\ x_i H(x_i, \mu^*) &= \mu^* x_i^2 + \frac{1-\mu^*}{T} \\ &\quad \times \operatorname{sgn}\left(\int_0^T \left[\omega_{0i}^2 + \alpha_i(t)\right]dt\right) \\ &\quad \times \int_0^T x_i^2\left[\omega_{0i}^2 + \alpha_i(t)\right]dt > 0, \\ &\qquad \lambda_i \equiv 0. \end{aligned} \tag{31}$$

It follows from (31) that $H(x, \mu^*)$ is homotopic and that

$$\begin{aligned} \deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} &= \deg\{H(\cdot, 0), \Omega \cap \operatorname{Ker} L, 0\} \\ &= \deg\{H(\cdot, 1), \Omega \cap \operatorname{Ker} L, 0\} \neq 0. \end{aligned} \tag{32}$$

Thus, the last condition of Lemma 1 is also satisfied. □

Applying Lemma 1, it can be concluded that the equation $Lx = Nx$ has at least one T-periodic solution $x(t)$ on $\overline{\Omega}$ with $|x|_\infty \leqslant M_1$. Furthermore, it is obvious that the T-periodic solution $x(t)$ is nontrivial. Otherwise, there is a constant vector $d = \{d_1, d_2, \ldots, d_n\}$ satisfying (11), that is,

$$\left[\omega_0^2 + \alpha(t)\right] d + \boldsymbol{\beta}\left(d_1^3, d_2^3, \ldots, d_n^3\right)^T = e(t). \quad (33)$$

By simple computation, we obtain

$$\omega_{0i}^2 + \alpha_{i,\min} \leqslant |\lambda_i|_{\max} d_i^2, \quad (34)$$

which contradicts the condition in Theorem 2. Then, system (3) has at least one non-trivial T-periodic solution. Therefore, we complete the proof of Theorem 2.

Theorem 3. *Assume that* $0 \leqslant ((|\lambda_i|^{1/3}(\omega_{0i}^2 + |\alpha_i|_{\max})^{1/2} T)/\Delta) < 2$ *and* $\lambda_i < 0$ *hold for all* $i \in \{1, 2, \ldots, n\}$, *then, system* (3) *has at least one* T*-periodic solution.*

Proof. The same proof also works for this theorem. We only need to show that $\int_0^T |\dot{x}_i|^2 dt$ is bounded.

As $x(t) \in C_T^1$, multiplying both sides of the ith component of (11) by $x_i(t)$ and integrating from 0 to T yield

$$\begin{aligned} -\int_0^T |\dot{x}_i|^2 dt &= -\epsilon \int_0^T x_i^2 \left[\omega_{0i}^2 + \alpha_i(t) + \lambda_i x_i^2\right] dt \\ &\quad + \epsilon \int_0^T e_i x_i dt. \end{aligned} \quad (35)$$

It can be easily found that $\Delta > 0$ when using the condition $0 \leqslant ((|\lambda_i|^{1/3}(\omega_{0i}^2 + |\alpha_i|_{\max})^{1/2} T)/\Delta) < 2$. Combining (21) and (35), we obtain

$$\begin{aligned} \int_0^T |\dot{x}_i|^2 dt &\leqslant |x_i|_{\max}^2 \left(\omega_{0i}^2 + |\alpha_i|_{\max}\right) T + |x_i|_{\max} |e_i|_{\max} T \\ &\leqslant \frac{|\lambda_i|^{2/3} T^2 \left(\omega_{0i}^2 + |\alpha_i|_{\max}\right)}{4\Delta^2} \int_0^T |\dot{x}_i|^2 dt \\ &\quad + \left[\frac{C\left(\omega_{0i}^2 + |\alpha_i|_{\max}\right)}{\Delta^2} + \frac{|e_i|_{\max}}{\Delta}\right] |\lambda_i|^{1/3} T \\ &\quad \times \int_0^T |\dot{x}_i| dt \\ &\quad + \frac{C^2 T \left(\omega_{0i}^2 + |\alpha_i|_{\max}\right)}{\Delta^2} + \frac{CT |e_i|_{\max}}{\Delta}. \end{aligned} \quad (36)$$

Noticing that $0 \leqslant ((|\lambda_i|^{1/3}(\omega_{0i}^2 + |\alpha_i|_{\max})^{1/2} T)/\Delta) < 2$, there is a constant $M^* > 0$, such that $\int_0^T |\dot{x}_i|^2 dt \leqslant M^*$. Therefore, the proof of the boundedness is completed. The rest proof of the theorem is almost identical to that of Theorem 2. □

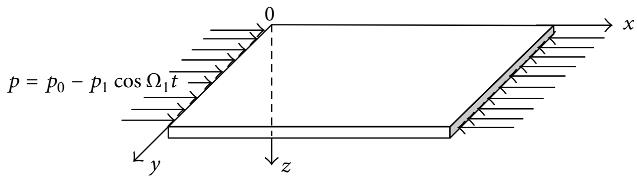

Figure 1: The model of a rectangular thin plate subjected to parametrical excitation.

Corollary 4. *If* $\int_0^T e(t)dt \neq 0$, *let* $\bar{e}(t) = e(t) - (1/T)\int_0^T e(s)ds$. *Then, we have* $\int_0^T \bar{e}(t)dt = 0$. *System* (11) *can be reduced to the following form*

$$\begin{aligned} &\ddot{x} + \mu_0 \dot{x} + \left[\omega_0^2 + \alpha(t)\right] x + \boldsymbol{\beta}\left(x_1^3, x_2^3, \ldots, x_n^3\right)^T \\ &\quad - \frac{1}{T}\int_0^T e(s)\, ds = \bar{e}(t). \end{aligned} \quad (37)$$

Thus, one only needs to study system (37) *by using the aforementioned results.*

3.2. Application. In this section, we apply some of the main results obtained in the previous section to a well-known model for practical engineering.

We now investigate periodic motions of the simply supported at the four-edged rectangular thin plate system subjected to parametrical excitation (see Figure 1).

According to [5, 19], we have the following partial differential governing equations:

$$\begin{aligned} &D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \phi}{\partial x^2} \\ &\quad + 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 \phi}{\partial x \partial y} + \mu \frac{\partial w}{\partial t} = 0, \\ &\nabla^4 \phi = Eh\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right], \end{aligned} \quad (38)$$

where ρ represents the density of the plate, $D = Eh^3/12(1-\nu^2)$ is the bending rigidity, E is Young's modulus, ν is the Poission ratio, ϕ is the stress function, and μ the damping coefficient.

By means of Galerkin's method, (38) can be reduced to the following dimensionless form:

$$\ddot{y} + \mu_* \dot{y} + \left[\omega_*^2 + \alpha_*(t)\right] y + \beta_* y^3 = 0, \quad (39)$$

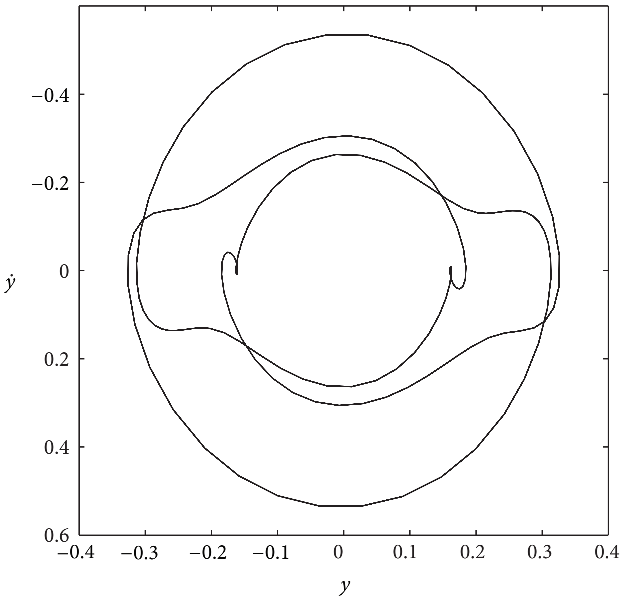

Figure 2: Phase portrait for $(y, \dot{y})$.

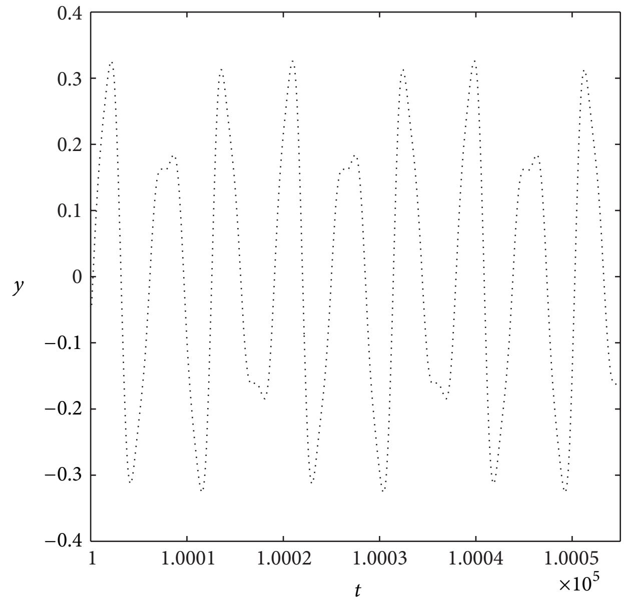

Figure 3: Time history curve (t, y).

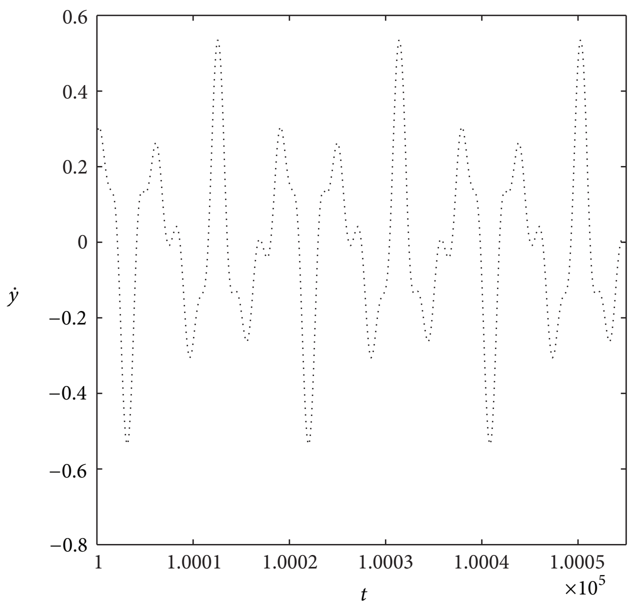

Figure 4: Time history curve $(t, \dot{y})$.

where

$$\begin{aligned} \mu_* &= \sqrt{\frac{12\left(1-\nu^2\right)}{\rho E}}\frac{ab\mu}{\pi^2 h^2}, \\ \alpha_*(t) &= \frac{12\left(1-\nu^2\right)h^2}{a}\frac{bp_1}{\pi^2 D}\cos\Omega_1 t, \\ \omega_*^2 &= \frac{\left(9a^2+b^2\right)^2}{b^4}-\frac{b^2 p_0}{\pi^2 D}, \\ \beta_* &= \frac{3\left(1-\nu^2\right)h^2\left(81a^2+b^2\right)}{4ab^3}. \end{aligned} \tag{40}$$

Obviously, system (11) can be reduced to the system (39), provided that $n = 1$, $\mu_0 = \mu_*$, $\omega_0 = \omega_*$, $\alpha = \alpha_*$, $\beta = \beta_*$, and $e(t) \equiv 0$.

In what follows, in order to illustrate the aforementioned theoretical results, several numerical simulations are carried out. For system (39), by Theorem 2, we take a set of initial values $(0.03, 0.77)$, let $\mu_* = 0.02$ and $\Omega_1 = 2$ and choose appropriate p_0, p_1 such that $\omega_* = 1$.

By using ode45 in MATLAB 7, three families of dynamic characteristics of the thin plate system are illustrated for different values of $\alpha_*(t)$ and β_*, respectively.

For $\alpha_*(t) = 2\cos 2t$, a centrosymmetric periodic solution in the $(y, \dot{y})$ plane is shown in Figure 2, keeping β_* fixed at 15. Moreover, time history curves with respect to the displacement and velocity of the plate are also shown in Figures 3 and 4 for the same condition. It is straightforward to see that the curves are also periodic.

If we set $\alpha_*(t) = 7\cos 2t$ and $\beta_* = 10$, a group of dynamics behavior of the thin plate system in the planes $(y, \dot{y})$, (t, y), and $(t, \dot{y})$ is obtained in Figures 5, 6, and 7, respectively. It can be observed from these figures that the phase portrait is also centrosymmetric and the time history curves are periodic too. Furthermore, according to the corresponding power spectrum, there is a dominant peak at the frequency that is approximately equal to 2.3 with symmetric sidebands surrounding it.

For $\alpha_*(t) = 13.05\cos 2t$, when β_* is gradually increased to 20, a set of dynamics characteristic of the thin plate system is addressed in Figures 8, 9 and 10, respectively.

In Figure 8, phase portrait in the $(y, \dot{y})$ plane is also illustrated to be centrosymmetric, though it seems to be more complex than the previous ones. The corresponding periodic time history curves with respect to the displacement and

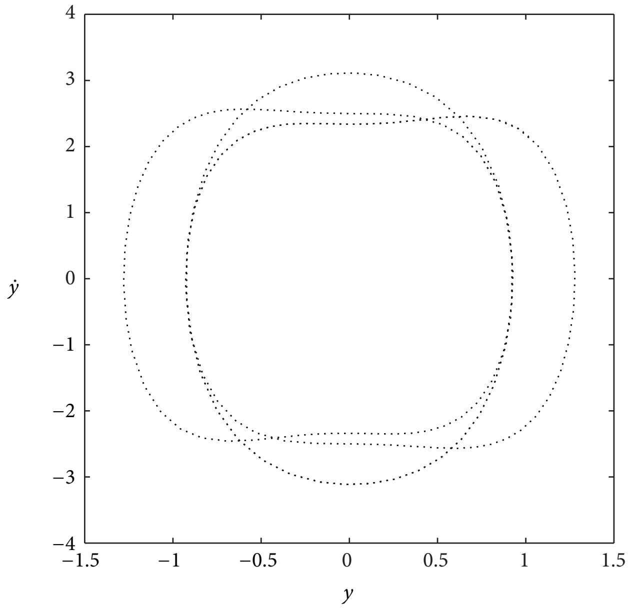

FIGURE 5: Phase portrait $(y, \dot{y})$.

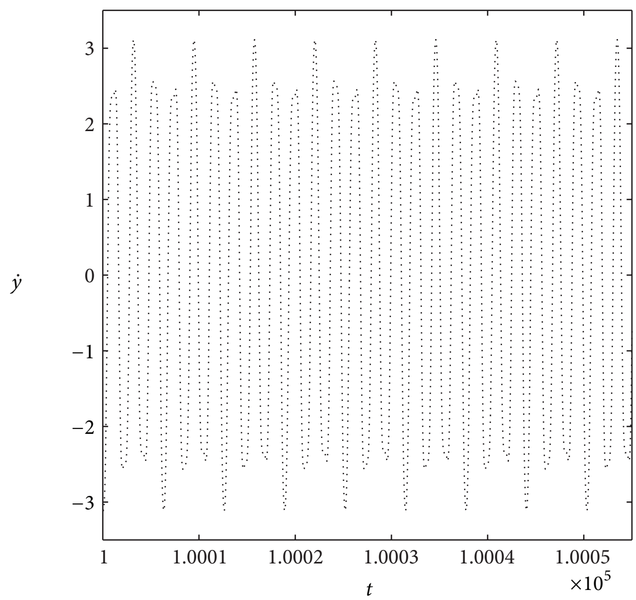

FIGURE 7: Time history curve $(t, \dot{y})$.

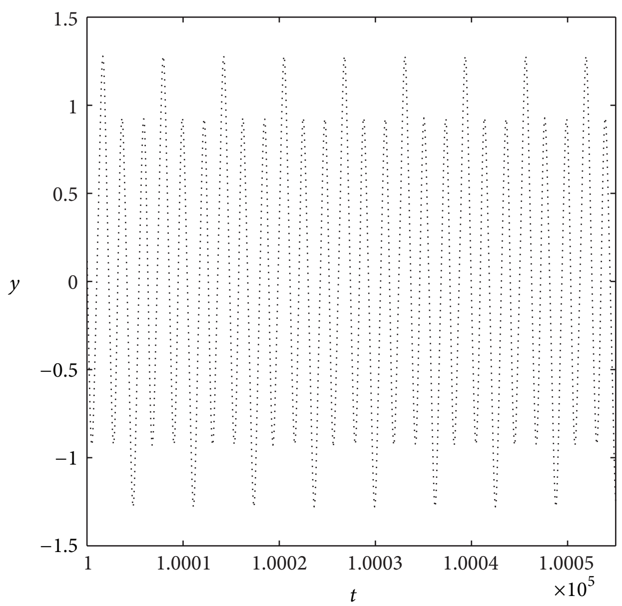

FIGURE 6: Time history curve (t, y).

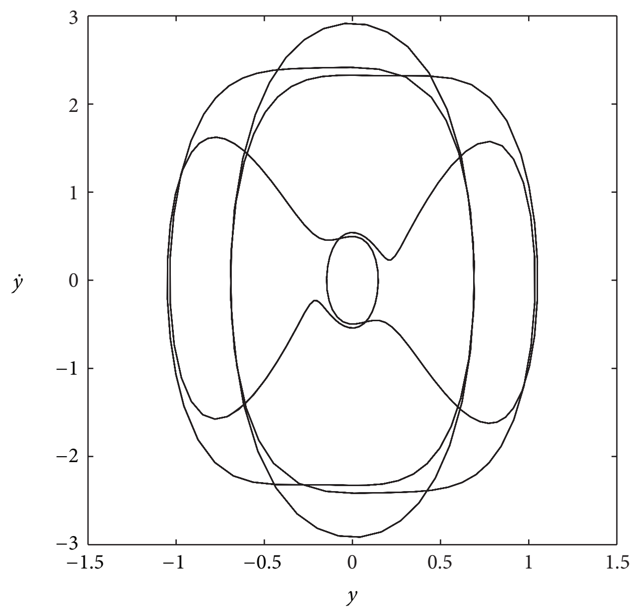

FIGURE 8: Phase portrait $(y, \dot{y})$.

velocity of the plate are depicted in Figures 9 and 10. In addition, the power spectrum in this case associated with the periodic solution admits a distinctive broadband character.

4. Conclusions

This paper primarily deals with the existence of nontrivial periodic solutions for a second-order n-dimensional differential system. Moreover, the simply supported at the four-edged rectangular thin plate system subjected to parametrical excitation is investigated as an application. Theoretical analysis and numerical validation produce several important results as follows.

(i) From the conditions of the proved theorems, it is easy to find that the nontrivial periodic solutions of the system are mainly influenced by the system's damping coefficient, natural frequency, parametrical excitation, and the coefficient of the nonlinear term.

(ii) By substituting the variables μ_*, ω_*, α_*, and β_* into the condition $[C_1]$ of Theorem 2, and combining with the phase diagrams and time history curves displayed above, one can see that there exist a set of T-periodic

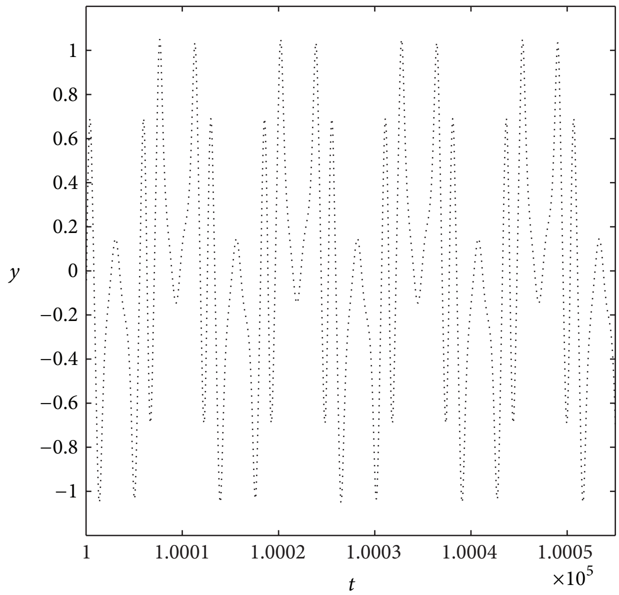

Figure 9: Time history curve (t, y).

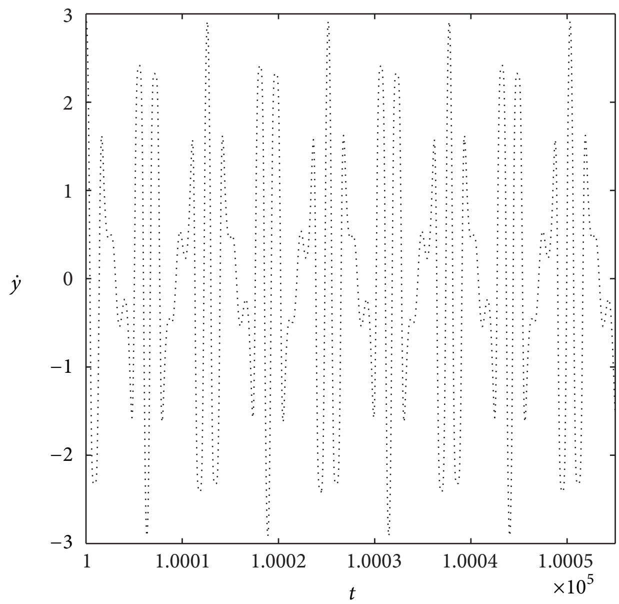

Figure 10: Time history curve $(t, \dot{y})$.

solutions at least for system (39) with $T < 8.3039e-003$, $2.1393e-004$, and $3.4035e-004$ under the three sets of different parameter values, respectively.

(iii) It is significant that the existent range of periodic solutions for system (39) increases along with the increase of the ratio of $|\beta_*|$ and $|\alpha_*|_{\max}$ through simple calculation.

(iv) In addition, the parametrical excitation term need not be periodic in accordance with the proof of Theorem 2, though it finds expression in periodic form for the above illustrated model.

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