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Research Article

On Generalized Bazilevic Functions Related with Conic Regions

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We define and study some generalized classes of Bazilevic functions associated with convex domains. These convex domains are formed by conic regions which are included in the right half plane. Such results as inclusion relationships and integral-preserving properties are proved. Some interesting special cases of the main results are also pointed out.

1. Introduction

Let A denote the class of analytic functions $f(z)$ defined in the unit disc $E = \{z : |z| < 1\}$ and satisfying the conditions $f(0) = 0$, $f'(0) = 1$. Let S denote the subclass of A consisting of univalent functions in E, and let S^* and C be the subclasses of S which contains, respectively, star-like and convex in Bazilevič [1] introduced the class $B(\alpha, \beta, h, g)$ as follows.

Let $f \in A$. Then, $f \in B(\alpha, \beta, h, g)$, α, β real and $\alpha > 0$ if

$$f(z) = \left[(\alpha + i\beta) \int_0^z h(z) g^{\alpha}(t) t^{i\beta - 1} dt\right]^{1/(\alpha + i\beta)}, \tag{1.1}$$

for some $g \in S^*$ and $\operatorname{Re} h(z) > 0, z \in E$.

The powers appearing in (1.1) are meant as principle values. The functions f in the class $B(\alpha, \beta, h, g)$ are shown to be analytic and univalent, see [1]. $B(\alpha, \beta, h, g)$ is the largest known subclass of univalent functions defined by an explicit formula and contains many of the heavily researched subclasses of S. We note the following:

(i) $B(1, 0, 1, g) = C$,

(ii) $B(1, 0, zg'/g, g) = S^*$,

(iii) $B(1, 0, h, g) = K$, where K is the class of close-to-convex functions introduced by Kaplan [2],

(iv) $B(\cos\gamma, \sin\gamma, \cos(zg'/(g+i\sin\gamma)), g)$ is the class of γ-spiral like functions which are univalent for $|\gamma| < \pi/2$.

For analytic functions $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=0}^{\infty} b_n z^n$, by $f * g$ we denote the Hadamard product (convolution) of f and g, defined by

$$(f * g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n. \tag{1.2}$$

For $k \in [0, \infty)$, the conic domain Ω_k is defined in [3] as follows:

$$\Omega_k = \left\{ u + iv : u > k\sqrt{(u-1)^2 + v^2} \right\}. \tag{1.3}$$

For fixed k, Ω_k represents the conic region bounded successively by the imaginary axis ($k = 0$), the right branch of hyperbola ($0 < k < 1$), a parabola ($k = 1$) and an ellipse ($k > 1$).

The following univalent functions, defined by $p_k(z)$ with $p_k(0) = 1$ and $p_k'(0) > 0$, map the unit disc E onto Ω_k :

$$p_k(z) = \begin{cases} \dfrac{1+z}{1-z}, & (k = 0), \\ 1 + \dfrac{2}{\pi^2}\left(\log \dfrac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2, & (k = 1), \\ 1 + \dfrac{2}{1-k^2} \sin\text{h}^2\left[A(k) \text{arc tanh} \sqrt{z}\right], & (0 < k < 1), \\ 1 + \dfrac{2}{k^2-1} \sin^2\left(\dfrac{\pi}{2K(t)} F\left(\sqrt{\dfrac{z}{t}}, t\right)\right), & (k > 1), \end{cases} \tag{1.4}$$

where $A(k) = (2/\pi)\text{arc } \cos k$, $F(w, t)$ is the Jacobi elliptic integral of the first kind:

$$F(w, t) = \int_0^w \frac{dx}{\sqrt{1-x^2}\sqrt{1-t^2x^2}}, \tag{1.5}$$

and $t \in (0, 1)$ is chosen such that $k = \cosh(\pi K'(t)/2K(t))$, where $K(t)$ is the complete elliptic integral of the first kind, $K(t) = F(1, t), K'(t) = K(\sqrt{1-t^2})$.

It is known that $p_k(z)$ are continuous as regards to k and have real coefficients for $k \in [0, \infty)$.

Let $P(p_k)$ be the subclass of the class P of Caratheodory functions $p(z)$, analytic in E with $p(0) = 1$ and such that $p(z)$ is subordinate to $p_k(z)$, written as $p(z) \prec p_k(z)$ in E.

We define the following.

Definition 1.1. Let $h(z)$ be analytic in E with $h(0) = 1$. Then, $h \in P_m(p_k)$ if and only if, for $m \geq 2, k \in [0, \infty), h_1, h_2 \in P(p_k)$ we can write

$$h(z) = \left(\frac{m}{4} + \frac{1}{2}\right) h_1(z) - \left(\frac{m}{4} - \frac{1}{2}\right) h_2(z), \quad z \in E. \tag{1.6}$$

We note that $P_2(p_k) = P(p_k)$, and $P_m(p_0) = P_m$, see [4].

Definition 1.2. Let $f \in A$. Then, $f(z)$ is said to belong to the class $k - \cup R_m$ if and only if $zf'/f \in P_m(p_k)$ for $k \in [0, \infty)$, $m \geq 2$, and $z \in E$.

For $m = 2, k = 0$, the class $0 - \cup R_2 = R_2$ coincides with the class S^* of starlike functions, and $0 - \cup R_m = R_m$ consists of analytic functions with bounded radius rotation, see [5, 6]. Also $k - \cup R_2$ is the class $\cup ST$ studied by several authors, see [7, 8].

Definition 1.3. Let $f \in A$. Then, $f \in k - \cup B_m(\alpha, \beta, h, g)$ if and only if $f(z)$ is as given by (1.1) for some $g \in k - \cup R_2$, $h \in P_m(p_k)$ in E with $k \in [0, \infty)$, $m \geq 2$, $\alpha > 0$ and β real.

When $m = 2$ and $k = 0$, we obtain the class $B(\alpha, \beta, h, g)$ of Bazilevic functions.

We shall assume throughout, unless otherwise stated, that $k \in [0, \infty), m \geq 2, \alpha > 0, \beta$ real and $z \in E$.

2. Preliminary Results

Lemma 2.1 (see [3]). *Let $0 \leq k < \infty$, and let β_0, δ be any complex numbers with $\beta_0 \neq 0$ and $\operatorname{Re}(\beta_0 k/(k+1) + \delta) > 0$. If $h(z)$ is analytic in $E, h(0) = 1$ and satisfies*

$$\left\{ h(z) + \frac{zh'(z)}{\beta_0 h(z) + \delta} \right\} \prec p_k(z) \tag{2.1}$$

and $q_k(z)$ is analytic solution of

$$\left\{ q_k(z) + \frac{zq_k'(z)}{\beta_0 q_k(z) + \delta} \right\} = p_k(z), \tag{2.2}$$

then $q_k(z)$ is univalent,

$$h(z) \prec q_k(z) \prec p_k(z), \tag{2.3}$$

and $q_k(z)$ is the best dominant of (2.1).

Lemma 2.2 (see [9]). *Let $q(z)$ be convex in E and $j : E \to \mathbb{C}$ with* $\operatorname{Re} j(z) > 0$, $z \in E$. *If $p(z)$ is analytic in E with $p(0) = 1$ and satisfies $\{p(z) + j(z) \cdot zp'(z)\} \prec q(z)$, then $p(z) \prec q(z)$.*

Lemma 2.3 (see [9]). *Let $u = u_1 + i\, u_2, v = v_1 + iv_2$, and let $\psi(u, v)$ be a complex-valued function satisfying the conditions*

(i) *$\psi(u, v)$ is continuous in a domain $D \subset \mathbb{C}^2$,*

(ii) $(0,1) \in D$ *and* $\operatorname{Re}\psi(1,0) > 0$,

(iii) $\operatorname{Re}\psi(i\,u_2, v_1) \leq 0$, *whenever* $(i\,u_2, v_1) \in D$ *and* $v_1 \leq -(1/2)(1+u_2^2)$.

If $h(z) = 1 + c_1 z + c_2 z^2 + \cdots$ *is a function analytic in* E *such that* $(h(z), zh'(z)) \in D$ *and* $\operatorname{Re}\psi\{h(z), zh'(z)\} > 0$ *for* $z \in E$*, then* $\operatorname{Re} h(z) > 0$ *in* E.

3. Main Results

Theorem 3.1. *Let* $(1/(1-\gamma))\{zf'(z)/f(z) - \gamma\} \in P_m(p_k)$*, for* $z \in E$ *and* $\gamma \in [0,1]$*. Define*

$$g(z) = \left[(c+1)z^{-c}\int_0^z t^{c-1} f^{\alpha}(t)dt\right]^{1/\alpha}, \quad \alpha > 0,\ c \in \mathbb{C}, \operatorname{Re} c \geq 0. \tag{3.1}$$

Then, $(1/(1-\gamma))\{zg'(z)/g(z) - \gamma\} \in P_m(p_k)$ *in* E*. In particular* $g \in k - \cup R_m$ *in* E.

Proof. Let

$$\frac{zg'(z)}{g(z)} = (1-\gamma)p(z) + \gamma, \tag{3.2}$$

where $p(z)$ is analytic in E with $p(0) = 1$, and let

$$p(z) = \left(\frac{m}{4} + \frac{1}{2}\right)p_1(z) - \left(\frac{m}{4} - \frac{1}{2}\right)p_2(z). \tag{3.3}$$

From (3.1) and (3.2), we have

$$g^{\alpha}(z) = [\alpha(1-\gamma)p(z) + c + \alpha\gamma] = f^{\alpha}(z). \tag{3.4}$$

Logarithmic differentiation of (3.4) and some computation yield

$$p(z) + \frac{zp'(z)}{\alpha(1-\gamma)p(z) + (c+\alpha\gamma)} = \frac{1}{1-\gamma}\left\{\frac{zf'(z)}{f(z)} - \gamma\right\}. \tag{3.5}$$

That is

$$p(z) + \frac{zp'(z)}{\alpha(1-\gamma)p(z) + (c+\alpha\gamma)} \in P_m(p_k) \quad \text{in } E. \tag{3.6}$$

Let $\phi_{a,b}(z) = z + \sum_{n=2}^{\infty} z^n/((n-1)a+b)$. Then,

$$\left(p(z) * \frac{\phi_{a,b}(z)}{z}\right) = p(z) + \frac{a(zp'(z))}{p(z)+b}. \tag{3.7}$$

Using convolution technique (3.7) with $a = 1/\alpha(1-\gamma), b = (c+\alpha\gamma)/\alpha(1-\gamma)$, we obtain, from (3.3) and (3.6),

$$\left\{p_i(z) + \frac{zp_i'(z)}{\alpha(1-\gamma)p_i(z) + (c+\alpha\gamma)}\right\} \prec p_k(z) \quad \text{in } E, \ i = 1,2. \tag{3.8}$$

Since $\operatorname{Re}\{(\alpha(1-\gamma)k/(k+1)) + c + \alpha\gamma\} \geq 0$, we apply Lemma 2.1 with $\beta_0 = \alpha(1-\gamma), \delta = c + \alpha\gamma$ to obtain $p_i(z) \prec q_k(z) \prec p_k(z)$, where $q_k(z)$ is the best dominant and is given as

$$q_k(z) = \left[\beta_0 \int_0^1 \left(t^{\beta_0+\delta-1} \exp \int_z^{tz} \frac{p_k(u)-1}{u} du\right)^{\beta_0} dt\right]^{-1} - \frac{\delta}{\beta_0}. \tag{3.9}$$

Consequently, $p \in P_m(p_k)$ in E, and this completes the result.

As a special case, we prove the following. □

Corollary 3.2. *Let $k = 0$ and let $(1/(1-\gamma_1))\{zf'(z)/f(z) - \gamma_1\} \in P_m$ in E. Then, for g defined by (3.1), $1/(1-\gamma)\{zg'(z)/g(z) - \gamma\} \in P_m$ in E where*

$$\gamma = \frac{2}{\left\{(2c - 2\alpha\gamma_1 + 1) + \sqrt{(2c - 2\alpha\gamma_1 + 1)^2 + 8\alpha}\right\}}. \tag{3.10}$$

Proof. We can write

$$\frac{zf'(z)}{f(z)} = (1-\gamma_1)h(z) + \gamma_1, \tag{3.11}$$

where $h \in P_m$ in E.

Now proceeding as before, we have, with

$$\frac{zg'(z)}{g(z)} = (1-\gamma)p(z) + \gamma = \left(\frac{m}{4} + \frac{1}{2}\right)\{(1-\gamma)p_1(z) + \gamma\} - \left(\frac{m}{4} - \frac{1}{2}\right)\{(1-\gamma)p_2(z) + \gamma\} \tag{3.12}$$

$$(1-\gamma)p(z) + \gamma + \frac{(1-\gamma)z\, p'(z)}{\alpha(1-\gamma)p(z) + (c+\alpha\gamma)} = \frac{zf'(z)}{f(z)}. \tag{3.13}$$

Using convolution technique together with (3.11), we obtain

$$\operatorname{Re}\left\{(1-\gamma)p_i(z) + (\gamma - \gamma_1) + \frac{(1-\gamma)z\, p_i'(z)}{\alpha(1-\gamma)p_i(z) + (c+\alpha\gamma)}\right\} > 0, \tag{3.14}$$

for $i = 1,2$.

We construct the functional $\psi(u,v)$ by taking $u = p_i(z), v = zp_i'(z)$ as

$$\psi(u,v) = (1-\gamma)u + (\gamma-\gamma_1) + \frac{(1-\gamma)v}{\alpha(1-\gamma)u + (c+\alpha\gamma)}. \tag{3.15}$$

The first two conditions of Lemma 2.3 are clearly satisfied. We verify condition (iii) as follows.

$$\begin{aligned} \operatorname{Re}\psi(i\ u_2, v_1) &= (\gamma-\gamma_1) + \operatorname{Re}\left\{\frac{(1-\gamma)v_1}{i\alpha(1-\gamma)u_2 + (c+\alpha\gamma)}\right\}, \\ &= (\gamma-\gamma_1) + \frac{(c+\alpha\gamma)(1-\gamma)v_1}{(c+\alpha\gamma)^2 + \alpha^2(1-\gamma)^2u_2^2}, \\ &\le (\gamma-\gamma_1) + \frac{(c+\alpha\gamma)(1-\gamma)(1+u_2^2)}{2\left[(c+\alpha\gamma)^2 + \alpha^2(1-\gamma)^2u_2^2\right]}, \quad \left(v_1 \le -\frac{1+u_2^2}{2}\right), \\ &= \frac{A+Bu_2^2}{2C}, \end{aligned} \tag{3.16}$$

where

$A = 2(\gamma-\gamma_1)(c+\alpha\gamma)^2 - (1-\gamma)(c+\alpha\gamma), B = 2\alpha^2(\gamma-\gamma_1)(1-\gamma)^2 - (1-\gamma)(c+\alpha\gamma), C = (c+\alpha\gamma)^2 + \alpha^2(1-\gamma)^2u_2^2 > 0$.

$\operatorname{Re}\psi(iu_2, v_1) \le 0$ if and only if $A \le 0, B \le 0$. From $A \le 0$, we obtain γ as given by (3.10) and $B \le 0$ ensures that $\gamma \in [0,1)$.

Now proceeding as before, it follows from (3.12) that $p \in P_m$, and this proves our result. □

By assigning certain permissible values to different parameters, we obtain several new and some known result.

Corollary 3.3. *Let $f \in k-\cup R_2 = k-\cup ST$. Then, it is known that $f \in S^*(\gamma_1), \gamma_1 = k/(k+1)$ and, form Corollary 3.2, it follows that $g \in S^*(\gamma)$ where γ is given by* (3.10). *Also a starlike function is k-uniformly convex for $|z| < r_k$,*

$$r_k = \frac{1}{2(k+1) + \sqrt{4k^2+6k+3}}, \textit{see } [8]. \tag{3.17}$$

Therefore, for $f \in k-\cup R_2$, it follows that $(1/(1-\gamma))\{(zg'(z))'/g'(z) - \gamma\} \prec p_k$ for $|z| < r_k$, where γ is given by (3.10).

As special cases we note the following.

(i) *For $k = 0$, we have $r_0 = 1/(2+\sqrt{3})$ and $f \in S^*(0)$ implies that $g \in C(\gamma_*)$, with*

$$\gamma_* = \frac{2}{\left\{2(c+1) + \sqrt{2(c+1)^2 + 8\alpha}\right\}}. \tag{3.18}$$

(ii) *When* $k = 1$, *we have* $\gamma_1 = 1/2, \gamma = 2/((2c-\alpha+1)+\sqrt{(2c-\alpha+1)^2+8\alpha})$ *and* $r_1 = 1/(4+\sqrt{13})$.

Theorem 3.4. *Let* $F \in k-\cup B_m(\alpha,\beta,p,f)$, $f \in k-\cup R_2$, $p \in P_m(p_k)$. *Define, for* $\mathrm{Re}[\alpha k/(k+1)+(c+i\beta)] > 0$,

$$G(z) = \left[(c+1)z^{-c}\int_0^z t^{c-1}F^{\alpha+i\beta}(t)dt\right]^{1/(\alpha+i\beta)}. \tag{3.19}$$

Then, $G \in k-\cup B_m(\alpha,\beta,h,g)$ *in* E, *where* $g(z)$ *is given by* (3.1), *and* $h(z)$ *is analytic in* E *with* $h(0)=1$.

Proof. Set

$$\frac{zG'(z)G^{\alpha+i\beta-1}(z)}{z^{i\beta}g^{\alpha}(z)} = h(z) = \left(\frac{m}{4}+\frac{1}{2}\right)h_1(z) - \left(\frac{m}{4}-\frac{1}{2}\right)h_2(z). \tag{3.20}$$

We note that $h(z)$ is analytic in E with $h(0)=1$. From (3.20), we have

$$z^{i\beta}g^{\alpha}(z)\left\{zh'(z)+h(z)\left[\alpha\frac{zg'(z)}{g(z)}+c+i\beta\right]\right\} = zF'(z)F^{\alpha+i\beta-1}(z), \tag{3.21}$$

using (3.1), we note that

$$\left(\frac{f(z)}{g(z)}\right)^{\alpha} = \alpha\frac{zg'(z)}{g(z)}+c+i\beta. \tag{3.22}$$

From (3.21) and (3.22), it follows that

$$\left\{h(z)+\frac{zh'(z)}{\alpha h_0(z)+(c+i\beta)}\right\} \in P_m(p_k), \tag{3.23}$$

where $h_0(z) = zg'(z)/g(z) \in P(p_k)$ since $g \in k-\cup R_2$ by Theorem 3.1.

It can easily be seen that $g \in S^*(k/(k+1))$ and $\mathrm{Re}\{\alpha zg'(z)/g(z)+c+i\beta\} > 0$.

Now, using (3.8), we can easily derive

$$\left\{h_i(z)+j(z)\left(zh_i'(z)\right)\right\} \prec p_k(z) \quad \text{in } E, \quad i=1,2, \tag{3.24}$$

where $1/j(z) = \{\alpha zg'(z)/g(z)+c+i\beta\}$ and $\mathrm{Re}\, j(z) > 0$.

Applying Lemma 2.2, it follows from (3.24) $h_i(z) \prec p_k(z)$ in E and therefore $h \in P_m(p_k)$ in E. This completes the proof. □

Theorem 3.5. *Let $f(z)$ be given by* (1.1) *with* $h(z) = 1, \{(\alpha^2+\beta^2)^{1/2} e^{i\gamma}(zg'/g)\} \in P_m(p_k)$ $(\alpha^2+\beta^2)^{1/2} e^{i\gamma} = \alpha + i\beta, |\gamma| < \pi/2$. *Then, for* $z \in E$

(i) $e^{i\gamma}(zf'(z)/f(z)) = \cos\gamma(p(z)) + i\sin\gamma, \quad p \in P_m(p_k)$,

(ii) *For* $\alpha' + i\beta' = t(\alpha + i\beta),\ t \geq 1$,

$$k - \cup B_m(\alpha, \beta, 1, g) \subset kt - \cup B_m(\alpha', \beta', 1, g). \tag{3.25}$$

Proof. (i) From (1.1), we have

$$1 + \frac{zf''(z)}{f'(z)} + (\alpha - 1 + i\beta)\frac{zf'(z)}{f(z)} = \left(\alpha\frac{zg'(z)}{g(z)} + i\beta\right) = H_2(z), \quad H_2 \in P_m(p_k) \text{ in } E. \tag{3.26}$$

Define a function $p(z)$ analytic in E by

$$e^{i\gamma}\frac{zf'(z)}{f(z)} = \cos\gamma(p(z)) + i\sin\gamma, \qquad \gamma = \tan^{-1}\frac{\beta}{\alpha}. \tag{3.27}$$

We can easily check that $p(0) = 1$.

Now, from (3.26) and (3.27), we have

$$\left[\frac{zp'(z)}{p(z) + i\tan\gamma} + \alpha p(z) + i\beta\right] \in P_m(p_k) \quad \text{in } E. \tag{3.28}$$

That is

$$\left[\frac{\alpha zp'(z)}{\alpha p(z) + i\beta} + \alpha p(z) + i\beta\right] \in P_m(p_k), \tag{3.29}$$

and, with $h(z) = \alpha p(z) + i\beta = (m/4 + 1/2)h_1(z) - (m/4 - 1/2)h_2(z)$, we apply convolution technique used before to have

$$\left\{h_i(z) + \frac{zh_i'(z)}{h_i(z)}\right\} \prec p_k(z) \quad \text{in } E. \tag{3.30}$$

Applying Lemma, it follows that

$$h_i(z) \prec q_k(z) \prec p_k(z), \quad z \in E, \tag{3.31}$$

where $q_k(z)$ is the best dominant and is given by

$$q_k(z) = \left[\int_0^1 \exp\left(\int_0^{tz} \frac{p_k(u) - 1}{u} du\right)\right]^{-1}. \tag{3.32}$$

From (3.31), we have $h(z) = (\alpha p(z) + i\beta) \in P_m(p_k)$ in E, and this proves part (i).

(ii) From part (i), we have

$$\left(\alpha^2+\beta^2\right)^{1/2} e^{i\gamma}\frac{zf'(z)}{f(z)} = H_1(z), \quad H_1 \in P_m(p_k) \quad \text{in } E. \tag{3.33}$$

Now,

$$\begin{aligned}
&1+\frac{zf''(z)}{f'(z)}+\left(\alpha'-1+i\beta'\right)\frac{zf'(z)}{f(z)}\\
&\quad=\left\{1+\frac{zf''(z)}{f'(z)}+\left(\alpha-1+i\beta\right)\frac{zf'(z)}{f(z)}\right\}\\
&\qquad+(t-1)\left(\alpha^2+\beta^2\right)^{1/2}e^{i\gamma}\frac{zf'(z)}{f(z)},\\
&\quad=H_2(z)+(t-1)H_1(z), \quad H_i\in P_m(p_k), \quad i=1,2,\\
&\quad=t\left[\left(1-\frac{1}{t}\right)H_1(z)+\frac{1}{t}H_2(z)\right],\\
&\quad=tH, \quad t\geq 1,
\end{aligned} \tag{3.34}$$

$H \in P_m(p_k)$, since $P_m(p_k)$ is convex set, see [8].

Therefore, $f \in kt-\cup B_m(\alpha',\beta',1,g)$ for $z\in E$. This completes the proof. □

As a special case, with $m=2, k=0$, we obtain a result proved in [10].

By assigning certain permissible values to the parameters α, β and m, we have several other new results.

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