Research Article

Cluster Synchronization of Time-Varying Delays Coupled Complex Networks with Nonidentical Dynamical Nodes

Shuguo Wang,[1,2] Hongxing Yao,[2] and Mingping Sun[2]

[1] *Department of Mathematics and Physics, Changzhou Campus, Hohai University, Jiangsu, Changzhou 213022, China*
[2] *Faculty of science, Jiangsu University, Zhenjiang, Jiangsu 212013, China*

Correspondence should be addressed to Shuguo Wang, wsg97@163.com

This paper investigates a new cluster synchronization scheme in the nonlinear coupled complex dynamical networks with nonidentical nodes. The controllers are designed based on the community structure of the networks; some sufficient criteria are derived to ensure cluster synchronization of the network model. Particularly, the weight configuration matrix is not assumed to be symmetric, irreducible. The numerical simulations are performed to verify the effectiveness of the theoretical results.

1. Introduction

Complex networks model is used to describe various interconnected systems of real world, which have become a focal research topic and have drawn much attention from researchers working in different fields; one of the most important reasons is that most practical systems can be modeled by complex dynamical networks. Recently, the research on synchronization and dynamical behavior analysis of complex network systems has become a new and important direction in this field [1–13]; many control approaches have been developed to synchronize complex networks such as feedback control, adaptive control, pinning control, impulsive control, and intermittent control [14–21].

Cluster synchronization means that nodes in the same group synchronize with each other, but there is no synchronization between nodes in different groups [22–25]; Belykh et al. [26] investigated systems of diffusively coupled identical chaotic oscillators; an effective method to determine the possible states of cluster synchronization and ensure their stability

is presented. The method, which may find applications in communication engineering and other fields of science and technology, is illustrated through concrete examples of coupled biological cell models. Wu and Lu [27] investigated cluster synchronization in the adaptive complex dynamical networks with nonidentical nodes by a local control method and a novel adaptive strategy for the coupling strengths of the networks. Ma et al. [28] proposed cluster synchronization scheme via dominant intracouplings and common intercluster couplings. Sorrentino and Ott [29] studied local cluster synchronization for bipartite systems, where no intracluster couplings (driving scheme) exist. Chen and Lu [30] investigated global cluster synchronization in networks of two clusters with inter- and intracluster couplings. Belykh et al. [31, 26] studied this problem in 1D and 2D lattices of coupled identical dynamical systems. Lu et al. [32] studied the cluster synchronization of general networks with nonidentical clusters and derived sufficient conditions for achieving local cluster synchronization of networks. Recently, Wang et al. [33] considered the cluster synchronization of dynamical networks with community structure and nonidentical nodes and with identical local dynamics for all individual nodes in each community by using pinning control schemes. However, there is few theoretical result on the cluster synchronization of nonlinear coupled complex networks with time-varying delays coupling and time-varying delays in nonidentical dynamical nodes.

Motivated by the above discussions, this paper investigates cluster synchronization in the nonlinear coupled complex dynamical networks with nonidentical nodes. The controllers are designed based on the community structure of the networks; some sufficient criteria are derived to ensure cluster synchronization in nonlinear coupled complex dynamical networks with time-varying delays coupling and time-varying delays in dynamic nodes. Particularly the weight configuration matrix is not assumed to be symmetric, irreducible.

The paper is organized as follows: the network model is introduced followed by some definitions, lemmas, and hypotheses in Section 2. The cluster synchronization of the complex coupled networks is discussed in Section 3. Simulations are obtained in Section 4. Finally, in Section 5 the various conclusions are discussed.

2. Model and Preliminaries

The network with nondelayed and time-varying delays coupling and adaptive coupling strengths can be described by

$$
\begin{aligned}
\dot{x}_i(t) = {} & f_{\phi_i}\big(t, x_i(t), x_i\big(t - \tau_{\phi_i}(t)\big)\big) + c\sum_{j=1}^{N} a_{ij} H_1\big(x_j(t)\big) \\
& + c\sum_{j=1}^{N} b_{ij} H_2\big(x_j\big(t - \eta_{\phi_i}(t)\big)\big), \quad i = 1, 2, \ldots, N,
\end{aligned}
\tag{2.1}
$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ is the state vector of node i; $f_{\phi_i} : R^n \rightarrow R^n$ describes the local dynamics of nodes in the ϕ_ith community. For any pair of nodes i and j, if $\phi_i \neq \phi_j$, that is, nodes i and j belong to different communities, then $f_{\phi_i} \neq f_{\phi_j} \cdot \eta_{\phi_i}(t)$, $\tau_{\phi_i}(t)$, is a time-varying delay. $H_1(\cdot)$ and $H_2(\cdot)$ are nonlinear functions. c is coupling strength. $A = (a_{ij})_{N \times N}$, $B = (b_{ij})_{N \times N}$ are the weight configuration matrices. If there is a connection from

node i to node j $(j \neq i)$, then the $a_{ij} > 0$, $b_{ij} > 0$ otherwise, $a_{ij} = a_{ji} = 0$, $b_{ij} = b_{ji} = 0$, and the diagonal elements of matrix A, B are defined as

$$a_{ii} = -\sum_{j=1,\, j \neq i}^{N} a_{ji}, \qquad b_{ii} = -\sum_{j=1,\, j \neq i}^{N} b_{ji}, \quad i = 1, 2, \ldots, N. \tag{2.2}$$

Particularly, the weight configuration matrix is not assumed to be symmetric, irreducible.

When the control inputs $u_i(t) \in R^n$ and $v_i(t) \in R^n$ $(i = 1, 2, \ldots, N)$ are introduced, the controlled dynamical network with respect to network (2.1) can be written as

$$\dot{x}_i(t) = f_{\phi_i}(t, x_i(t), x_i(t - \tau_{\phi_i}(t))) + c\sum_{j=1}^{N} a_{ij} H_1(x_j(t))$$

$$+ c\sum_{j=1}^{N} b_{ij} H_2(x_j(t - \eta_{\phi_i}(t))) + u_i(t), \quad \phi_i(t) \in \overline{J}_{\phi_i},$$

$$\dot{x}_i(t) = f_{\phi_i}(t, x_i(t), x_i(t - \tau_{\phi_i}(t))) + c\sum_{j=1}^{N} a_{ij} H_1(x_j(t)) \tag{2.3}$$

$$+ c\sum_{j=1}^{N} b_{ij} H_2(x_j(t - \eta_{\phi_i}(t))) - v_i(t), \quad \phi_i(t) \in J_{\phi_i} - \overline{J}_{\phi_i},$$

where J_{ϕ_i} denotes all the nodes in the ϕ_ith community and $\overline{J}_{\phi_i}$ represents the nodes in the ϕ_ith community which have direct links with the nodes in other communities.

The study presents the mathematical definition of the cluster synchronization.

Let $\{C_1, C_2, \ldots, C_m\}$ denote m $(2 \leq m \leq N)$ communities of the networks and $\bigcup_{i=1}^{m} C_i = \{1, 2, \ldots N\}$. If node i belongs to the jth community, then we denote $\phi_i = j$. We employ $f_i(\cdot)$ to represent the local dynamics of all nodes in the ith community. Let $s_i(t)$ be the solution of the system $\dot{s}_i(t) = f_{\phi_i}(t, s_i(t), s_i(t - \tau_{\phi_i}(t)))$, $(i = 1, 2, \ldots, m)$ where $\lim_{t \to \infty} \|s_i(t) - s_j(t)\| \neq 0$ $(i \neq j)$; the set $S = \{s_1(t), s_2(t), \ldots, s_m(t)\}$ is used as the cluster synchronization manifold for network (2.3). Cluster synchronization can be realized if and only if the manifold S is stable.

Definition 2.1 (see [19]). The error variables as $e_i(t) = x_i(t) - s_{\phi_i}(t)$ for $i = 1, 2, \ldots, N$, where $s_{\phi_i}(t)$ satisfies $\dot{s}_{\phi_i}(t) = f_{\phi_i}(t, s_{\phi_i}(t), s_{\phi_i}(t - \tau_{\phi_i}(t)))$.

Definition 2.2 (see [19]). Let $\{1, 2, \ldots, N\}$ be the N nodes of the network and $\{C_1, C_2, \ldots, C_m\}$ be the m communities, respectively. A network with m communities is said to realize cluster synchronization if $\lim_{t \to \infty} e_i(t) = 0$ and $\lim_{t \to \infty} \|x_i(t) - x_j(t)\| \neq 0$ for $\phi_i \neq \phi_j$.

Lemma 2.3. *For any two vectors x and y, a matrix $Q > 0$ with compatible dimensions, one has $2x^T y \leq x^T Q x + y^T Q^{-1} y$.*

Assumption 2.4. For the vector valued function $f_{\phi_i}(t, x_i(t), x_i(t-\tau_{\phi_i}))$, assuming that there exist positive constants $\alpha_{\phi_i} > 0$, $\gamma_{\phi_i} > 0$ such that f satisfies the semi-Lipschitz condition

$$
\begin{aligned}
(x_i(t) - y_i(t))^T &(f_{\phi_i}(t, x_i(t), x_i(t - \tau_{\phi_i})) - f_{\phi_i}(t, y_i(t), y_i(t - \tau_{\phi_i}))) \\
&\leq \alpha_{\phi_i} (x_i(t) - y_i(t))^T (x_i(t) - y_i(t)) \\
&\quad + \gamma_{\phi_i} (x_i(t - \tau_{\phi_i}) - y_i(t - \tau_{\phi_i}))^T (x_i(t - \tau_{\phi_i}) - y_i(t - \tau_{\phi_i})),
\end{aligned}
\tag{2.4}
$$

for all $x, y \in R^n$ and $\tau_{\phi_i}(t) \geq 0$. $i = 1, 2, \ldots, N$.

Assumption 2.5. $\eta_{\phi_i}(t)$ and $\tau_{\phi_i}(t)$ is a differential function with $0 \leq \dot{\eta}_{\phi_i}(t) \leq \varepsilon \leq 1$ and $0 \leq \dot{\tau}_{\phi_i}(t) \leq \varepsilon \leq 1$. Clearly, this assumption is certainly ensured if the delay $\eta_{\phi_i}(t)$ and $\tau_{\phi_i}(t)$ is constant.

Assumption 2.6. [34] (Global Lipschitz Condition)]
      Suppose that there exist nonnegative constants ϑ, β, for all $t \in R_+$, such that for any time-varying vectors $x(t), y(t) \in R^n$

$$
\|H_1(x) - H_1(y)\| \leq \vartheta \|x - y\|, \qquad \|H_2(x) - H_2(y)\| \leq \beta \|x - y\|, \tag{2.5}
$$

where $\| \, \|$ denotes the 2-norm throughout the paper.

3. Main Results

In this section, a control scheme is developed to synchronize a delayed complex network with nonidentical nodes to any smooth dynamics $s_{\phi_i}(t)$. Let synchronization errors $e_i(t) = x_i(t) - s_{\phi_i}(t)$ for $i = 1, 2, \ldots, N$, according to system (2.1), the error dynamical system can be derived as

$$
\begin{aligned}
\dot{e}_i(t) &= \widetilde{f}_{\phi_i}(t, x_i(t), x_i(t - \tau_{\phi_i}(t))) + c \sum_{j=1}^{N} a_{ij}[H_1(x_j(t)) - H_1(s_{\phi_i}(t))] \\
&\quad + c \sum_{j=1}^{N} b_{ij}[H_2(x_j(t - \eta_{\phi_i}(t))) - H_2(s_{\phi_i}(t - \eta_{\phi_i}(t)))] \ + \sum_{i=1}^{N} a_{ij} H_1(s_{\phi_i}(t)) \\
&\quad + \sum_{i=1}^{N} b_{ij} H_2(s_{\phi_i}(t - \eta_{\phi_i}(t))) + u_i(t), \quad \phi_i(t) \in \overline{J}_{\phi_i}, \\
\dot{e}_i(t) &= \widetilde{f}_{\phi_i}(t, x_i(t), x_i(t - \tau_{\phi_i}(t))) + c \sum_{j=1}^{N} a_{ij}[H_1(x_j(t)) - H_1(s_{\phi_i}(t))] \\
&\quad + c \sum_{j=1}^{N} b_{ij}[H_2(x_j(t - \eta_{\phi_i}(t))) - H_2(s_{\phi_i}(t - \eta_{\phi_i}(t)))] \\
&\quad - v_i(t), \quad \phi_i(t) \in J_{\phi_i} - \overline{J}_{\phi_i},
\end{aligned}
\tag{3.1}
$$

where $\widetilde{f}_{\phi_i}(t, x_i(t), x_i(t - \tau_{\phi_i}(t))) = f_{\phi_i}(t, x_i(t), x_i(t - \tau_{\phi_i}(t))) - f_{\phi_i}(t, s_{\phi_i}(t), s_{\phi_i}(t - \tau_{\phi_i}(t)))$ for $i = 1, 2, \ldots, N$.

According to the diffusive coupling condition (2.2) of the matrix A, B we have

$$c\sum_{i=1}^{N}a_{ij}H_1\big(s_{\phi_i}(t)\big) + c\sum_{i=1}^{N}b_{ij}H_2\big(s_{\phi_i}(t-\eta_{\phi_i}(t))\big) = 0, \quad i \in J_{\phi_i} - \overline{J}_{\phi_i}. \tag{3.2}$$

On the basis of this property, for achieving cluster synchronization, we design controllers as follows:

$$u_i(t) = \begin{cases} -c\sum_{i=1}^{N}a_{ij}H_1\big(s_{\phi_i}(t)\big) - c\sum_{i=1}^{N}b_{ij}H_2\big(s_{\phi_i}(t-\eta_{\phi_i}(t))\big) - d_ie_i(t), & i \in \overline{J}_{\phi_i}, \\ v_i(t) = d_ie_i(t), & i \in J_{\phi_i} - \overline{J}_{\phi_i}, \end{cases} \tag{3.3}$$

where $\dot{d}_i = k_ie_i^T(t)e_i(t)$.

Theorem 3.1. *Suppose assumptions 2.4–2.5 hold. Consider the network* (2.1) *via control law* (3.3). *If the following conditions hold:*

$$\alpha + \vartheta c\lambda_{\max}(Q) + \frac{1}{2}\beta^2c^2\lambda_{\max}\big(PP^T\big) + \frac{1}{1-\varepsilon}\left(\gamma + \frac{1}{2}\right) < d, \tag{3.4}$$

where $\alpha = \max(\alpha_{\phi_1}, \alpha_{\phi_2}, \ldots, \alpha_{\phi_m})$, $\gamma = \max(\gamma_{\phi_1}, \gamma_{\phi_2}, \ldots, \gamma_{\phi_m})$. *Then, the systems* (2.3) *is cluster synchronization.*

Proof. Construct the following Lyapunov functional:

$$\begin{aligned} V(t) = &\frac{1}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t) + \frac{\gamma}{1-\varepsilon}\int_{t-\tau_{\phi_i}(t)}^{t}\sum_{i=1}^{N}e_i^T(\theta)e_i(\theta)d\theta \\ &+ \frac{1}{2(1-\varepsilon)}\int_{t-\eta_{\phi_i}(t)}^{t}\sum_{i=1}^{N}e_i^T(\theta)e_i(\theta)d\theta + \frac{1}{2}\sum_{i=1}^{N}\frac{(d_i-d)^2}{k_i}. \end{aligned} \tag{3.5}$$

Calculating the derivative of $V(t)$, we have

$$\begin{aligned} \dot{V}(t) = &\sum_{i=1}^{N}e_i^T(t)\dot{e}_i(t) + \frac{1}{1-\varepsilon}\left(\gamma + \frac{1}{2}\right)\sum_{i=1}^{N}e_i^T(t)e_i(t) - \frac{\gamma(1-\dot{\tau}_{\phi_i}(t))}{1-\varepsilon}\sum_{i=1}^{N}e_i^T(t-\tau_{\phi_i}(t))e_i(t-\tau_{\phi_i}(t)) \\ &- \frac{1-\dot{\eta}_{\phi_i}(t)}{2(1-\varepsilon)}\sum_{i=1}^{N}e_i^T(t-\eta_{\phi_i}(t))e_i(t-\eta_{\phi_i}(t)) + \sum_{i=1}^{N}(d_i-d)e_i^T(t)e_i(t) \end{aligned}$$

$$= \sum_{i=1}^{N} e_i^T(t) \left\{ \tilde{f}_{\phi_i}(t, x_i(t), x_i(t - \tau_{\phi_i}(t))) + c \sum_{j=1}^{N} a_{ij} [H_1(x_j(t)) - H_1(s_{\phi_i}(t))] \right.$$

$$\left. + c \sum_{j=1}^{N} b_{ij} [H_2(x_j(t - \eta_{\phi_i}(t))) - H_2(s_{\phi_i}(t - \eta_{\phi_i}(t)))] - d_i e_i(t) \right\}$$

$$+ \frac{1}{1-\varepsilon} \left(\gamma + \frac{1}{2} \right) \sum_{i=1}^{N} e_i^T(t) e_i(t) - \frac{\gamma(1 - \dot{\tau}_{\phi_i}(t))}{1-\varepsilon} \sum_{i=1}^{N} e_i^T(t - \tau_{\phi_i}(t)) e_i(t - \tau_{\phi_i}(t))$$

$$- \frac{1 - \dot{\eta}_{\phi_i}(t)}{2(1-\varepsilon)} \sum_{i=1}^{N} e_i^T(t - \eta_{\phi_i}(t)) e_i(t - \eta_{\phi_i}(t)) + \sum_{i=1}^{N} (d_i - d) e_i^T(t) e_i(t). \tag{3.6}$$

By assumptions 2.4–2.6, we obtain

$$\leq \alpha \sum_{i=1}^{N} e_i^T(t) e_i(t) + \gamma \sum_{i=1}^{N} e_i^T(t - \tau_{\phi_i}(t)) e_i(t - \tau_{\phi_i}(t)) + \vartheta c \sum_{i=1}^{N} e_i^T(t) \sum_{j=1}^{N} a_{ij} e_j(t)$$

$$+ \beta c \sum_{i=1}^{N} e_i^T(t) \sum_{j=1}^{N} b_{ij} e_j(t - \eta_{\phi_i}(t)) - d_i \sum_{i=1}^{N} e_i^T(t) e_i(t)$$

$$+ \frac{1}{1-\varepsilon} \left(\gamma + \frac{1}{2} \right) \sum_{i=1}^{N} e_i^T(t) e_i(t) - \gamma \sum_{i=1}^{N} e_i^T(t - \tau_{\phi_i}(t)) e_i(t - \tau_{\phi_i}(t))$$

$$- \frac{1}{2} \sum_{i=1}^{N} e_i^T(t - \eta_{\phi_i}(t)) e_i(t - \eta_{\phi_i}(t)) + \sum_{i=1}^{N} (d_i - d) e_i^T(t) e_i(t)$$

$$\leq \alpha \sum_{i=1}^{N} e_i^T(t) e_i(t) + \vartheta c e^T (A \otimes I) e + \beta c e^T (B \otimes I) e(t - \eta_{\phi_i}(t)) + \frac{1}{1-\varepsilon} \left(\gamma + \frac{1}{2} \right) \sum_{i=1}^{N} e_i^T(t) e_i(t)$$

$$- \frac{1}{2} \sum_{i=1}^{N} e_i^T(t - \eta_{\phi_i}(t)) e_i(t - \eta_{\phi_i}(t)) - d e^T(t) e(t). \tag{3.7}$$

Let $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T \in R^{nN}$, $Q = (A \otimes I), P = (B \otimes I)$, where $\otimes$ represents the Kronecker product. Then

$$\dot{V}(t) \leq \alpha e^T(t) e(t) + \vartheta c e^T(t) Q e(t) + \beta c e^T(t) P e(t - \eta_{\phi_i}(t)) + \frac{1}{1-\varepsilon} \left(\gamma + \frac{1}{2} \right) e^T(t) e(t)$$

$$- \frac{1}{2} e^T(t - \eta_{\phi_i}(t)) e(t - \eta_{\phi_i}(t)) - d e^T(t) e(t). \tag{3.8}$$

By the Lemma 2.3, we have

$$\leq \alpha e^T(t) e(t) + \vartheta c e^T(t) Q e(t) + \frac{1}{2} (\beta c)^2 e^T(t) P P^T e(t) + \frac{1}{1-\varepsilon} \left(\gamma + \frac{1}{2} \right) e^T(t) e(t) - d e^T(t) e(t)$$

$$\leq \left(\alpha + \vartheta c \lambda_{\max}(Q) + \frac{1}{2} \beta^2 c^2 \lambda_{\max}(P P^T) + \frac{1}{1-\varepsilon} \left(\gamma + \frac{1}{2} \right) - d \right) e^T(t) e(t). \tag{3.9}$$

Therefore, if we have $\alpha + \vartheta c \lambda_{\max}(Q) + (1/2)\beta^2 c^2 \lambda_{\max}(PP^T) + (1/(1-\varepsilon))(\gamma + (1/2)) < d$ then

$$\dot{V}(t) \leq 0. \tag{3.10}$$

Theorem 3.1 is proved completely.

We can conclude that, for any initial values, the solutions $x_1(t), x_2(t), \ldots, x_N(t)$ of the system (2.3) satisfy $\lim_{t \to \infty} \sum_{k=1}^{m} \sum_{i \in C_k} \|x_i(t) - s_k(t)\| = 0$, that is, we get the global stability of the cluster synchronization manifold S. Therefore, cluster synchronization in the network (2.3) is achieved under the local controllers (3.3). This completes the proof. $\square$

Corollary 3.2. *When $A = 0$, network (2.1) is translated into*

$$\dot{x}_i(t) = f_{\phi_i}\big(t, x_i(t), x_i(t - \tau_{\phi_i}(t))\big) + c \sum_{j=1}^{N} b_{ij} H_2\big(x_j(t - \eta_{\phi_i}(t))\big), \quad i = 1, 2, \ldots, N. \tag{3.11}$$

We design the controllers, as follows, then the complex networks can also achieve synchronization, where

$$u_i(t) = \begin{cases} -c \sum_{i=1}^{N} b_{ij} H_2\big(s_{\phi_i}(t - \eta_{\phi_i}(t))\big) - d_i e_i(t), & i \in \overline{J}_{\phi_i}, \\ v_i(t) = d_i e_i(t), & i \in J_{\phi_i} - \overline{J}_{\phi_i}. \end{cases} \tag{3.12}$$

Corollary 3.3. *When $B = 0$, network (2.1) is translated into*

$$\dot{x}_i(t) = f_{\phi_i}\big(t, x_i(t), x_i(t - \tau_{\phi_i}(t))\big) + c \sum_{j=1}^{N} a_{ij} H_1(x_j(t)), \quad i = 1, 2, \ldots, N. \tag{3.13}$$

We design the controllers, as follows, then the complex networks can also achieve synchronization, where

$$u_i(t) = \begin{cases} -c \sum_{i=1}^{N} a_{ij} H_1\big(s_{\phi_i}(t)\big) - d_i e_i(t), & i \in \overline{J}_{\phi_i}, \\ v_i(t) = d_i e_i(t), & i \in J_{\phi_i} - \overline{J}_{\phi_i}, \end{cases} \tag{3.14}$$
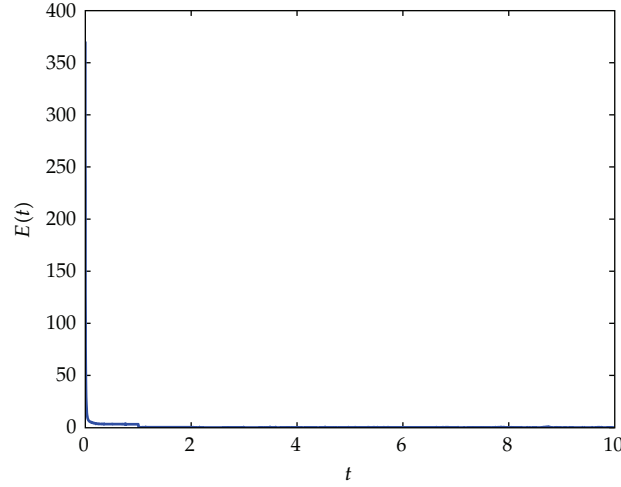
Figure 1: Time evolution of the synchronization errors $E(t)$.

4. Illustrative Examples

In this section, a numerical example will be given to demonstrate the validity of the synchronization criteria obtained in the previous sections. Considering the following network:

$$
\begin{aligned}
\dot{x}_i(t) &= f_{\phi_i}\big(t, x_i(t), x_i(t - \tau_{\phi_i}(t))\big) + c\sum_{j=1}^{N} a_{ij} H_1\big(x_j(t)\big) \\
&\quad + c\sum_{j=1}^{N} b_{ij} H_2\big(x_j(t - \eta_{\phi_i}(t))\big) + u_i(t), \quad \phi_i(t) \in \overline{J}_{\phi_i}, \\
\dot{x}_i(t) &= f_{\phi_i}\big(t, x_i(t), x_i(t - \tau_{\phi_i}(t))\big) + c\sum_{j=1}^{N} a_{ij} H_1\big(x_j(t)\big) \\
&\quad + c\sum_{j=1}^{N} b_{ij} H_2\big(x_j(t - \eta_{\phi_i}(t))\big) - v_i(t), \quad \phi_i(t) \in J_{\phi_i} - \overline{J}_{\phi_i}, \; i = 1, 2, \ldots, N,
\end{aligned}
\tag{4.1}
$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t))^T$, $f_1(t, x_i(t), x_i(t - \tau_1(t))) = D_1 x_i(t) + h_{11}(x_i(t)) + h_{12}(x_i(t - \tau_1(t)))$, $f_2(t, x_i(t), x_i(t - \tau_2(t))) = D_2 x_i(t) + h_{21}(x_i(t)) + h_{22}(x_i(t - \tau_2(t))) + V$, $f_3(t, x_i(t), x_i(t - \tau_3(t))) = D_3 x_i(t) + h_{31}(x_i(t)) + h_{32}(x_i(t - \tau_3(t)))$. $k_1 = k_2 = \cdots = k_N = 10$, $c = 1$, $H_1(x) = \sin x$, $H_2(x) = \cos x$.

In simulation, we choose $h_{11}(x_i) = (0, -x_{i1}x_{i3}, x_{i1}x_{i2})^T$, $h_{12}(x_i) = (0, 5x_{i2}, 0)^T$, $h_{21}(x_i) = (0, 0, x_{i1}x_{i3})^T$, $h_{22}(x_i) = (x_{i1}, 0, 0)^T$, $V = [0, 0, 0.2]^T$, $h_{31}(x_i) = (3.247(|x_{i1} + 1| - |x_{i1} - $
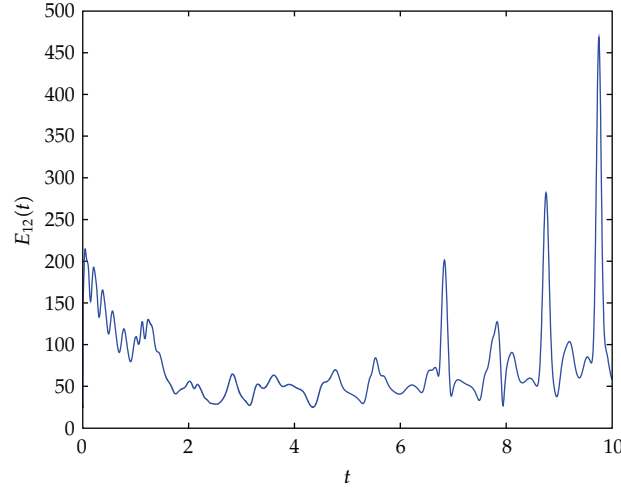
Figure 2: Time evolution of the synchronization errors $E_{12}(t)$.

$1|), 0, 0)^T$, $h_{32}(x_i) = (0, 0, -3.906 \ \sin(0.5x_{i1}))^T$, $\tau_1(t) = e^t/(1 + e^t)$, $\tau_2(t) = 2e^t/(1 + e^t)$, $\tau_3(t) = 0.5e^t/(1 + e^t)$,

$$D_1 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & 4 & 0 \\ 0 & 0 & -\dfrac{8}{3} \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 0 & 0 & -1.2 \end{bmatrix}, \quad D_3 = \begin{bmatrix} -2.169 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -19.53 & -0.1636 \end{bmatrix}. \quad (4.2)$$

Taking the weight configuration coupling matrices

$$A = B = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 1 \\ -1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \\ 1 & 0 & 0 & 0 & 1 & -2 \end{bmatrix}. \quad (4.3)$$

The following quantities are utilized to measure the process of cluster synchronization

$$E(t) = \sum_{i=1}^{N} \|x_i(t) - s_{\phi_i}(t)\|,$$

$$E_{12}(t) = \|x_u(t) - x_v(t)\|, \quad u \in C_1, \ v \in C_2, \quad (4.4)$$

$$E_{13}(t) = \|x_u(t) - x_v(t)\|, \quad u \in C_1, \ v \in C_3,$$

$$E_{23}(t) = \|x_u(t) - x_v(t)\|, \quad u \in C_2, \ v \in C_3,$$

where $E(t)$ is the error of cluster synchronization for this controlled network (2.2); $E_{12}(t)$, $E_{13}(t)$, and $E_{23}(t)$ are the errors between two communities; cluster synchronization is
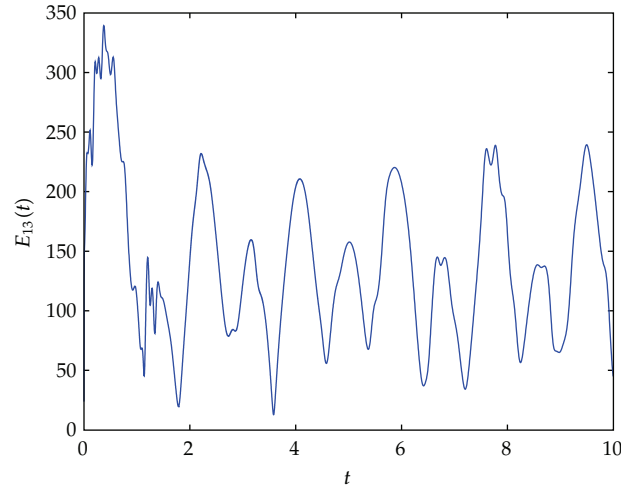
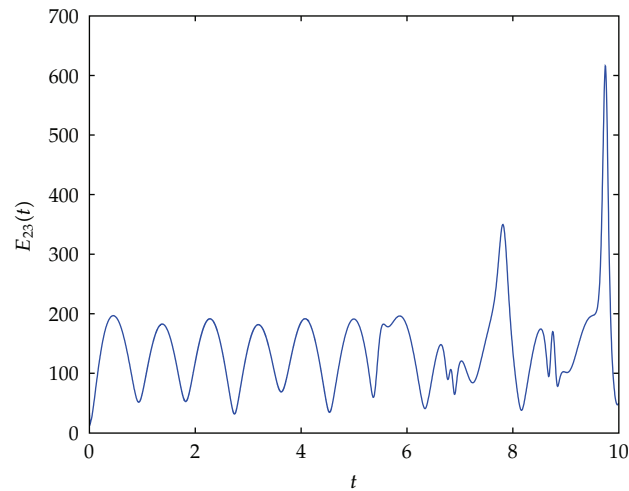Figure 3: Time evolution of the synchronization errors $E_{13}(t)$.

Figure 4: Time evolution of the synchronization errors $E_{23}(t)$.

achieved if the synchronization error $E(t)$ converges to zero and $E_{12}(t)$, $E_{13}(t)$ and $E_{23}(t)$ do not as $t \to \infty$. Simulation results are given in Figures 1, 2, 3, and 4. From the Figures 1–4, we see the time evolution of the synchronization errors. The numerical results show that Theorem 3.1 is effective.

5. Conclusions

The problems of cluster synchronization and adaptive feedback controller for the nonlinear coupled complex networks are investigated. The weight configuration matrix is not assumed to be symmetric, irreducible. It is shown that cluster synchronization can be realized via adaptive feedback controller. The study showed that the use of simple control law helps

to derive sufficient criteria which ensure that nodes in the same group synchronize with each other, but there is no synchronization between nodes in different groups is derived. Particularly the synchronization criteria are independent of time delay. The developed techniques are applied three complex community networks which are synchronized to different chaotic trajectories. Finally, the numerical simulations were performed to verify the effectiveness of the theoretical results.

Acknowledgments

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