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Research Article

Existence of Periodic Solutions for a Class of Asymptotically p-Linear Discrete Systems Involving p-Laplacian

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By applying Mountain Pass Theorem in critical point theory, two existence results are obtained for the following asymptotically p-linear p-Laplacian discrete system $\Delta(|\Delta u(t-1)|^{p-2}\Delta u(t-1)) + \nabla[-K(t,u(t)) + W(t,u(t))] = 0$. The results obtained generalize some known works.

1. Introduction

Consider the periodic solutions of the following ordinary p-Laplacian discrete system

$$\Delta\left(|\Delta u(t-1)|^{p-2}\Delta u(t-1)\right) + \nabla F(t,u(t)) = 0, \tag{1.1}$$

where Δ is the forward difference operator defined by $\Delta u(t) = u(t+1) - u(t)$, $\Delta^2 u(t) = \Delta(\Delta u(t))$, $p \in (1,+\infty)$, $t \in \mathbb{Z}, u \in \mathbb{R}^N$, $F : \mathbb{Z} \times \mathbb{R}^N \to \mathbb{R}$, $F(t,x)$ is continuously differentiable in x for every $t \in \mathbb{Z}$ and T-periodic in t for all $x \in \mathbb{R}^N$, and F satisfies (F) $F(t,x) = -K(t,x) + W(t,x)$, and K and W are T-periodic in t for all $x \in \mathbb{R}^N$ with $T > 1$.

Difference equations provide a natural description of many discrete models in real world. Discrete models exist in various fields of science and technology such as statistics, computer science, electrical circuit analysis, biology, neural network, and optimal control; so it is very important to study the solutions of difference equations. For more details about difference equations, we refer the readers to the books [1–3].

In some recent papers [4–17], the authors investigated the existence of periodic solutions and subharmonic solutions of difference equations by applying critical point theory. These papers imply that the critical point theory is a useful method to the study of periodic solutions for difference equations. Motivated by the above papers and the paper [18], we will generalize the results of [18] to p-Laplacian systems (1.1). Here are our main results.

Theorem 1.1. *Assume that F satisfies (F) and K and W satisfy the following conditions:*

(A1) *there exist constants $a_1 > 0$ and $\gamma \in (p-1, p]$ such that*

$$K(t,0) = 0, \qquad K(t,x) \geq a_1 |x|^{\gamma}, \qquad \forall (t,x) \in \mathbb{Z}[1,T] \times \mathbb{R}^N; \tag{1.2}$$

(A2) $(\nabla K(t,x), x) \leq pK(t,x)$ *for all* $(t,x) \in \mathbb{Z}[1,T] \times \mathbb{R}^N$, *where* $\mathbb{Z}[a,b] := \mathbb{Z} \cap [a,b]$ *for every* $a, b \in \mathbb{Z}$ *with* $a \leq b$;

(A3) $W(t,0) = 0$, $\limsup_{|x| \to 0}(W(t,x)/|x|^p) < a_1$ *uniformly for* $t \in \mathbb{Z}[1,T]$;

(A4) *there exists a function* $g \in l^1(\mathbb{Z}[1,T], \mathbb{R})$ *such that*

$$(\nabla W(t,x), x) - pW(t,x) \geq g(t) \quad \text{for } (t,x) \in \mathbb{Z}[1,T] \times \mathbb{R}^N, \tag{1.3}$$

$$\lim_{|x| \to \infty} \left[(\nabla W(t,x), x) - pW(t,x) \right] = +\infty \quad \forall t \in \mathbb{Z}[1,T]; \tag{1.4}$$

(A5) *there exist constants* $a_2 > 0$ *and* $d > 0$ *such that*

$$W(t,x) \leq a_2 |x|^p + d, \qquad \forall (t,x) \in \mathbb{Z}[1,T] \times \mathbb{R}^N; \tag{1.5}$$

(A6) *there exists* $x_0 \in \mathbb{R}^N$ *such that*

$$\sum_{t=1}^{T} \left[K(t,x_0) - W(t,x_0) - \frac{g(t)}{p} \right] < 0. \tag{1.6}$$

Then problem (1.1) *possesses one nontrivial periodic solution.*

Corollary 1.2. *Assuming that F satisfies (F) and that K and W satisfy* (A1)–(A4) *and*

(A7) *there exists a function* $V_\infty \in l^\infty(\mathbb{Z}[1,T], \mathbb{R})$ *such that*

$$\lim_{|x| \to \infty} \frac{\left| \nabla W(t,x) - V_\infty(t) |x|^{p-2} x \right|}{|x|^{p-1}} = 0 \quad \text{uniformly for } t \in \mathbb{Z}[1,T], \tag{1.7}$$

$$\sum_{t=1}^{T} \left[\max_{|x|=1} K(t,x) - \frac{V_\infty(t)}{p} \right] < 0. \tag{1.8}$$

Then problem (1.1) *possesses one nontrivial periodic solution.*

Remark 1.3. As far as we know, similar results of discrete system (1.1) which satisfies (F) and is asymptotically p-linear at infinity cannot be found in the literature. From this point, our results are new.

2. Preliminaries

Let E_T be the Sobolev space defined by

$$E_T = \left\{u : \mathbb{Z} \longrightarrow \mathbb{R}^N \mid u(t+T) = u(t),\ t \in \mathbb{Z}\right\}, \tag{2.1}$$

with the norm

$$\|u\| = \left(\sum_{t=1}^{T}\left(|\Delta u(t)|^p + |u(t)|^p\right)\right)^{1/p}, \quad u \in E_T, \tag{2.2}$$

where $|\cdot|$ denote the usual norm in $\mathbb{R}^N$. It is easy to see that $(E_T, \|\cdot\|)$ is a finite dimensional Banach space and linear homeomorphic to $\mathbb{R}^{NT}$. As usual, let

$$\|u\|_\infty = \sup\{|u(t)| : t \in \mathbb{Z}[1,T]\}, \quad \forall u \in l^\infty\left(\mathbb{Z}[1,T], \mathbb{R}^N\right). \tag{2.3}$$

Since E_T is finite dimensional Banach space, there exists a positive constant C_0 such that

$$\|u\|_\infty \le C_0\|u\|. \tag{2.4}$$

For any $u \in E_T$, let

$$\varphi(u) = \frac{1}{p}\sum_{t=1}^{T}|\Delta u(t)|^p + \sum_{t=1}^{T}[K(t,u(t)) - W(t,u(t))]. \tag{2.5}$$

We can compute the Fréchet derivative of (2.5) as

$$\frac{\partial\varphi(u)}{\partial u} = \Delta\left(|\Delta u(t-1)|^{p-2}\Delta u(t-1)\right) + \nabla[K(t,u(t)) - W(t,u(t))], \quad t \in \mathbb{Z}[1,T]. \tag{2.6}$$

Hence, u is a critical point of φ on E_T if and only if

$$\Delta\left(|\Delta u(t-1)\ |^{p-2}\Delta u(t-1)\right) + \nabla[K(t,u(t)) - W(t,u(t))] = 0, \quad t \in \mathbb{Z}[1,T],\ u \in \mathbb{R}^N. \tag{2.7}$$

So, the critical points of φ are classical solutions of (1.1). We will use the following lemma to prove our main results.

Lemma 2.1 (see [19]). *Let E be a real Banach space and $\varphi \in C^1(E, \mathbb{R})$ satisfying the (PS) condition. Suppose $\varphi(0) = 0$ and*

(a) *there exist constants $\rho, \alpha > 0$ such that $\varphi|_{\partial B_\rho(0)} \geq \alpha$;*

(b) *there exists an $e \in E \setminus \overline{B}_\rho(0)$ such that $\varphi(e) \leq 0$.*

Then φ possesses a critical value $c \geq \alpha$ which can be characterized as $c = \inf_{h\in\Gamma}\max_{s\in[0,1]}\varphi(h(s))$, where $\Gamma = \{h \in C([0,1], E) \mid h(0) = 0,\ h(1) = e\}$ and $B_\rho(0)$ is an open ball in E of radius ρ centered at 0.

It is well known that a deformation lemma can be proved with the weaker condition (C) replacing the usual (PS) condition. So Lemma 2.1 holds true under condition (C).

3. Proofs of Main Results

Proof of Theorem 1.1. The proof is divided into three steps.

Step 1. The functional φ satisfies condition (C). Let $\{u_n\} \subset E_T$ satisfying $(1+\|u_n\|)\|\varphi'(u_n)\| \to 0$ as $n \to \infty$ and $\varphi(u_n)$ is bounded. Hence, there exists a positive constant C_1 such that

$$\left|\varphi(u_n)\right| \leq C_1, \qquad (1+\|u_n\|)\left\|\varphi'(u_n)\right\| \leq C_1. \tag{3.1}$$

We prove $\{u_n\}$ is bounded by contradiction. If $\{u_n\}$ is unbounded, without loss of generality, we can assume that $\|u_n\| \to \infty$ as $n \to \infty$. Let $z_n = u_n/\|u_n\|$, then we have $\|z_n\| = 1$. Going to a subsequence if necessary, we may assume that $z_n \rightharpoonup z$ weakly in E_T and so $z_n \to z$ strongly in $l^1(\mathbb{Z}[1,T], \mathbb{R})$. It follows from (3.1) and (A2) that

$$\begin{aligned}(p+1)C_1 &\geq p\varphi(u_n) - \langle \varphi'(u_n), u_n\rangle \\ &= \sum_{t=1}^{T}\left[(\nabla W(t,u_n), u_n) - pW(t,u_n)\right] + \sum_{t=1}^{T}\left[pK(t,u_n) - (\nabla K(t,u_n), u_n)\right] \\ &\geq \sum_{t=1}^{T}\left[(\nabla W(t,u_n), u_n) - pW(t,u_n)\right].\end{aligned} \tag{3.2}$$

From (A1) and (A5), we obtain

$$\begin{aligned}\varphi(u_n) &= \frac{1}{p}\sum_{t=1}^{T}|\Delta u_n(t)|^p + \sum_{t=1}^{T}\left[K(t,u_n(t)) - W(t,u_n(t))\right] \\ &\geq \frac{1}{p}\sum_{t=1}^{T}|\Delta u_n(t)|^p - a_2\sum_{t=1}^{T}|u_n(t)|^p - dT \\ &= \frac{1}{p}\|u_n\| - \left(\frac{1}{p} + a_2\right)\sum_{t=1}^{T}|u_n(t)|^p - dT.\end{aligned} \tag{3.3}$$

Hence, we have

$$\frac{\varphi(u_n)}{\|u_n\|^p} \geq \frac{1}{p} - \left(\frac{1}{p} + a_2\right)\sum_{t=1}^{T}|z_n(t)|^p - \frac{dT}{\|u_n\|^p}. \tag{3.4}$$

Passing to the limit in the above inequality, by using the fact that $\varphi(u_n)$ is bounded and $\{z_n(t)\}$ converges uniformly to $z(t)$ on $\mathbb{Z}[1,T]$, we obtain

$$\left(\frac{1}{p} + a_2\right)\sum_{t=1}^{T}|z(t)|^p \geq \frac{1}{p}, \tag{3.5}$$

which implies that $z \neq 0$. Let $\Omega \subset \mathbb{Z}[1,T]$ be the set on which $z \neq 0$, then the measure of Ω is positive. Moreover, $|u_n(t)| \to \infty$ as $n \to \infty$ for $t \in \Omega$. Thus, from (A4), we get

$$\begin{aligned}\sum_{t=1}^{T}\left[(\nabla W(t,u_n),u_n) - pW(t,u_n)\right] &= \sum_{\Omega}\left[(\nabla W(t,u_n),u_n) - pW(t,u_n)\right] \\ &\quad + \sum_{\mathbb{Z}[1,T]\setminus\Omega}\left[(\nabla W(t,u_n),u_n) - pW(t,u_n)\right] \\ &\geq \sum_{\Omega}\left[(\nabla W(t,u_n),u_n) - pW(t,u_n)\right] + \sum_{\mathbb{Z}[1,T]\setminus\Omega} g(t).\end{aligned} \tag{3.6}$$

It follows from Fatou's lemma and (A4) that

$$\lim_{n\to\infty}\sum_{t=1}^{T}\left[(\nabla W(t,u_n),u_n) - pW(t,u_n)\right] = +\infty, \tag{3.7}$$

which contradicts with (3.2). Therefore, $\{u_n\}$ is bounded in E_T. Hence, there exists a subsequence, still denoted by $\{u_n\}$, such that

$$u_n \rightharpoonup u_0 \quad \text{weakly in } E_T. \tag{3.8}$$

Since E_T is finite dimensional space, we have $u_n \to u_0$ in E_T. Therefore, the functional φ satisfies condition (C).

Step 2. From (A3) and (A5), there exist constants $0 < \varepsilon < 1/p$, $q > p$ and $C_2 > 1/TC_0^p$ such that

$$W(t,u) \leq (a_1 - \varepsilon)|u|^p + C_2|u|^q \quad \text{for } u \in \mathbb{R}^N,\ t \in \mathbb{Z}[1,T]. \tag{3.9}$$

Let

$$\delta = \left(\frac{p\varepsilon}{qTC_2C_0^p}\right)^{1/(q-p)}. \tag{3.10}$$

Then $0 < \delta < 1$. If $u \in E_T$ and $\|u\| = \delta / C_0 := \rho$, then it follows from (2.4) that $|u(t)| \le \delta$ for $t \in \mathbb{Z}[1, T]$. Set

$$\alpha = \frac{(q-p)\varepsilon}{q} \rho^p. \tag{3.11}$$

Then from (A1), (2.4), (3.9), (3.10), and (3.11), we have

$$\begin{aligned}
\varphi(u) &= \frac{1}{p}\sum_{t=1}^{T}|\Delta u(t)|^p + \sum_{t=1}^{T}K(t,u(t)) - \sum_{t=1}^{T}W(t,u(t)) \\
&\ge \frac{1}{p}\sum_{t=1}^{T}|\Delta u(t)|^p + a_1\sum_{t=1}^{T}|u(t)|^\gamma - (a_1-\varepsilon)\sum_{t=1}^{T}|u(t)|^p - C_2\sum_{t=1}^{T}|u(t)|^q \\
&\ge \frac{1}{p}\sum_{t=1}^{T}|\Delta u(t)|^p + \varepsilon\sum_{t=1}^{T}|u(t)|^p - C_2\sum_{t=1}^{T}|u(t)|^q \\
&\ge \varepsilon\left(\sum_{t=1}^{T}|\Delta u(t)|^p + \sum_{t=1}^{T}|u(t)|^p\right) - C_2T\|u\|_\infty^q \ge \varepsilon\|u\|^p - C_2TC_0^q\|u\|^q \\
&= \frac{(q-p)\varepsilon}{q}\rho^p = \alpha \quad \forall u \in E_T \text{ with } \|u\| = \rho.
\end{aligned} \tag{3.12}$$

Step 3. Set $f(s) = s^{-p}W(t, sx_0)$ for $s > 0$. Then it follows from (A4) that

$$f'(s) = s^{-p-1}\left[-pW(t,sx_0) + (\nabla W(t,sx_0), sx_0)\right] \ge s^{-p-1}g(t) \quad \forall t \in \mathbb{Z}[1,T],\ s > 0. \tag{3.13}$$

Integrating the above inequality from 1 to $\xi > 1$, we have

$$W(t,\xi x_0) \ge \xi^p W(t,x_0) + \frac{g(t)}{p}(\xi^p - 1) \quad \forall t \in \mathbb{Z}[1,T],\ \xi > 1. \tag{3.14}$$

From (A2), it is easy to see that

$$K(t,\xi x_0) \le \xi^p K(t,x_0) \quad \forall t \in \mathbb{Z}[1,T],\ \xi > 1. \tag{3.15}$$

From (3.14), (3.15), and (A6), we have

$$\begin{aligned}
\varphi(\xi x_0) &= \sum_{t=1}^{T}[K(t,\xi x_0) - W(t,\xi x_0)] \le \xi^p\sum_{t=1}^{T}\left[K(t,x_0) - W(t,x_0) - \frac{g(t)}{p}\right] + \frac{1}{p}\sum_{t=1}^{T}g(t) \\
&\le 0 \quad \text{for large enough } \xi > 1.
\end{aligned} \tag{3.16}$$

Choose $\xi_1 > 1$ such that $T^{1/p}|\xi_1 x_0| > \rho$ and $\varphi(\xi_1 x_0) \le 0$. Let $e = \xi_1 x_0$, then $\|e\| = T^{1/p}|\xi_1 x_0| > \rho$ and $\varphi(e) \le 0$. It is easy to see that $\varphi(0) = 0$. Hence, by Lemma 2.1, there exists $u \in E_T$ such that

$$\varphi(u) = c, \qquad \varphi'(u) = 0. \tag{3.17}$$

Then the function u is a desired nontrivial T-periodic solution of (1.1). The proof is complete.

□

Proof of Corollary 1.2. Let $C_3 = -(1/3T)\sum_{t=1}^{T}[\max_{|x|=1}K(t,x) - (V_\infty(t)/p)]$. Then it follows from (A7) that $C_3 > 0$ and there exists a positive constant $C_4 > 0$ such that

$$\left|\nabla W(t,x) - V_\infty(t)|x|^{p-2}x\right| \le C_3|x|^{p-1} \quad \text{for } t \in \mathbb{Z}[1,T],\ |x| \ge C_4. \tag{3.18}$$

For any $x \in \mathbb{R}^N \setminus \{0\}$, let $x_* = C_4 x/|x|$. Then it follows from (3.18) that for all $t \in \mathbb{Z}[1,T]$ and $x \in \mathbb{R}^N$ with $|x| > C_4$

$$\begin{aligned} W(t,x) - \frac{V_\infty(t)}{p}|x|^p &= W(t,x_*) - \frac{1}{p}V_\infty(t)|x_*|^p \\ &\quad + \int_0^1 (\nabla W(t, x_* + s(x-x_*)) - V_\infty(t)(x_* + s(x-x_*)), x - x_*)ds \\ &\le W(t,x_*) - \frac{C_4^p}{p}V_\infty(t) + C_3\int_0^1 |x_* + s(x-x_*)||x-x_*|ds \\ &\le W(t,x_*) - \frac{C_4^p}{p}V_\infty(t) + 2C_3|x|^p, \end{aligned} \tag{3.19}$$

which implies that

$$W(t,x) \le \left(\frac{V_\infty(t)}{p} + 2C_3\right)|x|^p + W(t,x_*) - \frac{C_4^p}{p}V_\infty(t) \quad \text{for } t \in \mathbb{Z}[1,T],\ |x| > C_4, \tag{3.20}$$

which together with (A3) shows that (A5) holds. Similarly, we have

$$W(t,x) \ge \left(\frac{V_\infty(t)}{p} - 2C_3\right)|x|^p + W(t,x_*) - \frac{C_4^p}{p}V_\infty(t) \quad \text{for } t \in \mathbb{Z}[1,T],\ |x| > C_4. \tag{3.21}$$

From (A2), it is easy to show that

$$K(t,x) \le K\left(t, \frac{x}{|x|}\right)|x|^p \quad \text{for } t \in \mathbb{Z}[1,T],\ |x| > 1. \tag{3.22}$$

Choose $x_0 \in \mathbb{R}^N$ such that $|x_0| > C_4 + 1$ and

$$C_3 T|x_0|^p + \sum_{t=1}^{T}\left[\min_{|x|=C_4} W(t,x) + \frac{g(t)}{p} - \frac{C_4^p}{p}V_\infty(t)\right] > 0. \tag{3.23}$$

It follows from (3.21), (3.22), and (3.23) that

$$\begin{aligned}
&\sum_{t=1}^{T}\left[K(t,x_0) - W(t,x_0) - \frac{g(t)}{p}\right] \\
&\quad\le |x_0|^p \sum_{t=1}^{T}\left[K\left(t, \frac{x_0}{|x_0|}\right) - \frac{V_\infty(t)}{p} + 2C_3\right] - \sum_{t=1}^{T}\left[W(t,x_*) + \frac{g(t)}{p} - \frac{C_4^p}{p}V_\infty(t)\right] \\
&\quad\le |x_0|^p \sum_{t=1}^{T}\left[\max_{|x|=1} K(t,x) - \frac{V_\infty(t)}{p} + 2C_3\right] - \sum_{t=1}^{T}\left[\min_{|x|=C_4} W(t,x) + \frac{g(t)}{p} - \frac{C_4^p}{p}V_\infty(t)\right] \\
&\quad\le -C_3 T|x_0|^p - \sum_{t=1}^{T}\left[\min_{|x|=C_4} W(t,x) + \frac{g(t)}{p} - \frac{C_4^p}{p}V_\infty(t)\right] \\
&\quad< 0.
\end{aligned} \tag{3.24}$$

This implies that (A6) holds. By Theorem 1.1, the conclusion of Corollary 1.2 holds true. The proof is complete. □

4. An Example

In this section, we give an example to illustrate our results.

Example 4.1. In problem (1.1), let $p = 4/3$ and

$$W(t,x) = a(t)|x|^{4/3}\left(1 - \frac{1}{\ln(e+|x|)}\right), \qquad K(t,x) = b|x|^\theta + c(t)|x|^\sigma, \tag{4.1}$$

where $b > 0$, $a, c \in l^1(\mathbb{Z}, [0,+\infty))$, $1 < \theta < \sigma \le 4/3$, $a(t+T) = a(t)$, $c(t+T) = c(t)$. It is easy to check that (F), (A1)–(A3), and (A5) hold. On the one hand, we have

$$(\nabla W(t,x), x) - \frac{4}{3}W(t,x) = \frac{(4/3)a(t)|x|^{7/3}}{(e+|x|)(\ln(e+|x|))^2}. \tag{4.2}$$

Then, it is easy to check that condition (A4) holds. On the other hand, we have

$$\begin{aligned}
\sum_{t=1}^{T}\left[K(t,x) - W(t,x) - \frac{g(t)}{p}\right] &= \sum_{t=1}^{T}\left[b|x|^\theta + c(t)|x|^\sigma - a(t)|x|^{4/3}\left(1 - \frac{1}{\ln(e+|x|)}\right) - \frac{g(t)}{p}\right] \\
&= bT|x|^\theta + |x|^\sigma \sum_{t=1}^{T} c(t) - \frac{\|g\|_{l^1}}{p} - |x|^{4/3}\left(1 - \frac{1}{\ln(e+|x|)}\right)\sum_{t=1}^{T} a(t),
\end{aligned} \tag{4.3}$$

which implies that there exists $x_0 \in \mathbb{R}^N$ such that (A6) holds if

$$\sum_{t=1}^{T} a(t) > \sum_{t=1}^{T} c(t). \tag{4.4}$$

Hence, from Theorem 1.1, problem (1.1) with W and K as in (4.1) has one nontrivial T-periodic solution if (4.4) holds.

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