

Research Article

New Jacobi Elliptic Function Solutions for the Zakharov Equations

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A generalized (G'/G) -expansion method is proposed to seek the exact solutions of nonlinear evolution equations. Being concise and straightforward, this method is applied to the Zakharov equations. As a result, some new Jacobi elliptic function solutions of the Zakharov equations are obtained. This method can also be applied to other nonlinear evolution equations in mathematical physics.

1. Introduction

In recent years, with the development of symbolic computation packages like Maple and Mathematica, searching for solutions of nonlinear differential equations directly has become more and more attractive [1–7]. This is because of the availability of computers symbolic system, which allows us to perform some complicated and tedious algebraic calculation and help us find new exact solutions of nonlinear differential equations.

In 2008, Wang et al. [8] introduced a new direct method called the (G'/G) -expansion method to look for travelling wave solutions of nonlinear evolution equations (NLEEs). The method is based on the homogeneous balance principle and linear ordinary differential equation (LODE) theory. It is assumed that the traveling wave solutions can be expressed by a polynomial in (G'/G) , and that $G = G(\xi)$ satisfies a second-order LODE $G'' + \lambda G' + \mu G = 0$. The degree of the polynomial can be determined by the homogeneous balance between the highest order derivative and nonlinear terms appearing in the given NPDEs. The coefficients of the polynomial can be obtained by solving a set of algebraic equations. Many literatures have shown that the (G'/G) -expansion method is very effective, and many nonlinear equations have been successfully solved. Later, the further developed methods named the generalized (G'/G) -expansion method [9], the modified (G'/G) -expansion method [10],

the extended (G'/G) -expansion method [11], the improved (G'/G) -expansion method [12], and the $(G'/G, 1/G)$ -expansion method [13] have been proposed.

As we know, when using the direct method, the choice of an appropriate auxiliary LODE is of great importance. In this paper, by introducing a new auxiliary LODE of different literature [8], we propose the generalized (G'/G) -expansion method, which can be used to obtain travelling wave solutions of NLEEs.

In our contribution, we will seek exact solutions of the Zakharov equations [14]:

$$n_{tt} - c_s^2 n_{xx} = \beta (|E|^2)_{xx}, \quad (1.1)$$

$$iE_t + \alpha E_{xx} = \delta nE, \quad (1.2)$$

which are one of the classical models on governing the dynamics of nonlinear waves and describing the interactions between high- and low-frequency waves, where n is the perturbed number density of the ion (in the low-frequency response), E is the slow variation amplitude of the electric field intensity, c_s is the thermal transportation velocity of the electron ion, and $\alpha \neq 0$, $\beta \neq 0$, $\delta \neq 0$ and c_s are constants.

Recently, many exact solutions of (1.1)-(1.2) have been successfully obtained by using the extended tanh-expansion method, the extended hyperbolic function method, the F -expansion method, the $(G'/G, 1/G)$ -expansion method [13–19], and so on.

In this paper, we construct the exact solutions to (1.1)-(1.2) by using the generalized (G'/G) -expansion method. Furthermore, we show that the exact solutions are expressed by the Jacobi elliptic function.

2. The Generalized (G'/G) -Expansion Method

Suppose that we have a nonlinear partial differential equation (PDE) for $u(x, t)$ in the form

$$N(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \dots) = 0, \quad (2.1)$$

where N is a polynomial in its arguments.

Step 1. By taking $u(x, t) = u(\xi)$, $\xi = x - ct$, we look for traveling wave solutions of (2.1) and transform it to the ordinary differential equation (ODE)

$$N(u, -cu', u', c^2 u'', -cu'', u'', \dots) = 0. \quad (2.2)$$

Step 2. Suppose the solution u of (2.2) can be expressed as a finite series in the form

$$u = a_0 + \sum_{i=1}^m a_i \left(\frac{f'}{f} \right)^i, \quad (2.3)$$

where a_0, a_i ($i = 1, 2, \dots, m$) are constants to be determined later; $f = f(\xi)$ is a solution of the auxiliary LODE

$$f'^2 = Pf^4 + Qf^2 + R, \quad (2.4)$$

where P, Q , and R are constants.

Step 3. Determine the parameter m by balancing the highest order nonlinear term and the highest order partial derivative of u in (2.2).

Step 4. Substituting (2.3) and (2.4) into (2.2), setting all the coefficients of all terms with the same powers of $(f'/f)^k$ ($k = 1, 2, \dots$) to zero, we obtain a system of nonlinear algebraic equations (NAEs) with respect to the parameters c, a_0, a_i ($i = 1, 2, \dots, m$). By solving the NAEs if available, we can determine those parameters explicitly.

Step 5. Assuming that the constants c, a_0, a_i ($i = 1, 2, \dots, m$) can be obtained by solving the algebraic equations in Step 4, then substituting these constants and the known general solutions into (2.3), we can obtain the explicit solutions of (2.1) immediately.

3. Exact Solutions of the Zakharov Equations

In this section, we mainly apply the method proposed in Section 2 to seek the exact solutions of the Zakharov equations.

Since $E(x, t)$ in (1.2) is a complex function and we are looking for the traveling wave solutions, thus we introduce a gauge transformation:

$$E(x, t) = \varphi(\xi)e^{i(kx - \omega t + \xi_0)}, \quad n = n(\xi), \quad \xi = x - c_g t + \xi_1, \quad (3.1)$$

where $\varphi(x, t)$ is a real-valued function, c_g, k , and ω are constants to be determined later, and ξ_0 and ξ_1 are constants. Substituting (3.1) into (1.1)-(1.2), we have

$$(c_g^2 - c_s^2)n'' = \beta(\varphi^2)'', \quad (3.2)$$

$$\alpha\varphi'' + i(2\alpha k - c_g)\varphi' + (\omega - \alpha k^2)\varphi - \delta n\varphi = 0. \quad (3.3)$$

Integrating (3.2) twice with respect to ξ , we have

$$n = \frac{\beta}{c_g^2 - c_s^2}\varphi^2 + C_1\xi + C_2, \quad c_g^2 - c_s^2 \neq 0, \quad (3.4)$$

where C_1 and C_2 are integration constants. Substituting (3.4) into (3.3), we have

$$\alpha\varphi'' + i(2\alpha k - c_g)\varphi' + (\omega - \alpha k^2)\varphi - \delta\left(\frac{\beta}{c_g^2 - c_s^2}\varphi^2 + C_1\xi + C_2\right)\varphi = 0. \quad (3.5)$$

In (3.5), we assume that

$$c_g = 2\alpha k, \quad k_1 = \frac{\beta}{c_g^2 - c_s^2}. \quad (3.6)$$

Then (3.5) becomes the nonlinear ODE

$$\alpha\varphi'' + (\omega - \alpha k^2)\varphi - \delta k_1 \varphi^3 + \delta(C_1 \xi + C_2)\varphi = 0. \quad (3.7)$$

According to the homogeneous balance between φ'' and φ^3 in (3.7), we obtain $m = 1$. So we assume that φ can be expressed as

$$\varphi(\xi) = a_0 + a_1 \left(\frac{f'}{f} \right), \quad (3.8)$$

where $f = f(\xi)$ satisfies (2.4). By using (2.4) and (3.8), it is easily derived that

$$\varphi'' = 2a_1 \left(\frac{f'}{f} \right) \left(\left(\frac{f'}{f} \right)^2 - Q \right). \quad (3.9)$$

Substituting (3.8) and (3.9) into (3.7), the left-hand side of (3.7) becomes a polynomial in (f'/f) and ξ . Setting their coefficients to zero yields a system of algebraic equations in a_0 , a_1 , k , k_1 , and ω . Solving the overdetermined algebraic equations by Maple, we can obtain the following results:

$$a_0 = 0, \quad a_1 = \sqrt{\frac{2\alpha}{k_1\delta}}, \quad \omega = \alpha(2Q + k^2) + \delta C_2, \quad C_1 = 0. \quad (3.10)$$

Substituting (3.10) into (3.8), we obtain

$$\varphi = \sqrt{\frac{2\alpha}{k_1\delta}} \left(\frac{f'}{f} \right). \quad (3.11)$$

Substituting (3.11) into (3.1) and (3.4), we have the following formal solution of (1.1)-(1.2):

$$E = \sqrt{\frac{2\alpha}{k_1\delta}} \left(\frac{f'}{f} \right) e^{i(kx - \omega t + \xi_0)}, \quad (3.12)$$

$$n = \frac{2\alpha}{\delta} \left(\frac{f'}{f} \right)^2 + C_2,$$

where $\xi = x - 2\alpha kt + \xi_1$, $\omega = \alpha(2Q + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

With the aid of the appendix [20] and from the formal solution (3.12), we get the following set of exact solutions of (1.1)-(1.2).

Case 1. Choosing $P = m^2$, $Q = -(1 + m^2)$, $R = 1$, and $f(\xi) = \text{sn}(\xi)$ and inserting them into (3.12), we obtain the Jacobi elliptic function solution of (1.1)-(1.2)

$$E_1 = \sqrt{\frac{2\alpha}{k_1\delta}} \text{cs}(\xi) \text{dn}(\xi) e^{[i(kx - \omega t + \xi_0)]},$$

$$n_1 = \frac{2\alpha}{\delta} \text{cs}^2(\xi) \text{dn}^2(\xi) + C_2,$$
(3.13)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(-2(1 + m^2) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 2. Choosing $P = m^2$, $Q = -(1 + m^2)$, $R = 1$, and $f(\xi) = \text{cd}(\xi)$ and inserting them into (3.12), we obtain the Jacobi elliptic function solution of (1.1)-(1.2)

$$E_2 = -\sqrt{\frac{2\alpha}{k_1\delta}} (1 - m^2) \text{sd}(\xi) \text{nc}(\xi) e^{[i(kx - \omega t + \xi_0)]},$$

$$n_2 = \frac{2\alpha(1 - m^2)^2}{\delta} \text{sd}^2(\xi) \text{nc}^2(\xi) + C_2,$$
(3.14)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(-2(1 + m^2) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 3. Choosing $P = -m^2$, $Q = 2m^2 - 1$, $R = 1 - m^2$, and $f(\xi) = \text{cn}(\xi)$, we obtain

$$E_3 = -\sqrt{\frac{2\alpha}{k_1\delta}} \text{dc}(\xi) \text{sn}(\xi) e^{[i(kx - \omega t + \xi_0)]},$$

$$n_3 = \frac{2\alpha}{\delta} \text{dc}^2(\xi) \text{sn}^2(\xi) + C_2,$$
(3.15)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2m^2 - 1) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 4. Choosing $P = -1$, $Q = 2 - m^2$, $R = m^2 - 1$, and $f(\xi) = \text{dn}(\xi)$, we obtain

$$E_4 = -\sqrt{\frac{2\alpha}{k_1\delta}} m^2 \text{cd}(\xi) \text{sn}(\xi) e^{[i(kx - \omega t + \xi_0)]},$$

$$n_4 = \frac{2\alpha m^4}{\delta} \text{cd}^2(\xi) \text{sn}^2(\xi) + C_2,$$
(3.16)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2 - m^2) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 5. Choosing $P = 1$, $Q = -(1 + m^2)$, $R = m^2$, and $f(\xi) = \text{ns}(\xi)$, we obtain

$$\begin{aligned} E_5 &= -\sqrt{\frac{2\alpha}{k_1\delta}} \text{cs}(\xi) \text{dn}(\xi) e^{i(kx - \omega t + \xi_0)}, \\ n_5 &= \frac{2\alpha}{\delta} \text{cs}^2(\xi) \text{dn}^2(\xi) + C_2, \end{aligned} \quad (3.17)$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(-2(1 + m^2) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 6. Choosing $P = 1$, $Q = -(1 + m^2)$, $R = m^2$, and $f(\xi) = \text{dc}(\xi)$, we obtain

$$\begin{aligned} E_6 &= \sqrt{\frac{2\alpha}{k_1\delta}} (1 - m^2) \text{sc}(\xi) \text{nd}(\xi) e^{i(kx - \omega t + \xi_0)}, \\ n_6 &= \frac{2\alpha(1 - m^2)^2}{\delta} \text{sc}^2(\xi) \text{nd}^2(\xi) + C_2, \end{aligned} \quad (3.18)$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(-2(1 + m^2) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 7. Choosing $P = 1 - m^2$, $Q = 2m^2 - 1$, $R = -m^2$, and $f(\xi) = \text{nc}(\xi)$, we obtain

$$\begin{aligned} E_7 &= \sqrt{\frac{2\alpha}{k_1\delta}} \text{dc}(\xi) \text{sn}(\xi) e^{i(kx - \omega t + \xi_0)}, \\ n_7 &= \frac{2\alpha}{\delta} \text{dc}^2(\xi) \text{sn}^2(\xi) + C_2, \end{aligned} \quad (3.19)$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2m^2 - 1) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 8. Choosing $P = m^2 - 1$, $Q = 2 - m^2$, $R = -1$, and $f(\xi) = \text{nd}(\xi)$, we obtain

$$\begin{aligned} E_8 &= \sqrt{\frac{2\alpha}{k_1\delta}} m^2 \text{cd}(\xi) \text{sn}(\xi) e^{i(kx - \omega t + \xi_0)}, \\ n_8 &= \frac{2\alpha m^4}{\delta} \text{cd}^2(\xi) \text{sn}^2(\xi) + C_2, \end{aligned} \quad (3.20)$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2 - m^2) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 9. Choosing $P = 1 - m^2$, $Q = 2 - m^2$, $R = 1$, and $f(\xi) = \text{sc}(\xi)$, we obtain

$$\begin{aligned} E_9 &= \sqrt{\frac{2\alpha}{k_1\delta}} \text{dc}(\xi) \text{ns}(\xi) e^{i(kx - \omega t + \xi_0)}, \\ n_9 &= \frac{2\alpha}{\delta} \text{dc}^2(\xi) \text{ns}^2(\xi) + C_2, \end{aligned} \quad (3.21)$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2 - m^2) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 10. Choosing $P = -m^2(1 - m^2)$, $Q = 2m^2 - 1$, $R = 1$, and $f(\xi) = \text{sd}(\xi)$, we obtain

$$\begin{aligned} E_{10} &= \sqrt{\frac{2\alpha}{k_1\delta}} \text{cd}(\xi) \text{ns}(\xi) e^{[i(kx - \omega t + \xi_0)]}, \\ n_{10} &= \frac{2\alpha}{\delta} \text{cd}^2(\xi) \text{ns}^2(\xi) + C_2, \end{aligned} \quad (3.22)$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2m^2 - 1) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 11. Choosing $P = 1$, $Q = 2 - m^2$, $R = 1 - m^2$, and $f(\xi) = \text{cs}(\xi)$, we obtain

$$\begin{aligned} E_{11} &= \sqrt{\frac{2\alpha}{k_1\delta}} \text{ds}(\xi) \text{nc}(\xi) e^{[i(kx - \omega t + \xi_0)]}, \\ n_{11} &= \frac{2\alpha}{\delta} \text{ds}^2(\xi) \text{nc}^2(\xi) + C_2, \end{aligned} \quad (3.23)$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2 - m^2) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 12. Choosing $P = 1$, $Q = 2m^2 - 1$, $R = -m^2(1 - m^2)$, and $f(\xi) = \text{ds}(\xi)$, we obtain

$$\begin{aligned} E_{12} &= -\sqrt{\frac{2\alpha}{k_1\delta}} \text{cs}(\xi) \text{nd}(\xi) e^{[i(kx - \omega t + \xi_0)]}, \\ n_{12} &= \frac{2\alpha}{\delta} \text{cs}^2(\xi) \text{nd}^2(\xi) + C_2, \end{aligned} \quad (3.24)$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2m^2 - 1) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 13. Choosing $P = 1/4$, $Q = (1 - 2m^2)/2$, $R = 1/4$, and $f(\xi) = \text{ns}(\xi) \pm \text{cs}(\xi)$, we obtain

$$\begin{aligned} E_{13} &= \mp \sqrt{\frac{2\alpha}{k_1\delta}} \text{ds}(\xi) e^{[i(kx - \omega t + \xi_0)]}, \\ n_{13} &= \frac{2\alpha}{\delta} \text{ds}^2(\xi) + C_2, \end{aligned} \quad (3.25)$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(1 - 2m^2 + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 14. Choosing $P = (1 - m^2)/4$, $Q = (1 + m^2)/2$, $R = (1 - m^2)/4$, and $f(\xi) = \text{nc}(\xi) \pm \text{sc}(\xi)$, we obtain

$$\begin{aligned} E_{14} &= \pm \sqrt{\frac{2\alpha}{k_1\delta}} \text{dc}(\xi) e^{[i(kx - \omega t + \xi_0)]}, \\ n_{14} &= \frac{2\alpha}{\delta} \text{dc}^2(\xi) + C_2, \end{aligned} \quad (3.26)$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(1 + m^2 + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 15. Choosing $P = 1/4$, $Q = (m^2 - 2)/2$, $R = m^2/4$, and $f(\xi) = \text{ns}(\xi) \pm \text{ds}(\xi)$, we obtain

$$E_{15} = \mp \sqrt{\frac{2\alpha}{k_1\delta}} \text{cs}(\xi) e^{[i(kx - \omega t + \xi_0)]},$$

$$n_{15} = \frac{2\alpha}{\delta} \text{cs}^2(\xi) + C_2,$$
(3.27)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(m^2 - 2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 16. Choosing $P = m^2/4$, $Q = (m^2 - 2)/2$, $R = m^2/4$, and $f(\xi) = \text{sn}(\xi) \pm \text{icn}(\xi)$, we obtain

$$E_{16} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{\text{dn}(\xi)(\text{cn}(\xi) \mp \text{isn}(\xi)) e^{[i(kx - \omega t + \xi_0)]}}{\text{sn}(\xi) \pm \text{icn}(\xi)},$$

$$n_{16} = \frac{2\alpha \text{dn}^2(\xi)(\text{cn}(\xi) \mp \text{isn}(\xi))^2}{\delta(\text{sn}(\xi) \pm \text{icn}(\xi))^2} + C_2,$$
(3.28)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(m^2 - 2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 17. Choosing $P = m^2/4$, $Q = (m^2 - 2)/2$, $R = m^2/4$, and $f(\xi) = \sqrt{m^2 - 1} \text{sd}(\xi) \pm \text{cd}(\xi)$, we obtain

$$E_{17} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{(\sqrt{m^2 - 1} \text{cn}(\xi) \pm m^2 \text{sn}(\xi) \mp \text{sn}(\xi)) e^{[i(kx - \omega t + \xi_0)]}}{\text{dn}(\xi)(\sqrt{m^2 - 1} \text{sn}(\xi) \pm \text{cn}(\xi))},$$

$$n_{17} = \frac{2\alpha (\sqrt{m^2 - 1} \text{cn}(\xi) \pm m^2 \text{sn}(\xi) \mp \text{sn}(\xi))^2}{\delta \text{dn}^2(\xi) (\sqrt{m^2 - 1} \text{sn}(\xi) \pm \text{cn}(\xi))^2} + C_2,$$
(3.29)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(m^2 - 2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 18. Choosing $P = 1/4$, $Q = (1 - 2m^2)/2$, $R = 1/4$, and $f(\xi) = \text{mcd}(\xi) \pm i\sqrt{1 - m^2} \text{nd}(\xi)$, we obtain

$$E_{18} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{m \text{sn}(\xi) (\pm m^2 \mp 1 + im\sqrt{1 - m^2} \text{cn}(\xi)) e^{[i(kx - \omega t + \xi_0)]}}{\text{dn}(\xi) (i\sqrt{1 - m^2} \pm m \text{cn}(\xi))},$$

$$n_{18} = \frac{2\alpha m^2 \text{sn}^2(\xi) (\pm m^2 \mp 1 + im\sqrt{1 - m^2} \text{cn}(\xi))^2}{\delta \text{dn}^2(\xi) (i\sqrt{1 - m^2} \pm m \text{cn}(\xi))^2} + C_2,$$
(3.30)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(1 - 2m^2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 19. Choosing $P = 1/4$, $Q = (1 - 2m^2)/2$, $R = 1/4$, and $f(\xi) = msn(\xi) \pm idn(\xi)$, we obtain

$$E_{19} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{m\text{cn}(\xi)(\text{dn}(\xi) \mp im\text{sn}(\xi))e^{[i(kx-\omega t+\xi_0)]}}{msn(\xi) \pm idn(\xi)},$$

$$n_{19} = \frac{2\alpha m^2 \text{cn}^2(\xi)(\text{dn}(\xi) \mp im\text{sn}(\xi))^2}{\delta(msn(\xi) \pm idn(\xi))^2} + C_2,$$
(3.31)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(1 - 2m^2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 20. Choosing $P = 1/4$, $Q = (1 - 2m^2)/2$, $R = 1/4$, and $f(\xi) = \sqrt{m^2 - 1}\text{sc}(\xi) \pm idc(\xi)$, we obtain

$$E_{20} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{(\sqrt{m^2 - 1}\text{dn}(\xi) \mp im^2\text{sn}(\xi) \pm isn(\xi))e^{[i(kx-\omega t+\xi_0)]}}{\text{cn}(\xi)(\sqrt{m^2 - 1}\text{sn}(\xi) \pm idn(\xi))},$$

$$n_{20} = \frac{2\alpha(\sqrt{m^2 - 1}\text{dn}(\xi) \mp im^2\text{sn}(\xi) \pm isn(\xi))^2}{\delta \text{cn}^2(\xi)(\sqrt{m^2 - 1}\text{sn}(\xi) \pm idn(\xi))^2} + C_2,$$
(3.32)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(1 - 2m^2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 21. Choosing $P = (m^2 - 1)/4$, $Q = (m^2 + 1)/2$, $R = (m^2 - 1)/4$, and $f(\xi) = m\text{sd}(\xi) \pm nd(\xi)$, we obtain

$$E_{21} = \pm \sqrt{\frac{2\alpha}{k_1\delta}} m\text{cd}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$

$$n_{21} = \frac{2\alpha m^2}{\delta} \text{cd}^2(\xi) + C_2,$$
(3.33)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(m^2 + 1 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 22. Choosing $P = m^2/4$, $Q = (m^2 - 2)/2$, $R = 1/4$, and $f(\xi) = \text{sn}(\xi)/(1 \pm \text{dn}(\xi))$, we obtain

$$E_{22} = \pm \sqrt{\frac{2\alpha}{k_1\delta}} \text{cs}(\xi)e^{[i(kx-\omega t+\xi_0)]},$$

$$n_{22} = \frac{2\alpha}{\delta} \text{cs}^2(\xi) + C_2,$$
(3.34)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(m^2 - 2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 23. Choosing $P = -1/4$, $Q = (m^2 + 1)/2$, $R = (1 - m^2)^2/4$, and $f(\xi) = m\text{cn}(\xi) \pm \text{dn}(\xi)$, we obtain

$$E_{23} = \mp \sqrt{\frac{2\alpha}{k_1\delta}} m \text{sn}(\xi) e^{[i(kx - \omega t + \xi_0)]},$$

$$n_{23} = \frac{2\alpha m^2}{\delta} \text{sn}^2(\xi) + C_2,$$
(3.35)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(m^2 + 1 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 24. Choosing $P = (1 - m^2)^2/4$, $Q = (m^2 + 1)/2$, $R = 1/4$, and $f(\xi) = \text{ds}(\xi) \pm \text{cs}(\xi)$, we obtain

$$E_{24} = \mp \sqrt{\frac{2\alpha}{k_1\delta}} \text{ns}(\xi) e^{[i(kx - \omega t + \xi_0)]},$$

$$n_{24} = \frac{2\alpha}{\delta} \text{ns}^2(\xi) + C_2,$$
(3.36)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(m^2 + 1 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 25. Choosing $P = 1/4$, $Q = (m^2 - 2)/2$, $R = m^4/4$, and $f(\xi) = \text{dc}(\xi) \pm \sqrt{1 - m^2} \text{nc}(\xi)$, we obtain

$$E_{25} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{\text{sn}(\xi) \left(\mp m^2 \pm 1 + \sqrt{1 - m^2} \text{dn}(\xi) \right) e^{[i(kx - \omega t + \xi_0)]}}{\text{cn}(\xi) \left(\sqrt{1 - m^2} \pm \text{dn}(\xi) \right)},$$

$$n_{25} = \frac{2\alpha \text{sn}^2(\xi) \left(\mp m^2 \pm 1 + \sqrt{1 - m^2} \text{dn}(\xi) \right)^2}{\delta \text{cn}^2(\xi) \left(\sqrt{1 - m^2} \pm \text{dn}(\xi) \right)^2} + C_2,$$
(3.37)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(m^2 - 2 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 26. Choosing $R = m^2 Q^2 / (m^2 + 1)^2 P$, $Q < 0$, $P > 0$, and $f(\xi) = \sqrt{-m^2 Q / (m^2 + 1) P} \text{sn}(\sqrt{(-Q / (m^2 + 1))} \xi)$, we obtain

$$E_{26} = \sqrt{\frac{2\alpha}{k_1\delta}} \sqrt{\frac{-Q}{m^2 + 1}} \text{cs} \left(\sqrt{\frac{-Q}{m^2 + 1}} \xi \right) \text{dn} \left(\sqrt{\frac{-Q}{m^2 + 1}} \xi \right) e^{[i(kx - \omega t + \xi_0)]},$$

$$n_{26} = -\frac{2\alpha Q}{\delta (m^2 + 1)} \text{cs}^2 \left(\sqrt{\frac{-Q}{m^2 + 1}} \xi \right) \text{dn}^2 \left(\sqrt{\frac{-Q}{m^2 + 1}} \xi \right) + C_2,$$
(3.38)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2Q + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2 k^2 - c_s^2)$.

Case 27. Choosing $R = (1 - m^2)Q^2 / (m^2 - 2)^2 P$, $Q > 0$, $P < 0$, and $f(\xi) = \sqrt{-Q / (2 - m^2)P} \operatorname{dn}(\sqrt{Q / (2 - m^2)}\xi)$, we obtain

$$E_{27} = -\sqrt{\frac{2\alpha}{k_1\delta}} \sqrt{\frac{Q}{2 - m^2}} m^2 \operatorname{cd} \left(\sqrt{\frac{Q}{2 - m^2}} \xi \right) \operatorname{sn} \left(\sqrt{\frac{Q}{2 - m^2}} \xi \right) e^{[i(kx - \omega t + \xi_0)]},$$

$$n_{27} = \frac{2\alpha Q m^4}{\delta(2 - m^2)} \operatorname{cd}^2 \left(\sqrt{\frac{Q}{2 - m^2}} \xi \right) \operatorname{sn}^2 \left(\sqrt{\frac{Q}{2 - m^2}} \xi \right) + C_2,$$
(3.39)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2Q + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 28. Choosing $R = m^2(m^2 - 1)Q^2 / (2m^2 - 1)^2 P$, $Q > 0$, $P < 0$, and $f(\xi) = \sqrt{-(m^2 Q / (2m^2 - 1)P)} \operatorname{cn}(\sqrt{Q / (2m^2 - 1)}\xi)$, we obtain

$$E_{28} = -\sqrt{\frac{2\alpha}{k_1\delta}} \sqrt{\frac{Q}{2m^2 - 1}} \operatorname{dc} \left(\sqrt{\frac{Q}{2m^2 - 1}} \xi \right) \operatorname{sn} \left(\sqrt{\frac{Q}{2m^2 - 1}} \xi \right) e^{[i(kx - \omega t + \xi_0)]},$$

$$n_{28} = \frac{2\alpha Q}{\delta(2m^2 - 1)} \operatorname{dc}^2 \left(\sqrt{\frac{Q}{2m^2 - 1}} \xi \right) \operatorname{sn}^2 \left(\sqrt{\frac{Q}{2m^2 - 1}} \xi \right) + C_2,$$
(3.40)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2Q + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 29. Choosing $P = 1$, $Q = 2 - 4m^2$, $R = 1$, and $f(\xi) = \operatorname{sn}(\xi) \operatorname{dn}(\xi) / \operatorname{cn}(\xi)$, we obtain

$$E_{29} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{(1 - 2m^2 \operatorname{sn}^2(\xi) + m^2 \operatorname{sn}^4(\xi)) e^{[i(kx - \omega t + \xi_0)]}}{\operatorname{cn}(\xi) \operatorname{sn}(\xi) \operatorname{dn}(\xi)},$$

$$n_{29} = \frac{2\alpha (1 - 2m^2 \operatorname{sn}^2(\xi) + m^2 \operatorname{sn}^4(\xi))^2}{\delta \operatorname{cn}^2(\xi) \operatorname{sn}^2(\xi) \operatorname{dn}^2(\xi)} + C_2,$$
(3.41)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2 - 4m^2) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 30. Choosing $P = m^4$, $Q = 2m^2 - 4$, $R = 1$, and $f(\xi) = \operatorname{sn}(\xi) \operatorname{cn}(\xi) / \operatorname{dn}(\xi)$, we obtain

$$E_{30} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{(1 - 2\operatorname{sn}^2(\xi) + m^2 \operatorname{sn}^4(\xi)) e^{[i(kx - \omega t + \xi_0)]}}{\operatorname{dn}(\xi) \operatorname{sn}(\xi) \operatorname{cn}(\xi)},$$

$$n_{30} = \frac{2\alpha (1 - 2\operatorname{sn}^2(\xi) + m^2 \operatorname{sn}^4(\xi))^2}{\delta \operatorname{dn}^2(\xi) \operatorname{sn}^2(\xi) \operatorname{cn}^2(\xi)} + C_2,$$
(3.42)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2m^2 - 4) + k^2) + \delta C_2$, and $k_1 = \beta / (4\alpha^2 k^2 - c_s^2)$.

Case 31. Choosing $P = 1$, $Q = 2m^2 + 2$, $R = 1 - 2m^2 + m^4$, and $f(\xi) = \text{cn}(\xi)\text{dn}(\xi)/\text{sn}(\xi)$, we obtain

$$E_{31} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{(m^2\text{sn}^4(\xi) - 1)e^{i(kx - \omega t + \xi_0)}}{\text{sn}(\xi)\text{cn}(\xi)\text{dn}(\xi)},$$

$$n_{31} = \frac{2\alpha(m^2\text{sn}^4(\xi) - 1)^2}{\delta\text{sn}^2(\xi)\text{cn}^2(\xi)\text{dn}^2(\xi)} + C_2,$$
(3.43)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2m^2 + 2) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 32. Choosing $P = A^2(m - 1)^2/4$, $Q = (m^2 + 1)/2 + 3m$, $R = (m - 1)^2/4A^2$, and $f(\xi) = \text{dn}(\xi)\text{cn}(\xi)/A(1 + \text{sn}(\xi))(1 + m\text{sn}(\xi))$, we obtain

$$E_{32} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{(m^2\text{sn}^2(\xi) + m\text{sn}^2(\xi) - 1 - m)e^{i(kx - \omega t + \xi_0)}}{\text{dn}(\xi)\text{cn}(\xi)},$$

$$n_{32} = \frac{2\alpha(m^2\text{sn}^2(\xi) + m\text{sn}^2(\xi) - 1 - m)^2}{\delta\text{dn}^2(\xi)\text{cn}^2(\xi)} + C_2,$$
(3.44)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(m^2 + 1 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 33. Choosing $P = A^2(m + 1)^2/4$, $Q = (m^2 + 1)/2 - 3m$, $R = (m + 1)^2/4A^2$, and $f(\xi) = \text{dn}(\xi)\text{cn}(\xi)/A(1 + \text{sn}(\xi))(1 - m\text{sn}(\xi))$, we obtain

$$E_{33} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{(m^2\text{sn}^2(\xi) - m\text{sn}^2(\xi) + m - 1)e^{i(kx - \omega t + \xi_0)}}{\text{dn}(\xi)\text{cn}(\xi)},$$

$$n_{33} = \frac{2\alpha(m^2\text{sn}^2(\xi) - m\text{sn}^2(\xi) + m - 1)^2}{\delta\text{dn}^2(\xi)\text{cn}^2(\xi)} + C_2,$$
(3.45)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(m^2 + 1 + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 34. Choosing $P = -4/m$, $Q = 6m - m^2 - 1$, $R = -2m^3 + m^4 + m^2$, and $f(\xi) = m\text{cn}(\xi)\text{dn}(\xi)/(m\text{sn}^2(\xi) + 1)$, we obtain

$$E_{34} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{\text{sn}(\xi)(m\text{sn}^2(\xi) - 1 + 2m^2\text{sn}^2(\xi) + m^3\text{sn}^2(\xi) - m^2 - 2m)e^{i(kx - \omega t + \xi_0)}}{(m\text{sn}^2(\xi) + 1)\text{cn}(\xi)\text{dn}(\xi)},$$

$$n_{34} = \frac{2\alpha\text{sn}^2(\xi)(m\text{sn}^2(\xi) - 1 + 2m^2\text{sn}^2(\xi) + m^3\text{sn}^2(\xi) - m^2 - 2m)^2}{\delta(m\text{sn}^2(\xi) + 1)^2\text{cn}^2(\xi)\text{dn}^2(\xi)} + C_2,$$
(3.46)

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(6m - m^2 - 1) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 35. Choosing $P = 4/m$, $Q = -6m - m^2 - 1$, $R = 2m^3 + m^4 + m^2$, and $f(\xi) = m\text{cn}(\xi)\text{dn}(\xi)/(m\text{sn}^2(\xi) - 1)$, we obtain

$$E_{35} = \sqrt{\frac{2\alpha}{k_1\delta}} \frac{\text{sn}(\xi)(m\text{sn}^2(\xi) + 1 - 2m^2\text{sn}^2(\xi) + m^3\text{sn}^2(\xi) + m^2 - 2m)e^{[i(kx - \omega t + \xi_0)]}}{(m\text{sn}^2(\xi) - 1)\text{cn}(\xi)\text{dn}(\xi)}, \quad (3.47)$$

$$n_{35} = \frac{2\alpha\text{sn}^2(\xi)(m\text{sn}^2(\xi) + 1 - 2m^2\text{sn}^2(\xi) + m^3\text{sn}^2(\xi) + m^2 - 2m)^2}{\delta(m\text{sn}^2(\xi) - 1)^2\text{cn}^2(\xi)\text{dn}^2(\xi)} + C_2,$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(-2(6m + m^2 + 1) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 36. Choosing $P = -(m^2 + 2m + 1)B^2$, $Q = 2m^2 + 2$, $R = (2m - m^2 - 1)/B^2$, and $f(\xi) = (m\text{sn}^2(\xi) - 1)/B(m\text{sn}^2(\xi) + 1)$, we obtain

$$E_{36} = 4\sqrt{\frac{2\alpha}{k_1\delta}} m \frac{\text{sn}(\xi)\text{cn}(\xi)\text{dn}(\xi)}{m^2\text{sn}^4(\xi) - 1} e^{[i(kx - \omega t + \xi_0)]}, \quad (3.48)$$

$$n_{36} = \frac{32\alpha m^2\text{sn}^2(\xi)\text{cn}^2(\xi)\text{dn}^2(\xi)}{\delta(m^2\text{sn}^4(\xi) - 1)^2} + C_2,$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2m^2 + 2) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

Case 37. Choosing $P = -(m^2 - 2m + 1)B^2$, $Q = 2m^2 + 2$, $R = -(2m + m^2 + 1)/B^2$, and $f(\xi) = (m\text{sn}^2(\xi) + 1)/B(m\text{sn}^2(\xi) - 1)$, we obtain

$$E_{37} = -4\sqrt{\frac{2\alpha}{k_1\delta}} m \frac{\text{sn}(\xi)\text{cn}(\xi)\text{dn}(\xi)}{m^2\text{sn}^4(\xi) - 1} e^{[i(kx - \omega t + \xi_0)]}, \quad (3.49)$$

$$n_{37} = \frac{32\alpha m^2\text{sn}^2(\xi)\text{cn}^2(\xi)\text{dn}^2(\xi)}{\delta(m^2\text{sn}^4(\xi) - 1)^2} + C_2,$$

where $\xi = x - 2akt + \xi_1$, $\omega = \alpha(2(2m^2 + 2) + k^2) + \delta C_2$, and $k_1 = \beta/(4\alpha^2k^2 - c_s^2)$.

4. Conclusions

In this paper, by using the generalized (G'/G) -expansion method, we have successfully obtained some exact solutions of Jacobi elliptic function form of the Zakharov equations. When the modulus of the Jacobi elliptic function $m \rightarrow 0$ or 1 , the corresponding solitary wave solutions and trigonometric function solutions are also obtained. This work shows that the generalized (G'/G) -expansion method provides a very effective and powerful tool for solving nonlinear equations in mathematical physics.

Table 1: Relations between the coefficients (P, Q, and R) and corresponding $f(\xi)$ in $f'^2 = Pf^4 + Qf^2 + R$.

Case	P	Q	R	$f(\xi)$
Case 1	m^2	$-(1+m^2)$	1	$\operatorname{sn}\xi$
Case 2	m^2	$-(1+m^2)$	1	$\operatorname{cd}\xi$
Case 3	$-m^2$	$2m^2-1$	$1-m^2$	$\operatorname{cn}\xi$
Case 4	-1	$2-m^2$	m^2-1	$\operatorname{dn}\xi$
Case 5	1	$-(1+m^2)$	m^2	$\operatorname{ns}\xi$
Case 6	1	$-(1+m^2)$	m^2	$\operatorname{dc}\xi$
Case 7	$1-m^2$	$2m^2-1$	$-m^2$	$\operatorname{nc}\xi$
Case 8	m^2-1	$2-m^2$	-1	$\operatorname{nd}\xi$
Case 9	$1-m^2$	$2-m^2$	1	$\operatorname{sc}\xi$
Case 10	$-m^2(1-m^2)$	$2m^2-1$	1	$\operatorname{sd}\xi$
Case 11	1	$2-m^2$	1	$\operatorname{cs}\xi$
Case 12	1	$2m^2-1$	$1-m^2$	$\operatorname{ds}\xi$
Case 13	$1/4$	$(1-2m^2)/2$	$-m^2(1-m^2)$	$\operatorname{ns}\xi \pm \operatorname{cs}\xi$
Case 14	$(1-m^2)/4$	$(1+m^2)/2$	$1/4$	$\operatorname{nc}\xi \pm \operatorname{sc}\xi$
Case 15	$1/4$	$(m^2-2)/2$	$(1-m^2)/4$	$\operatorname{ns}\xi \pm \operatorname{ds}\xi$
Case 16	$m^2/4$	$(m^2-2)/2$	$m^2/4$	$\operatorname{sn}\xi \pm \operatorname{icn}\xi$
Case 17	$m^2/4$	$(m^2-2)/2$	$m^2/4$	$\sqrt{m^2-1}\operatorname{sd}\xi \pm \operatorname{cd}\xi$
Case 18	$1/4$	$(1-2m^2)/2$	$1/4$	$\operatorname{mcd}\xi \pm i\sqrt{1-m^2}\operatorname{nd}\xi$
Case 19	$1/4$	$(1-2m^2)/2$	$1/4$	$\operatorname{msn}\xi \pm \operatorname{idn}\xi$
Case 20	$1/4$	$(1-2m^2)/2$	$1/4$	$\sqrt{m^2-1}\operatorname{sc}\xi \pm \operatorname{idc}\xi$
Case 21	$(m^2-1)/4$	$(m^2+1)/2$	$(m^2-1)/4$	$\operatorname{msd}\xi \pm \operatorname{nd}\xi$
Case 22	$m^2/4$	$(m^2-2)/2$	$1/4$	$\operatorname{sn}\xi/(1 \pm \operatorname{dn}\xi)$
Case 23	-1/4	$(m^2+1)/2$	$(1-m^2)^2/4$	$\operatorname{mcn}\xi \pm \operatorname{dn}\xi$
Case 24	$(1-m^2)^2/4$	$(m^2+1)/2$	$1/4$	$\operatorname{ds}\xi \pm \operatorname{cs}\xi$
Case 25	$1/4$	$(m^2-2)/2$	$m^4/4$	$\operatorname{dc}\xi \pm \sqrt{1-m^2}\operatorname{nc}\xi$
Case 26	$P > 0$	$Q < 0$	$m^2Q^2/(m^2+1)^2P$	$\sqrt{-m^2Q/(m^2+1)}\operatorname{Psn}(\sqrt{(-Q/(m^2+1))}\xi)$
Case 27	$P < 0$	$Q > 0$	$(1-m^2)Q^2/(m^2-2)^2P$	$\sqrt{-Q/(2-m^2)}\operatorname{Pdn}(\sqrt{(Q/(2-m^2))}\xi)$
Case 28	$P < 0$	$Q > 0$	$m^2(m^2-1)Q^2/(2m^2-1)^2P$	$\sqrt{-m^2Q/(2m^2-1)}\operatorname{Pcn}(\sqrt{(Q/(2m^2-1))}\xi)$
Case 29	1	$2-4m^2$	1	$\operatorname{sn}\xi\operatorname{dn}\xi/\operatorname{cn}\xi$
Case 30	m^4	$2m^2-4$	1	$\operatorname{sn}\xi\operatorname{cn}\xi/\operatorname{dn}\xi$
Case 31	1	$2m^2+2$	$1-2m^2+m^4$	$\operatorname{cn}\xi\operatorname{dn}\xi/\operatorname{sn}\xi$
Case 32	$A^2(m-1)^2/4$	$(m^2+1)/2+3m$	$(m-1)^2/4A^2$	$\operatorname{dn}\xi\operatorname{cn}\xi/A(1+\operatorname{sn}\xi)(1+\operatorname{msn}\xi)$
Case 33	$A^2(m+1)^2/4$	$(m^2+1)/2-3m$	$(m+1)^2/4A^2$	$\operatorname{dn}\xi\operatorname{cn}\xi/A(1+\operatorname{sn}\xi)(1-\operatorname{msn}\xi)$
Case 34	$-4/m$	$6m-m^2-1$	$-2m^3+m^4+m^2$	$\operatorname{mcn}\xi\operatorname{dn}\xi/(\operatorname{msn}\xi^2+1)$
Case 35	$4/m$	$-6m-m^2-1$	$2m^3+m^4+m^2$	$\operatorname{mcn}\xi\operatorname{dn}\xi/(\operatorname{msn}\xi^2-1)$
Case 36	$-(m^2+2m+1)B^2$	$2m^2+2$	$(2m-m^2-1)/B^2$	$(\operatorname{msn}\xi-1)/B(\operatorname{msn}\xi+1)$
Case 37	$-(m^2-2m+1)B^2$	$2m^2+2$	$-(2m+m^2+1)/B^2$	$(\operatorname{msn}\xi+1)/B(\operatorname{msn}\xi-1)$

Appendix

For more details see Table 1 and [20].

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