Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 829594, 12 pages
doi:10.1155/2012/829594

Research Article

Robust Stochastic Stability Analysis for Uncertain Neutral-Type Delayed Neural Networks Driven by Wiener Process

Weiwei Zhang[1] and Linshan Wang[2]

[1] *College of Information Science and Engineering, Ocean University of China, Qingdao 266100, China*
[2] *Department of Mathematics, Ocean University of China, Qingdao 266100, China*

Correspondence should be addressed to Linshan Wang, wangls@ouc.edu.cn

Received 9 July 2011; Revised 20 September 2011; Accepted 27 September 2011

Academic Editor: Shiping Lu

The robust stochastic stability for a class of uncertain neutral-type delayed neural networks driven by Wiener process is investigated. By utilizing the Lyapunov-Krasovskii functional and inequality technique, some sufficient criteria are presented in terms of linear matrix inequality (LMI) to ensure the stability of the system. A numerical example is given to illustrate the applicability of the result.

1. Introduction

In the past few years, neural networks and their various generalizations have drawn much research attention owing to their promising potential applications in a variety of areas, such as robotics, aerospace, telecommunications, pattern recognition, image processing, associative memory, signal processing, and combinatorial optimization [1–3]. In such applications, it is of prime importance to ensure the asymptotic stability of the designed neural networks. Because of this, the stability of neural networks has been deeply investigated in the literature [4–14].

It is known that time delays and stochastic perturbations are commonly encountered in the implementation of neural networks, and may result in instability or oscillation. So it is essential to investigate the stability of delayed stochastic neural networks [15, 16]. Moreover, uncertainties are unavoidable in practical implementation of neural networks due to modeling errors and parameter fluctuation, which also cause instability and poor performance [15, 17, 18]. Therefore, it is significant to introduce such uncertainties into delayed stochastic neural networks.

On the other hand, because of the complicated dynamic properties of the neural cells in the real world, it is natural and important that systems will contain some information about the derivative of the past state. Practically, such phenomenon always appears in the study of automatic control, circuit analysis, chemical process simulation, and population dynamics, and so forth. Recently, there has been increasing interest in the study of delayed neural networks of neutral type, see [6–15, 18–24]. In [6, 8], the authors developed the global asymptotic stability of neutral-type neural networks with delays by utilizing the Lyapunov stability theory and LMI technique. In [9, 10], the global exponential stability of neutral-type neural networks with distributed delays is studied. However, the stochastic perturbations were not taken into account in those delayed neural networks [6–10].

In [23, 24], the authors discussed the robust stability for uncertain stochastic neural networks of neutral-type with time-varying delays. However, the distributed delays were not taken into account in the models. So far, there are only a few papers that not only deal with the stochastic stability analysis for delayed neural networks of neutral-type, but also consider the parameter uncertainties.

To the best of our knowledge, there are very few results on the stochastic stability analysis for uncertain neutral-type neural networks with both discrete and distributed delays driven by Wiener process. This motivates the research in this paper.

In this paper, a class of uncertain neutral-type delayed neural networks driven by Wiener process is considered. By constructing a suitable Lyapunov functional, some new stability criteria to guarantee the system to be stochastically asymptotically stable in the mean square are given, which are less conservative than some existing reports. The structure of the addressed system is more general than in the other papers. The criteria can be checked easily by the LMI control toolbox in MATLAB. Moreover, a numerical example is given to illustrate the effectiveness and improvement over some existing results.

2. Preliminaries

Notations. $\mathbf{A} < \mathbf{0}$ denotes that $\mathbf{A}$ is a negative definite matrix. The superscript "T" stands for the transpose of a matrix. (Ω, F, P) denotes a complete probability space, $\mathrm{E}(\cdot)$ stands for the mathematical expectation operator. $\|\cdot\|$ stands for the Euclidean norm. $\mathbf{I}$ is the identity matrix of appropriate dimension, and the symmetric terms in a symmetric matrix are denoted by $*$.

Consider the following class of uncertain neutral-type delayed neural networks driven by Wiener process:

$$\begin{aligned} \mathrm{d}[\mathbf{x}(t) - \mathbf{C}\mathbf{x}(t-h(t))] &= \left[-\mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{f}(\mathbf{x}(t-\tau(t))) + \mathbf{D}(t)\int_{t-r(t)}^{t} \mathbf{f}(\mathbf{x}(s))\mathrm{d}s\right]\mathrm{d}t \\ &\quad + [\mathbf{H}_0(t)\mathbf{x}(t) + \mathbf{H}_1(t)\mathbf{x}(t-\tau(t))]\mathrm{d}\mathbf{w}(t), \\ \mathbf{x}(t_0+s) &= \varphi(s), \quad s \in [t_0-\rho, t_0], \end{aligned} \tag{2.1}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ is the neuron state vector, $\mathbf{A}(t) = \overline{\mathbf{A}} + \Delta\mathbf{A}(t)$, $\mathbf{B}(t) = \overline{\mathbf{B}} + \Delta\mathbf{B}(t)$, $\mathbf{D}(t) = \overline{\mathbf{D}} + \Delta\mathbf{D}(t)$, $\mathbf{H}_0(t) = \overline{\mathbf{H}}_0 + \Delta\mathbf{H}_0(t)$, $\mathbf{H}_1(t) = \overline{\mathbf{H}}_1 + \Delta\mathbf{H}_1(t)$, $\overline{\mathbf{A}} = \mathrm{diag}(a_i)_{n\times n}$ is a positive diagonal matrix, $\overline{\mathbf{B}}, \mathbf{C}, \overline{\mathbf{D}} \in \mathbf{R}^{n\times n}$ are the connection weight matrices, $\overline{\mathbf{H}}_0, \overline{\mathbf{H}}_1 \in \mathbf{R}^{n\times n}$ are known real constant matrices, $\Delta\mathbf{A}(t)$, $\Delta\mathbf{B}(t)$, $\Delta\mathbf{D}(t)$, $\Delta\mathbf{H}_0(t)$, $\Delta\mathbf{H}_1(t)$ represent the time-varying parameter uncertain terms. $\mathbf{f}(\mathbf{x}) = (f_1(x_1), f_2(x_2), \ldots, f_n(x_n))^T$ is the neuron activation function with $\mathbf{f}(0) = 0$. $\mathbf{w}(t) = (w_1(t), w_2(t), \ldots, w_n(t))^T$ is an n-dimensional Wiener process defined on a complete probability space (Ω, F, P). $r(t)$, $\tau(t)$, $h(t)$ are nonnegative, bounded, and differentiable time varying delays satisfying

$$\begin{gathered} 0 < r(t) \leq \overline{r} < \infty, \\ 0 < \tau(t) \leq \overline{\tau} < \infty, \quad \dot{\tau}(t) \leq \eta_1 < \infty, \\ 0 < h(t) \leq \overline{h} < \infty, \quad \dot{h}(t) \leq \eta_2 < \infty. \end{gathered} \tag{2.2}$$

The admissible parameter uncertain terms are assumed to be the following form:

$$[\Delta\mathbf{A}(t), \Delta\mathbf{B}(t), \Delta\mathbf{D}(t), \Delta\mathbf{H}_0(t), \Delta\mathbf{H}_1(t)] = \mathbf{U}\mathbf{F}(t)[\mathbf{M}_1, \mathbf{M}_2, \mathbf{M}_3, \mathbf{M}_4, \mathbf{M}_5], \tag{2.3}$$

where $\mathbf{U}, \mathbf{M}_i$, $i = 1, \ldots, 5$ are known real constant matrices, $\mathbf{F}(t)$ is the time-varying uncertain matrix satisfying

$$\mathbf{F}^T\mathbf{F} \leq \mathbf{I}. \tag{2.4}$$

Suppose that $\mathbf{f}(\cdot)$ is bounded and satisfies the following condition:

$$\|\mathbf{f}(\mathbf{x})\| \leq \|\mathbf{G}\mathbf{x}\|, \tag{2.5}$$

where $\mathbf{G} \in \mathbf{R}^{n\times n}$ is a known constant matrix.

Assume that the initial value $\varphi : [-\rho, 0] \rightarrow \mathbf{R}^n$ is F_0-measurable and continuously differentiable, we introduce the following norm:

$$\|\varphi\|_\rho^2 = \max\left\{ \sup_{-\alpha\leq s\leq 0} \mathrm{E}\left|\varphi_i(s)\right|^2, \sup_{-\overline{h}\leq s\leq 0} \mathrm{E}\left|\varphi_i'(s)\right|^2 \right\} < \infty, \tag{2.6}$$

where $\rho = \max\{\overline{\tau}, \overline{h}, \overline{r}\}$, $\alpha = \max\{\overline{\tau}, \overline{r}\}$.

Under the above assumptions, it is easy to verify that there exists a unique equilibrium point of system (2.1) (see [25]).

Definition 2.1. The equilibrium point of (2.1) is said to be globally robustly stochastically asymptotically stable in the mean square, if the following condition holds:

$$\lim_{t\rightarrow +\infty} \mathrm{E}\left|\mathbf{x}(t, t_0, \varphi)\right|^2 = 0, \quad t \geq t_0, \tag{2.7}$$

where $\mathbf{x}(t, t_0, \varphi)$ is any solution of model (2.1) with initial value φ.

Lemma 2.2 (Schur complement [26]). *Given constant matrices $\mathbf{\Omega}_1$, $\mathbf{\Omega}_2$, $\mathbf{\Omega}_3$ with appropriate dimensions, where $\mathbf{\Omega}_1^T = \mathbf{\Omega}_1$ and $\mathbf{\Omega}_2^T = \mathbf{\Omega}_2 > 0$, then*

$$\mathbf{\Omega}_1 + \mathbf{\Omega}_3^T\mathbf{\Omega}_2^{-1}\mathbf{\Omega}_3 < 0, \tag{2.8}$$

if and only if

$$\begin{pmatrix} \mathbf{\Omega}_1 & \mathbf{\Omega}_3^T \\ * & -\mathbf{\Omega}_2 \end{pmatrix} < 0, \quad \text{or} \quad \begin{pmatrix} -\mathbf{\Omega}_2 & \mathbf{\Omega}_3 \\ * & \mathbf{\Omega}_1 \end{pmatrix} < 0. \tag{2.9}$$

Lemma 2.3 (see [26]). *Given matrices* $\mathbf{D}$, $\mathbf{E}$, *and* $\mathbf{F}$ *with* $\mathbf{F}^T\mathbf{F} \leq \mathbf{I}$ *and a scalar* $\varepsilon > 0$, *then*

$$\mathbf{DFE} + \mathbf{E}^T\mathbf{F}^T\mathbf{D}^T \leq \varepsilon\mathbf{DD}^T + \varepsilon^{-1}\mathbf{E}^T\mathbf{E}. \tag{2.10}$$

Lemma 2.4 (see [27]). *For any constant matrix* $\mathbf{M} \in \mathbf{R}^{n\times n}$, $\mathbf{M} = \mathbf{M}^T > 0$, *a scalar* $\gamma > 0$, *vector function* $x(t) : [0,\gamma] \to \mathbf{R}^n$ *such that the integrations are well defined, then*

$$\left[\int_0^\gamma x(s)\mathrm{d}s\right]^T \mathbf{M}\left[\int_0^\gamma x(s)\mathrm{d}s\right] \leq \gamma \int_0^\gamma x^T(s)\mathbf{M}x(s)\mathrm{d}s. \tag{2.11}$$

3. Main Results

Theorem 3.1. *System* (2.1) *is globally robustly stochastically asymptotically stable in the mean square, if there exist symmetric positive definite matrices* $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{S}$, $\mathbf{U}_1$, $\mathbf{U}_2$ *and positive scalars* δ, ε_1, $\varepsilon_2 > 0$ *such that LMI holds:*

$$\Lambda = \begin{pmatrix} \Gamma_1 & \overline{\mathbf{A}}^T\mathbf{PC} & \mathbf{P}\overline{\mathbf{B}} - \varepsilon_1\mathbf{M}_1^T\mathbf{M}_2 & \mathbf{P}\overline{\mathbf{D}} - \varepsilon_1\mathbf{M}_1^T\mathbf{M}_3 & \varepsilon_2\mathbf{M}_4^T\mathbf{M}_5 & \overline{\mathbf{H}}_0^T\mathbf{P} & -\mathbf{PU} & 0 \\ * & \Gamma_2 & -\mathbf{C}^T\mathbf{P}\overline{\mathbf{B}} & -\mathbf{C}^T\mathbf{P}\overline{\mathbf{D}} & 0 & 0 & -\mathbf{C}^T\mathbf{PU} & 0 \\ * & * & \Gamma_3 & \varepsilon_1\mathbf{M}_2^T\mathbf{M}_3 & 0 & 0 & 0 & 0 \\ * & * & * & \Gamma_4 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Gamma_5 & \overline{\mathbf{H}}_1^T\mathbf{P} & 0 & 0 \\ * & * & * & * & * & -\mathbf{P} & 0 & \mathbf{PU} \\ * & * & * & * & * & * & -\varepsilon_1\mathbf{I} & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2\mathbf{I} \end{pmatrix} < 0, \tag{3.1}$$

where $\Gamma_1 = -\mathbf{P}\overline{\mathbf{A}} - \overline{\mathbf{A}}^T\mathbf{P} + \mathbf{Q} + \mathbf{R} + \overline{r}\mathbf{G}^T\mathbf{SG} + \varepsilon_1\mathbf{M}_1^T\mathbf{M}_1 + \varepsilon_2\mathbf{M}_4^T\mathbf{M}_4$, $\Gamma_2 = -\mathbf{U}_1 - (1-\eta_2)\mathbf{R}$, $\Gamma_3 = -\delta\mathbf{I} + \varepsilon_1\mathbf{M}_2^T\mathbf{M}_2$, $\Gamma_4 = -\overline{r}^{-1}\mathbf{S} + \varepsilon_1\mathbf{M}_3^T\mathbf{M}_3$, $\Gamma_5 = -\mathbf{U}_2 - (1-\eta_1)\mathbf{Q} + \delta\mathbf{G}^T\mathbf{G} + \varepsilon_2\mathbf{M}_5^T\mathbf{M}_5$.

Proof. Using Lemma 2.2, the matrix $\Lambda < 0$ implies that

$$
\begin{pmatrix}
\Gamma_1 & \overline{\mathbf{A}}^T\mathbf{PC} & \mathbf{P}\overline{\mathbf{B}} - \varepsilon_1\mathbf{M}_1^T\mathbf{M}_2 & \mathbf{P}\overline{\mathbf{D}} - \varepsilon_1\mathbf{M}_1^T\mathbf{M}_3 & \varepsilon_2\mathbf{M}_4^T\mathbf{M}_5 & \overline{\mathbf{H}}_0^T\mathbf{P} \\
* & \Gamma_2 & -\mathbf{C}^T\mathbf{P}\overline{\mathbf{B}} & -\mathbf{C}^T\mathbf{P}\overline{\mathbf{D}} & 0 & 0 \\
* & * & \Gamma_3 & \varepsilon_1\mathbf{M}_2^T\mathbf{M}_3 & 0 & 0 \\
* & * & * & \Gamma_4 & 0 & 0 \\
* & * & * & * & \Gamma_5 & \overline{\mathbf{H}}_1^T\mathbf{P} \\
* & * & * & * & * & -\mathbf{P}
\end{pmatrix}
+ \begin{pmatrix}
\mathbf{PU} & 0 \\
-\mathbf{C}^T\mathbf{PU} & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & \mathbf{PU}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_1^{-1}\mathbf{I} & 0 \\
* & \varepsilon_2^{-1}\mathbf{I}
\end{pmatrix}
\begin{pmatrix}
\mathbf{U}^T\mathbf{P} & -\mathbf{U}^T\mathbf{PC} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mathbf{U}^T\mathbf{P}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
\Phi_1 & \overline{\mathbf{A}}^T\mathbf{PC} & \mathbf{P}\overline{\mathbf{B}} & \mathbf{P}\overline{\mathbf{D}} & 0 & \overline{\mathbf{H}}_0^T\mathbf{P} \\
* & \Gamma_2 & -\mathbf{C}^T\mathbf{P}\overline{\mathbf{B}} & -\mathbf{C}^T\mathbf{P}\overline{\mathbf{D}} & 0 & 0 \\
* & * & -\delta\mathbf{I} & 0 & 0 & 0 \\
* & * & * & -\overline{r}^{-1}\mathbf{S} & 0 & 0 \\
* & * & * & * & \Phi_2 & \overline{\mathbf{H}}_1^T\mathbf{P} \\
* & * & * & * & * & -\mathbf{P}
\end{pmatrix}
+ \varepsilon_1
\begin{pmatrix} -\mathbf{M}_1^T \\ 0 \\ \mathbf{M}_2^T \\ \mathbf{M}_3^T \\ 0 \\ 0 \end{pmatrix}
\begin{pmatrix} -\mathbf{M}_1^T \\ 0 \\ \mathbf{M}_2^T \\ \mathbf{M}_3^T \\ 0 \\ 0 \end{pmatrix}^T
\tag{3.2}
$$

$$
+ \varepsilon_2
\begin{pmatrix} \mathbf{M}_4^T \\ 0 \\ 0 \\ 0 \\ \mathbf{M}_5^T \\ 0 \end{pmatrix}
\begin{pmatrix} \mathbf{M}_4^T \\ 0 \\ 0 \\ 0 \\ \mathbf{M}_5^T \\ 0 \end{pmatrix}^T
+ \varepsilon_1^{-1}
\begin{pmatrix} \mathbf{PU} \\ -\mathbf{C}^T\mathbf{PU} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
\begin{pmatrix} \mathbf{PU} \\ -\mathbf{C}^T\mathbf{PU} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T
+ \varepsilon_2^{-1}
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{PU} \end{pmatrix}
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{PU} \end{pmatrix}^T
$$

$$
< 0,
$$

where $\Phi_1 = -\mathbf{P}\overline{\mathbf{A}} - \overline{\mathbf{A}}^T\mathbf{P} + \mathbf{Q} + \mathbf{R} + \overline{r}\mathbf{G}^T\mathbf{S}\mathbf{G}$, $\Phi_2 = -\mathbf{U}_2 - (1 - \eta_1)\mathbf{Q} + \delta\mathbf{G}^T\mathbf{G}$.

From (2.3), (2.4), using Lemma 2.3, we have

$$\begin{pmatrix} -\mathbf{P}\Delta\mathbf{A}(t)-\Delta\mathbf{A}^T(t)\mathbf{P} & \Delta\mathbf{A}^T(t)\mathbf{PC} & \mathbf{P}\Delta\mathbf{B}(t) & \mathbf{P}\Delta\mathbf{D}(t) & 0 & \Delta\mathbf{H}_0^T(t)\mathbf{P} \\ * & 0 & -\mathbf{C}^T\mathbf{P}\Delta\mathbf{B}(t) & -\mathbf{C}^T\mathbf{P}\Delta\mathbf{D}(t) & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & \Delta\mathbf{H}_1^T(t)\mathbf{P} \\ * & * & * & * & * & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -\mathbf{M}_1^T \\ 0 \\ \mathbf{M}_2^T \\ \mathbf{M}_3^T \\ 0 \\ 0 \end{pmatrix} \mathbf{F}^T(t) \begin{pmatrix} \mathbf{PU} \\ -\mathbf{C}^T\mathbf{PU} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T + \begin{pmatrix} \mathbf{PU} \\ -\mathbf{C}^T\mathbf{PU} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \mathbf{F}(t) \begin{pmatrix} -\mathbf{M}_1^T \\ 0 \\ \mathbf{M}_2^T \\ \mathbf{M}_3^T \\ 0 \\ 0 \end{pmatrix}^T$$

$$+ \begin{pmatrix} \mathbf{M}_4^T \\ 0 \\ 0 \\ 0 \\ \mathbf{M}_5^T \\ 0 \end{pmatrix} \mathbf{F}^T(t) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{PU} \end{pmatrix}^T + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{PU} \end{pmatrix} \mathbf{F}(t) \begin{pmatrix} \mathbf{M}_4^T \\ 0 \\ 0 \\ 0 \\ \mathbf{M}_5^T \\ 0 \end{pmatrix}^T \tag{3.3}$$

$$\leq \varepsilon_1 \begin{pmatrix} -\mathbf{M}_1^T \\ 0 \\ \mathbf{M}_2^T \\ \mathbf{M}_3^T \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} -\mathbf{M}_1^T \\ 0 \\ \mathbf{M}_2^T \\ \mathbf{M}_3^T \\ 0 \\ 0 \end{pmatrix}^T + \varepsilon_1^{-1} \begin{pmatrix} \mathbf{PU} \\ -\mathbf{C}^T\mathbf{PU} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \mathbf{PU} \\ -\mathbf{C}^T\mathbf{PU} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T$$

$$+ \varepsilon_2 \begin{pmatrix} \mathbf{M}_4^T \\ 0 \\ 0 \\ 0 \\ \mathbf{M}_5^T \\ 0 \end{pmatrix} \begin{pmatrix} \mathbf{M}_4^T \\ 0 \\ 0 \\ 0 \\ \mathbf{M}_5^T \\ 0 \end{pmatrix}^T + \varepsilon_2^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{PU} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{PU} \end{pmatrix}^T.$$

Together with (3.2), we get

$$\begin{pmatrix} \Psi & \mathbf{A}^T(t)\mathbf{PC} & \mathbf{PB}(t) & \mathbf{PD}(t) & 0 & \mathbf{H}_0^T(t)\mathbf{P} \\ * & \Gamma_2 & -\mathbf{C}^T\mathbf{PB}(t) & -\mathbf{C}^T\mathbf{PD}(t) & 0 & 0 \\ * & * & -\delta\mathbf{I} & 0 & 0 & 0 \\ * & * & * & -\overline{r}^{-1}\mathbf{S} & 0 & 0 \\ * & * & * & * & \Phi_2 & \mathbf{H}_1^T(t)\mathbf{P} \\ * & * & * & * & * & -\mathbf{P} \end{pmatrix} < 0, \tag{3.4}$$

where $\Psi = -\mathbf{PA}(t) - \mathbf{A}^T(t)\mathbf{P} + \mathbf{Q} + \mathbf{R} + \overline{r}\mathbf{G}^T\mathbf{SG}$.

Utilizing Lemma 2.2 again, we obtain

$$\Sigma = \begin{pmatrix} \Psi & \mathbf{A}^T(t)\mathbf{PC} & \mathbf{PB}(t) & \mathbf{PD}(t) & 0 \\ * & \Gamma_2 & -\mathbf{C}^T\mathbf{PB}(t) & -\mathbf{C}^T\mathbf{PD}(t) & 0 \\ * & * & -\delta\mathbf{I} & 0 & 0 \\ * & * & * & -\overline{r}^{-1}\mathbf{S} & 0 \\ * & * & * & * & \Phi_2 \end{pmatrix} + \begin{pmatrix} \mathbf{H}_0^T(t) \\ 0 \\ 0 \\ 0 \\ \mathbf{H}_1^T(t) \end{pmatrix} \mathbf{P} \begin{pmatrix} \mathbf{H}_0^T(t) \\ 0 \\ 0 \\ 0 \\ \mathbf{H}_1^T(t) \end{pmatrix}^T < 0. \tag{3.5}$$

Constructing a positive definite Lyapunov-Krasovskii functional as follows:

$$\begin{aligned} V(t,\mathbf{x}(t)) &= \mathbf{y}^T(t)\mathbf{P}\mathbf{y}(t) + \int_{t-\tau(t)}^{t} \mathbf{x}^T(s)\mathbf{Q}\mathbf{x}(s)\mathrm{d}s + \int_{t-h(t)}^{t} \mathbf{x}^T(s)\mathbf{R}\mathbf{x}(s)\mathrm{d}s \\ &\quad + \int_{-r(t)}^{0}\int_{t+\theta}^{t} \mathbf{f}^T(\mathbf{x}(s))\mathbf{S}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s\mathrm{d}\theta + \int_{t}^{T} \mathbf{x}^T(s-h(s))\mathbf{U}_1\mathbf{x}(s-h(s))\mathrm{d}s \\ &\quad + \int_{t}^{T} \mathbf{x}^T(s-\tau(s))\mathbf{U}_2\mathbf{x}(s-\tau(s))\mathrm{d}s, \end{aligned} \tag{3.6}$$

where $y(t) = \mathbf{x}(t) - \mathbf{C}\mathbf{x}(t-h(t))$, $T > t$ is a constant.

By Ito's differential formula, we get

$$\begin{aligned} \mathrm{d}V(t,\mathbf{x}(t)) \le \Bigg\{ & 2\mathbf{y}^T(t)\mathbf{P}\left[-\mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{f}(\mathbf{x}(t-\tau(t))) + \mathbf{D}(t)\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s\right] \\ & + \mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) - (1-\dot{\tau}(t))\mathbf{x}^T(t-\tau(t))\mathbf{Q}\mathbf{x}(t-\tau(t)) + \mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t) \\ & - \left(1-\dot{h}(t)\right)\mathbf{x}^T(t-h(t))\mathbf{R}\mathbf{x}(t-h(t)) + \overline{r}\mathbf{f}^T(\mathbf{x}(t))\mathbf{S}\mathbf{f}(\mathbf{x}(t)) \\ & - \int_{t-r(t)}^{t}\mathbf{f}^T(\mathbf{x}(s))\mathbf{S}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s \\ & - \mathbf{x}^T(t-h(t))\mathbf{U}_1\mathbf{x}(t-h(t)) - \mathbf{x}^T(t-\tau(t))\mathbf{U}_2\mathbf{x}(t-\tau(t)) \end{aligned}$$

$$
\begin{aligned}
&+[\mathbf{H}_0(t)\mathbf{x}(t)+\mathbf{H}_1(t)\mathbf{x}(t-\tau(t))]^T\mathbf{P}[\mathbf{H}_0(t)\mathbf{x}(t)+\mathbf{H}_1(t)\mathbf{x}(t-\tau(t))]\Big\}\mathrm{d}t\\
&+2\mathbf{y}^T(t)\mathbf{P}[\mathbf{H}_0(t)\mathbf{x}(t)+\mathbf{H}_1(t)\mathbf{x}(t-\tau(t))]\mathrm{d}\mathbf{w}(t)\\
\le&\Big\{2[\mathbf{x}(t)-\mathbf{C}\mathbf{x}(t-h(t))]^T\\
&\times\mathbf{P}\left[-\mathbf{A}(t)\mathbf{x}(t)+\mathbf{B}(t)\mathbf{f}(\mathbf{x}(t-\tau(t)))+\mathbf{D}(t)\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s\right]\\
&+\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t)-(1-\eta_1)\mathbf{x}^T(t-\tau(t))\mathbf{Q}\mathbf{x}(t-\tau(t))+\mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t)\\
&-(1-\eta_2)\mathbf{x}^T(t-h(t))\mathbf{R}\mathbf{x}(t-h(t))+\bar{r}\mathbf{f}^T(\mathbf{x}(t))\mathbf{S}\mathbf{f}(\mathbf{x}(t))\\
&-\int_{t-r(t)}^{t}\mathbf{f}^T(\mathbf{x}(s))\mathbf{S}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s\\
&-\mathbf{x}^T(t-h(t))\mathbf{U}_1\mathbf{x}(t-h(t))-\mathbf{x}^T(t-\tau(t))\mathbf{U}_2\mathbf{x}(t-\tau(t))\\
&+[\mathbf{H}_0(t)\mathbf{x}(t)+\mathbf{H}_1(t)\mathbf{x}(t-\tau(t))]^T\mathbf{P}[\mathbf{H}_0(t)\mathbf{x}(t)+\mathbf{H}_1(t)\mathbf{x}(t-\tau(t))]\Big\}\mathrm{d}t\\
&+2\mathbf{y}^T(t)\mathbf{P}[\mathbf{H}_0(t)\mathbf{x}(t)+\mathbf{H}_1(t)\mathbf{x}(t-\tau(t))]\mathrm{d}\mathbf{w}(t).
\end{aligned}
\tag{3.7}
$$

From (2.5), for a scalar $\delta>0$, we have

$$
-\delta\left[\mathbf{f}^T(\mathbf{x}(t-\tau(t)))\mathbf{f}(\mathbf{x}(t-\tau(t)))-\mathbf{x}^T(t-\tau(t))\mathbf{G}^T\mathbf{G}\mathbf{x}(t-\tau(t))\right]\ge 0. \tag{3.8}
$$

Using Lemma 2.4, we have

$$
\left(\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s\right)^T\bar{r}^{-1}\mathbf{S}\left(\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s\right)\le\int_{t-r(t)}^{t}\mathbf{f}^T(\mathbf{x}(s))\mathbf{S}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s. \tag{3.9}
$$

Together (3.8), (3.9) with $\mathrm{d}V(t,\mathbf{x}(t))$, we obtain

$$
\begin{aligned}
\mathrm{d}V(t,\mathbf{x}(t))\le&\Big\{\mathbf{x}^T(t)\left[-\mathbf{P}\mathbf{A}(t)-\mathbf{A}^T(t)\mathbf{P}+\mathbf{Q}+\mathbf{R}+\bar{r}\mathbf{G}^T\mathbf{S}\mathbf{G}\right]\mathbf{x}(t)+\mathbf{x}^T(t)\mathbf{A}^T(t)\mathbf{P}\mathbf{C}\mathbf{x}(t-h(t))\\
&+\mathbf{x}^T(t-h(t))\mathbf{C}^T\mathbf{P}\mathbf{A}(t)\mathbf{x}(t)+\mathbf{x}^T(t)\mathbf{P}\mathbf{B}(t)\mathbf{f}(\mathbf{x}(t-\tau(t)))\\
&+\mathbf{f}^T(\mathbf{x}(t-\tau(t)))\mathbf{B}^T(t)\mathbf{P}\mathbf{x}(t)\\
&+\mathbf{x}^T(t)\mathbf{P}\mathbf{D}(t)\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s+\left(\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s\right)^T\mathbf{D}^T(t)\mathbf{P}\mathbf{x}(t)\\
&+\mathbf{x}^T(t-h(t))\left[-\mathbf{U}_1-(1-\eta_2)\mathbf{R}\right]\mathbf{x}(t-h(t))-\mathbf{x}^T(t-h(t))\mathbf{C}^T\mathbf{P}\mathbf{B}(t)\mathbf{f}(\mathbf{x}(t-\tau(t)))
\end{aligned}
$$

$$
\begin{aligned}
&- \mathbf{f}^T(\mathbf{x}(t-\tau(t)))\mathbf{B}^T(t)\mathbf{P}\mathbf{C}\mathbf{x}(t-h(t)) - \mathbf{x}^T(t-h(t))\mathbf{C}^T\mathbf{P}\mathbf{D}(t)\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s \\
&- \left(\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s\right)^T\mathbf{D}^T(t)\mathbf{P}\mathbf{C}\mathbf{x}(t-h(t)) + \mathbf{x}^T(t-\tau(t)) \\
&\times\left[-\mathbf{U}_2 - (1-\eta_1)\mathbf{Q}\right]\mathbf{x}(t-\tau(t)) \\
&- \left(\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s\right)^T\overline{r}^{-1}\mathbf{S}\left(\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s\right) \\
&+[\mathbf{H}_0(t)\mathbf{x}(t)+\mathbf{H}_1(t)\mathbf{x}(t-\tau(t))]^T\mathbf{P}[\mathbf{H}_0(t)\mathbf{x}(t)+\mathbf{H}_1(t)\mathbf{x}(t-\tau(t))]\Big\}\mathrm{d}t \\
&+ 2\mathbf{y}^T(t)\mathbf{P}[\mathbf{H}_0(t)\mathbf{x}(t)+\mathbf{H}_1(t)\mathbf{x}(t-\tau(t))]\mathrm{d}\mathbf{w}(t).
\end{aligned}
\tag{3.10}
$$

That is,

$$
\mathrm{d}V(t,\mathbf{x}(t)) \le \xi^T(t)\Sigma\xi(t)\mathrm{d}t + 2\mathbf{y}^T(t)\mathbf{P}[\mathbf{H}_0(t)\mathbf{x}(t)+\mathbf{H}_1(t)\mathbf{x}(t-\tau(t))]\mathrm{d}\mathbf{w}(t), \tag{3.11}
$$

where $\xi^T(t) = (\mathbf{x}^T(t), \mathbf{x}^T(t-h(t)),\ \mathbf{f}^T(\mathbf{x}(t-\tau(t))),\ (\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s)^T,\ \mathbf{x}^T(t-\tau(t)))$, and the matrix Σ is given in (3.5).

Taking the mathematical expectation, we get

$$
\mathrm{E}\left(\frac{\mathrm{d}V(t,\mathbf{x}(t))}{\mathrm{d}t}\right) \le \mathrm{E}\left(\xi^T(t)\Sigma\xi(t)\right) \le \lambda_{\max}(\Sigma)\mathrm{E}\|\mathbf{x}(t)\|^2. \tag{3.12}
$$

From (3.5), we know $\Sigma < 0$, that is, $\lambda_{\max}(\Sigma) < 0$. By Lyapunov-Krasovskii stability theorems, the system (2.1) is globally robustly asymptotically stable. The proof is completed. □

Remark 3.2. To the best of our knowledge, few authors have considered the stochastically asymptotic stability for uncertain neutral-type neural networks driven by Wiener process. We can find recent papers [18, 22–24]. However, it is assumed in [18] that the system is a linear model and all delays are constants. In [22], it is assumed that the time-varying delays satisfying $\dot{\tau}(t) \le \rho_\tau < 1$, $\dot{h}(t) \le \rho_h < 1$, in this paper, we relax it to $\dot{\tau}(t) \le \rho_\tau < \infty$, $\dot{h}(t) \le \rho_h < \infty$. In [23, 24], the authors discussed the robust stability for uncertain stochastic neural networks of neutral-type with time-varying delays. However, the distributed delays were not taken into account in the models. Hence, our results in this paper have wider adaptive range.

Remark 3.3. Suppose that $\mathbf{C} = 0$, $\mathbf{D}(t) = 0$ (i.e., without neutral-type and distributed delays), then the system (2.1) becomes the one investigated in [15].

Remark 3.4. In [17], the authors studied the global stability for uncertain stochastic neural networks with time-varying delay by Lyapunov functional method and LMI technique. However, the neutral term and distributed delays were not taken into account in the models. Therefore, our developed results in this paper are more general than those reported in [17].

Remark 3.5. It should be noted that the condition is given as linear matrix inequalities LMIs, therefore, by using the MATLAB LMI Toolbox, it is straightforward to check the feasibility of the condition.

4. Numerical Example

Consider the following uncertain neutral-type delayed neural networks:

$$\begin{aligned} \mathrm{d}[\mathbf{x}(t) - \mathbf{C}\mathbf{x}(t-h(t))] = \Bigg[&-(\mathbf{A}+\mathbf{U}\mathbf{F}(t)\mathbf{M}_1)\mathbf{x}(t) + (\mathbf{B}+\mathbf{U}\mathbf{F}(t)\mathbf{M}_2)\mathbf{f}(\mathbf{x}(t-\tau(t))) \\ &+(\mathbf{D}+\mathbf{U}\mathbf{F}(t)\mathbf{M}_3)\int_{t-r(t)}^{t}\mathbf{f}(\mathbf{x}(s))\mathrm{d}s\Bigg]\mathrm{d}t \\ &+[\mathbf{U}\mathbf{F}(t)\mathbf{M}_4\mathbf{x}(t) + \mathbf{U}\mathbf{F}(t)\mathbf{M}_5\mathbf{x}(t-\tau(t))]\mathrm{d}\mathbf{w}(t), \end{aligned} \tag{4.1}$$

where $n = 2$, $f_i(x_i) = \sin x_i$, $i = 1, 2$, $\eta_1 = 0.7$, $\eta_2 = 0.5$, $0 < r(t) \le \bar{r} = 3$, $\mathbf{F}^T(t)\mathbf{F}(t) \le \mathbf{I}$.

The constant matrices are

$$\begin{gathered} \mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0.2 & 0.16 \\ 0.04 & 0.08 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}, \\ \mathbf{D} = \begin{pmatrix} 0.04 & 0.03 \\ -0.02 & 0.05 \end{pmatrix}, \quad \mathbf{U} = \begin{pmatrix} 0.1 & 0.5 \\ 0.5 & 0.3 \end{pmatrix}, \quad \mathbf{M}_1 = \begin{pmatrix} 0.6 & 0 \\ 0 & 0.6 \end{pmatrix}, \\ \mathbf{M}_2 = \mathbf{M}_3 = \mathbf{M}_4 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.2 \end{pmatrix}, \quad \mathbf{M}_5 = \begin{pmatrix} 0.4 & 0 \\ 0 & 0.4 \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \end{gathered} \tag{4.2}$$

By using the MATLAB LMI Control Toolbox, we obtain the feasible solution as follows: $\delta = 2.0876$, $\varepsilon_1 = 5.0486$, $\varepsilon_2 = 8.0446$,

$$\begin{gathered} \mathbf{P} = \begin{pmatrix} 6.8465 & -0.7257 \\ -0.7257 & 6.6012 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 9.9371 & -0.2792 \\ -0.2792 & 10.4388 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 8.0104 & -2.3936 \\ -2.3936 & 5.4991 \end{pmatrix}, \\ \mathbf{S} = \begin{pmatrix} 3.4984 & -0.8143 \\ -0.8143 & 1.9588 \end{pmatrix}, \quad \mathbf{U}_1 = \begin{pmatrix} 1.1111 & 0.3889 \\ 0.3889 & 1.1060 \end{pmatrix}, \quad \mathbf{U}_2 = \begin{pmatrix} 2.4193 & -0.6561 \\ -0.6561 & 2.6270 \end{pmatrix}. \end{gathered} \tag{4.3}$$

That is the system (4.1) is globally robustly stochastically asymptotically stable in the mean square.

5. Conclusion

In this paper, the stochastically asymptotic stability problem has been studied for a class of uncertain neutral-type delayed neural networks driven by Wiener process by utilizing the

Lyapunov-Krasovskii functional and linear matrix inequality (LMI) approach. A numerical example is given to illustrate the applicability of the result.

Acknowledgment

This paper was fully supported by the National Natural Science Foundation of China (no. 10771199 and no. 10871117).

References

[1] A. Bouzerdoum and T. R. Pattison, "Neural network for quadratic optimization with bound constraints," *IEEE Transactions on Neural Networks*, vol. 4, no. 2, pp. 293–304, 1993.
[2] M. Forti and A. Tesi, "New conditions for global stability of neural networks with application to linear and quadratic programming problems," *IEEE Transactions on Circuits and Systems I*, vol. 42, no. 7, pp. 354–366, 1995.
[3] M. P. Kennedy and L. O. Chua, "Neural networks for nonlinear programming," *IEEE Transactions on Circuits and Systems*, vol. 35, no. 5, pp. 554–562, 1988.
[4] S. Xu, J. Lam, D. W. C. Ho, and Y. Zou, "Delay-dependent exponential stability for a class of neural networks with time delays," *Journal of Computational and Applied Mathematics*, vol. 183, no. 1, pp. 16–28, 2005.
[5] R. Zhang and L. Wang, "Global exponential robust stability of interval cellular neural networks with S-type distributed delays," *Mathematical and Computer Modelling*, vol. 50, no. 3-4, pp. 380–385, 2009.
[6] J. H. Park, O. M. Kwon, and S. M. Lee, "LMI optimization approach on stability for delayed neural networks of neutral-type," *Applied Mathematics and Computation*, vol. 196, no. 1, pp. 236–244, 2008.
[7] R. Samli and S. Arik, "New results for global stability of a class of neutral-type neural systems with time delays," *Applied Mathematics and Computation*, vol. 210, no. 2, pp. 564–570, 2009.
[8] R. Rakkiyappan and P. Balasubramaniam, "LMI conditions for global asymptotic stability results for neutral-type neural networks with distributed time delays," *Applied Mathematics and Computation*, vol. 204, no. 1, pp. 317–324, 2008.
[9] R. Rakkiyappan and P. Balasubramaniam, "New global exponential stability results for neutral type neural networks with distributed time delays," *Neurocomputing*, vol. 71, no. 4–6, pp. 1039–1045, 2008.
[10] L. Liu, Z. Han, and W. Li, "Global stability analysis of interval neural networks with discrete and distributed delays of neutral type," *Expert Systems with Applications*, vol. 36, no. 3, pp. 7328–7331, 2009.
[11] R. Samidurai, S. M. Anthoni, and K. Balachandran, "Global exponential stability of neutral-type impulsive neural networks with discrete and distributed delays," *Nonlinear Analysis: Hybrid Systems*, vol. 4, no. 1, pp. 103–112, 2010.
[12] R. Rakkiyappan, P. Balasubramaniam, and J. Cao, "Global exponential stability results for neutral-type impulsive neural networks," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 1, pp. 122–130, 2010.
[13] J. H. Park and O. M. Kwon, "Further results on state estimation for neural networks of neutral-type with time-varying delay," *Applied Mathematics and Computation*, vol. 208, no. 1, pp. 69–75, 2009.
[14] J. Qiu and J. Cao, "Delay-dependent robust stability of neutral-type neural networks with time delays," *Journal of Mathematical Control Science and Applications*, vol. 1, pp. 179–188, 2007.
[15] J. Zhang, P. Shi, and J. Qiu, "Novel robust stability criteria for uncertain stochastic Hopfield neural networks with time-varying delays," *Nonlinear Analysis: Real World Applications*, vol. 8, no. 4, pp. 1349–1357, 2007.
[16] L. Wang, Z. Zhang, and Y. Wang, "Stochastic exponential stability of the delayed reaction-diffusion recurrent neural networks with Markovian jumping parameters," *Physics Letters A*, vol. 372, no. 18, pp. 3201–3209, 2008.
[17] Y. Wu, Y. Wu, and Y. Chen, "Mean square exponential stability of uncertain stochastic neural networks with time-varying delay," *Neurocomputing*, vol. 72, no. 10–12, pp. 2379–2384, 2009.
[18] M. H. Jiang, Y. Shen, and X. X. Liao, "Robust stability of uncertain neutral linear stochastic differential delay system," *Applied Mathematics and Mechanics*, vol. 28, no. 6, pp. 741–748, 2007.

[19] H. Zhang, M. Dong, Y. Wang, and N. Sun, “Stochastic stability analysis of neutral-type impulsive neural networks with mixed time-varying delays and Markovian jumping,” *Neurocomputing*, vol. 73, no. 13–15, pp. 2689–2695, 2010.

[20] Q. Zhu and J. Cao, “Stability analysis for stochastic neural networks of neutral type with both Markovian jump parameters and mixed time delays,” *Neurocomputing*, vol. 73, no. 13–15, pp. 2671–2680, 2010.

[21] H. Bao and J. Cao, “Stochastic global exponential stability for neutral-type impulsive neural networks with mixed time-delays and Markovian jumping parameters,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 9, pp. 3786–3791, 2011.

[22] X. Li, “Global robust stability for stochastic interval neural networks with continuously distributed delays of neutral type,” *Applied Mathematics and Computation*, vol. 215, no. 12, pp. 4370–4384, 2010.

[23] H. Chen, Y. Zhang, and P. Hu, “Novel delay-dependent robust stability criteria for neutral stochastic delayed neural networks,” *Neurocomputing*, vol. 73, no. 13–15, pp. 2554–2561, 2010.

[24] G. Liu, S. X. Yang, Y. Chai, W. Feng, and W. Fu, “Robust stability criteria for uncertain stochastic neural networks of neutral-type with interval time-varying delays,” *Neural Computing and Applications*. In press.

[25] X. Mao, *Stochastic Differential Equations and Their Applications*, Horwood Publishing Series in Mathematics & Applications, Horwood Publishing, Chichester, UK, 1997.

[26] Y. Y. Wang, L. Xie, and C. E. de Souza, “Robust control of a class of uncertain nonlinear systems,” *Systems & Control Letters*, vol. 19, no. 2, pp. 139–149, 1992.

[27] K. Gu, “An integral inequality in the stability problem of time-delay systems,” in *Proceedings of the 39th IEEE Confernce on Decision and Control*, pp. 2805–2810, Sydney, Australia, December 2000.