Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 762768, 13 pages
doi:10.1155/2012/762768

Research Article

Periodic Solutions of a Lotka-Volterra System with Delay and Diffusion

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Received 29 January 2012; Accepted 15 June 2012

Academic Editor: Josef Diblík

Our purpose is to prove the existence of periodic solutions for a competition Lotka-Volterra system on time scales, and one example is given to illustrate our results.

1. Introduction

Denote $\mathbb{T}$ as an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. The Lotka-Volterra system is mainly devoted to the study of population dynamics in mathematics. The classical two classes of species can be modeled as

$$\begin{aligned} x_1'(t) &= x_1(t)(r_{11}(t) + r_{12}(t)x_1(t) + r_{13}(t)x_2(t)), \\ x_2'(t) &= x_2(t)(r_{21}(t) + r_{22}(t)x_1(t) + r_{23}(t)x_2(t)), \end{aligned} \tag{1.1}$$

which are viewed in terms of different situations. For example, it is named with predator-prey system if $r_{13}(t)r_{22}(t) < 0$, while competition system when $r_{13}(t) < 0$, $r_{22}(t) < 0$, also a reciprocal system if $r_{13}(t) > 0$, $r_{22}(t) > 0$. Moreover, in order to reflect the seasonal fluctuations, the Lotka-Volterra system with periodic coefficients is also considered in [1]. The time delay effect, density regulation, and diffusion between patches in many ecological systems have been investigated for its ecological significance in [2–4].

The recently interest on ratio-dependent predator functional response calls for detailed qualitative study on ratio-dependent predator-prey differential systems. Predator-prey models where one or more terms involve ratios of the predator and prey populations may not be valid mathematically unless it can be shown that solutions with positive initial conditions never get arbitrarily close to the axis in question, that is, that persistence holds. By means of a transformation of variables, criteria for persistence are derived for two classes of such models, thereby leading to their validity. Ratio-dependent predator-prey models are favored by many animal ecologists recently involving a searching process.

Our concern in this paper is to consider both the periodic variations of the environment and the density regulation of the predators by considering account delay effect and diffusion between patches. The environments of most natural populations undergo temporal variation, causing changes in the growth characteristics of populations. One method of incorporating temporal nonuniformity of the environments in models is to assume that the parameters are periodic with the same period of the time variable. It can be modeled with the following dynamic system:

$$
\begin{aligned}
x_1^{\delta}(t) &= r_1(t) - f_1(t)e^{x_1(t)} - \frac{g_1(t)e^{y_1(t-\tau)}}{e^{x_1(t-\tau)} + \beta_1(t)e^{y_1(t-\tau)}} + p_1(t)\left(e^{x_2(t)-x_1(t)} - 1\right), \\
x_2^{\delta}(t) &= r_2(t) - f_2(t)e^{x_2(t)} + p_2(t)\left(e^{x_1(t)-x_2(t)} - 1\right), \\
y_1^{\delta}(t) &= r_3(t) - f_3(t)e^{y_1(t)} - \frac{g_2(t)e^{x_1(t-\tau)}}{e^{x_1(t-\tau)} + \beta_1(t)e^{y_1(t-\tau)}} - \frac{h_1(t)e^{y_2(t)}}{e^{y_1(t)} + \beta_2(t)e^{y_2(t)}}, \\
y_2^{\delta}(t) &= r_4(t) - f_4(t)e^{y_2(t)} + \frac{h_2(t)e^{y_1(t-\tau)}}{e^{y_1(t-\tau)} + \beta_2(t)e^{y_2(t-\tau)}},
\end{aligned}
\tag{1.2}
$$

where $y_1(t)$ and $x_1(t)$ denote the population density of species y and species x in patch 1, $y_2(t)$ and $x_2(t)$ represent the density of species y and species x in patch 2. Species x and y can be diffused between two patches and species y is confined to compete with species x. $\tau > 0$ is a delay due to gestation. $p_i(t) > 0$ $(i = 1, 2)$ are rd-continuous ω-periodic functions and denote the dispersal rate of species y in the ith patch $(i = 1, 2)$, respectively. $r_i(t) > 0$, $f_i(t) > 0$ $(i = 1, 2, 3, 4)$, $g_j(t) > 0$, $h_j(t) > 0$, $\beta_j(t) > 0$ $(j = 1, 2)$ are rd-continuous ω-periodic functions.

With the transition variable $X_i = e^{x_i(t)}$, $Y(t) = e^{y_1(t)}$, and $y_2(t) \equiv 0$, the system (1.2) reduces to

$$
\begin{aligned}
X_1'(t) &= X_1(t)\left(r_1(t) - f_1(t)X_1(t) - \frac{g_1(t)Y(t-\tau)}{X_1(t-\tau) + \beta(t)Y(t-\tau)}\right) \\
&\quad - p_1(t)X_1(t) - p_1(t)X_2(t), \\
X_2'(t) &= X_2(t)r_2(t) - f_2(t)X_2^2(t) + p_2(t)X_1(t) - p_2(t)X_2(t), \\
Y'(t) &= Y(t)r_3(t) - f_3(t)Y^2(t) - \frac{g_2(t)X_1(t-\tau)Y(t)}{X_1(t-\tau) + \beta(t)Y(t-\tau)},
\end{aligned}
\tag{1.3}
$$

which was introduced by Hilger in [5] who firstly proposed the theory of time scales. There are many related studies of positive solutions for delayed equation [6–8], dynamic equation

(1.3) on time scales [5, 9–17], and the uniform persistence, global asymptotic stability, and periodicity of system (1.3); see [18–23].

Recently, various continuation theorems in coincidence degree have played an important role in study the existence of periodic solutions of the Lotka-Volterra system (see, e.g., [11, 23–28]). In this paper, by using the well-known Gains and Mawhin's theorem, we prove the existence of periodic solutions of competition Lotka-Volterra dynamic system (1.2) with time delay and diffusion on time scales.

This paper is organized as follows. In Section 2, we present some basic definitions and results of topological degree theory. Section 3 is contributed to the proof of the main results while the last section goes to one example.

2. Preliminaries

Several definitions and results will be presented in this section. For more details, refer to [9, 12].

Let $\omega > 0$. Throughout this paper, the time scales we considered are always assumed to be ω-periodic (i.e., $t \in \mathbb{T} = \mathbb{Z}$ or $\mathbb{R}$ implies $t \pm \omega \in \mathbb{T}$) and unbounded above and below (may be represented by $\bigcup_{k\in\mathbb{T}}[2(k-1)\omega, 2k\omega]$). We denote $\epsilon = \min \mathbb{T} \cap (\mathbb{R} - \mathbb{R}^-)$, $\mathbb{I}_\omega = [\epsilon, \epsilon + \omega] \cap \mathbb{T}$.

Definition 2.1. The forward jump operator $\sigma : \mathbb{T} \to \mathbb{T}$ and the backward jump operator $\rho : \mathbb{T} \to \mathbb{T}$ are defined by

$$\sigma(t) := \inf\{s \in \mathbb{T} : s \geq t\}, \qquad \rho(t) := \sup\{s \in \mathbb{T} : s \leq t\}, \tag{2.1}$$

respectively, for any $t \in \mathbb{T}$: If $\sigma(t) = t$, then t is called right dense (otherwise: right scattered), and if $\rho(t) = t$, then t is called left dense (otherwise left scattered).

Definition 2.2. Suppose that $f : \mathbb{T} \to \mathbb{R}$ and fix $t \in \mathbb{T}^\kappa$. Then f is called differential at $t \in \mathbb{T}^\kappa$ if there exists a constant $c \in \mathbb{R}$ such that, for any given $\epsilon > 0$, there is an open neighborhood U of t such that

$$\left| f(\rho(t)) - f(s) - c(\rho(t) - s) \right| \leq \epsilon \left|\rho(t) - s\right|, \quad s \in U. \tag{2.2}$$

c is named with the delta (or Hilger) derivative of f at $t \in \mathbb{T}^\kappa$ and is denoted by $c = f^\delta(t)$. Here, $[a,b]^\kappa = [a,b]$ if b is left dense and $[a,b]^\kappa = [a,b)$ if b is left scattered.

As far as $\mathbb{T} = \mathbb{Z}$ or $\mathbb{R}$ is considered, $\mathbb{T}^\kappa = \mathbb{T}$. We say that f is delta (Hilger) differential on $\mathbb{T}$ if $f(t)$ exists for all $t \in \mathbb{T}$. A function $F : \mathbb{T} \to \mathbb{R}$ is called an antiderivative of $f : \mathbb{T} \to \mathbb{R}$ provided that $F^\delta(t) = f(t)$ for all $t \in \mathbb{T}$. Then we define

$$\int_r^s f(t)\ \delta t = F(s) - F(r), \quad r, s \in \mathbb{T}. \tag{2.3}$$

Definition 2.3. A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous if it is continuous at right dense points in $\mathbb{T}$ and its left-sided limits exist (finite) at left dense points in $\mathbb{T}$. The set of rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ will be denoted by $C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$.

It is easy to see that every rd-continuous function has an antiderivative and every continuous function is rd-continuous.

Lemma 2.4 (see [11]). *If $s, t \in \mathbb{T}$, $\alpha, \beta \in \mathbb{R}$ and $f, g \in C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$, then*

(1) $\int_s^t [\alpha f(u) + \beta g(u)]\delta u = \alpha \int_s^t f(u)\delta u + \beta \int_s^t g(u)\delta u$;

(2) *if* $f(u) \geq 0$ *for all* $s \leq u < t$, *then* $\int_s^t f(u)\delta u \geq 0$;

(3) *if* $|f(u)| \leq g(u)$ *on* $[s, t) := \{u \in \mathbb{T} : s \leq u < t\}$, *then* $|\int_s^t f(u)\delta u| \leq \int_s^t g(u)\delta u$.

Next, we introduce some results related to the topology degree theories which are crucial in our arguments [20].

Let X and Y be two Banach spaces. Consider an operator equation:

$$\mathcal{L}x = \lambda \mathcal{N}x, \quad \lambda \in (0,1), \tag{2.4}$$

where $\mathcal{L} : \operatorname{Dom}\mathcal{L} \cap X \to Y$ is a linear operator, $\mathcal{N} : X \to Y$ is continuous, and λ is a parameter. Let P and Q be two projections $P : X \to X$ and $Q : Y \to Y$ such that $\operatorname{Im} P = \ker \mathcal{L}$ and $\operatorname{Im}\mathcal{L} = \ker Q = \operatorname{Im}(I - Q)$. It is easy to see that $\mathcal{L}|_{\operatorname{Dom}\mathcal{L} \cap \ker P} : (I - P)X \to \operatorname{Im}\mathcal{L}$ is invertible, and thus we denote the inverse of this map by Φ. If Ω is a bounded open subset of X, the mapping $\mathcal{N}$ is called *$\mathcal{L}$-compact* on $\overline{\Omega}$ if $Q \circ \mathcal{N}(\overline{\Omega})$ is bounded and $\Phi \circ (I - Q) \circ \mathcal{N} : \overline{\Omega} \to X$ is compact. Since $\operatorname{Im} Q$ is isomorphic to $\ker \mathcal{L}$, there exists an isomorphism $\Psi : \operatorname{Im} Q \to \ker \mathcal{L}$.

Note that operator $\mathcal{L}$ is called a *Fredholm operator* of index zero if $\dim(\ker \mathcal{L}) = \operatorname{codim}(\operatorname{Im}\mathcal{L}) < \infty$ and $\operatorname{Im}\mathcal{L}$ is closed in Y.

Lemma 2.5 (Gains and Mawhin's theorem [20]). *Let $\mathcal{L}$ be a Fredholm mapping of index zero, and let $\mathcal{N}$ be $\mathcal{L}$-compact on $\overline{\Omega}$. Suppose that*

(C1) *for each* $\lambda \in (0,1)$, *every solution* $x \in \partial\Omega \cap \operatorname{Dom}\mathcal{L}$ *of* $\mathcal{L}x = \lambda\mathcal{N}(x, \lambda)$ *is such that* $x \notin \partial\Omega$;

(C2) $Q \circ \mathcal{N}x \neq 0$ *for each* $x \in \partial\Omega \cap \ker \mathcal{L}$;

(C3) $\deg(\Psi \circ Q \circ \mathcal{N}, \Omega \cap \ker \mathcal{L}, 0) \neq 0$.

Then equation $\mathcal{L}x = \mathcal{N}x$ *has at least one solution lying in* $\operatorname{Dom}\mathcal{L} \cap \overline{\Omega}$.

Throughout this paper, we take the following notations for convenience, and all the other notations are defined analogously:

$$\overline{f} = \frac{1}{\omega}\int_{\mathbb{I}_\omega} f(t)\delta t, \qquad f^s = \min_{t \in \mathbb{I}_\omega} f(t), \qquad f^M = \max_{t \in \mathbb{I}_\omega} f(t), \tag{2.5}$$

where $f \in C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$ is an ω-periodic function.

3. Periodic Solution

The main result is stated as follows about the existence of ω-periodic solutions.

Theorem 3.1. *Suppose that*

(1) $(r_1\beta_1 - g_1)^s > 0$;

(2) $\bar{r}_1\beta_1^s - g_1^M > 0$;

(3) $(r_3\beta_2 - g_2\beta_2 - h_1)^s > 0$;

(4) $\bar{r}_3\beta_2^s - g_2^M\beta_2^s - h_1^M > 0$.

Then the dynamic system (1.2) *has at least one ω-periodic solution.*

Before proving Theorem 3.1, we first give some useful lemmas.

Lemma 3.2. *Suppose $\lambda \in (0,1)$ is a parameter, $(r_1\beta_1 - g_1)^s > 0$, and $(r_3\beta_2 - g_2\beta_2 - h_1)^s > 0$. If $(x_1(t), x_2(t), y_1(t), y_2(t))^T$ is an ω-periodic solution of the system* (1.2)*, then $|x_1(t)| + |x_2(t)| \leq 2C_1$ and $|y_1(t)| + |y_2(t)| \leq 2C_2$, where*

$$C_1 := \max\left\{\left|\left(\ln\frac{r_1}{f_1}\right)^M\right|, \left|\left(\ln\frac{r_2}{f_2}\right)^M\right|, \left|\left(\ln\frac{r_2}{f_2}\right)^s\right|, \left|\left(\ln\frac{r_1\beta_1 - g_1}{\beta_1 f_1}\right)^s\right|\right\}, \tag{3.1}$$

$$C_2 := \max\left\{\left|\left(\ln\frac{r_3}{f_3}\right)^M\right|, \left|\left(\ln\frac{r_4 + h_2}{f_4}\right)^M\right|, \left|\left(\ln\frac{r_4}{f_4}\right)^s\right|, \left|\left(\ln\frac{r_3\beta_2 - g_2\beta_2 - h_1}{\beta_2 f_3}\right)^s\right|\right\}. \tag{3.2}$$

Proof. Corresponding to the operator equation (2.4), we have

$$\begin{aligned}
x_1^\delta(t) &= \lambda\left[r_1(t) - f_1(t)e^{x_1(t)} - \frac{g_1(t)e^{y_1(t-\tau)}}{e^{x_1(t-\tau)} + \beta_1(t)e^{y_1(t-\tau)}} + p_1(t)\left(e^{x_2(t)-x_1(t)} - 1\right)\right], \\
x_2^\delta(t) &= \lambda\left[r_2(t) - f_2(t)e^{x_2(t)} + p_2(t)\left(e^{x_1(t)-x_2(t)} - 1\right)\right], \\
y_1^\delta(t) &= \lambda\left[r_3(t) - f_3(t)e^{y_1(t)} - \frac{g_2(t)e^{x_1(t-\tau)}}{e^{x_1(t-\tau)} + \beta_1(t)e^{y_1(t-\tau)}} - \frac{h_1(t)e^{y_2(t)}}{e^{y_1(t)} + \beta_2(t)e^{y_2(t)}}\right], \\
y_2^\delta(t) &= \lambda\left[r_4(t) - f_4(t)e^{y_2(t)} + \frac{h_2(t)e^{y_1(t-\tau)}}{e^{y_1(t-\tau)} + \beta_2(t)e^{y_2(t-\tau)}}\right].
\end{aligned} \tag{3.3}$$

Define

$$\Gamma = \left\{\mathbf{u} = (x_1(t), x_2(t), y_1(t), y_2(t))^T \in C\left(\mathbb{T}, \mathbb{R}^4\right) : x_i(t+\omega) = x_i(t),\ y_i(t+\omega) = y_i(t)\right\}, \tag{3.4}$$

with the norm

$$\|\mathbf{u}\| = \sum_{i=1}^{2}\max_{t\in\mathbb{I}_\omega}|x_i(t)| + \sum_{i=1}^{2}\max_{t\in\mathbb{I}_\omega}|y_i(t)|. \tag{3.5}$$

Then Γ is a Banach space. Take $X = Y = \Gamma$. Assume that $\mathbf{u} = (x_1(t), x_2(t), y_1(t), y_2(t))^T \in \Gamma$ is a solution of the system (3.3) for $\lambda \in (0,1)$. It only needs to be proven that there exists a $M_1 > 0$ such that $\|\mathbf{u}\| < M_1$.

In fact, since $\mathbf{u} \in \Gamma$, there exists $t_i \in \mathbb{I}_\omega$ $(i = 1, 2, 3, 4)$ such that $x_i(t_i) = \max_{t\in\mathbb{I}_\omega} x_i(t)$ and $y_i(t_{i+2}) = \max_{t\in\mathbb{I}_\omega} y_i(t)$ $(i = 1, 2)$. Thus $x_i^\delta(t_i) = y_i^\delta(t_{i+2}) = 0$, for $i = 1, 2$. Consequently, it follows from the system (3.3) that

$$r_1(t_1) - f_1(t_1)e^{x_1(t_1)} - p_1(t_1) + p_1(t_1)e^{x_2(t_1)-x_1(t_1)} - \frac{g_1(t_1)e^{y_1(t_1-\tau)}}{e^{x_1(t_1-\tau)} + \beta_1(t_1)e^{y_1(t_1-\tau)}} = 0, \tag{3.6a}$$

$$r_2(t_2) - f_2(t_2)e^{x_2(t_2)} - p_2(t_2) + p_2(t_2)e^{x_1(t_2)-x_2(t_2)} = 0, \tag{3.6b}$$

$$r_3(t_3) - f_3(t_3)e^{y_1(t_3)} - \frac{g_2(t_3)e^{x_1(t_3-\tau)}}{e^{x_1(t_3-\tau)} + \beta_1(t_3)e^{y_1(t_3-\tau)}} - \frac{h_1(t_3)e^{y_2(t_3)}}{e^{y_1(t_3)} + \beta_2(t_3)e^{y_2(t_3)}} = 0, \tag{3.6c}$$

$$r_4(t_4) - f_4(t_4)e^{y_2(t_4)} + \frac{h_2(t_4)e^{y_1(t_4-\tau)}}{e^{y_1(t_4-\tau)} + \beta_2(t_4)e^{y_2(t_4-\tau)}} = 0. \tag{3.6d}$$

When $x_1(t_1) \geq x_2(t_2)$, then $x_1(t_1) \geq x_2(t_1)$. From (3.6a) we get

$$\begin{aligned} f_1(t_1)e^{x_1(t_1)} &= r_1(t_1) - p_1(t_1) + p_1(t_1)e^{x_2(t_1)-x_1(t_1)} - \frac{g_1(t_1)e^{y_1(t_1-\tau)}}{e^{x_1(t_1-\tau)} + \beta_1(t_1)e^{y_1(t_1-\tau)}} \\ &\leq r_1(t_1). \end{aligned} \tag{3.7}$$

It follows that

$$x_2(t_2) \leq x_1(t_1) \leq \ln\frac{r_1(t_1)}{f_1(t_1)} \leq \left(\ln\frac{r_1}{f_1}\right)^M. \tag{3.8}$$

When $x_1(t_1) < x_2(t_2)$, then $x_1(t_2) < x_2(t_2)$. From (3.6b) we obtain

$$f_2(t_2)e^{x_2(t_2)} = r_2(t_2) - p_2(t_2) + p_2(t_2)e^{x_1(t_2)-x_2(t_2)} \leq r_2(t_2), \tag{3.9}$$

which means

$$x_1(t_1) \leq x_2(t_2) \leq \ln\frac{r_2(t_2)}{f_2(t_2)} \leq \left(\ln\frac{r_2}{f_2}\right)^M. \tag{3.10}$$

As far as (3.6c) and (3.6d) are concerned, with analogue arguments above, we get

$$\begin{aligned} y_1(t_3) &\leq \frac{r_3(t_3)}{f_3(t_3)} \leq \left(\ln\frac{r_3}{f_3}\right)^M, \\ y_2(t_4) &\leq \ln\frac{r_4(t_4) + h_2(t_4)}{f_4(t_4)} \leq \left(\ln\frac{r_4 + h_2}{f_4}\right)^M. \end{aligned} \tag{3.11}$$

Now choose $\kappa_i \in \mathbb{I}_\omega$ $(i = 1, 2)$ such that $x_1(\kappa_1) = \min_{t\in\mathbb{I}_\omega} x_1(t)$, $x_2(\kappa_2) = \min_{t\in\mathbb{I}_\omega} x_2(t)$, then $x_i^\delta(\kappa_i) = 0$. Thus we obtain that

$$r_1(\kappa_1) - f_1(\kappa_1)e^{x_1(\kappa_1)} - p_1(\kappa_1) + p_1(\kappa_1)e^{x_2(\kappa_1)-x_1(\kappa_1)} - \frac{g_1(\kappa_1)e^{y_1(\kappa_1-\tau)}}{e^{x_1(\kappa_1-\tau)} + \beta_1(\kappa_1)e^{y_1(\kappa_1-\tau)}} = 0, \tag{3.12a}$$

$$r_2(\kappa_2) - f_2(\kappa_2)e^{x_2(\kappa_2)} - p_2(\kappa_2) + p_2(\kappa_2)e^{x_1(\kappa_2)-x_2(\kappa_2)} = 0. \tag{3.12b}$$

When $x_1(\kappa_1) < x_2(\kappa_2)$, then $x_1(\kappa_1) < x_2(\kappa_2) \le x_2(\kappa_1)$. From (3.12a), we have

$$\begin{aligned} f_1(\kappa_1)e^{x_1(\kappa_1)} &= r_1(\kappa_1) - p_1(\kappa_1) + p_1(\kappa_1)e^{x_2(\kappa_1)-x_1(\kappa_1)} - \frac{g_1(\kappa_1)e^{y_1(\kappa_1-\tau)}}{e^{x_1(\kappa_1-\tau)} + \beta_1(\kappa_1)e^{y_1(\kappa_1-\tau)}} \\ &\ge r_1(\kappa_1) - \frac{g_1(\kappa_1)}{\beta_1(\kappa_1)}, \end{aligned} \tag{3.13}$$

and thus

$$x_2(\kappa_2) > x_1(\kappa_1) \ge \ln\frac{r_1(\kappa_1)\beta(\kappa_1) - g_1(\kappa_1)}{f_1(\kappa_1)\beta_1(\kappa_1)} \ge \left(\ln\frac{r_1\beta_1 - g_1}{\beta_1 f_1}\right)^s, \tag{3.14}$$

according to the hypothesis $(r_1\beta_1 - g_1)^s > 0$.

When $x_1(\kappa_1) \ge x_2(\kappa_2)$, then $x_1(\kappa_2) \ge x_1(\kappa_1) \ge x_2(\kappa_2)$. From (3.12b), we get

$$f_2(\kappa_2)e^{x_2(\kappa_2)} = r_2(\kappa_2) - p_2(\kappa_2) + p_2(\kappa_2)e^{x_1(\kappa_2)-x_2(\kappa_2)} \ge r_2(\kappa_2), \tag{3.15}$$

which yields

$$x_1(\kappa_1) \ge x_2(\kappa_2) \ge \ln\frac{r_2(\kappa_2)}{f_2(\kappa_2)} \ge \left(\ln\frac{r_2}{f_2}\right)^s. \tag{3.16}$$

Combing the inequalities (3.8) and (3.10) with (3.14) and (3.16), from (3.1) it easily gets that

$$|x_1(t)| + |x_2(t)| \le 2C_1. \tag{3.17}$$

On the other hand, choose $\kappa_{i+2} \in \mathbb{I}_\omega$ $(i = 1, 2)$ such that $y_1(\kappa_3) = \min_{t\in\mathbb{I}_\omega} y_1(t)$, $y_2(\kappa_4) = \min_{t\in\mathbb{I}_\omega} y_2(t)$, we have $y_i^\delta(\kappa_{i+2}) = 0$ and then

$$r_3(\kappa_3) - f_3(\kappa_3)e^{y_1(\kappa_3)} - \frac{g_2(\kappa_3)e^{x_1(\kappa_3-\tau)}}{e^{x_1(\kappa_3-\tau)} + \beta_1(\kappa_3)e^{y_1(\kappa_3-\tau)}} - \frac{h_1(\kappa_3)e^{y_2(\kappa_3)}}{e^{y_1(\kappa_3)} + \beta_2(\kappa_3)e^{y_2(\kappa_3)}} = 0, \tag{3.18a}$$

$$r_4(\kappa_4) - f_4(\kappa_4)e^{y_2(\kappa_4)} + \frac{h_2(\kappa_4)e^{y_1(\kappa_4-\tau)}}{e^{y_1(\kappa_4-\tau)} + \beta_2(\kappa_4)e^{y_2(\kappa_4-\tau)}} = 0. \tag{3.18b}$$

When $y_1(\kappa_3) < y_2(\kappa_4)$, then $y_1(\kappa_3) < y_2(\kappa_4) \le y_2(\kappa_3)$. From (3.18a), we get

$$\begin{aligned} f_3(\kappa_3)e^{y_1(\kappa_3)} &= r_3(\kappa_3) - \frac{g_2(\kappa_3)e^{x_1(\kappa_3-\tau)}}{e^{x_1(\kappa_3-\tau)} + \beta_1(\kappa_3)e^{y_1(\kappa_3-\tau)}} - \frac{h_1(\kappa_3)e^{y_2(\kappa_3)}}{e^{y_1(\kappa_3)} + \beta_2(\kappa_3)e^{y_2(\kappa_3)}} \\ &\ge r_3(\kappa_3) - g_2(\kappa_3) - \frac{h_1(\kappa_3)}{\beta_2(\kappa_3)}, \end{aligned} \tag{3.19}$$

which means

$$\begin{aligned} y_2(\kappa_4) > y_1(\kappa_3) &\ge \ln\frac{r_3(\kappa_3)\beta_2(\kappa_3) - g_2(\kappa_3)\beta_2(\kappa_3) - h_1(\kappa_3)}{\beta_2(\kappa_3)f_3(\kappa_3)} \\ &\ge \left(\ln\frac{r_3\beta_2 - g_2\beta_2 - h_1}{\beta_2 f_3}\right)^s, \end{aligned} \tag{3.20}$$

under the hypothesis that $(r_3f_3 - g_2f_3 - h_1)^s > 0$.

When $y_1(\kappa_3) \ge y_2(\kappa_4)$, then $y_1(\kappa_4) \ge y_1(\kappa_3) \ge y_2(\kappa_4)$. From (3.18b), we have

$$f_4(\kappa_4)e^{y_2(\kappa_4)} = r_4(\kappa_4) + \frac{h_2(\kappa_4)e^{y_1(\kappa_4-\tau)}}{e^{y_1(\kappa_4-\tau)} + \beta_2(\kappa_4)e^{y_2(\kappa_4-\tau)}} \ge r_4(\kappa_4), \tag{3.21}$$

which implies

$$y_1(\kappa_3) \ge y_2(\kappa_4) \ge \ln\frac{r_4(\kappa_4)}{f_4(\kappa_4)} \ge \left(\ln\frac{r_4}{f_4}\right)^s. \tag{3.22}$$

Combing the inequalities (3.11) with (3.20) and (3.22), from (3.2) it easily follows that

$$\left|y_1(n)\right| + \left|y_2(n)\right| \le 2C_2. \tag{3.23}$$

Set $M_1 = 2C_1 + 2C_2 + 1$, then $\|\mathbf{u}\| < M_1$. M_1 is independent on $\lambda \in (0,1)$. □

Lemma 3.3. *Suppose $\mu \in (0,1)$ is a parameter, $\overline{r}_1\beta_1^s - g_1^M > 0$ and $\overline{r}_3\beta_2^s - g_2^M\beta_2^s - h_1^M > 0$. Then any solution $\mathbf{v} = (v_1, v_2, v_3, v_4)^T$ of the algebraic system*

$$\begin{aligned} 0 &= \overline{r}_1 - \overline{f}_1e^{v_1} + \mu\left(-\overline{p}_1 + \overline{p}_1e^{v_2-v_1}\right) - \frac{\mu}{\omega}\int_{\mathbb{I}_\omega}\frac{g_1(t)e^{v_3}}{e^{v_1} + \beta_1(t)e^{v_3}}\delta t, \\ 0 &= \overline{r}_2 - \overline{f}_2e^{v_2} + \mu\left(-\overline{p}_2 + \overline{p}_2e^{v_1-v_2}\right), \\ 0 &= \overline{r}_3 - \overline{f}_3e^{v_3} - \frac{\mu}{\omega}\int_{\mathbb{I}_\omega}\frac{g_2(t)e^{v_1}}{e^{v_1} + \beta_1(t)e^{v_3}}\delta t - \frac{\mu}{\omega}\int_{\mathbb{I}_\omega}\frac{h_1(t)e^{v_4}}{e^{v_3} + \beta_2(t)e^{v_4}}\delta t, \\ 0 &= \overline{r}_4 - \overline{f}_4e^{v_4} + \frac{\mu}{\omega}\int_{\mathbb{I}_\omega}\frac{h_2(t)e^{v_3}}{e^{v_3} + \beta_2(t)e^{v_4}}\delta t \end{aligned} \tag{3.24}$$

satisfies $\|\mathbf{v}\| \le 2C_3 + 2C_4$, *where*

$$C_3 := \max\left\{\left|\ln \frac{\overline{r}_1\beta_1^s - g_1^M}{\overline{f}_1\beta_1^s}\right|, \left|\ln \frac{\overline{r}_1}{\overline{f}_1}\right|, \left|\ln \frac{\overline{r}_2}{\overline{f}_2}\right|\right\}, \tag{3.25}$$

$$C_4 := \max\left\{\left|\ln \frac{\overline{r}_3\beta_2^s - g_2^M\beta_2^s - h_1^M}{\overline{f}_3\beta_2^s}\right|, \left|\ln \frac{\overline{r}_3}{\overline{f}_3}\right|, \left|\ln \frac{\overline{r}_4}{\overline{f}_4}\right|, \left|\ln \frac{\overline{r}_4 + h_2^M}{\overline{f}_4}\right|\right\}. \tag{3.26}$$

Proof. When $v_2 \le v_1$, from the first two equations of (3.24) and Lemma 2.4, we obtain that

$$\begin{aligned} \overline{f}_1 e^{v_1} &= \overline{r}_1 + \mu\left(-\overline{p}_1 + \overline{p}_1 e^{v_2 - v_1}\right) - \frac{\mu e^{v_3}}{\omega}\int_{\mathbb{I}_\omega} \frac{g_1(t)}{e^{v_1} + \beta_1(t)e^{v_3}}\delta t \le \overline{r}_1, \\ \overline{f}_2 e^{v_2} &= \overline{r}_2 + \mu\left(-\overline{p}_2 + \overline{p}_2 e^{v_1 - v_2}\right) \ge \overline{r}_2, \end{aligned} \tag{3.27}$$

which implies that

$$\ln \frac{\overline{r}_2}{\overline{f}_2} \le v_2 \le v_1 \le \ln \frac{\overline{r}_1}{\overline{f}_1}. \tag{3.28}$$

Analogously, when $v_1 < v_2$, we have

$$\begin{aligned} \overline{f}_1 e^{v_1} &= \overline{r}_1 + \mu\left(-\overline{p}_1 + \overline{p}_1 e^{v_2 - v_1}\right) - \frac{\mu e^{v_3}}{\omega}\int_{\mathbb{I}_\omega} \frac{g_1(t)}{e^{v_1} + \beta_1(t)e^{v_3}}\delta t \ge \overline{r}_1 - \frac{g_1^M}{\beta_1^s}, \\ \overline{f}_2 e^{v_2} &= \overline{r}_2 + \mu\left(-\overline{p}_2 + \overline{p}_2 e^{v_1 - v_2}\right) \le \overline{r}_2; \end{aligned} \tag{3.29}$$

it follows that

$$\ln \frac{\overline{r}_1\beta_1^s - g_1^M}{\overline{f}_1\beta_1^s} \le v_1 < v_2 \le \ln \frac{\overline{r}_2}{\overline{f}_2}, \tag{3.30}$$

by the assumption that $\overline{r}_1\beta_1^s - g_1^M > 0$.

Hence, from (3.25) we have $|v_1| + |v_2| \le 2M_3$.

On the other hand, with similar discussion above, from the last two of (3.24), we obtain

$$\begin{aligned} \ln \frac{\overline{r}_3\beta_2^s - g_2^M\beta_2^s - h_1^M}{\overline{f}_3\beta_2^s} &\le v_3 \le \ln \frac{\overline{r}_3}{\overline{f}_3}, \\ \ln \frac{\overline{r}_4}{\overline{f}_4} &\le v_4 \le \ln \frac{\overline{r}_4 + h_2^M}{\overline{f}_4}, \end{aligned} \tag{3.31}$$

which imply that $|v_3| + |v_4| \le 2C_4$ from (3.26). So, $\|v\| \le 2C_3 + 2C_4$. □

With the preparations above, we can complete the proof of Theorem 3.1 as follows.

Proof of Theorem 3.1. Set $M_2 = 2C_3 + 2C_4 + 1$. By Lemma 3.3, we know that any solution $\mathbf{v}$ of the system (3.24) satisfies $\|\mathbf{u}\| < M_2$. Take $C = M_1 + M_2$, and define $\Omega = \{\mathbf{u} \in \Gamma : \|\mathbf{u}\| < C\}$. Due to Lemmas 3.2 and 3.3, condition (C1) in Lemma 2.5 is satisfied.

Let

$$\mathcal{L} : \operatorname{Dom}\mathcal{L} \cap \Gamma \longrightarrow \Gamma,$$
$$\mathcal{L}\mathbf{u}(t) = \begin{pmatrix} x_1^{\delta}(t) \\ x_2^{\delta}(t) \\ y_1^{\delta}(t) \\ y_2^{\delta}(t) \end{pmatrix}, \tag{3.32}$$

where $\operatorname{Dom}\mathcal{L} = \{(x_1(t), x_2(t), y_1(t), y_2(t))^T \in C(\mathbb{T}, \mathbb{R}^4)\}$ and

$$\mathcal{N} : \Gamma \longrightarrow \Gamma,$$
$$\mathcal{N}\mathbf{u}(t) = \begin{pmatrix} N_1(t) \\ N_2(t) \\ N_3(t) \\ N_4(t) \end{pmatrix}, \tag{3.33}$$

where

$$\begin{aligned}
N_1(t) &= r_1(t) - f_1(t)e^{x_1(t)} - \frac{g_1(t)e^{y_1(t-\tau)}}{e^{x_1(t-\tau)} + \beta_1(t)e^{y_1(t-\tau)}} - p_1(t) + p_1(t)e^{x_2(t)-x_1(t)}, \\
N_2(t) &= r_2(t) - f_2(t)e^{x_2(t)} - p_2(t) + p_2(t)e^{x_1(t)-x_2(t)}, \\
N_3(t) &= r_3(t) - f_3(t)e^{y_1(t)} - \frac{g_2(t)e^{x_1(t-\tau)}}{e^{x_1(t-\tau)} + \beta_1(t)e^{y_1(t-\tau)}} - \frac{h_1(t)e^{y_2(t)}}{e^{y_1(t)} + \beta_2(t)e^{y_2(t)}}, \\
N_4(t) &= r_4(t) - f_4(t)e^{y_2(t)} + \frac{h_2(t)e^{y_1(t-\tau)}}{e^{y_1(t-\tau)} + \beta_2(t)e^{y_2(t-\tau)}}.
\end{aligned} \tag{3.34}$$

With the definitions above, we obtain that $\mathcal{L}\mathbf{u} = \mathcal{N}\mathbf{u}$ for $\mathbf{u} \in \operatorname{Dom}\mathcal{L} \cap \Gamma$ with $\operatorname{Im}\mathcal{L} = \{\mathbf{u} \in \Gamma : \int_{\mathbb{I}_\omega} x_i(t)\delta t = 0, \int_{\mathbb{I}_\omega} y_i(t)\delta t = 0,\ t \in \mathbb{T},\ i = 1,2\}$ and $\ker\mathcal{L} = \mathbb{R}^4$ which is closed in Γ, and $\dim(\ker\mathcal{L}) = \operatorname{codim}(\operatorname{Im}\mathcal{L}) = 4$. Therefore, $\mathcal{L}$ is a Fredholm mapping of index zero. Moreover, define two projections P, Q such that $\operatorname{Im}P = \ker\mathcal{L}$ and $\operatorname{Im}\mathcal{L} = \ker Q = \operatorname{Im}(I - Q)$, where

$$P = Q : \Gamma \longrightarrow \Gamma,$$
$$P\mathbf{u} = Q\mathbf{u} = \begin{pmatrix} \dfrac{1}{\omega}\displaystyle\int_{\mathbb{I}_\omega} x_1(t)\delta t \\ \dfrac{1}{\omega}\displaystyle\int_{\mathbb{I}_\omega} x_2(t)\delta t \\ \dfrac{1}{\omega}\displaystyle\int_{\mathbb{I}_\omega} y_1(s)\delta s \\ \dfrac{1}{\omega}\displaystyle\int_{\mathbb{I}_\omega} y_2(s)\delta s \end{pmatrix}. \tag{3.35}$$

Then $\Gamma = \ker \mathcal{L} \oplus \ker P = \ker \mathcal{L} \oplus \ker Q$, and choose Ψ as the identity isomorphism of $\operatorname{Im} Q$ to $\ker P$. Furthermore, the generalized inverse (to $\mathcal{L}$) exists and is given by

$$\Phi : \operatorname{Im} \mathcal{L} \longrightarrow \operatorname{Dom} \mathcal{L} \cap \ker P,$$

$$\Phi \mathbf{u} = \begin{pmatrix} \displaystyle\int_\eta^s x_1(t)\delta t - \frac{1}{\omega}\int_{\mathbb{I}_\omega}\int_\eta^t x_1(s)\delta s\delta t \\ \displaystyle\int_\eta^s x_2(t)\delta t - \frac{1}{\omega}\int_{\mathbb{I}_\omega}\int_\eta^t x_2(s)\delta s\delta t \\ \displaystyle\int_\eta^t y_1(s)\delta s - \frac{1}{\omega}\int_{\mathbb{I}_\omega}\sum_\eta^t y_1(s)\delta s \\ \displaystyle\int_\eta^t y_2(s)\delta s - \frac{1}{\omega}\int_{\mathbb{I}_\omega}\sum_\eta^t y_2(s)\delta s \end{pmatrix}. \tag{3.36}$$

Thus

$$Q \circ \mathcal{N}\mathbf{u} = \left(\frac{1}{\omega}\int_{\mathbb{I}_\omega} N_1(t)\delta t, \frac{1}{\omega}\int_{\mathbb{I}_\omega} N_2(t)\delta t, \frac{1}{\omega}\int_{\mathbb{I}_\omega} N_3(t)\delta t, \frac{1}{\omega}\int_{\mathbb{I}_\omega} N_4(t)\delta t\right)^T. \tag{3.37}$$

Clearly, $Q \circ \mathcal{N}$ and $\Phi \circ (I - Q)$ are well defined. By the Lebesgue convergence theorem and the Arzela-Ascoli theorem, $\Phi \circ (I - Q)(\overline{\Omega})$ is relatively compact for any open-bounded set $\Omega \subset \Gamma$. Moreover, $Q \circ \mathcal{N}(\overline{\Omega})$ is bounded. Therefore, $\mathcal{N}$ is $\mathcal{L}$-compact on $\overline{\Omega}$ for any open-bounded set $\Omega \subset \Gamma$. When $\mathbf{u} \in \partial\Omega \cap \mathbb{R}^4$ is a constant vector in $\mathbb{R}^4$, then $Q \circ \mathcal{N}\mathbf{u} \neq 0$ since $Q \circ \mathcal{N}\mathbf{u} = 0$ is the system (3.24) with $\epsilon = 1$. Condition (C2) in Lemma 2.5 is also satisfied.

Finally, we claim that $\deg(\Psi \circ Q \circ \mathcal{N}, \Omega, O) \neq 0$, where $O := (0,0,0,0)^T$. In fact, consider the homotopy

$$H_\mu \mathbf{v} = \mu Q \circ \mathcal{N}\mathbf{v} + (1-\mu)G\mathbf{v}, \quad \mu \in [0,1], \tag{3.38}$$

where $G\mathbf{v} = (\overline{r}_1 - \overline{f}_1 e^{v_1}, \overline{r}_2 - \overline{f}_2 e^{v_2}, \overline{r}_3 - \overline{f}_3 e^{v_3}, \overline{r}_4 - \overline{f}_4 e^{v_4})^T$.

When $\mathbf{v} \in \Omega \cap \ker \mathcal{L} = \Omega \cap \mathbb{R}^4$ is a constant vector with $\|\mathbf{v}\| = C$, from Lemma 2.5, we get that $H_\mu \mathbf{v} \neq O$ on $\partial\Omega \cap \ker \mathcal{L}$. Since $\operatorname{Im} Q = \ker \mathcal{L}$ and $(v_1^*, v_2^*, v_3^*, v_4^*)^T \in \Omega \cap \ker \mathcal{L}$ is the unique solution of the algebraic equations $G\mathbf{v} = (0,0,0,0)^T$, by the homotopy invariance of Brouwer degree, we obtain

$$\deg(\Psi \circ Q \circ \mathcal{N}, \partial\Omega \cap \ker \mathcal{L}, O) = \operatorname{sign}\left(-\overline{f}_1 \overline{f}_2 \overline{f}_3 \overline{f}_4 e^{v_1^* + v_2^* + v_3^* + v_4^*}\right) \neq 0. \tag{3.39}$$

Therefore, all the conditions in Lemma 2.5 are fulfilled and the dynamic system (1.2) has at least one ω-periodic solution lying in $\operatorname{Dom} \mathcal{L} \cap \overline{\Omega}$. □

4. Example

Consider the following system with 20π-periodic time scale:

$$\begin{aligned}
x_1^{\delta}(t) &= 4 - 2\cos\frac{t}{10} - \left(5 - \sin\frac{t}{10}\right)e^{x_1(t)} - \frac{(5-\cos(t/10)+2\sin(t/10))e^{y_1(t-1/3)}}{e^{x_1(t-1/3)}+(5-\cos(t/10))e^{y_1(t-1/3)}} \\
&\quad + \left(\frac{5}{3} - \frac{\sin t}{10}\right)\left(e^{x_2(t)-x_1(t)} - 1\right), \\
x_2^{\delta}(t) &= 5 - 2\sin\frac{t}{10} - \left(4 + \cos\frac{t}{10}\right)e^{x_2(t)} + \left(\frac{5}{4} - \cos\left(\frac{t}{10}\right)\right)\left(e^{x_1(t)-x_2(t)} - 1\right), \\
y_1^{\delta}(t) &= 5 + 2\sin\frac{t}{10} + \cos\frac{t}{10} - \frac{(2-\cos(t/10))e^{y_2(t)}}{e^{y_1(t)}+(4-\sin(t/10))e^{y_2(t)}} \\
&\quad - \left(\frac{7}{3} - \cos\frac{t}{10}\right)e^{y_1(t)} - \frac{(2+\sin(t/10))e^{x_1(t-1/3)}}{e^{x_1(t-1/3)}+(5-\cos(t/10))e^{y_1(t-1/3)}}, \\
y_2^{\delta}(t) &= 5 + 2\sin\frac{t}{10} - \left(5 + \cos\frac{t}{10}\right)e^{y_2(t)} + \frac{(3+\sin(t/10))e^{y_1(t-1/3)}}{e^{y_1(t-1/3)}+(4-\sin t/10)e^{y_2(t-1/3)}}.
\end{aligned} \tag{4.1}$$

From the definition of $\mathbb{I}_{\omega}$, we obtain that $\mathbb{I}_{\omega} = [0, 20\pi]$. It is straight to check that $(r_1\beta_1 - g_1)^s = \min_t\{r_1(t)\beta_1(t) - g_1(t)\} = 15 - 13\cos(t/10) + 2\sin(t/10) + 2\cos^2(t/10) > 0$, and other inequalities $\overline{r}_1\beta_1^s - g_1^M > 0$, $(r_3\beta_2 - g_2\beta_2 - h_1)^s > 0$ and $\overline{r}_3\beta_2^s - g_2^M\beta_2^s - h_1^M > 0$. Hence, from Theorem 3.1, the dynamic system (4.1) has at least one 20-periodic solution on the time scale $\mathbb{T}$.

Acknowledgments

The authors would like to thank the referee for many helpful suggestions. This work is supported by the National Natural Science Foundation of China (no. 61003287) and the Fundamental Research Funds for the Central Universities (no. SWJT U11BR174).

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