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Research Article

Statistical Behavior of a Financial Model by Lattice Fractal Sierpinski Carpet Percolation

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The lattice fractal Sierpinski carpet and the percolation theory are applied to develop a new random stock price for the financial market. Percolation theory is usually used to describe the behavior of connected clusters in a random graph, and Sierpinski carpet is an infinitely ramified fractal. In this paper, we consider percolation on the Sierpinski carpet lattice, and the corresponding financial price model is given and investigated. Then, we analyze the statistical behaviors of the Hong Kong Hang Seng Index and the simulative data derived from the financial model by comparison.

1. Introduction

Financial fluctuation system is one of complex systems, and the statistical behavior of fluctuation of stock price changes has long been a focus of financial research. With the flourishing research of complex systems, it becomes more and more attractive to find universal rules and principles of these systems and further to answer the origination of financial complex system. Recent research is no longer restricted to the traditional areas but concentrated on the more comprehensive domains, leading to the birth of many burgeoning disciplines through the interaction and amalgamation of mathematics and other fields such as finance, biology, and sociology. For example, the theory of stochastic interacting particle systems (see [1–6]) recently has been applied to study the behaviors of market fluctuations, see [7–15]. And the study of financial market prices has been found to exhibit some universal properties similar to those observed in interacting particle systems with a large number of interacting units.

Percolation theory, as a model (in interacting particle systems) for a disordered medium, has brought new understanding and techniques to a broad range of topics in nature and society. First we consider the bond percolation on $\mathbb{Z}^d$, that is, for $x, y \in \mathbb{Z}^d$, the distance $\delta(x,y)$ from x to y is defined by $\delta(x,y) = \sum_{i=1}^{d} |x_i - y_i|$, where $x = (x_1, \ldots, x_d)$ and $y = (y_1, \ldots, y_d)$. By adding edges (or bonds) between all pairs x, y of points of $\mathbb{Z}^d$ with $\delta(x,y) = 1$, we establish the d-dimensional lattice $\mathbb{L}^d = (\mathbb{Z}^d, \mathbb{E}^d)$, and we write $\mathbb{E}^d$ for the set of the edges. Suppose that each bond of lattice $\mathbb{L}^d$ is either open (occupied) with probability p or closed (empty) with probability $1-p$, then connected components of this graph are called open clusters. Let $C(x)$ denote the open cluster containing the vertex x, and $\theta(p) = P(|C(0)| = \infty)$ be the probability that the origin belongs to an infinite open cluster. When the intensity p increases from zero to one, at some sharp percolation threshold (or critical point) p_c, for the first time, one infinite cluster appears; for all $p > p_c$ we have exactly one infinite cluster, for all $p < p_c$ we have no infinite cluster, and at critical value $p = p_c$ the incipient infinite clusters are supposed to be fractal.

A lattice fractal is a graph which corresponds to a fractal, all of them have a self-similarity, but most of them have no translation invariance, see [1, 16–19]. The Sierpinski gasket and the Sierpinski carpet are well-known examples of fractals. The former is a finitely ramified fractal (i.e., it can be disconnected by removing a finite number of points) and the latter is an infinitely ramified fractal. Fractals also have close relations to financial markets [17], electrical conductivity, superconductivity, and mechanical properties of percolating systems, and so forth. In [1], it shows that the Ising model on the lattice Sierpinski carpet does exhibit the phase transition in any dimension, but the Ising model on the lattice Sierpinski gasket has no phase transition in any dimension (because of the character of the finitely ramified fractal). Similar results of phase transitions can be obtained for percolation on the lattice Sierpinski carpet and on the lattice Sierpinski gasket, see [18].

In the present paper, a new method is introduced to model and describe the fluctuations of market prices, namely, we use the lattice fractal Sierpinski carpet percolation to establish a new random market price in a financial market. In this financial model, the local interaction or influence among traders in one stock market is constructed, and a cluster of percolation is used to define the cluster of traders sharing the same opinion about the market. For the comparison, we also consider the most important index of Hong Kong financial market, the Hong Kong Hang Seng Index. We analyze the statistical properties of Hong Kong Hang Seng Index and the simulative data derived from the price model by comparison, which including the sharp peak and the fat-tail distribution for the price changes, the distribution of returns decays with power law in the tails, the price fluctuations are not invariant against time reversal (i.e., they show a forward-backward asymmetry), and so forth. Moreover, the behaviors of long memory and long-range correlation in volatility series of market returns are exhibited.

2. Description of Price Model on Lattice Sierpinski Carpet Percolation

First we give a brief description of percolation on the lattice Sierpinski carpet $\mathbb{S}^{(d)}$ (for $d = 2$), which is defined as follows: consider $\mathbb{Z}^2$ as a graph in the usual sense and set

$$\widetilde{\mathbb{S}}_0^{(2)} = \mathbb{Z}^2 \cap [0,3]^2, \qquad \widetilde{\mathbb{S}}_{n+1}^{(2)} = \bigcup_{\substack{i_1, i_2 \in \{0,1,2\} \\ (i_1, i_2) \neq (1,1)}} \left\{ \left(i_1 3^{n+1}, i_2 3^{n+1} \right) + \widetilde{\mathbb{S}}_n^{(2)} \right\}, \tag{2.1}$$

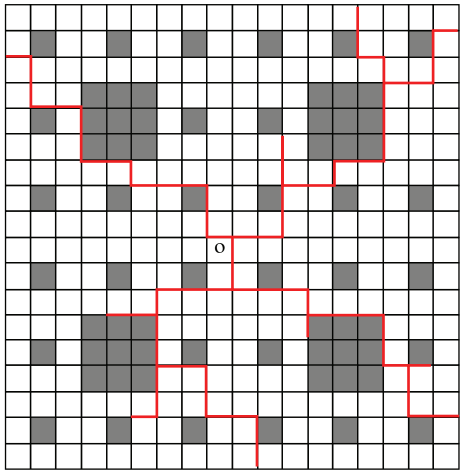


Figure 1: Lattice percolation on lattice Sierpinski carpet.

where $u + \widetilde{\mathbb{S}}_n^{(2)} = \{u + v : v \in \widetilde{\mathbb{S}}_n^{(2)}\}$. To make the graph more symmetric, let $\mathbb{S}_n^{(2)}$ be the union of $\widetilde{\mathbb{S}}_n^{(2)}$ and its reflections in every coordinate hyperplane. Then we define the lattice Sierpinski carpet as

$$\mathbb{S}^{(2)} = \bigcup_{n=0}^{\infty} \mathbb{S}_n^{(2)}. \tag{2.2}$$

Similarly to Section 1, we define the corresponding edges set of $\mathbb{S}^{(2)}$ as $\mathbb{E}(\mathbb{S}^{(2)})$. Next we consider random graph (bond percolation) on the lattice $\mathbb{L}(\mathbb{S}^{(2)}) = (\mathbb{S}^{(2)}, \ \mathbb{E}(\mathbb{S}^{(2)}))$, see Figure 1. Let p (the intensity value) satisfies $0 \le p \le 1$, each edge of $\mathbb{L}(\mathbb{S}^{(2)})$ is declared to be open with probability p and closed with probability $1 - p$ independently. We denote the product probability by P_p (or P), and define $\theta(p) = P(|C(0)| = \infty)$, where $C(0)$ is the open cluster containing the origin on $\mathbb{L}(\mathbb{S}^{(2)})$, and $|C(0)|$ is the number of vertices in $C(0)$. Let $p_c(\mathbb{S}^{(2)}) = \inf\{p : \theta(p) > 0\}$, then percolation on the Sierpinski carpet $\mathbb{S}^{(2)}$ exhibits the existence of a phase transition, that is, $\theta(p) > 0$ for $p > p_c(\mathbb{S}^{(2)})$, for details see [1, 18].

Next we consider a price model of auctions for a stock in a stock market. Assume that each trader can trade the stock several times at each day $t \in \{1, 2, \ldots, T\}$, but at most one unit number of the stock at each time. Let $S(t)$ denote the daily closing price of tth trading day. And let Λ_n be a subset of $\mathbb{S}^{(2)}$, where

$$\Lambda_n = \left\{ (x_1, x_2) \in \mathbb{S}^{(2)} : -3^n \le x_1 \le 3^n, -3^n \le x_2 \le 3^n \right\} \tag{2.3}$$

and $C_t(0)$ be a random open cluster on Λ_n. Suppose that this stock consists of $|\Lambda_n|$ (n is large enough) investors, who are located in Λ_n lattice. And $C_t(0)$ is a random set of the selected traders who receive the information. At the beginning of trading in each day, suppose that the investors receive some news. We define a random variable ζ_t for these investors, suppose that these investors taking buying positions ($\zeta_t = 1$) selling positions ($\zeta_t = -1$), or neutral positions ($\zeta_t = 0$) with probability q_1, q_{-1} or $1 - (q_1 + q_{-1})(q_1, q_2 > 0, q_1 + q_2 \le 1)$, respectively. Then these investors send bullish, bearish or neutral signal to the market. According to bond percolation on $\mathbb{S}^{(2)}$, investors can affect each other or the news can be spread, which is assumed as the main factor of price fluctuations. For a fixed $t \in \{1, 2, \ldots, T\}$, let

$$B_t = \frac{\zeta_t |C_t(0)|}{|\Lambda_n|}. \tag{2.4}$$

From the above definitions and mathematical finance theory [20–24], we define the stock price at tth trading day as

$$S(t) = e^{\alpha(t) B_t} S(t-1), \tag{2.5}$$

where $S(0)$ is the initial stock price at time 0, and $\alpha(t)(>0)$ represents the depth function of the market at trading day t. Then we have

$$S(t) = S(0) \exp\left\{ \alpha(t) \sum_{k=1}^{t} B_k \right\}, \quad t \in \{1, 2, \ldots, T\}. \tag{2.6}$$

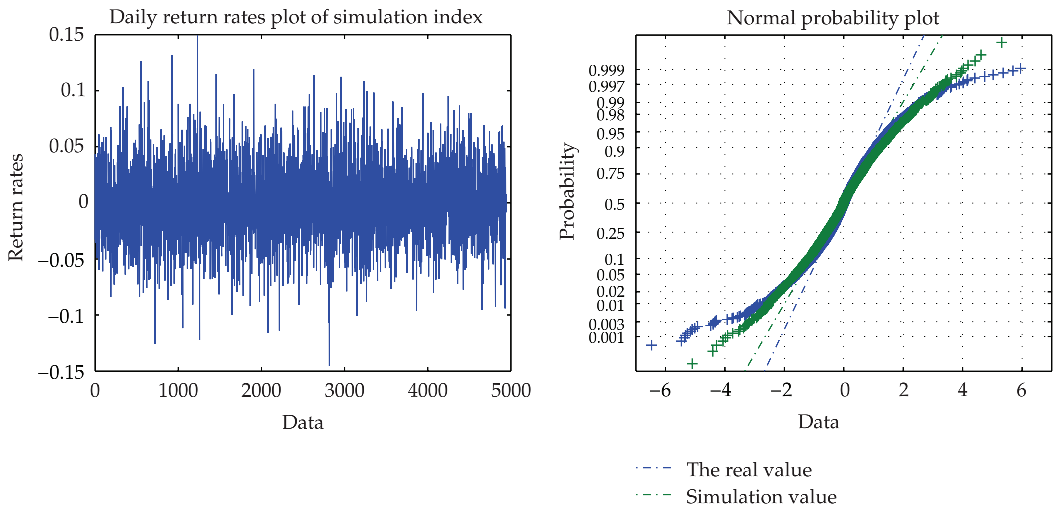
The formula of the single-period stock logarithmic returns from t to $t+1$ is given as follows:

$$r(t) = \ln S(t+1) - \ln S(t), \quad t \in \{1, 2, \ldots, T\}. \tag{2.7}$$

3. Experiment Analysis of Market Return Distribution

In order to make empirical research on the financial price model and an actual stock market by comparison, we select the daily closing prices of Hang Seng Index in the 20-year period from September 3, 1990 to September 3, 2010, the total number of observed data is about 4942. Recent research shows that returns on financial markets are not Gaussian, but exhibit excess kurtosis and fatter tails than the normal distribution, which is usually called the “fat-tail” phenomenon, see [21, 25–30]. The general explanation for this phenomenon is thought to be the “herd effect” of investors in the market. The time series of returns by simulating the price model which is developed on the Sierpinski carpet percolation is plotted in Figure 2(a). The returns distributions of Hang Seng Index and the financial model are plotted in Figure 2(b), the part (where the probability is above the 75th or below 25th percentiles of the samples) deviates from the dash line. This implies that the probability distributions of returns deviate from the corresponding normal distributions at the tail parts.

For further analyzing the character of returns distributions for the simulative data and Hang Seng Index, we make the single-sample Kolmogorov-Smirnov test by the statistical method, the basic statistics of the corresponding returns is displayed in Table 1. The value of two-tail test P is 0.000, thus the hypothesis is denied that the distribution of returns follows the Gaussian distribution.



(a) The returns time series of the price model

(b) The plot of returns distributions

Figure 2: (a) The returns time series of simulation data for the price model with the intensity $p = 0.49$. (b) The comparison of returns distributions for 20-year period Hang Seng index and the simulative data with $p = 0.49$.

Table 1: The Kolmogorov-Smirnov test.

	The financial model	Hang Seng Index
Capability	4942	4942
The H value	1	1
The P value of double tail	0.0000	0.0000
K-S statistics to measure	0.4633	0.4736
The CV value	0.0193	0.0193

In this part, we study the properties of skewness and kurtosis on the returns for the simulative data and Hang Seng Index. Next we give the definitions of skewness and kurtosis as follows:

$$\text{Skewness} = \sum_{i=1}^{n} \frac{(r_i - u_r)^3}{(n-1)\delta^3}, \quad \text{Kurtosis} = \sum_{i=1}^{n} \frac{(r_i - u_r)^4}{(n-1)\delta^4}, \tag{3.1}$$

where r_i denotes the return of ith trading day, u_r is the mean of r, n is the total number of trading dates, and δ is the corresponding standard variance. Kurtosis shows the centrality of data, and the skewness shows the symmetry of the data; it is a measure of the "peakedness" of the probability distribution of a real-valued random variable, and the infrequent extreme deviations lead higher kurtosis. Skewness is important because kurtosis is not independent of skewness, and the latter may "induce" the former. It is known that the skewness of standard normal distribution is 0 and the kurtosis is 3. Next we investigate the statistical behaviors of the returns for different intensity values p , where the value p changes from 0.39 to 0.55 with the interval length 0.01.

Table 2: The analysis of the price model for different intensity values p.

p	Kurtosis	Skewness	Mean	Variance	Min	Max
0.55	5.144784	−0.09809	$-9.88E-06$	$2.19E-07$	−0.00242	0.002424
0.54	5.131352	−0.02482	$-7.67E-06$	$2.80E-07$	−0.0029	0.003056
0.53	4.893179	0.02581	$2.39E-06$	$3.35E-07$	−0.00316	0.003425
0.52	5.145840	−0.12553	$1.41E-06$	$4.17E-07$	−0.00411	0.003056
0.51	4.923338	−0.03637	$1.16E-05$	$5.47E-07$	−0.00379	0.003741
0.50	4.417513	0.061596	$6.37E-06$	$6.88E-07$	−0.00374	0.004057
0.49	5.114434	−0.10168	$-6.15E-06$	$8.47E-07$	−0.00559	0.004321
0.48	4.697023	0.030538	$-1.25E-05$	$1.11E-06$	−0.00495	0.005954
0.47	4.484873	−0.07156	$-1.90E-05$	$1.47E-06$	−0.00669	0.005427
0.46	5.045552	0.13255	$1.03E-05$	$1.92E-06$	−0.00801	0.00743
0.45	5.066152	−0.07007	$-3.35E-05$	$2.46E-06$	−0.01112	0.008536
0.44	4.491076	0.076139	$2.90E-05$	$3.11E-06$	−0.00843	0.008325
0.43	4.905725	−0.14384	$5.43E-05$	$4.61E-06$	−0.01565	0.008115
0.42	5.029006	−0.03238	$-1.70E-06$	$6.18E-06$	−0.01655	0.013437
0.41	4.905482	−0.08227	$-2.31E-05$	$8.84E-06$	−0.01813	0.013806
0.40	4.539316	−0.05757	$1.30E-05$	$1.23E-05$	−0.01739	0.01702
0.39	4.431923	0.042044	$-6.44E-06$	$1.92E-05$	−0.02176	0.019338

Table 2 gives a description of the statistics for 17 group data of the price model. This shows that the distribution of the returns deviates from the Gaussian distribution with the intensity values p increasing, and the kurtosis distribution of the returns has a sharper peak, longer and fatter tails for larger p. From the definitions in Section 2, p is the intensity for the Sierpinski carpet lattice percolation and represents the strength of information spread in the price model. The wider the information spread, the larger the value of p is. In the following, we hope to exhibit that the numerical characteristics of simulation results for some intensity p are very close to those of the real data. We analyze the probability distributions of the logarithmic returns and the cumulative distributions of the normalized returns for these data in Figure 3, where the intensity values of the model are $p = 0.485$, $p = 0.49$, and $p = 0.495$, respectively.

4. Long Memory Test of the Model and Hang Seng Index

We analyze the long memory of the returns by using Lo's modified rescaled range statistic [31]. The long memory is measured by the Hurst exponent H, calculated by Lo's modified rescaled range statistic. For $0.5 < H < 1$, the series exhibits the long-term persistence, with the maintenance of tendency; for $0 < H < 0.5$, the series is the antipersistent, presenting reversion to the mean; and for $H = 0.5$, the series corresponds to a random walk. We consider a sample of series $X_1, X_2, \ldots, X_n$ and let $\overline{X}_n$ denote the sample mean. Then the modified rescaled range statistic, denoted by Q_n, is defined by

$$Q_n = \frac{1}{\widehat{\sigma}_n(q)} \left[\max_{1\le k\le n} \sum_{j=1}^{k} \left(X_j - \overline{X}_n \right) - \min_{1\le k\le n} \sum_{j=1}^{k} \left(X_j - \overline{X}_n \right) \right], \tag{4.1}$$

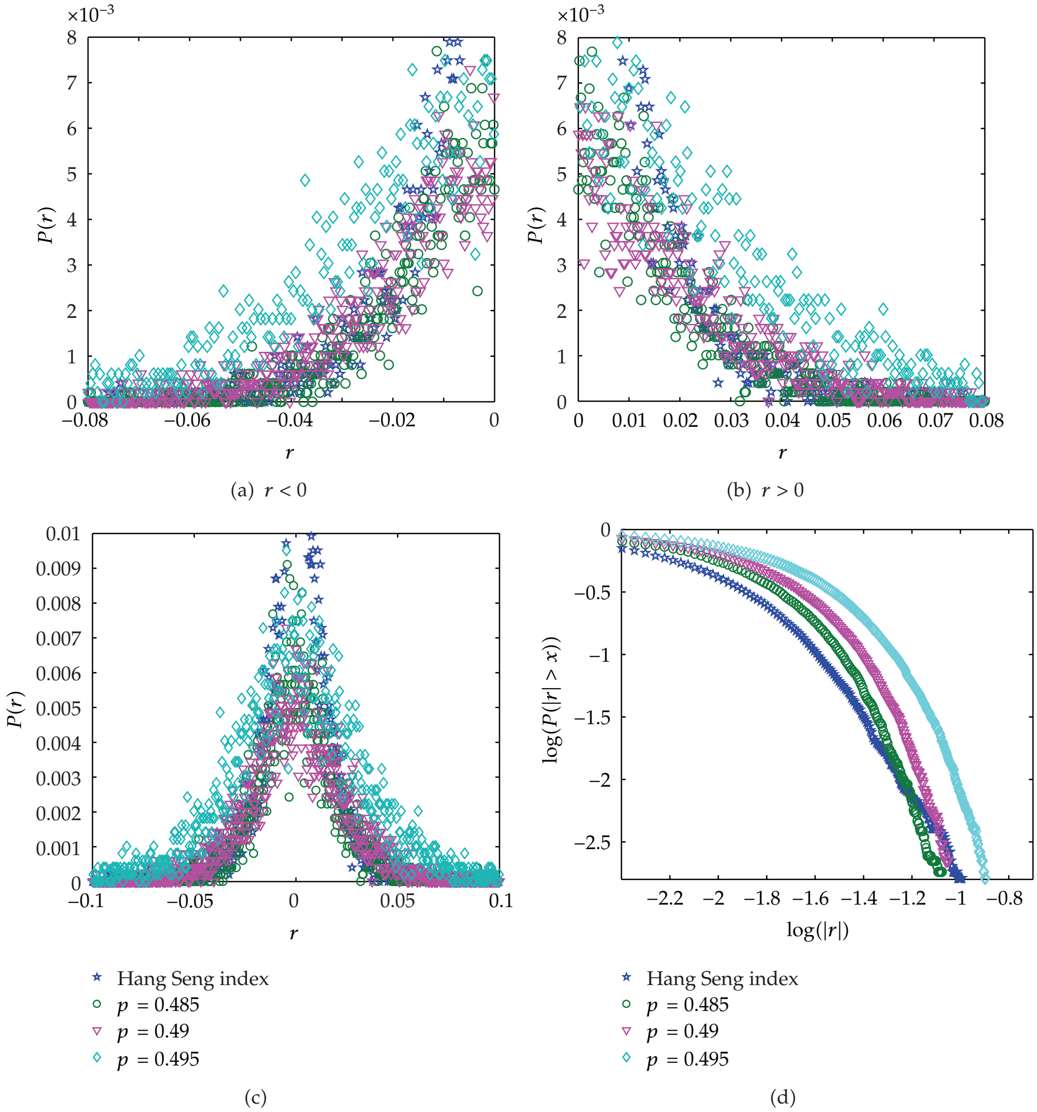


Figure 3: The plots (a), (b), and (c) are the probability distributions of the logarithmic returns, and the plot (d) is the cumulative distributions of the normalized price returns. The data is selected from Hang Seng Index and from the simulation data with the different values p, $p = 0.485$, $p = 0.49$ and $p = 0.495$.

where

$$\begin{aligned}
\widehat{\sigma}_n^2(q) &= \frac{1}{n}\sum_{j=1}^{n}\left(X_j - \overline{X}_n\right)^2 + \frac{2}{n}\sum_{j=1}^{q}\omega_j(q)\left[\sum_{i=j+1}^{n}\left(X_i - \overline{X}_n\right)\left(X_{i-j} - \overline{X}_n\right)\right] \\
&= \widehat{\sigma}_X^2 + 2\sum_{j=1}^{q}\omega_j(q)\widehat{\gamma}_j, \\
\omega_j(q) &= 1 - \frac{j}{q+1}(q < n)
\end{aligned} \tag{4.2}$$

$\widehat{\sigma}_X^2$ and $\widehat{\gamma}_j$ denote the sample variance and the autocovariance estimators of X.

Table 3: Statistics of returns for V and H.

Return series	V statistic results				Hurst index results	
	n	First-order autocorrelation	q	V	Intercept c	H
The financial model	4942	0.018461152	2.1624	1.3901	0.129960913	0.5175
Hang Seng Index	4942	0.009450232	1.3835	1.2361	−0.357078997	0.5893

In order to make the statistical inference for the above-modified rescaled range statistics, Lo derived that $V_n(q) = n^{1/2}Q_n$ converges weakly to a random variable V, where V is the range of a Brownian bridge on the unit interval. Then the corresponding distribution function of V is given by

$$F(v) = 1 + 2\sum_{k=1}^{\infty}\left(1 - 4k^2v^2\right)e^{-2(kv)^2}. \tag{4.3}$$

Form this function $F(v)$, we can get test threshold for any level of significance (by examining significant of $V_n(q)$), this reflects the long memory behavior for the time series. It is important to select the window wide q; we take the experience value

$$q = \left(\frac{3T}{2}\right)^{1/3} \cdot \left(\frac{2\hat{\rho}_1}{1 - \hat{\rho}_1^2}\right)^{2/3}, \tag{4.4}$$

where $\hat{\rho}_1$ is the estimation of first-order autocorrelation coefficient of the time series. Then the Hurst exponent H is defined as the limit of the ratio $\log Q_n / \log n$. At the same time, it shows a linear growth trend between modified R/S statistic and sample size n, by using regression

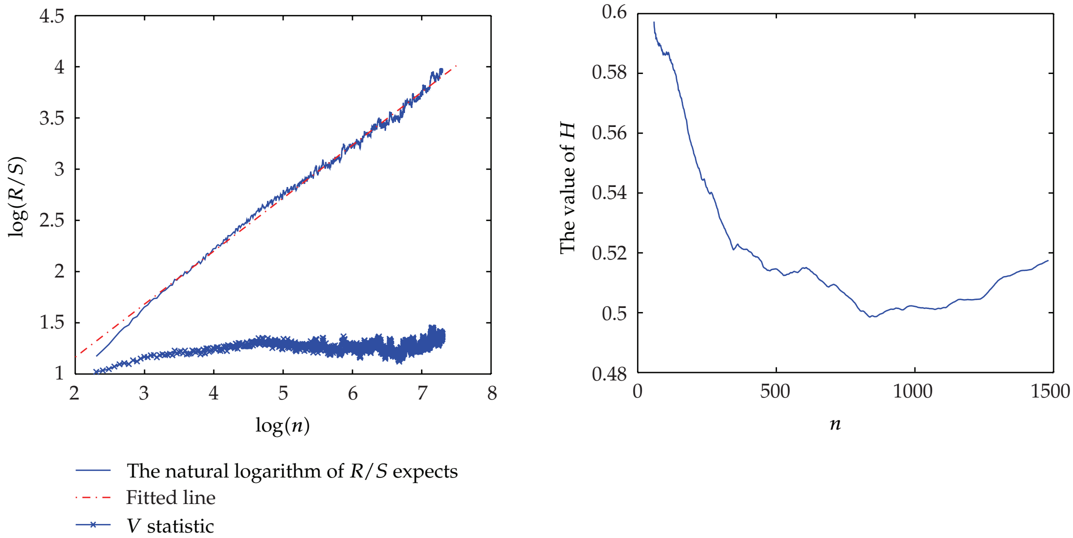
$$\ln Q_n = \ln c + H \ln n. \tag{4.5}$$

With some optimal q value, the statistics of returns by the modified R/S statistic is given in Table 3 and Figure 4. Figure 4 also shows the fluctuations of exponent H of returns for the price model and Hang Seng Index.

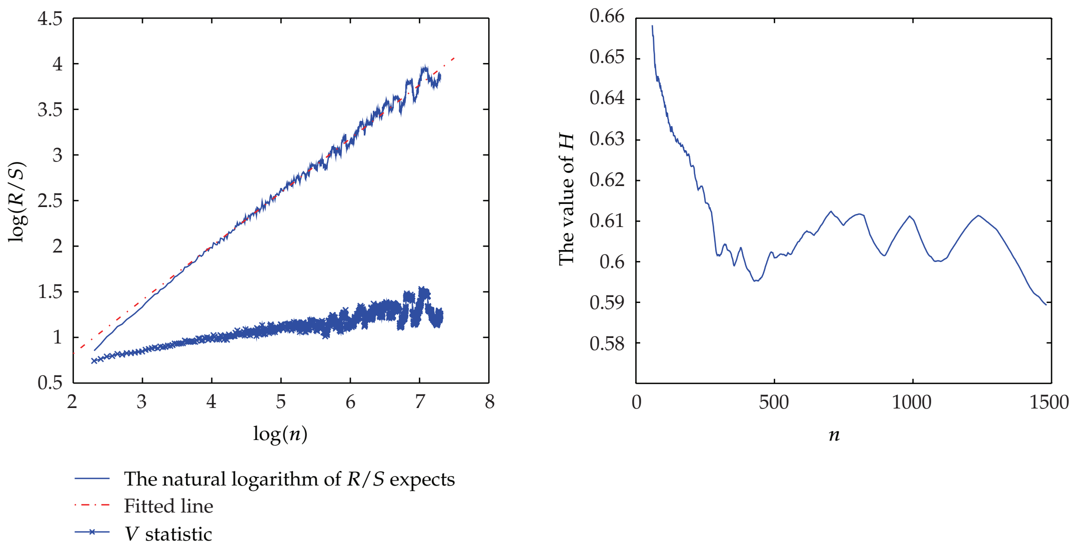
5. Long-Range Correlation of the Model and Hang Seng Index

In this section, detrended fluctuation analysis (DFA) method is applied on the lattice Sierpinski carpet percolation. The DFA is a technique used to estimate a scaling exponent from the behavior of the average fluctuation of a random variable around its local trend, for the details see [26]. The cumulative deviation of time series $\{x_t,\ t = 1, \ldots, N\}$ is given by

$$Y(i) = \sum_{k=1}^{i}(x_k - \overline{x}), \quad \text{for } i = 1, \ldots, N. \tag{5.1}$$



(a)



(b)

Figure 4: (a) The plots of modified *R/S* statistics and the fluctuation of exponent H for the price model. (b) The corresponding plots for the actual data from Hang Seng Index.

We divide $Y(i)$ into intervals of nonoverlapping and equal length of time (n). Then the root mean square fluctuation for all such length interval is defined as

$$F(n) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[Y(i) - Y_n(i)]^2}, \tag{5.2}$$

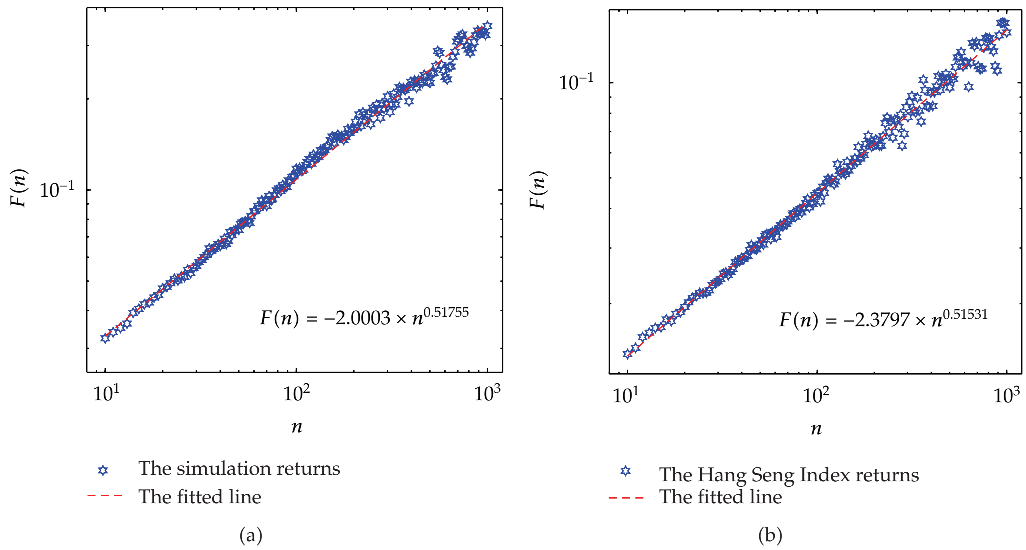


Figure 5: (a) DFA analysis of the returns for the simulation data with the intensity $p = 0.49$. (b) DFA analysis of the returns for Hang Seng Index.

where $Y_n(i)$ is the fitting polynomial of the interval. The above definition is repeated for all the divided intervals. There is a power-law relation between $F(n)$ and n, namely,

$$F(n) \sim n^{\alpha}. \tag{5.3}$$

The parameter α is the scaling exponent or the correlation exponent, which exhibits the long-range correlation of the time series. For $\alpha = 0.5$, it indicates that the time series is uncorrelated (white noise); for the value $0 < \alpha < 0.5$, it indicates the anticorrelations; for $0.5 < \alpha < 1$, the time series has the persistent long-range correlation. According to DFA method and computer simulation, the scaling exponents of the returns of the price model and Hang Seng Index are 0.51755 and 0.51531, respectively, in Figure 5. Although both the exponent values are larger than 0.5, they are very close to 0.5. This shows that there is some strong indication of long-range correlations for the returns.

6. Conclusion

A new random stock price model is developed by the lattice Sierpinski carpet percolation in the present paper, and a cluster of carpet percolation is applied to describe the cluster of traders sharing the same opinion about the market. The statistical properties of the returns are investigated and analyzed for different intensity values, and the behaviors of long memory and long-range correlation in volatility series are exhibited. Further, Hang Seng Index is also introduced and investigated by comparison; the empirical results show that the price model is accord with the real market to some degree.

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