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Research Article

Coderivations of Ranked Bigroupoids

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The notion of (co)derivations of ranked bigroupoids is discussed by Alshehri et al. (in press), and their generalized version is studied by Jun et al. (under review press). In particular, Jun et al. (under review press) studied coderivations of ranked bigroupoids. In this paper, the generalization of coderivations of ranked bigroupoids is discussed. The notion of generalized coderivations in ranked bigroupoids is introduced, and new generalized coderivations of ranked bigroupoids are obtained by combining a generalized self-coderivation with a rankomorphism. From the notion of $(X, *, \&)$-derivation, the existence of a rankomorphism of ranked bigroupoids is established.

1. Introduction

Several authors [1–4] have studied derivations in rings and near-rings. Jun and Xin [5] applied the notion of derivation in ring and near-ring theory to BCI-algebras, and as a result they introduced a new concept, called a (regular) derivation, in BCI-algebras. Alshehri [6] applied the notion of derivations to incline algebras. Alshehri et al. [7] introduced the notion of ranked bigroupoids and discussed $(X, *, \&)$-self-(co)derivations. Jun et al. [8] discussed generalized derivations on ranked bigroupoids. They studied coderivations of ranked bigroupoids. In this paper, we discuss the generalization of coderivations of ranked bigroupoids. We introduce the notion of generalized coderivations in ranked bigroupoids. Combining a generalized self-coderivation with a rankomorphism, we obtain new generalized coderivations of ranked bigroupoids. From the notion of $(X, *, \&)$-derivation, we induce the existence of a rankomorphism of ranked bigroupoids.

2. Preliminaries

Let X be a nonempty set with a distinguished element 0. For any binary operation $\natural$ on X, we consider the following axioms:

(a1) $((x\natural y)\natural(x\natural z))\natural(z\natural y) = 0$,

(a2) $(x\natural(x\natural y))\natural y = 0$,

(a3) $x\natural x = 0$,

(a4) $x\natural y = 0$ and $y\natural x = 0$ imply $x = y$,

(b1) $x\natural 0 = x$,

(b2) $(x\natural y)\natural z = (x\natural z)\natural y$,

(b3) $((x\natural z)\natural(y\natural z))\natural(x\natural y) = 0$,

(b4) $x\natural(x\natural(x\natural y)) = x\natural y$.

If X satisfies axioms (a1), (a2), (a3), and (a4) under the binary operation $*$, then we say that $(X, *, 0)$ is a BCI-algebra. If a BCI-algebra $(X, *, 0)$ satisfies the identity $0 * x = 0$ for all $x \in X$, we say that $(X, *, 0)$ is a BCK-*algebra*. Note that a BCI-algebra $(X, *, 0)$ satisfies conditions (b1), (b2), (b3), and (b4) under the binary operation $*$ (see [9]).

In a p-semisimple BCI-algebra $(X, *, 0)$, the following hold

(b5) $(x * z) * (y * z) = x * y$.

(b6) $0 * (0 * x) = x$.

(b7) $x * (0 * y) = y * (0 * x)$.

(b8) $x * y = 0$ implies $x = y$.

(b9) $x * a = x * b$ implies $a = b$.

(b10) $a * x = b * x$ implies $a = b$.

(b11) $a * (a * x) = x$.

(b12) $((x * y) * z) * (x * (y * z)) = 0$.

A *ranked bigroupoid* (see [7]) is an algebraic system $(X, *, \bullet)$ where X is a nonempty set and "$*$" and "$\bullet$" are binary operations defined on X. We may consider the first binary operation $*$ as the major operation and the second binary operation $\bullet$ as the minor operation.

Given a ranked bigroupoid $(X, *, \&)$, a map $d : X \to X$ is called an $(X, *, \&)$-*self-derivation* (see [7]) if for all $x, y \in X$,

$$d(x * y) = (d(x) * y)\&(x * d(y)). \tag{2.1}$$

In the same setting, a map $d : X \to X$ is called an $(X, *, \&)$-*self-coderivation* (see [7]) if for all $x, y \in X$,

$$d(x * y) = (x * d(y))\&(d(x) * y). \tag{2.2}$$

Note that if $(X, *)$ is a commutative groupoid, then $(X, *, \&)$-self-derivations are $(X, *, \&)$-self-coderivations. A map $d : X \to X$ is called an *abelian*-$(X, *, \&)$-*self-derivation* (see [7]) if it is both an $(X, *, \&)$-self-derivation and an $(X, *, \&)$-self-coderivation.

Given ranked bigroupoids $(X, *, \&)$ and $(Y, \bullet, \omega)$, a map $f : (X, *, \&) \to (Y, \bullet, \omega)$ is called a *rankomorphism* (see [7]) if it satisfies $f(x * y) = f(x) \bullet f(y)$ and $f(x \& y) = f(x) \omega f(y)$ for all $x, y \in X$.

3. Coderivations of Ranked Bigroupoids

Definition 3.1 (see [8]). Let $(X, *, \&)$ be a ranked bigroupoid. A mapping $D : X \to X$ is called a *generalized* $(X, *, \&)$*-self-derivation* if there exists an $(X, *, \&)$-self-derivation $d : X \to X$ such that $D(x * y) = (D(x) * y) \& (x * d(y))$ for all $x, y \in X$. If there exists an $(X, *, \&)$-self-coderivation $d : X \to X$ such that $D(x * y) = (x * D(y)) \& (d(x) * y)$ for all $x, y \in X$, the mapping $D : X \to X$ is called a *generalized* $(X, *, \&)$*-self-coderivation*. If D is both a generalized $(X, *, \&)$-self-derivation and a generalized $(X, *, \&)$-self-coderivation, we say that D is a *generalized abelian* $(X, *, \&)$*-self-derivation*.

Definition 3.2 (see [7]). Let $(X, *, \&)$ and $(Y, \bullet, \omega)$ be ranked bigroupoids. A map $\delta : (X, *, \&) \to (Y, \bullet, \omega)$ is called an $(X, *, \&)$*-derivation* if there exists a rankomorphism $f : X \to Y$ such that

$$\delta(x * y) = (\delta(x) \bullet f(y)) \omega (f(x) \bullet \delta(y)), \tag{3.1}$$

for all $x, y \in X$.

Theorem 3.3. *Let $(X, *, \&)$ and $(Y, \bullet, \omega)$ be ranked bigroupoids with distinguished element 0 in which the following items are valid.*

(1) *The axioms (a3) and (b1) are valid under the major operation $*$.*

(2) *The axioms (b1), (b2), (b3), (a3) and (a4) are valid under the major operation $\bullet$.*

(3) *The minor operation ω is defined by $a \omega b = b \bullet (b \bullet a)$ for all $a, b \in Y$.*

If $\delta : (X, *, \&) \to (Y, \bullet, \omega)$ is an $(X, *, \&)$-derivation, then there exists a rankomorphism $f : (X, *, \&) \to (Y, \bullet, \omega)$ such that $\delta(x) = \delta(x) \omega f(x)$ for all $x \in X$. In particular, $\delta(0) = 0 \bullet (0 \bullet \delta(0))$.

Proof. Assume that $\delta : (X, *, \&) \to (Y, \bullet, \omega)$ is an $(X, *, \&)$-derivation. Then there exists a rankomorphism $f : (X, *, \&) \to (Y, \bullet, \omega)$ such that

$$\delta(x * y) = (\delta(x) \bullet f(y)) \omega (f(x) \bullet \delta(y)), \tag{3.2}$$

for all $x, y \in X$. Since the axiom (a3) is valid under the major operations $*$ and $\bullet$, we get $f(0) = 0$. Hence

$$\begin{aligned}
\delta(x) = \delta(x * 0) &= (\delta(x) \bullet f(0)) \omega (f(x) \bullet \delta(0)) \\
&= (\delta(x) \bullet 0) \omega (f(x) \bullet \delta(0)) \\
&= \delta(x) \omega (f(x) \bullet \delta(0)) \\
&= (f(x) \bullet \delta(0)) \bullet ((f(x) \bullet \delta(0)) \bullet \delta(x)) \\
&= (f(x) \bullet \delta(0)) \bullet ((f(x) \bullet \delta(x)) \bullet \delta(0)),
\end{aligned} \tag{3.3}$$

for all $x \in X$. It follows from (b3) that

$$\begin{aligned}\delta(x) \bullet (\delta(x)\omega f(x)) &= \delta(x) \bullet (f(x) \bullet (f(x) \bullet \delta(x))) \\ &= ((f(x) \bullet \delta(0)) \bullet ((f(x) \bullet \delta(x)) \bullet \delta(0))) \bullet (f(x) \bullet (f(x) \bullet \delta(x))) \\ &= 0,\end{aligned} \tag{3.4}$$

for all $x \in X$. Note from (b2) and (a3) that

$$\begin{aligned}(\delta(x)\omega f(x)) \bullet \delta(x) &= (f(x) \bullet (f(x) \bullet \delta(x))) \bullet \delta(x) \\ &= (f(x) \bullet \delta(x)) \bullet (f(x) \bullet \delta(x)) \\ &= 0,\end{aligned} \tag{3.5}$$

for all $x \in X$. Using (a4), we have $\delta(x) = \delta(x)\omega f(x)$ for all $x \in X$. If we let $x := 0$, then $\delta(0) = 0 \bullet (0 \bullet \delta(0))$. □

Corollary 3.4. *Let $(X, *, \&)$ and $(Y, \bullet, \omega)$ be ranked bigroupoids with distinguished element 0 in which the following items are valid.*

(1) *The axioms (a3) and (b1) are valid under the major operation $*$.*

(2) *$(Y, \bullet, 0)$ is a BCI-algebra.*

(3) *The minor operation ω is defined by $a\omega b = b \bullet (b \bullet a)$ for all $a, b \in Y$.*

If $\delta : (X, *, \&) \to (Y, \bullet, \omega)$ is an $(X, *, \&)$-derivation, then there exists a rankomorphism $f : (X, *, \&) \to (Y, \bullet, \omega)$ such that $\delta(x) = \delta(x)\omega f(x)$ for all $x \in X$. In particular, $\delta(0) = 0 \bullet (0 \bullet \delta(0))$.

Theorem 3.5. *Let $(X, *, \&)$ and $(Y, \bullet, \omega)$ be ranked bigroupoids with distinguished element 0 in which the following items are valid.*

(1) *The axiom (a3) is valid under the major operation $*$.*

(2) *The axioms (a3), (b2), (b5), and (b6) are valid under the major operation $\bullet$.*

(3) *The minor operation ω is defined by $a\omega b = b \bullet (b \bullet a)$ for all $a, b \in Y$.*

If $\delta : (X, *, \&) \to (Y, \bullet, \omega)$ is an $(X, *, \&)$-derivation, then there exists a rankomorphism $f : (X, *, \&) \to (Y, \bullet, \omega)$ such that

$$(\forall a \in X)(a = 0 * (0 * a) \Longrightarrow \delta(a) = \delta(0) \bullet (0 \bullet f(a))). \tag{3.6}$$

Proof. Assume that $\delta : (X, *, \&) \to (Y, \bullet, \omega)$ is an $(X, *, \&)$-derivation. Then there exists a rankomorphism $f : (X, *, \&) \to (Y, \bullet, \omega)$ such that

$$\delta(x * y) = (\delta(x) \bullet f(y))\omega(f(x) \bullet \delta(y)) \tag{3.7}$$

for all $x, y \in X$. Let $a \in X$ be such that $a = 0 * (0 * a)$. Then

$$f(a) = f(0 * (0 * a)) = f(0) \bullet \left(f(0) \bullet f(a)\right) = 0 \bullet \left(0 \bullet f(a)\right), \tag{3.8}$$

and so

$$\begin{aligned}
\delta(a) = \delta(0 * (0 * a)) &= \left(\delta(0) \bullet f(0 * a)\right)\omega\left(f(0) \bullet \delta(0 * a)\right) \\
&= \left(\delta(0) \bullet f(0 * a)\right)\omega(0 \bullet \delta(0 * a)) \\
&= (0 \bullet \delta(0 * a)) \bullet \left((0 \bullet \delta(0 * a)) \bullet \left(\delta(0) \bullet f(0 * a)\right)\right) \\
&= (0 \bullet \delta(0 * a)) \bullet \left((0 \bullet \left(\delta(0) \bullet f(0 * a)\right)) \bullet \delta(0 * a)\right) \\
&= 0 \bullet \left(0 \bullet \left(\delta(0) \bullet f(0 * a)\right)\right) \\
&= 0 \bullet \left(0 \bullet \left(\delta(0) \bullet \left(f(0) \bullet f(a)\right)\right)\right) \\
&= 0 \bullet \left(0 \bullet \left(\delta(0) \bullet \left(0 \bullet f(a)\right)\right)\right) \\
&= \delta(0) \bullet \left(0 \bullet f(a)\right).
\end{aligned} \tag{3.9}$$

This completes the proof. □

Corollary 3.6. *Let $(X, *, \&)$ and $(Y, \bullet, \omega)$ be ranked bigroupoids with distinguished element 0 in which the following items are valid.*

(1) *The axiom (a3) is valid under the major operation $*$.*

(2) *$(Y, \bullet, 0)$ is a p-semisimple BCI-algebra.*

(3) *The minor operation ω is defined by $a\omega b = b \bullet (b \bullet a)$ for all $a, b \in Y$.*

If $\delta : (X, *, \&) \to (Y, \bullet, \omega)$ is an $(X, *, \&)$-derivation, then there exists a rankomorphism $f : (X, *, \&) \to (Y, \bullet, \omega)$ such that

$$(\forall a \in X)\left(a = 0 * (0 * a) \Longrightarrow \delta(a) = \delta(0) \bullet \left(0 \bullet f(a)\right)\right). \tag{3.10}$$

Definition 3.7 (see [8]). Given ranked bigroupoids $(X, *, \&)$ and $(Y, \bullet, \omega)$, a map $\varphi : (X, *, \&) \to (Y, \bullet, \omega)$ is called an $(X, *, \&)$-*coderivation* if there exists a rankomorphism $f : X \to Y$ such that

$$\varphi(x * y) = \left(f(x) \bullet \varphi(y)\right)\omega\left(\varphi(x) \bullet f(y)\right), \tag{3.11}$$

for all $x, y \in X$.

Definition 3.8. A map $\Psi : (X, *, \&) \to (Y, \bullet, \omega)$ is called a *generalized* $(X, *, \&)$-*coderivation* if there exist both a rankomorphism $f : X \to Y$ and an $(X, *, \&)$-coderivation $\delta : (X, *, \&) \to (Y, \bullet, \omega)$ such that

$$\Psi(x * y) = \left(f(x) \bullet \Psi(y)\right)\omega\left(\delta(x) \bullet f(y)\right), \tag{3.12}$$

for all $x, y \in X$.

Lemma 3.9 (see [8]). *If $f : (X, *, \&) \to (Y, \bullet, \omega)$ is a rankomorphism of ranked bigroupoids and $d : X \to X$ is an $(X, *, \&)$-self-coderivation, then $f \circ d : (X, *, \&) \to (Y, \bullet, \omega)$ is an $(X, *, \&)$-coderivation.*

Theorem 3.10. *Let $D : (X, *, \&) \to (X, *, \&)$ be a generalized $(X, *, \&)$-self-coderivation and $f : (X, *, \&) \to (Y, \bullet, \omega)$ a rankomorphism. Then $f \circ D$ is a generalized $(X, *, \&)$-coderivation.*

Proof. Since D is a generalized $(X, *, \&)$-self-coderivation, there exists an $(X, *, \&)$-self-coderivation $d : X \to X$ such that

$$D(x * y) = (x * D(y)) \& (d(x) * y), \tag{3.13}$$

for all $x, y \in X$. It follows that

$$\begin{aligned}(f \circ D)(x * y) &= f(D(x * y)) = f((x * D(y)) \& (d(x) * y)) \\ &= f(x * D(y)) \omega f(d(x) * y) \\ &= (f(x) \bullet (f \circ D)(y)) \omega ((f \circ d)(x) \bullet f(y)),\end{aligned} \tag{3.14}$$

for all $x, y \in X$. Note from Lemma 3.9 that $f \circ d$ is an $(X, *, \&)$-coderivation. Hence $f \circ D$ is a generalized $(X, *, \&)$-coderivation. □

Lemma 3.11 (see [8]). *If $f : (X, *, \&) \to (Y, \bullet, \omega)$ is a rankomorphism of ranked bigroupoids and $\delta : (Y, \bullet, \omega) \to (Y, \bullet, \omega)$ is a $(Y, \bullet, \omega)$-self-coderivation, then $\delta \circ f : (X, *, \&) \to (Y, \bullet, \omega)$ is an $(X, *, \&)$-coderivation.*

Theorem 3.12. *Let $D : (Y, \bullet, \omega) \to (Y, \bullet, \omega)$ be a generalized $(Y, \bullet, \omega)$-self-coderivation. If $f : (X, *, \&) \to (Y, \bullet, \omega)$ is a rankomorphism, then $D \circ f : (X, *, \&) \to (Y, \bullet, \omega)$ is a generalized $(X, *, \&)$-coderivation.*

Proof. Since D is a generalized $(Y, \bullet, \omega)$-self-coderivation, there exists a $(Y, \bullet, \omega)$-self-coderivation $\delta : (Y, \bullet, \omega) \to (X, *, \&)$ such that

$$D(a \bullet b) = (a \bullet D(b)) \omega (\delta(a) \bullet b), \tag{3.15}$$

for all $a, b \in Y$. It follows that

$$\begin{aligned}(D \circ f)(x * y) &= D(f(x * y)) = D(f(x) \bullet f(y)) \\ &= (f(x) \bullet D(f(y))) \omega (\delta(f(x)) \bullet f(y)) \\ &= (f(x) \bullet (D \circ f)(y)) \omega ((\delta \circ f)(x) \bullet f(y)),\end{aligned} \tag{3.16}$$

for all $x, y \in X$. By Lemma 3.11, $\delta \circ f : (X, *, \&) \to (Y, \bullet, \omega)$ is an $(X, *, \&)$-coderivation. Hence $D \circ f : (X, *, \&) \to (Y, \bullet, \omega)$ is a generalized $(X, *, \&)$-coderivation. □

Lemma 3.13 (see [8]). *For ranked bigroupoids $(X, *, \&)$, $(Y, \bullet, \omega)$ and $(Z, \square, \pi)$, consider a rankomorphism $f : (X, *, \&) \to (Y, \bullet, \omega)$. If $\varphi : (Y, \bullet, \omega) \to (Z, \square, \pi)$ is a $(Y, \bullet, \omega)$-coderivation, then $\varphi \circ f : (X, *, \&) \to (Z, \square, \pi)$ is an $(X, *, \&)$-coderivation.*

Theorem 3.14. *Given ranked bigroupoids $(X, *, \&)$, $(Y, \bullet, \omega)$, and $(Z, \square, \pi)$, consider a rankomorphism $f : (X, *, \&) \to (Y, \bullet, \omega)$. If $\Psi : (Y, \bullet, \omega) \to (Z, \square, \pi)$ is a generalized $(Y, \bullet, \omega)$-coderivation, then $\Psi \circ f : (X, *, \&) \to (Z, \square, \pi)$ is a generalized $(X, *, \&)$-coderivation.*

Proof. If $\Psi : (Y, \bullet, \omega) \to (Z, \square, \pi)$ is a generalized $(Y, \bullet, \omega)$-coderivation, then there exist both a rankomorphism $g : (Y, \bullet, \omega) \to (Z, \square, \pi)$ and a $(Y, \bullet, \omega)$-coderivation $\varphi : (Y, \bullet, \omega) \to (Z, \square, \pi)$ such that

$$\Psi(a \bullet b) = \big(g(a) \square \Psi(b)\big) \pi \big(\varphi(a) \square g(b)\big), \tag{3.17}$$

for all $a, b \in Y$. It follows that

$$\begin{aligned}
(\Psi \circ f)(x * y) &= \Psi\big(f(x * y)\big) = \Psi\big(f(x) \bullet f(y)\big) \\
&= \Big(g\big(f(y)\big) \omega \Psi\big(f(y)\big)\Big) \pi \Big(\varphi\big(f(x)\big) \square g\big(f(y)\big)\Big) \\
&= \big((g \circ f)(x) \square (\varphi \circ f)(y)\big) \pi \big((\varphi \circ f)(x) \square (g \circ f)(y)\big),
\end{aligned} \tag{3.18}$$

for all $x, y \in X$. Obviously, $g \circ f$ is a rankomorphism. By Lemma 3.13, $\varphi \circ f$ is an $(X, *, \&)$-coderivation. Therefore $\Psi \circ f : (X, *, \&) \to (Z, \square, \pi)$ is a generalized $(X, *, \&)$-coderivation. □

Lemma 3.15 (see [8]). *For ranked bigroupoids $(X, *, \&)$, $(Y, \bullet, \omega)$, and $(Z, \square, \pi)$, consider a rankomorphism $g : (Y, \bullet, \omega) \to (Z, \square, \pi)$. If $\eta : (X, *, \&) \to (Y, \bullet, \omega)$ is an $(X, *, \&)$-coderivation, then $g \circ \eta : (X, *, \&) \to (Z, \square, \pi)$ is an $(X, *, \&)$-coderivation.*

Theorem 3.16. *Given ranked bigroupoids $(X, *, \&)$,$(Y, \bullet, \omega)$, and $(Z, \square, \pi)$, consider a rankomorphism $g : (Y, \bullet, \omega) \to (Z, \square, \pi)$. If $\Phi : (X, *, \&) \to (Y, \bullet, \omega)$ is a generalized $(X, *, \&)$-coderivation, then $g \circ \Phi : (X, *, \&) \to (Z, \square, \pi)$ is a generalized $(X, *, \&)$-coderivation.*

Proof. If $\Phi : (X, *, \&) \to (Y, \bullet, \omega)$ is a generalized $(X, *, \&)$-coderivation, then there exist both a rankomorphism $f : X \to Y$ and an $(X, *, \&)$-coderivation $\eta : (X, *, \&) \to (Y, \bullet, \omega)$ such that

$$\Phi(x * y) = \big(f(x) \bullet \Phi(y)\big) \omega \big(\eta(x) \bullet f(y)\big), \tag{3.19}$$

for all $x, y \in X$. It follows that

$$\begin{aligned}
(g \circ \Phi)(x * y) &= g\big(\Phi(x * y)\big) = g\Big(\big(f(x) \bullet \Phi(y)\big) \omega \big(\eta(x) \bullet f(y)\big)\Big) \\
&= g\big(f(x) \bullet \Phi(y)\big) \pi g\big(\eta(x) \bullet f(y)\big) \\
&= \Big(g\big(f(x)\big) \square g\big(\Phi(y)\big)\Big) \pi \Big(g\big(\eta(x)\big) \square g\big(f(y)\big)\Big) \\
&= \big((g \circ f)(x) \square (g \circ \Phi)(y)\big) \pi \big((g \circ \eta)(x) \square (g \circ f)(y)\big),
\end{aligned} \tag{3.20}$$

for all $x, y \in X$. Obviously, $g \circ f$ is a rankomorphism and $g \circ \eta$ is an $(X, *, \&)$-coderivation by Lemma 3.15. This shows that $g \circ \Phi : (X, *, \&) \to (Z, \square, \pi)$ is a generalized $(X, *, \&)$-coderivation. □

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