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Research Article

Mean Square Exponential Stability of Stochastic Switched System with Interval Time-Varying Delays

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This paper is concerned with mean square exponential stability of switched stochastic system with interval time-varying delays. The time delay is any continuous function belonging to a given interval, but not necessary to be differentiable. By constructing a suitable augmented Lyapunov-Krasovskii functional combined with Leibniz-Newton's formula, a switching rule for the mean square exponential stability of switched stochastic system with interval time-varying delays and new delay-dependent sufficient conditions for the mean square exponential stability of the switched stochastic system are first established in terms of LMIs. Numerical example is given to show the effectiveness of the obtained result.

1. Introduction

Stability analysis of linear systems with time-varying delays $\dot{x}(t) = Ax(t) + Dx(t - h(t))$ is fundamental to many practical problems and has received considarable attention [1–11]. Most of the known results on this problem are derived assuming only that the time-varing delay $h(t)$ is a continuously differentiable function, satisfying some boundedness condition on its derivative: $\dot{h}(t) \leq \delta < 1$. In delay-dependent stability criteria, the main concern is to enlarge the feasible region of stability criteria in given time-delay interval. Interval time-varying delay means that a time delay varies in an interval in which the lower bound is not restricted to be zero. By constructing a suitable augmented Lyapunov functionals and utilizing free weight matrices, some less conservative conditions for asymptotic stability are derived in [12–21] for systems with time delay varying in an interval. However, the shortcoming of the method used in these works is that the delay function is assumed to be differential and its derivative is still bounded: $\dot{h}(t) \leq \delta$. This paper gives the improved results for the mean square exponential stability of switched stochastic system with interval time-varying delay. The time delay is assumed to be a time-varying continuous function belonging to a given interval, but not necessary to be differentiable. Specifically, our goal is to develop a constructive way to design switching rule to the mean square exponential stability of switched stochastic system with interval time-varying delay. By constructing argumented Lyapunov functional combined with LMI technique, we propose new criteria for the mean square exponential stability of the switched stochastic system. The delay-dependent stability conditions are formulated in terms of LMIs.

The paper is organized as follows: Section 2 presents definitions and some well-known technical propositions needed for the proof of the main results. Delay-dependent mean square exponential stability conditions of the switched stochastic system and numerical example showing the effectiveness of proposed method are presented in Section 3.

2. Preliminaries

The following notations will be used in this paper. R^+ denotes the set of all real nonnegative numbers; R^n denotes the n-dimensional space with the scalar product $\langle \cdot, \cdot \rangle$ and the vector norm $\| \cdot \|$; $M^{n\times r}$ denotes the space of all matrices of $(n \times r)$-dimensions; A^T denotes the transpose of matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\min/\max}(A) = \min/\max\{\operatorname{Re}\lambda;\ \lambda \in \lambda(A)\}$; $x_t := \{x(t+s) : s \in [-h, 0]\}$, $\|x_t\| = \sup_{s\in[-h,0]}\|x(t+s)\|$; $C([0,t], R^n)$ denotes the set of all R^n-valued continuous functions on $[0,t]$; matrix A is called semipositive definite $(A \geq 0)$ if $\langle Ax, x\rangle \geq 0$, for all $x \in R^n$; A is positive definite $(A > 0)$ if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$. $*$ denotes the symmetric term in a matrix.

Consider a switched stochastic system with interval time-varying delay of the form

$$\begin{aligned} \dot{x}(t) &= A_\gamma x(t) + D_\gamma x(t-h(t)) + \sigma_\gamma(x(t), x(k-h(t)), t)\omega(t), \quad t \in R^+, \\ x(t) &= \phi(t), \quad t \in [-h_2, 0], \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is the state; $\gamma(\cdot) : R^n \to \mathcal{N} := \{1, 2, \ldots, N\}$ is the switching rule, which is a function depending on the state at each time and will be designed. A switching function is a rule which determines a switching sequence for a given switching system. Moreover, $\gamma(x(t)) = i$ implies that the system realization is chosen as the ith system, $i = 1, 2, \ldots, N$. It is seen that the system (2.1) can be viewed as an autonomous switched system in which the effective subsystem changes when the state $x(t)$ hits predefined boundaries. A_i, $D_i \in M^{n\times n}$, $i = 1, 2, \ldots, N$ are given constant matrices, and $\phi(t) \in C([-h_2, 0], R^n)$ is the initial function with the norm $\|\phi\| = \sup_{s\in[-h_2,0]}\|\phi(s)\|$.

$\omega(k)$ is a scalar Wiener process (Brownian Motion) on $(\Omega, \mathcal{F}, \mathcal{P})$ with

$$E\{\omega(t)\} = 0, \quad E\left\{\omega^2(t)\right\} = 1, \quad E\{\omega(i)\omega(j)\} = 0 \quad (i \neq j), \tag{2.2}$$

and σ_i: $R^n \times R^n \times R \to R^n$, $i = 1, 2, \ldots, N$ is the continuous function and is assumed to satisfy that

$$\begin{aligned} \sigma_i^T(x(t), x(t-h(t)), t)\sigma_i(x(t), x(t-h(t)),\ t) \leq \rho_{i1}x^T(t)x(t) + \rho_{i2}x^T(t-h(t))x(t-h(t)), \\ x(t),\ x(t-h(t)) \in R^n, \end{aligned} \tag{2.3}$$

where $\rho_{i1} > 0$ and $\rho_{i2} > 0,\ i = 1, 2, \ldots, N$ are known constant scalars. For simplicity, we denote $\sigma_i(x(t),\ x(t-h(t)),\ t)$ by σ_i, respectively.

The time-varying delay function $h(t)$ satisfies

$$0 \le h_1 \le h(t) \le h_2, \quad t \in R^+. \tag{2.4}$$

The stability problem for switched stochastic system (2.1) is to construct a switching rule that makes the system exponentially stable.

Definition 2.1. Given $\alpha > 0$, the switched stochastic system (2.1) is α-exponentially stable in the mean square if there exists a switching rule $\gamma(\cdot)$ such that every solution $x(t, \phi)$ of the system satisfies the following condition:

$$\exists N > 0 : E\{\|x(t, \phi)\|\} \le E\{Ne^{-\alpha t}\|\phi\|\}, \quad \forall t \in R^+. \tag{2.5}$$

We end this section with the following technical well-known propositions, which will be used in the proof of the main results.

Proposition 2.2 (Cauchy inequality). *For any symmetric positive definite marix $N \in M^{n\times n}$ and $a, b \in R^n$ one has*

$$\pm a^T b \le a^T N a + b^T N^{-1} b. \tag{2.6}$$

Proposition 2.3 (see [22]). *For any symmetric positive definite matrix $M \in M^{n\times n}$, scalar $\gamma > 0$ and vector function $\omega : [0, \gamma] \to R^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$\left(\int_0^\gamma \omega(s)ds\right)^T M\left(\int_0^\gamma \omega(s)ds\right) \le \gamma\left(\int_0^\gamma \omega^T(s)M\omega(s)ds\right). \tag{2.7}$$

Proposition 2.4 (see [23]). *Let E, H, and F be any constant matrices of appropriate dimensions and $F^T F \le I$. For any $\epsilon > 0$, one has*

$$EFH + H^T F^T E^T \le \epsilon E E^T + \epsilon^{-1} H^T H. \tag{2.8}$$

Proposition 2.5 (Schur complement lemma [24]). *Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T,\ Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \quad or \quad \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0. \tag{2.9}$$

3. Main Results

Let us set

$$\mathcal{M}_i = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & 0 & M_{24} & S_2 \\ * & * & M_{33} & M_{34} & S_3 \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{pmatrix},$$

$$\begin{aligned}
&\lambda_1 = \lambda_{\min}(P), \\
&\lambda_2 = \lambda_{\max}(P) + 2h_2^2\lambda_{\max}(R), \\
&M_{11} = A_i^T P + PA_i - S_1A_i - A_i^T S_1^T + 2\alpha P \\
&\qquad - e^{-2\alpha h_1}R - e^{-2\alpha h_2}R + 2\rho_{i1}, \\
&M_{12} = e^{-2\alpha h_1}R - S_2A_i, \\
&M_{13} = e^{-2\alpha h_2}R - S_3A_i, \\
&M_{14} = PD_i - S_1D_i - S_4A_i, \\
&M_{15} = S_1 - S_5A_i, \\
&M_{22} = -e^{-2\alpha h_1}R, \\
&M_{24} = S_2D_i, \\
&M_{33} = -e^{-2\alpha h_2}R, \\
&M_{34} = -S_3D_i, \\
&M_{44} = -S_4D_i + 2\rho_{i2}, \\
&M_{45} = S_4 - S_5D_i, \\
&M_{55} = S_5 + S_5^T + h_1^2R + h_2^2R.
\end{aligned} \tag{3.1}$$

The main result of this paper is summarized in the following theorem.

Theorem 3.1. *Given $\alpha > 0$, the zero solution of the switched stochastic system* (2.1) *is α-exponentially stable in the mean square if there exist symmetric positive definite matrices P, R, and matrices S_i, $i = 1, 2, \ldots, 5$ satisfying the following conditions:*

(i) $\mathcal{M}_i < 0$, $i = 1, 2, \ldots, N$.

The switching rule is chosen as $\gamma(x(t)) = i$. Moreover, the solution $x(t,\phi)$ of the switched stochastic system satisfies

$$E\{\|x(t,\phi)\|\} \le E\left\{\sqrt{\frac{\lambda_2}{\lambda_1}}e^{-\alpha t}\|\phi\|\right\}, \quad \forall t \in R^+. \tag{3.2}$$

Proof. We consider the following Lyapunov-Krasovskii functional for the system (2.1):

$$E\{V(t,x_t)\} = \sum_{i=1}^{3} E\{V_i\}, \tag{3.3}$$

where

$$\begin{aligned}
V_1 &= x^T(t)Px(t), \\
V_2 &= h_1 \int_{-h_1}^{0} \int_{t+s}^{t} e^{2\alpha(\tau-t)} \dot{x}^T(\tau)R\dot{x}(\tau)d\tau\, ds, \\
V_3 &= h_2 \int_{-h_2}^{0} \int_{t+s}^{t} e^{2\alpha(\tau-t)} \dot{x}^T(\tau)R\dot{x}(\tau)d\tau\, ds.
\end{aligned} \tag{3.4}$$

It easy to check that

$$E\left\{\lambda_1\|x(t)\|^2\right\} \le E\{V(t,x_t)\} \le E\left\{\lambda_2\|x_t\|^2\right\}, \quad \forall t \ge 0, \tag{3.5}$$

Taking the derivative of Lyapunov-Krasovskii functional along the solution of system (2.1) and taking the mathematical expectation, we obtained

$$\begin{aligned}
E\{\dot{V}_1\} &= E\left\{2x^T(t)P\dot{x}(t)\right\} \\
&= E\left\{x^T(t)\left[A_i^T P + A_i P\right]x(t) + 2x^T(t)PD_i x(t-h(t)) + 2x^T(t)P\sigma_i\omega(t)\right\}, \\
E\{\dot{V}_2\} &= E\left\{h_1^2\dot{x}^T(t)R\dot{x}(t) - h_1 e^{-2\alpha h_1}\int_{t-h_1}^{t} \dot{x}^T(s)R\dot{x}(s)ds - 2\alpha V_2\right\}, \\
E\{\dot{V}_3\} &= E\left\{h_2^2\dot{x}^T(t)R\dot{x}(t) - h_2 e^{-2\alpha h_2}\int_{t-h_2}^{t} \dot{x}^T(s)R\dot{x}(s)ds - 2\alpha V_3\right\}.
\end{aligned} \tag{3.6}$$

Applying Proposition 2.3 and the Leibniz-Newton formula, we have

$$\begin{aligned}
E\left\{-h_i\int_{t-h_i}^{t} \dot{x}^T(s)R\dot{x}(s)ds\right\} &\le E\left\{-\left[\int_{t-h_i}^{t} \dot{x}(s)ds\right]^T R\left[\int_{t-h_i}^{t} \dot{x}(s)ds\right]\right\} \\
&\le E\left\{-[x(t)-x(t-h_i)]^T R[x(t)-x(t-h_i)]\right\} \\
&= E\left\{-x^T(t)Rx(t) + 2x^T(t)Rx(t-h_i) - x^T(t-h_i)Rx(t-h_i)\right\},
\end{aligned} \tag{3.7}$$

Therefore, we have

$$\begin{aligned}
E\{\dot{V}(\cdot)+2\alpha V(\cdot)\} \le\ & E\left\{x^T(t)\left[A_i^T P + A_i P + 2\alpha P\right]x(t)\right\} \\
&+ E\left\{2x^T(t)PD_i x(t-h(t)) + 2x^T(t)P\sigma_i\omega(t)\right\} \\
&+ E\left\{\dot{x}^T(t)\left[\left(h_1^2+h_2^2\right)R\right]\dot{x}(t)\right\} \\
&- E\left\{e^{-2\alpha h_1}[x(t)-x(t-h_1)]^T R[x(t)-x(t-h_1)]\right\} \\
&- E\left\{e^{-2\alpha h_2}[x(t)-x(t-h_2)]^T R[x(t)-x(t-h_2)]\right\}.
\end{aligned} \tag{3.8}$$

By using the following identity relation

$$\dot{x}(t) - A_i x(t) - D_i x(t-h(t)) = 0, \tag{3.9}$$

we have

$$\begin{aligned}
&2x^T(t)S_1\dot{x}(t) - 2x^T(t)S_1A_ix(t) - 2x^T(t)S_1D_ix(t-h(t)) - 2x^T(t)S_1\sigma_i\omega(t) = 0, \\
&2x^T(t-h_1)S_2\dot{x}(t) - 2x^T(t-h_1)S_2A_ix(t) - 2x^T(t-h_1)S_2D_ix(t-h(t)) \\
&\quad - 2x^T(t-h_1)S_2\sigma_i\omega(t) = 0, \\
&2x^T(t-h_2)S_3\dot{x}(t) - 2x^T(t-h_2)S_3A_ix(t) - 2x^T(t-h_2)S_3D_ix(t-h(t)) \\
&\quad - 2x^T(t-h_2)S_3\sigma_i\omega(t) = 0, \\
&2x^T(t-h(t))S_4\dot{x}(t) - 2x^T(t-h(t))S_4A_ix(t) - 2x^T(t-h(t))S_4D_ix(t-h(t)) \\
&\quad - 2x^T(t-h(t))S_4\sigma_i\omega(t) = 0, \\
&2\dot{x}^T(t)S_5\dot{x}(t) - 2\dot{x}^T(t)S_5A_ix(t) - 2\dot{x}^T(t)S_5D_ix(t-h(t)) - 2\dot{x}^T(t)S_5\sigma_i\omega(t) = 0, \\
&2\omega^T(t)\sigma_i^T\dot{x}(t) - 2\omega^T(t)\sigma_i^TA_ix(t) - 2\omega^T(t)\sigma_i^TD_ix(t-h(t)) - 2\omega^T(t)\sigma_i^T\sigma_i\omega(t) = 0.
\end{aligned} \tag{3.10}$$

Adding all the zero items of (3.10) into (3.8), we obtain

$$\begin{aligned}
E\{\dot{V}(\cdot)+2\alpha V(\cdot)\} \le\ & E\left\{x^T(t)\left[A_i^T P + PA_i + 2\alpha P - e^{-2\alpha h_1}R\right]x(t)\right\} \\
&- E\left\{x^T(t)\left[e^{-2\alpha h_2}R - S_1A_i - A_i^TS_1^T\right]x(t)\right\} \\
&+ E\left\{2x^T(t)\left[e^{-2\alpha h_1}R - S_2A_i\right]x(t-h_1)\right\} \\
&+ E\left\{2x^T(t)\left[e^{-2\alpha h_2}R - S_3A_i\right]x(t-h_2)\right\} \\
&+ E\left\{2x^T(t)[PD_i - S_1D_i - S_4A_i]x(t-h(t))\right\}
\end{aligned}$$

$$
\begin{aligned}
&+ E\left\{2x^T(t)[S_1 - S_5 A_i]\dot{x}(t)\right\} \\
&+ E\left\{2x^T(t)\left[P\sigma_i - S_1\sigma_i - A_i^T\sigma_i\right]\omega(t)\right\} \\
&+ E\left\{x^T(t-h_1)\left[-e^{-2\alpha h_1}R\right]x(t-h_1)\right\} \\
&+ E\left\{2x^T(t-h_1)[-S_2 D_i]x(t-h(t))\right\} \\
&+ E\left\{2x^T(t-h_1)S_2\dot{x}(t) + 2x^T(t-h_1)[-S_2\sigma_i]\omega(t)\right\} \\
&+ E\left\{x^T(t-h_2)\left[-e^{-2\alpha h_2}R\right]x(t-h_2)\right\} \\
&+ E\left\{x^T(t-h_2)[-S_3 D_i]x(t-h(t))\right\} \\
&+ E\left\{2x^T(t-h_2)S_3\dot{x}(t) + 2x^T(t-h_2)[-S_3\sigma_i]\omega(t)\right\} \\
&+ E\left\{x^T(t-h(t))[-S_4 D_i]x(t-h(t))\right\} \\
&+ E\left\{2x^T(t-h(t))[S_4 - S_5 D_i]\dot{x}(t)\right\} \\
&+ E\left\{2x^T(t-h(t))\left[-S_4\sigma_i - D_i^T\sigma_i\right]\omega(t)\right\} \\
&+ E\left\{\dot{x}^T(t)\left[S_5 + S_5^T + h_1^2 R + h_2^2 R\right]\dot{x}(t)\right\} \\
&+ E\left\{2\dot{x}^T(t)\left[\sigma_i^T - S_5\sigma_i\right]\omega(t)\right\} \\
&+ E\left\{2\omega^T(t)[-\sigma_i\sigma_i]\omega(t)\right\}.
\end{aligned}
\tag{3.11}
$$

By assumption (2.2), we have

$$
\begin{aligned}
E\{\dot{V}(\cdot) + 2\alpha V(\cdot)\} \leq\; & E\left\{x^T(t)\left[A_i^T P + P A_i + 2\alpha P - e^{-2\alpha h_1}R\right]x(t)\right\} \\
&- E\left\{x^T(t)\left[e^{-2\alpha h_2}R - S_1 A_i - A_i^T S_1^T\right]x(t)\right\} \\
&+ E\left\{2x^T(t)\left[e^{-2\alpha h_1}R - S_2 A_i\right]x(t-h_1)\right\} \\
&+ E\left\{2x^T(t)\left[e^{-2\alpha h_2}R - S_3 A_i\right]x(t-h_2)\right\} \\
&+ E\left\{2x^T(t)[P D_i - S_1 D_i - S_4 A_i]x(t-h(t))\right\} \\
&+ E\left\{2x^T(t)[S_1 - S_5 A_i]\dot{x}(t)\right\} \\
&+ E\left\{x^T(t-h_1)\left[-e^{-2\alpha h_1}R\right]x(t-h_1)\right\}
\end{aligned}
$$

$$
\begin{aligned}
&+ E\left\{2x^T(t-h_1)[-S_2D_i]x(t-h(t))\right\} \\
&+ E\left\{x^T(t-h_2)\left[-e^{-2\alpha h_2}R\right]x(t-h_2)\right\} \\
&+ E\left\{x^T(t-h_2)[-S_3D_i]x(t-h(t))\right\} \\
&+ E\left\{2x^T(t-h_2)S_3\dot{x}(t)\right\} \\
&+ E\left\{x^T(t-h(t))[-S_4D_i]x(t-h(t))\right\} \\
&+ E\left\{2x^T(t-h(t))[S_4-S_5D_i]\dot{x}(t)\right\} \\
&+ E\left\{\dot{x}^T(t)\left[S_5+S_5^T+h_1^2R+h_2^2R\right]\dot{x}(t)\right\} \\
&+ E\left\{2\left[-\sigma_i^T\sigma_i\right]\right\}.
\end{aligned}
\tag{3.12}
$$

Applying assumption (2.3), the following estimations hold:

$$
\begin{aligned}
E\{\dot{V}(\cdot)+2\alpha V(\cdot)\} \le{}& E\left\{x^T(t)\left[A_i^TP+PA_i+2\alpha P-e^{-2\alpha h_1}R\right]x(t)\right\} \\
&- E\left\{x^T(t)\left[e^{-2\alpha h_2}R-S_1A_i-A_i^TS_1^T+2\rho_{i1}I\right]x(t)\right\} \\
&+ E\left\{2x^T(t)\left[e^{-2\alpha h_1}R-S_2A_i\right]x(t-h_1)\right\} \\
&+ E\left\{2x^T(t)\left[e^{-2\alpha h_2}R-S_3A_i\right]x(t-h_2)\right\} \\
&+ E\left\{2x^T(t)[PD_i-S_1D_i-S_4A_i]x(t-h(t))\right\} \\
&+ E\left\{2x^T(t)[S_1-S_5A_i]\dot{x}(t)\right\} \\
&+ E\left\{x^T(t-h_1)\left[-e^{-2\alpha h_1}R\right]x(t-h_1)\right\} \\
&+ E\left\{2x^T(t-h_1)[-S_2D_i]x(t-h(t))\right\} \\
&+ E\left\{x^T(t-h_2)\left[-e^{-2\alpha h_2}R\right]x(t-h_2)\right\} \\
&+ E\left\{x^T(t-h_2)[-S_3D_i]x(t-h(t))\right\} \\
&+ E\left\{2x^T(t-h_2)S_3\dot{x}(t)\right\} \\
&+ E\left\{x^T(t-h(t))[-S_4D_i+2\rho_{i2}I]x(t-h(t))\right\} \\
&+ E\left\{2x^T(t-h(t))[S_4-S_5D_i]\dot{x}(t)\right\}
\end{aligned}
$$

$$
\begin{aligned}
&+ E\left\{\dot{x}^T(t)\left[S_5 + S_5^T + h_1^2 R + h_2^2 R\right]\dot{x}(t)\right\} \\
&= E\left\{\zeta^T(t)\mathcal{M}_i\zeta(t)\right\},
\end{aligned}
\tag{3.13}
$$

where

$$
\begin{aligned}
\zeta(t) &= [x(t), x(t-h_1), x(t-h_2), x(t-h(t)), \dot{x}(t)], \\
\mathcal{M}_i &= \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ * & M_{22} & 0 & M_{24} & S_2 \\ * & * & M_{33} & M_{34} & S_3 \\ * & * & * & M_{44} & M_{45} \\ * & * & * & * & M_{55} \end{bmatrix}, \\
M_{11} &= A_i^T P + P A_i - S_1 A_i - A_i^T S_1^T \\
&\quad + 2\alpha P - e^{-2\alpha h_1} R - e^{-2\alpha h_2} R + 2\rho_{i1}, \\
M_{12} &= e^{-2\alpha h_1} R - S_2 A_i, \\
M_{13} &= e^{-2\alpha h_2} R - S_3 A_i, \\
M_{14} &= P D_i - S_1 D_i - S_4 A_i, \\
M_{15} &= S_1 - S_5 A_i, \\
M_{22} &= -e^{-2\alpha h_1} R, \\
M_{24} &= -S_2 D_i, \\
M_{33} &= -e^{-2\alpha h_2} R, \\
M_{34} &= -S_3 D_i, \\
M_{44} &= -S_4 D_i + 2\rho_{i2} I, \\
M_{45} &= S_4 - S_5 D_i, \\
M_{55} &= S_5 + S_5^T + h_1^2 R + h_2^2 R.
\end{aligned}
\tag{3.14}
$$

Therefore, we finally obtain from (3.13) and the condition (i) that

$$
E\left\{\dot{V}(\cdot) + 2\alpha V(\cdot)\right\} < 0, \quad \forall i = 1, 2, \ldots, N, \; t \in R^+, \tag{3.15}
$$

and hence

$$
E\left\{\dot{V}(t, x_t)\right\} \leq -E\{2\alpha V(t, x_t)\}, \quad \forall t \in R^+. \tag{3.16}
$$

Integrating both sides of (3.16) from 0 to t, we obtain

$$E\{V(t,x_t)\} \le E\left\{V(\phi)e^{-2\alpha t}\right\}, \quad \forall t \in R^+. \tag{3.17}$$

Furthermore, taking condition (3.5) into account, we have

$$E\left\{\lambda_1\|x(t,\phi)\|^2\right\} \le E\{V(x_t)\} \le E\left\{V(\phi)e^{-2\alpha t}\right\} \le E\left\{\lambda_2 e^{-2\alpha t}\|\phi\|^2\right\}, \tag{3.18}$$

then

$$E\{\|x(t,\phi)\|\} \le E\left\{\sqrt{\frac{\lambda_2}{\lambda_1}}e^{-\alpha t}\|\phi\|\right\}, \quad t \in R^+, \tag{3.19}$$

By Definition 2.1, the system (2.1) is exponentially stable in the mean square. The proof is complete. □

To illustrate the obtained result, let us give the following numerical examples.

Example 3.2. Consider the following the switched stochastic systems with interval time-varying delay (2.1), where the delay function $h(t)$ is given by

$$\begin{gathered} h(t) = 0.1 + 0.8311\sin^2 3t, \\ A_1 = \begin{pmatrix} -1 & 0.01 \\ 0.02 & -2 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} -1.1 & 0.02 \\ 0.01 & -2 \end{pmatrix}, \\ D_1 = \begin{pmatrix} -0.1 & 0.01 \\ 0.02 & -0.3 \end{pmatrix}, \qquad D_2 = \begin{pmatrix} -0.1 & 0.02 \\ 0.01 & -0.2 \end{pmatrix}. \end{gathered} \tag{3.20}$$

It is worth noting that the delay function $h(t)$ is nondifferentiable. Therefore, the methods used is in [2–15] are not applicable to this system. By LMI toolbox of MATLAB, by using LMI Toolbox in MATLAB, the LMI (i) is feasible with $h_1 = 0.1,\ h_2 = 0.9311,\ \alpha = 0.1,\ \rho_{11} = 0.01,\ \rho_{12} = 0.01,\ \rho_{21} = 0.01,\ \rho_{22} = 0.01$, and

$$\begin{gathered} P = \begin{pmatrix} 2.0788 & -0.0135 \\ -0.0135 & 1.5086 \end{pmatrix}, \qquad R = \begin{pmatrix} 1.0801 & -0.0042 \\ -0.0042 & 0.8450 \end{pmatrix}, \\ S_1 = \begin{pmatrix} -0.6210 & -0.0335 \\ 0.0499 & -0.3576 \end{pmatrix}, \qquad S_2 = \begin{pmatrix} -0.3602 & 0.0170 \\ 0.0298 & -0.3550 \end{pmatrix}, \\ S_3 = \begin{pmatrix} -0.3602 & 0.0170 \\ 0.0298 & -0.3550 \end{pmatrix}, \qquad S_4 = \begin{pmatrix} 0.6968 & -0.0401 \\ -0.0525 & 0.7040 \end{pmatrix}, \qquad S_5 = \begin{pmatrix} -1.4043 & 0.0265 \\ -0.0028 & -0.9774 \end{pmatrix} \end{gathered} \tag{3.21}$$

By Theorem 3.1 the switched stochastic systems (2.1) are 0.1-exponentially stable in the mean square and the switching rule is chosen as $\gamma(x(t)) = i$. Moreover, the solution $x(t, \phi)$ of the system satisfies

$$E\{\|x(t,\phi)\|\} \leq E\left\{1.8731 e^{-0.1t} \|\phi\|\right\}, \quad \forall t \in R^{+}. \tag{3.22}$$

4. Conclusions

In this paper, we have proposed new delay-dependent conditions for the mean square exponential stability of switched stochastic system with non-differentiable interval time-varying delay. By constructing a set of improved Lyapunov-Krasovskii functionals and Newton-Leibniz formula, the conditions for the exponential stability of the systems have been established in terms of LMIs.

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