Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 491695, 8 pages
doi:10.1155/2012/491695

Research Article

Mixed Convection Flow along a Stretching Cylinder in a Thermally Stratified Medium

Swati Mukhopadhyay[1] and Anuar Ishak[2]

[1] *Department of Mathematics, The University of Burdwan, Burdwan 713104, India*
[2] *School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 Bangi, Malaysia*

Correspondence should be addressed to Anuar Ishak, anuarishak@yahoo.com

Received 15 February 2012; Accepted 22 November 2012

Academic Editor: Subhas Abel

An analysis for the axisymmetric laminar boundary layer mixed convection flow of a viscous and incompressible fluid towards a stretching cylinder immersed in a thermally stratified medium is presented in this paper. Similarity transformation is employed to convert the governing partial differential equations into highly nonlinear ordinary differential equations. Numerical solutions of these equations are obtained by a shooting method. It is found that the heat transfer rate at the surface is lower for flow in a thermally stratified medium compared to that of an unstratified medium. Moreover, both the skin friction coefficient and the heat transfer rate at the surface are larger for a cylinder compared to that for a flat plate.

1. Introduction

Many convection processes occur in environments with stratification. Good examples are closed containers and environmental chambers with heated walls. Also the convection flow associated with heat-rejection systems for long-duration deep ocean power modules where the ocean environment is stratified. Thermally stratified flows are also of great interest in various buoyant flow systems including geothermal systems, geological transport, power plant condensation systems, lake thermohydraulics, and volcanic flows and also in industrial thermal treatment processes. Stratification of the medium may arise due to a temperature variation, which gives rise to a density variation in the medium. This is known as thermal stratification and usually arises due to thermal energy input into the medium from heated bodies and thermal sources. Another situation of interest is the one in which stratification arises due to concentration differences. This is relevant in many natural processes such as transport processes in the sea where stratification exists due to salinity variation. Stratification may arise due to the presence of different fluids so that a stable situation arises when the lighter fluid overlies the denser one. The temperature difference varies from layer to layer and these types of flows have wider applications in oceanography, industry, and agriculture [1].

The flow due to a heated surface immersed in a stable stratified viscous fluid has been investigated experimentally and analytically in several studies such as Yang et al. [2], Jaluria and Gebhart [3], Chen and Eichhorn [4], and Ishak et al. [5]. The study of the hydrodynamic flow and heat transfer over a stretching cylinder or a flat plate has gained considerable attention due to its wide applications in industries and important bearings on several technological processes. Crane [6] investigated the flow caused by a stretching plate. Other researchers such as P. S. Gupta and A. S. Gupta [7], Dutta et al. [8], and Chen and Char [9] extended the work of Crane [6] by including the heat and mass transfer analysis under different physical situations. Recently, various aspects of similar problem have been investigated by many authors such as Xu and Liao [10], Cortell [11, 12], Hayat et al. [13], Hayat and Sajid [14], and Ishak et al. [15, 16].

Lin and Shih [17, 18] considered the laminar boundary layer and heat transfer along cylinders moving horizontally and vertically with constant velocity and found no similarity solutions due to the curvature effect of the cylinder. Ishak and Nazar [19] showed that the similarity solutions could be obtained by assuming that the cylinder is stretched with a linear velocity in the axial direction. In fact, the study by Ishak and Nazar [19] is an extension of the problem considered by Grubka and Bobba [20] and Ali [21], that is, from a stretching sheet to a stretching cylinder. By considering the effects of mixed convection and thermal stratification parameters, a new dimension is added to the above-mentioned study of Ishak and Nazar [19].

Since no attempt has been made to analyse the effects of thermal stratification on boundary layer axisymmetric mixed convection flow along a stretching cylinder, it is considered in this paper. Using a similarity transformation, a third-order ordinary differential equation corresponding to the momentum equation and a second-order ordinary differential equation corresponding to the heat equation are derived. Using a shooting method, up to a desired level of accuracy, the numerical computations are carried out for different values of the dimensionless parameters. The analysis of the results obtained shows that the flow field is influenced appreciably by the mixed convection parameter and the thermal stratification parameter. Estimations of the skin friction and heat transfer coefficients which are very important due to their applications in industries are also presented in the analysis. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2. Problem Formulation

Let us consider the steady axisymmetric mixed convection flow of an incompressible viscous fluid along a stretching cylinder embedded in a thermally stratified fluid-saturated medium of variable ambient temperature $T_\infty(x)$, where $T_w(x) > T_\infty(x)$ (heated surface). The continuity, momentum, and energy equations governing such type of flow are

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + g\beta(T - T_\infty), \tag{2.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\kappa}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{2.3}$$

where u and v are the components of velocity in the x and r directions, respectively, $\nu = \mu/\rho$ is the kinematic viscosity, ρ is the fluid density, μ is the coefficient of fluid viscosity, κ is the thermal diffusivity of the fluid, and T is the fluid temperature. It is assumed that the convecting fluid and the medium are in local thermodynamic equilibrium.

The boundary conditions for the problem are given by

$$\begin{gathered} u = U(x), \quad v = 0, \qquad T = T_w(x), \quad \text{at } r = R, \\ u \longrightarrow 0, \qquad T \longrightarrow T_\infty(x), \quad \text{as } r \longrightarrow \infty, \end{gathered} \tag{2.4}$$

where R is the radius of the cylinder, $U(x) = U_0 x/L$ is the stretching velocity, $T_w(x) = T_0 + b(x/L)$ is the prescribed surface temperature, and $T_\infty(x) = T_0 + c(x/L)$ is the variable ambient temperature. Further, U_0, is the reference velocity, T_0, the reference temperature, L the characteristic length, and b and c are positive constants.

To get similarity solutions of (2.1)–(2.3) subject to the boundary conditions (2.4), we introduce the following similarity transformation:

$$\eta = \frac{r^2 - R^2}{2R}\left(\frac{U}{\nu x}\right)^{1/2}, \quad \psi = (U\nu x)^{1/2} R f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_0}, \tag{2.5}$$

where ψ is the stream function defined as $u = r^{-1}(\partial\psi/\partial r)$ and $v = -r^{-1}(\partial\psi/\partial x)$ which identically satisfies the continuity equation (2.1). Substituting (2.5) into (2.2) and (2.3) gives

$$\begin{gathered} (1 + 2M\eta) f''' + 2M f'' + f f'' - f'^2 + \lambda\theta = 0, \\ (1 + 2M\eta)\theta'' + 2M\theta' + \Pr(f\theta' - f'\theta - f'S) = 0, \end{gathered} \tag{2.6}$$

subject to the boundary conditions

$$\begin{gathered} f' = 1, \qquad f = 0, \qquad \theta = 1 - S, \quad \text{at } \eta = 0, \\ f' \longrightarrow 0, \qquad \theta \longrightarrow 0, \quad \text{as } \eta \longrightarrow \infty, \end{gathered} \tag{2.7}$$

where prime denotes differentiation with respect to η, $S = c/b$ is the stratification parameter, $\lambda = g\beta Lb/U_0^2$ is the mixed convection parameter, and $M = (\nu L/U_0 R^2)^{1/2}$ is the curvature parameter. We note that $S = 0$ is for an unstratified environment and $M = 0$ is for a flat plate.

One can note that if $M = 0$ (i.e., $R \to \infty$), the problem under consideration (with $\lambda = 0$, $S = 0$) reduces to the boundary layer flow along a stretching flat plate considered by Ali [21], with $m = 1$ in that paper. Moreover, when $M = 0$, $\lambda = 0$, and $S = 0$, the analytical solution for the flow field was given by Crane [6] and that of the thermal field was given by Grubka and Bobba [20].

3. Results and Discussion

In order to get a clear insight of the physical problem, numerical results are displayed graphically. The results are given through a parametric study showing the influence of several

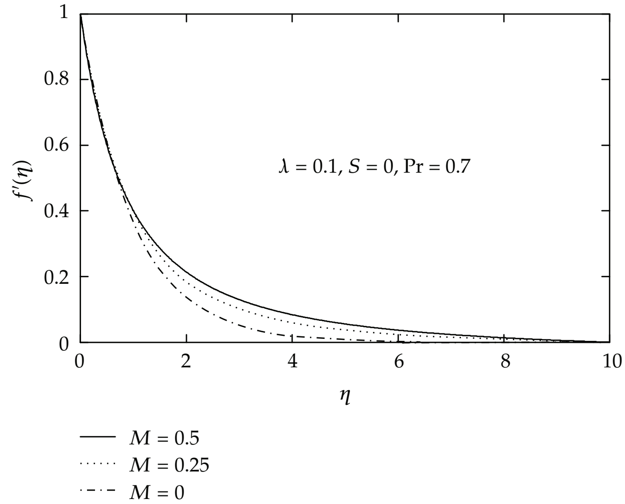

Figure 1: Variation of velocity $f'(\eta)$ with η for several values of the curvature parameter M when $\lambda = 0.1$, $S = 0$, and $\Pr = 0.7$.

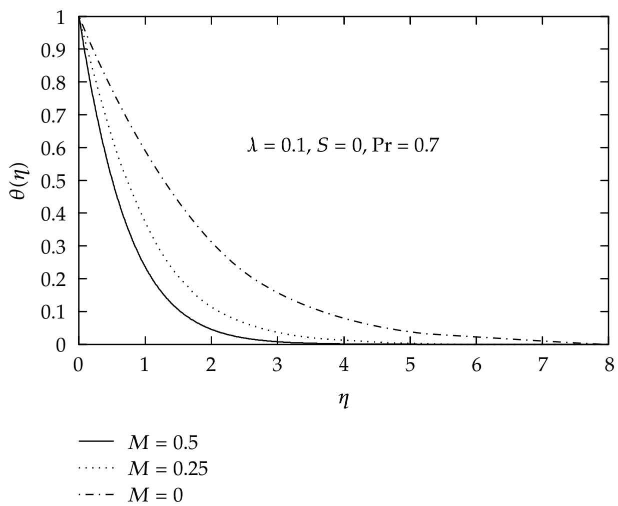

Figure 2: Variation of temperature $\theta(\eta)$ with η for several values of the curvature parameter M when $\lambda = 0.1$, $S = 0$, and $\Pr = 0.7$.

nondimensional parameters, namely, curvature parameter (M), mixed convection parameter (λ), thermal stratification parameter (S), and Prandtl's number $(\Pr)$.

Let us first concentrate on the effects of the curvature parameter M on the velocity and temperature distributions. In Figure 1, velocity profiles are shown for different values of M. The velocity curves show that the rate of transport decreases with increasing distance (η) from the surface and vanishes asymptotically at some large distance. The velocity gradient at the surface is larger for larger values of M, which produces larger skin friction coefficient. On the other hand, the temperature is found to decrease with increasing the values of the curvature parameter M (Figure 2). The thermal boundary layer thickness decreases as M

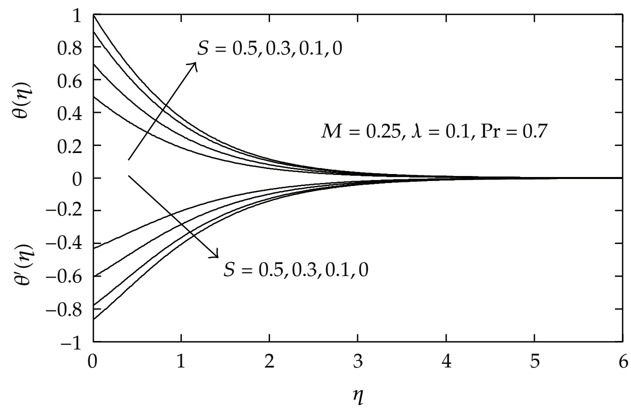

Figure 3: Variation of temperature $\theta(\eta)$ and temperature gradient $\theta'(\eta)$ with η for several values of the stratification parameter S when $M = 0.25$, $\lambda = 0.1$, and $\mathrm{Pr} = 0.7$.

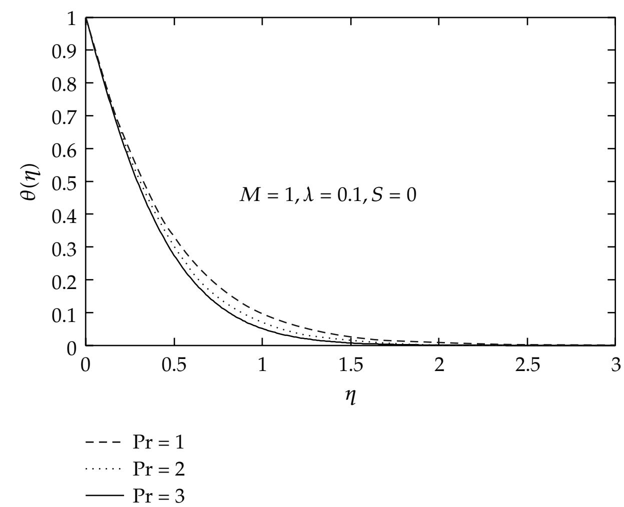

Figure 4: Variation of temperature $\theta(\eta)$ with η for several values of the Prandtl number Pr when $M = 1$, $\lambda = 0.1$, and $S = 0$.

increases, which implies an increase in the wall temperature gradient and in turn increases the heat transfer rate at the surface $-\theta'(0)$.

The effects of the stratification parameter S on the temperature and the temperature gradient are exhibited in Figure 3. Due to stratification, the temperature in the boundary layer decreases, which results in a decreasing manner of the temperature gradient (in absolute sense). The thermal boundary layer thickness also decreases with an increase in the stratification parameter S. With the increase in the stratification parameter, the buoyancy factor $(T_w - T_\infty)$ reduces within the boundary layer. Ambient thermal stratification causes a significant decrease the local buoyancy levels, which reduces the velocities in the boundary layer. All temperature profiles decay from the maximum value at the wall to zero in the free stream, that is, converge at the outer edge of the boundary layer.

The effect of the Prandtl number (Pr) on the temperature profiles is exhibited in Figure 4. Temperature is found to decrease with increasing Pr. An increase in the Prandtl

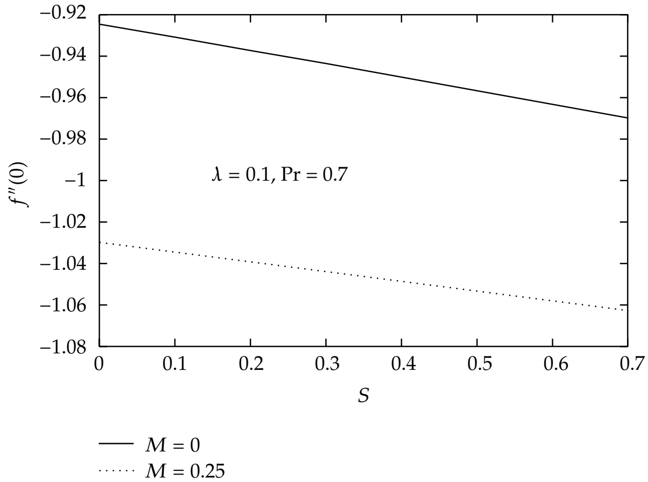

Figure 5: Skin friction coefficient $f''(0)$ against the stratification parameter S for two values of the curvature parameter M when $\lambda = 0.1$ and $\mathrm{Pr} = 0.7$.

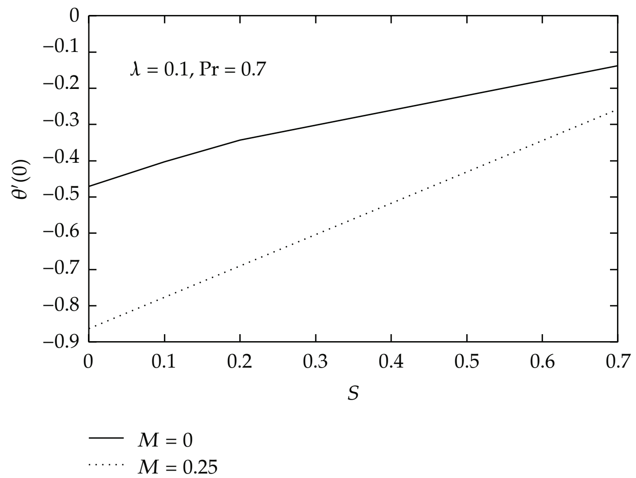

Figure 6: Heat transfer coefficient $\theta'(0)$ against the stratification parameter S for two values of the curvature parameter M when $\lambda = 0.1$ and $\mathrm{Pr} = 0.7$.

number reduces the thermal boundary layer thickness. The Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. Fluids with lower Prandtl number possess higher thermal conductivities (and thicker thermal boundary layer structures) so that heat can diffuse from the wall faster than higher Pr fluids (thinner boundary layers). Hence, the Prandtl number can control the rate of cooling in conducting flows.

The variation of the velocity gradient at the surface $f''(0)$ which is proportional to the skin friction coefficient is presented in Figure 5. It is seen in this figure that the skin friction coefficient is higher (in absolute sense) for a cylinder (with $M = 0.25$) compared to a flat plate ($M = 0$). The negative value of $f''(0)$ presented in Figure 5 means the solid surface exerts a drag force on the ambient fluid. In the presence of a buoyancy force ($\lambda \neq 0$), the magnitude of the skin friction coefficient $|f''(0)|$ increases as the stratification parameter S increases.

Finally, Figure 6 shows that the quantity $\theta'(0)$ increases with increasing values of S for both $M = 0$ and $M = 0.25$. Thus, the heat transfer rate at the surface $-\theta'(0)$ decreases as S increases. Moreover, the heat transfer rate at the surface is higher for a cylinder compared to a flat plate.

4. Conclusions

The present study gives the numerical solutions for steady mixed convection boundary layer flow and heat transfer along a stretching cylinder in a thermally stratified medium. In the presence of the buoyancy force, the magnitude of the skin friction coefficient increases, but the heat transfer rate at the surface decreases as the stratification parameter increases. In the absence of the curvature parameter, the present problem reduces to that of a flat plate. Both the magnitude of the skin friction coefficient and the heat transfer rate at the surface are higher for a cylinder compared to a flat plate.

Acknowledgment

The financial support received from the Universiti Kebangsaan Malaysia (Project code UKM-GUP-2011-202) is gratefully acknowledged.

References

[1] R. Kandasamy and A. Khamis, "Thermal stratification effects on nonlinear hydromagnetic flow over a vertical stretching surface with a power-law velocity," *International Journal of Applied Mechanics and Engineering*, vol. 12, pp. 47–54, 2007.

[2] K. T. Yang, J. L. Novotny, and Y. S. Cheng, "Laminar free convection from a non-isothermal plate immersed in a temperature stratified medium," *International Journal of Heat and Mass Transfer*, vol. 15, pp. 1097–1109, 1972.

[3] Y. Jaluria and B. Gebhart, "Stability and transition of buoyancy-induced flows in a stratified medium," *Journal of Fluid Mechanics*, vol. 66, pp. 593–612, 1974.

[4] C. C. Chen and R. Eichhorn, "Natural convection from simple bodies immersed in thermally stratified fluids," *The ASME Journal of Heat Transfer*, vol. 98, pp. 446–451, 1976.

[5] A. Ishak, R. Nazar, and I. Pop, "Mixed convection boundary layer flow adjacent to a vertical surface embedded in a stable stratified medium," *International Journal of Heat and Mass Transfer*, vol. 51, pp. 3693–3695, 2008.

[6] L. J. Crane, "Flow past a stretching plate," *Zeitschrift für Angewandte Mathematik und Physik*, vol. 21, pp. 645–647, 1970.

[7] P. S. Gupta and A. S. Gupta, "Heat and mass transfer on a stretching sheet with suction or blowing," *The Canadian Journal of Chemical Engineering*, vol. 55, pp. 744–746, 1977.

[8] B. K. Datta, P. Roy, and A. S. Gupta, "Temperature field in the flow over a stretching sheet with uniform heat flux," *International Communications in Heat and Mass Transfer*, vol. 12, pp. 89–94, 1985.

[9] C. K. Chen and M. I. Char, "Heat transfer of a continuous, stretching surface with suction or blowing," *Journal of Mathematical Analysis and Applications*, vol. 135, no. 2, pp. 568–580, 1988.

[10] H. Xu and S. J. Liao, "Series solutions of unsteady magnetohydrodynamics flows of non-Newtonian fluids caused by an impulsively stretching plate," *Journal of Non-Newtonian Fluid Mechanics*, vol. 159, pp. 46–55, 2005.

[11] R. Cortell, "Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing," *Fluid Dynamics Research*, vol. 37, pp. 231–245, 2005.

[12] R. Cortell, "Effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet," *Physics Letters A*, vol. 357, pp. 298–305, 2006.

[13] T. Hayat, Z. Abbas, and M. Sajid, "Series solution for the upper-convected Maxwell fluid over a porous stretching plate," *Physics Letters A*, vol. 358, pp. 396–403, 2006.

[14] T. Hayat and M. Sajid, "Analytic solution for axi-symmetric flow and heat transfer of a second grade fluid past a stretching sheet," *International Journal of Heat and Mass Transfer*, vol. 50, pp. 75–84, 2007.

[15] A. Ishak, R. Nazar, and I. Pop, "Unsteady mixed convection boundary layer flow due to a stretching vertical surface," *The Arabian Journal for Science and Engineering*, vol. 31, no. 2, pp. 165–182, 2006.

[16] A. Ishak, R. Nazar, and I. Pop, "MHD boundary-layer flow due to a moving extensible surface," *Journal of Engineering Mathematics*, vol. 62, no. 1, pp. 23–33, 2008.

[17] H. T. Lin and Y. P. Shih, "Laminar boundary layer heat transfer along static and moving cylinders," *Journal of the Chinese Institute of Engineers*, vol. 3, pp. 73–79, 1980.

[18] H. T. Lin and Y. P. Shih, "Buoyancy effects on the laminar boundary layer heat transfer along vertically moving cylinders," *Journal of the Chinese Institute of Engineers*, vol. 4, pp. 47–51, 1981.

[19] A. Ishak and R. Nazar, "Laminar boundary layer flow along a stretching cylinder," *European Journal of Scientific Research*, vol. 36, pp. 22–29, 2009.

[20] L. G. Grubka and K. M. Bobba, "Heat transfer characteristics of a continuous stretching surface with variable temperature," *The ASME Journal of Heat Transfer*, vol. 107, pp. 248–250, 1985.

[21] M. E. Ali, "Heat transfer characteristics of a continuous stretching surface," *Heat Mass Transfer*, vol. 29, pp. 227–234, 1994.