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Research Article

Exponential Admissibility and H_∞ Control of Switched Singular Time-Delay Systems: An Average Dwell Time Approach

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This paper deals with the problems of exponential admissibility and H_∞ control for a class of continuous-time switched singular systems with time-varying delay. The H_∞ controllers to be designed include both the state feedback (SF) and the static output feedback (SOF). First, by using the average dwell time scheme, the piecewise Lyapunov function, and the free-weighting matrix technique, an exponential admissibility criterion, which is not only delay-range-dependent but also decay-rate-dependent, is derived in terms of linear matrix inequalities (LMIs). A weighted H_∞ performance criterion is also provided. Then, based on these, the solvability conditions for the desired SF and SOF controllers are established by employing the LMI technique, respectively. Finally, two numerical examples are given to illustrate the effectiveness of the proposed approach.

1. Introduction

Many real-world engineering systems always exhibit several kinds of dynamic behavior in different parts of the system (e.g., continuous dynamics, discrete dynamics, jump phenomena, and logic commands) and are more appropriately modeled by hybrid systems. As an important class of hybrid systems, switched systems consist of a collection of continuous-time or discrete-time subsystems and a switching rule orchestrating the switching between them and are of great current interest; see, for example, Decarlo et al. [1], Liberzon [2], Lin and Antsaklis [3], and Sun and Ge [4] for some recent survey and monographs. Switched systems have great flexibility in modeling parameter-varying or structure-varying systems, event-driven systems, logic-based systems, and so forth. Also, multiple-controller switching technique offers an effective mechanism to cope with highly complex systems and/or systems with large uncertainties, particularly in the adaptive context [5]. Many effective methods have been developed for switched systems, for example, the multiple Lyapunov function approach [6, 7], the piecewise Lyapunov function approach [8, 9], the switched Lyapunov function method [10], convex combination technique [11], and the dwell-time or average dwell-time scheme [12–15]. Among them, the average dwell-time scheme provides a simple yet efficient tool for stability analysis of switched systems, especially when the switching is restricted and has been more and more favored [16].

On the other hand, time delay is a common phenomenon in various engineering systems and the main sources of instability and poor performance of a system. Hence, control of switched time-delay systems has been an attractive field in control theory and application in the past decade. Some of the aforementioned approaches for nondelayed switched systems have been successfully adopted to hand the switched time-delay systems; see, for example, Du et al. [17], Kim et al. [18], Mahmoud [19], Phat [20], Sun et al. [21], Sun et al. [22], Wang et al. [23], Wu and Zheng [24], Xie et al. [25], Zhang and Yu [26], and the references therein.

Recently, a more general class of switched time-delay systems described by the singular form was considered in Ma et al. [27] and Wang and Gao [28]. It is known that a singular model describes dynamic systems better than the standard state-space system model [29]. The singular form provides a convenient and natural representation of economic systems, electrical networks, power systems, mechanical systems, and many other systems which have to be modeled by additional algebraic constraints [29]. Meanwhile, it endows the aforementioned systems with several special features, such as regularity and impulse behavior, that are not found in standard state-space systems. Therefore, it is both worthwhile and challenging to investigate the stability and control problems of switched singular time-delay systems. In the past few years, some fundamental results based on the aforementioned approaches for standard state-space switched time-delay systems have been successfully extended to switched singular time-delay systems. For example, by using the switched Lyapunov function method, the robust stability, stabilization, and H_∞ control problems for a class of discrete-time uncertain switched singular systems with constant time delay under arbitrary switching were investigated in Ma et al. [27]; H_∞ filters were designed in Lin et al. [30] for discrete-time switched singular systems with time-varying time delay. In Wang and Gao [28], based on multiple Lyapunov function approach, a switching signal was constructed to guarantee the asymptotic stability of a class of continuous-time switched singular time-delay systems. With the help of average dwell time scheme, some initial results on the exponential admissibility (regularity, nonimpulsiveness, and exponential stability) were obtained in Lin and Fei [31] for continuous-time switched singular time-delay systems. However, to the best of our knowledge, few work has been conducted regarding the H_∞ control for continuous-time switched singular time-delay systems via the dwell time or average dwell time scheme, which constitutes the main motivation of the present study.

In this paper, we aim to solve the problem of H_∞ control for a class of continuous-time switched singular systems with interval time-varying delay via the average dwell time scheme. Both the state feedback (SF) control and the static output feedback (SOF) control are considered. Firstly, based on the average dwell time scheme, the piecewise Lyapunov function, as well as the free-weighting technique, a class of slow switching signals is identified to guarantee the unforced systems to be exponentially admissible with a weighted H_∞ performance γ, and several corresponding criteria, which are not only delay-range-dependent but also decay-rate-dependent, are derived in terms of linear matrix inequalities (LMIs). Next, the LMI-based approaches are proposed to design an SF controller and an SOF controller, respectively, such that the resultant closed-loop system is exponentially admissible and satisfies a weighted H_∞ performance γ. Finally, two illustrative examples are given to show the effectiveness of the proposed approach.

Notation 1. Throughout this paper, the superscript T represents matrix transposition. $\mathbf{R}^n$ denotes the real n-dimensional Euclidean space, and $\mathbf{R}^{n\times n}$ denotes the set of all $n \times n$ real matrices. I is an appropriately dimensioned identity matrix. $P > 0$ ($P \geq 0$) means that matrix P is positive definite (semi positive definite). $\operatorname{diag}\{\cdot,\cdot,\cdot\}$ stands for a block diagonal matrix. $\lambda_{\min}(P)$ ($\lambda_{\max}(P)$) denotes the minimum (maximum) eigenvalue of symmetric matrix P, $L_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$, $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix, and $\operatorname{Sym}\{A\}$ is the shorthand notation for $A + A^T$. In symmetric block matrices, we use an asterisk ($*$) to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Preliminaries and Problem Formulation

Consider a class of switched singular time-delay system of the form

$$\begin{gathered}
E\dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d(t)) + B_{\sigma(t)}u(t) + B_{w\sigma(t)}w(t), \\
z(t) = C_{\sigma(t)}x(t) + C_{d\sigma(t)}x(t-d(t)) + D_{\sigma(t)}u(t) + D_{w\sigma(t)}w(t), \\
y(t) = L_{\sigma(t)}x(t), \\
x(\theta) = \phi(\theta), \quad \theta \in [-d_2, 0],
\end{gathered} \tag{2.1}$$

where $x(t) \in \mathbf{R}^n$ is the system state, $u(t) \in \mathbf{R}^m$ is the control input, $z(t) \in \mathbf{R}^q$ is the controlled output, $y(t) \in \mathbf{R}^p$ is the measured output, and $w(t) \in \mathbf{R}^l$ is the disturbance input that belongs to $L_2[0,\infty)$; $\sigma(t) : [0,+\infty) \to \mathcal{I} = \{1,2,\ldots,I\}$ with integer $I > 1$ is the switching signal; $E \in \mathbf{R}^{n\times n}$ is a singular matrix with $\operatorname{rank} E = r \leq n$; for each possible value, $\sigma(t) = i$, $i \in \mathcal{I}$, A_i, A_{di}, B_i, B_{wi}, C_i, C_{di}, D_i, D_{wi}, and L_i are constant real matrices with appropriate dimensions; $\phi(\theta)$ is a compatible continuous vector-valued initial function on $[-d_2, 0]$; $d(t)$ denotes interval time-varying delay satisfying

$$d_1 \leq d(t) \leq d_2, \qquad \dot{d}(t) \leq \mu < 1, \tag{2.2}$$

where $0 \leq d_1 < d_2$ and μ are constants. Note that d_1 may not be equal to 0.

Since $\operatorname{rank} E = r \leq n$, there exist nonsingular matrices $P, Q \in \mathbf{R}^{n\times n}$ such that

$$PEQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \tag{2.3}$$

In this paper, without loss of generality, let

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \tag{2.4}$$

Corresponding to the switching signal $\sigma(t)$, we denote the switching sequence by $\mathcal{S} := \{(i_0, t_0), \ldots, (i_k, t_k) \mid i_k \in \mathcal{I}, k = 0, 1, \ldots\}$ with $t_0 = 0$, which means that the i_k subsystem is activated when $t \in [t_k, t_{k+1})$. To present the objective of this paper more precisely, the following definitions are introduced.

Definition 2.1 (see [2]). For any $T_2 > T_1 \geq 0$, let $N_\sigma(T_1, T_2)$ denote the number of switching of $\sigma(t)$ over (T_1, T_2). If $N_\sigma(T_1, T_2) \leq N_0 + (T_2 - T_1)/T_a$ holds for $T_a > 0$, $N_0 \geq 0$, then T_a is called average dwell time. As commonly used in the literature [21, 26], we choose $N_0 = 0$.

Definition 2.2 (see [21, 29, 32]). For any delay $d(t)$ satisfying (2.2), the unforced part of system (2.1) with $w(t) = 0$

$$
\begin{aligned}
E\dot{x}(t) &= A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t - d(t)), \\
x_{t_0}(\theta) &= x(t_0 + \theta) = \phi(\theta), \quad \theta \in [-d_2, 0]
\end{aligned}
\tag{2.5}
$$

is said to be

(1) regular if $\det(sE - A_i)$ is not identically zero for each $\sigma(t) = i$, $i \in \mathcal{I}$,

(2) impulse if $\deg(\det(sE - A_i)) = \operatorname{rank} E$ for each $\sigma(t) = i$, $i \in \mathcal{I}$,

(3) exponentially stable under the switching signal $\sigma(t)$ if the solution $x(t)$ of system (2.5) satisfies

$$
\|x(t)\| \leq \iota e^{-\lambda(t - t_0)} \|x_{t_0}\|_c, \quad \forall t \geq t_0,
\tag{2.6}
$$

where $\lambda > 0$ and $\iota > 0$ are called the decay rate and decay coefficient, respectively, and $\|x_{t_0}\|_c = \sup_{-d_2 \leq \theta \leq 0}\{\|x(t_0 + \theta)\|\}$,

(4) exponentially admissible under the switching signal $\sigma(t)$ if it is regular, impulse free, and exponentially stable under the switching signal $\sigma(t)$.

Remark 2.3. The regularity and nonimpulsiveness of the switched singular time-delay system (2.5) ensure that its every subsystem has unique solution for any compatible initial condition. However, even if a switched singular system is regular and causal, it still has inevitably finite jumps due to the incompatible initial conditions caused by subsystem switching [33]. For more details about the impulsiveness effects on the stability of systems, we refer readers to Chen and Sun [34], Li et al. [35], and the references therein. In this paper, without loss of generality, we assume that such jumps cannot destroy the stability of system (2.1). Nevertheless, how to suppress or eliminate the finite jumps in switched singular systems is a challenging problem which deserves further investigation.

Definition 2.4. For the given $\alpha > 0$ and $\gamma > 0$, system (2.1) is said to be exponentially admissible with a weighted H_∞ performance γ under the switching signal $\sigma(t)$, if it is exponentially admissible with $u(t) = 0$ and $w(t) = 0$, and under zero initial condition, that is, $\phi(\theta) = 0$, $\theta \in [-d_2, 0]$, for any nonzero $w(t) \in L_2[0, \infty)$, it holds that

$$
\int_0^t e^{-\alpha s} z^T(s) z(s) ds \leq \gamma^2 \int_0^t w^T(s) w(s) ds.
\tag{2.7}
$$

Remark 2.5. For switched systems with the average dwell time switching, the Lyapunov function values at switching instants are often allowed to increase β times ($\beta > 1$) to reduce the conservatism in system stability analysis, which will lead to the normal disturbance attenuation performance hard to compute or check, even in linear setting [15, 36]. Therefore, the weighted H_∞ performance criterion (2.7) [15, 21, 24] is adopted here to evaluate disturbance attenuation while obtaining the expected exponential stability.

This paper considers both SF control law

$$u(t) = K_{\sigma(t)}x(t) \tag{2.8}$$

and SOF control law

$$u(t) = F_{\sigma(t)}y(t), \tag{2.9}$$

where K_i and F_i, $\sigma(t) = i$, $i \in \mathcal{I}$, are appropriately dimensioned constant matrices to be determined.

Then, the problem to be addressed in this paper can be formulated as follows. Given the switched singular time-delay system (2.1) and a prescribed scalar $\gamma > 0$, identify a class of switching signal $\sigma(t)$ and design an SF controller of the form (2.8) and an SOF controller of the form (2.9) such that the resultant closed-loop system is exponentially admissible with a weighted H_∞ performance γ under the switching signal $\sigma(t)$.

3. Exponential Admissibility and H_∞ Performance Analysis

First, we apply the average dwell time approach and the piecewise Lyapunov function technique to investigate the exponential admissibility for the switched singular time-delay system (2.5) and give the following result.

Theorem 3.1. *For prescribed scalars $\alpha > 0$, $0 \le d_1 \le d_2$ and $0 < \mu < 1$, if for each $i \in \mathcal{I}$, there exist matrices $Q_{il} > 0$, $l = 1, 2, 3$, $Z_{iv} > 0$, M_{iv}, N_{iv}, S_{iv}, $v = 1, 2$, and P_i of the following form:*

$$P_i = \begin{bmatrix} P_{i11} & 0 \\ P_{i21} & P_{i22} \end{bmatrix}, \tag{3.1}$$

with $P_{i11} \in \mathbf{R}^r$, $P_{i11} > 0$, and P_{i22} being invertible, such that

$$\begin{bmatrix} \Phi_{i11} & \Phi_{i12} & \Phi_{i13} & -S_{i1}E & c_1 N_{i1} & c_{12} S_{i1} & c_{12} M_{i1} & A_i^T U_i \\ * & \Phi_{i22} & \Phi_{i23} & -S_{i2}E & c_1 N_{i2} & c_{12} S_{i2} & c_{12} M_{i2} & A_{di}^T U_i \\ * & * & \Phi_{i33} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{i44} & 0 & 0 & 0 & 0 \\ * & * & * & * & -c_1 Z_{i1} & 0 & 0 & 0 \\ * & * & * & * & * & -c_{12} Z_{i2} & 0 & 0 \\ * & * & * & * & * & * & -c_{12} Z_{i2} & 0 \\ * & * & * & * & * & * & * & -U_i \end{bmatrix} < 0, \tag{3.2}$$

where

$$\Phi_{i11} = \operatorname{Sym}\left\{P_i^T A_i + N_{i1} E\right\} + \sum_{l=1}^{3} Q_{il} + \alpha E^T P_i,$$
$$\Phi_{i12} = P_i^T A_{di} + (N_{i2}E)^T + S_{i1}E - M_{i1}E, \qquad \Phi_{i13} = M_{i1}E - N_{i1}E,$$
$$\Phi_{i22} = -(1-\mu)e^{-\alpha d_2} Q_{i3} + \operatorname{Sym}\{S_{i2}E - M_{i2}E\},$$
$$\Phi_{i23} = M_{i2}E - N_{i2}E, \qquad \Phi_{i33} = -e^{-\alpha d_1} Q_{i1}, \qquad \Phi_{i44} = -e^{-\alpha d_2} Q_{i2}, \tag{3.3}$$
$$c_1 = \frac{1}{\alpha}\left(e^{\alpha d_1} - 1\right), \qquad c_{12} = \frac{1}{\alpha}\left(e^{\alpha d_2} - e^{\alpha d_1}\right),$$
$$d_{12} = d_2 - d_1, \qquad U_i = d_1 Z_{i1} + d_{12} Z_{i2}.$$

Then, system (2.5) *with* $d(t)$ *satisfying* (2.2) *is exponentially admissible for any switching sequence* $\mathcal{S}$ *with average dwell time* $T_a \geq T_a^* = (\ln \beta)/\alpha$, *where* $\beta \geq 1$ *satisfies*

$$P_{i11} \leq \beta P_{j11}, \quad Q_{il} \leq \beta Q_{jl}, \quad Z_{iv} \leq \beta Z_{jv}, \quad l = 1,2,3, \ v = 1,2, \ \forall i, j \in \mathcal{I}. \tag{3.4}$$

Moreover, an estimate on the exponential decay rate is $\lambda = (1/2)(\alpha - (\ln \beta)/T_a)$.

Proof. The proof is divided into three parts: (i) to show the regularity and nonimpulsiveness; (ii) to show the exponential stability of the differential subsystem; (iii) to show the exponential stability of the algebraic subsystem.

(i) Regularity and nonimpulsiveness. According to (2.4), for each $i \in \mathcal{I}$, denote

$$A_i = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix}, \tag{3.5}$$

where $A_{i11} \in \mathbf{R}^r$. From (3.2), it is easy to see that $\Phi_{i11} < 0$, $i \in \mathcal{I}$. Noting $Q_{il} > 0$, $l = 1,2,3$, we get

$$\operatorname{Sym}\left\{P_i^T A_i + N_{i1} E\right\} + \alpha E^T P_i < 0. \tag{3.6}$$

Substituting P_i and E given as (3.1) and (2.4) into this inequality yields

$$\begin{bmatrix} \star & \star \\ \star & A_{i22}^T P_{i22} + P_{i22}^T A_{i22} \end{bmatrix} < 0, \tag{3.7}$$

where $\star$ denotes a matrix which is not relevant to the discussion. This implies that A_{i22}, $i \in \mathcal{I}$, is nonsingular. Then, by Dai [29] and Definition 2.1, system (2.5) is regular and impulse free.

(ii) Exponential stability of differential subsystem. Define the piecewise Lyapunov functional candidate for system (2.5) as the following:

$$\begin{aligned} V(x_t) &= V_{\sigma(t)}(x_t) \\ &= x^T(t)E^T P_{\sigma(t)}x(t) + \sum_{v=1}^{2}\int_{t-d_v}^{t} x^T(s)e^{\alpha(s-t)}Q_{\sigma(t)v}x(s)ds \\ &\quad + \int_{t-d(t)}^{t} x^T(s)e^{\alpha(s-t)}Q_{\sigma(t)3}x(s)ds \\ &\quad + \int_{-d_1}^{0}\int_{t+\theta}^{t} (E\dot{x}(s))^T e^{\alpha(s-t)}Z_{\sigma(t)1}(E\dot{x}(s))ds\,d\theta \\ &\quad + \int_{-d_2}^{-d_1}\int_{t+\theta}^{t} (E\dot{x}(s))^T e^{\alpha(s-t)}Z_{\sigma(t)2}(E\dot{x}(s))ds\,d\theta. \end{aligned} \tag{3.8}$$

Then, along the solution of system (2.5) for a fixed $\sigma(t) = i$, $i \in \mathcal{I}$, we have

$$\begin{aligned} \dot{V}_i(x_t) \le{}& 2x^T(t)P_i^T E\dot{x}(t) + \sum_{v=1}^{2}\left[x^T(t)Q_{iv}x(t) - x^T(t-d_v)e^{-\alpha d_v}Q_{iv}x(t-d_v)\right] + x^T(t)Q_{i3}x(t) \\ &- (1-\mu)x^T(t-d(t))e^{-\alpha d_2}Q_{i3}x(t-d(t)) + (E\dot{x}(t))^T(d_1 Z_{i1} + d_{12}Z_{i2})(E\dot{x}(t)) \\ &- \int_{t-d_1}^{t} (E\dot{x}(s))^T e^{\alpha(s-t)}Z_{i1}(E\dot{x}(s))ds - \int_{t-d_2}^{t-d_1} (E\dot{x}(s))^T e^{\alpha(s-t)}Z_{i2}(E\dot{x}(s))ds \\ &- \alpha\sum_{v=1}^{2}\int_{t-d_v}^{t} x^T(s)e^{\alpha(s-t)}Q_{iv}\xi(s)ds - \alpha\int_{t-d(t)}^{t} x^T(s)e^{\alpha(s-t)}Q_{i3}x(s)ds \\ &- \alpha\int_{-d_1}^{0}\int_{t+\theta}^{t} (E\dot{x}(s))^T e^{\alpha(s-t)}Z_{i1}(E\dot{x}(s))ds\,d\theta \\ &- \alpha\int_{-d_2}^{-d_1}\int_{t+\theta}^{t} (E\dot{x}(s))^T e^{\alpha(s-t)}Z_{i2}(E\dot{x}(s))ds\,d\theta. \end{aligned} \tag{3.9}$$

From the Leibniz-Newton formula, the following equations are true for any matrices N_{iv}, S_{iv}, and M_{iv}, $v = 1, 2$, with appropriate dimensions

$$2\left[x^T(t)N_{i1} + x^T(t-d(t))N_{i2}\right]\left[Ex(t) - Ex(t-d_1) - \int_{t-d_1}^{t} E\dot{x}(s)ds\right],$$

$$2\left[x^T(t)S_{i1} + x^T(t-d(t))S_{i2}\right]\left[Ex(t-d(t)) - Ex(t-d_2) - \int_{t-d_2}^{t-d(t)} E\dot{x}(s)ds\right],$$
$$2\left[x^T(t)M_{i1} + x^T(t-d(t))M_{i2}\right]\left[Ex(t-d_1) - Ex(t-d(t)) - \int_{t-d(t)}^{t-d_1} E\dot{x}(s)ds\right]. \tag{3.10}$$

On the other hand, the following equation is also true:

$$\begin{aligned}&-\int_{t-d_2}^{t-d_1} (E\dot{x}(s))^T e^{\alpha(s-t)} Z_{i2}(E\dot{x}(s))ds \\ &\quad = -\int_{t-d_2}^{t-d(t)} (E\dot{x}(s))^T e^{\alpha(s-t)} Z_{i2}(E\dot{x}(s))ds - \int_{t-d(t)}^{t-d_1} (E\dot{x}(s))^T e^{\alpha(s-t)} Z_{i2}(E\dot{x}(s))ds.\end{aligned} \tag{3.11}$$

By (3.8)–(3.11), we have

$$\begin{aligned}&\dot{V}_i(x_t) + \alpha V_i(x_t) \\ &\quad \leq \eta^T(t)\left[\Phi_i + \widetilde{A}_i^T(d_1 Z_{i1} + d_{12} Z_{i2})\widetilde{A}_i + c_1 \widetilde{N}_i Z_{i1}^{-1} \widetilde{N}_i^T + c_{12} \widetilde{S}_i Z_{i2}^{-1} \widetilde{S}_i^T + c_{12} \widetilde{M}_i Z_{i2}^{-1} \widetilde{M}_i^T\right]\eta(t) \\ &\quad - \int_{t-d_1}^{t} \left[\eta^T(t)\widetilde{N}_i + (E\dot{x}(s))^T e^{\alpha(s-t)} Z_{i1}\right] e^{\alpha(t-s)} Z_{i1}^{-1} \left[\eta^T(t)\widetilde{N}_i + (E\dot{x}(s))^T e^{\alpha(s-t)} Z_{i1}\right]^T ds \\ &\quad - \int_{t-d_2}^{t-d(t)} \left[\eta^T(t)\widetilde{S}_i + (E\dot{x}(s))^T e^{\alpha(s-t)} Z_{i2}\right] e^{\alpha(t-s)} Z_{i2}^{-1} \left[\eta^T(t)\widetilde{S}_i + (E\dot{x}(s))^T e^{\alpha(s-t)} Z_{i2}\right]^T ds \\ &\quad - \int_{t-d(t)}^{t-d_1} \left[\eta^T(t)\widetilde{M}_i + (E\dot{x}(s))^T e^{\alpha(s-t)} Z_{i2}\right] e^{\alpha(t-s)} Z_{i2}^{-1} \left[\eta^T(t)\widetilde{M}_i + (E\dot{x}(s))^T e^{\alpha(s-t)} Z_{i2}\right]^T ds,\end{aligned} \tag{3.12}$$

where $\eta(t) = [x^T(t) \;\; x^T(t-d(t)) \;\; x^T(t-d_1) \;\; x^T(t-d_2)]^T$, $\widetilde{A}_i = [A_i \;\; A_{di} \;\; 0 \;\; 0]$, and

$$\Phi_i = \begin{bmatrix} \Phi_{i11} & \Phi_{i12} & \Phi_{i13} & -S_{i1}E \\ * & \Phi_{i22} & \Phi_{i23} & -S_{i2}E \\ * & * & \Phi_{i33} & 0 \\ * & * & * & \Phi_{i44} \end{bmatrix}, \quad \widetilde{N}_i = \begin{bmatrix} N_{i1} \\ N_{i2} \\ 0 \\ 0 \end{bmatrix}, \quad \widetilde{M}_i = \begin{bmatrix} M_{i1} \\ M_{i2} \\ 0 \\ 0 \end{bmatrix}, \quad \widetilde{S}_i = \begin{bmatrix} S_{i1} \\ S_{i2} \\ 0 \\ 0 \end{bmatrix}. \tag{3.13}$$

By Schur complement, LMI (3.2) implies

$$\Phi_i + \widetilde{A}_i^T(d_1 Z_{i1} + d_{12} Z_{i2})\widetilde{A}_i + c_1 \widetilde{N}_i Z_{i1}^{-1} \widetilde{N}_i^T + c_{12} \widetilde{S}_i Z_{i2}^{-1} \widetilde{S}_i^T + c_{12} \widetilde{M}_i Z_{i2}^{-1} \widetilde{M}_i^T < 0. \tag{3.14}$$

Notice that the last three parts in (3.12) are all less than 0. So, if (3.14) holds, then

$$\dot{V}_i(x_t) + \alpha V_i(x_t) < 0. \tag{3.15}$$

For an arbitrary piecewise constant switching signal $\sigma(t)$, and for any $t > 0$, we let $0 = t_0 < t_1 < \cdots < t_k < \cdots$, $k = 1, 2, \cdots$, denote the switching points of $\sigma(t)$ over the interval $(0, t)$. As mentioned earlier, the i_kth subsystem is activated when $t \in [t_k, t_{k+1})$. Integrating (3.15) from t_k to t_{k+1} gives

$$V(x_t) = V_{\sigma(t)}(x_t) \le e^{-\alpha(t-t_k)} V_{\sigma(t_k)}(x_{t_k}), \quad t \in [t_k, t_{k+1}). \tag{3.16}$$

Let $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, where $x_1(t) \in \mathbf{R}^r$ and $x_2(t) \in \mathbf{R}^{n-r}$. From (2.4) and (3.1), it can be deduced that for each $\sigma(t) = i$, $i \in \mathcal{I}$

$$x^T(t) E^T P_i x(t) = x_1^T(t) P_{i11} x_1(t). \tag{3.17}$$

In view of this, and using (3.4) and (3.8), at switching instant t_i, we have

$$V_{\sigma(t_i)}(x_{t_i}) \le \beta V_{\sigma(t_i^-)}\left(x_{t_i^-}\right), \quad i = 1, 2, \ldots, \tag{3.18}$$

where t_i^- denotes the left limitation of t_i. Therefore, it follows from (3.16), (3.18), and the relation $k = N_\sigma(t_0, t) \le (t - t_0)/T_a$ that

$$\begin{aligned} V_{\sigma(t)}(x_t) &\le e^{-\alpha(t-t_k)} \beta V_{\sigma(t_i^-)}\left(x_{t_i^-}\right) \\ &\le \cdots \le e^{-\alpha(t-t_0)} \beta^k V_{\sigma(t_0)}(t_0) \\ &\le e^{-(\alpha - (\ln\beta)/T_a)(t-t_0)} V_{\sigma(t_0)}(x_{t_0}). \end{aligned} \tag{3.19}$$

According to (3.8) and (3.19), we obtain

$$\lambda_1 \|x_1(t)\|^2 \le V_{\sigma(t)}(t), \qquad V_{\sigma(t_0)}(x_{t_0}) \le \lambda_2 \|x_{t_0}\|_c^2, \tag{3.20}$$

where

$$\begin{aligned} \lambda_1 &= \min_{\forall i \in \mathcal{I}} \lambda_{\min}(P_{i11}), \\ \lambda_2 &= \max_{\forall i \in \mathcal{I}} \lambda_{\max}(P_{i11}) + \frac{1}{\alpha}\left(1 - e^{-\alpha d_1}\right) \max_{\forall i \in \mathcal{I}} \lambda_{\max}(Q_{i1}) + \frac{1}{\alpha}\left(1 - e^{-\alpha d_2}\right) \max_{\forall i \in \mathcal{I}} (\lambda_{\max}(Q_{i2}) + \lambda_{\max}(Q_{i3})) \\ &\quad + \frac{1}{\alpha^2}\left(\alpha d_1 - 1 + e^{-\alpha d_1}\right) \max_{\forall i \in \mathcal{I}} (2\lambda_{\max}(Z_{i1})(\|A_i\| + \|A_{di}\|)) \\ &\quad + \frac{\alpha d_{12} - e^{-\alpha d_1} + e^{-\alpha d_2}}{\alpha^2} \max_{\forall i \in \mathcal{I}} (2\lambda_{\max}(Z_{i2})(\|A_i\| + \|A_{di}\|)). \end{aligned} \tag{3.21}$$

Considering (3.19) and (3.20) yields

$$\|x_1(t)\|^2 \le \frac{1}{\lambda_1} V_{\sigma(t)}(x_t) \le \frac{\lambda_2}{\lambda_1} e^{-(\alpha-(\ln\beta)/T_a)(t-t_0)} \|x_{t_0}\|_c^2 \tag{3.22}$$

which implies

$$\|x_1(t)\| \le \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)} \|x_{t_0}\|_c. \tag{3.23}$$

(iii) Exponential stability of algebraic subsystem. Since A_{i22}, $i \in \mathcal{I}$, is nonsingular, we choose

$$G_i = \begin{bmatrix} I_r & -A_{i12}A_{i22}^{-1} \\ 0 & A_{i22}^{-1} \end{bmatrix}, \qquad H = \begin{bmatrix} I_r & 0 \\ 0 & I_{n-r} \end{bmatrix}. \tag{3.24}$$

Then, it is easy to get

$$\widehat{E} := G_i E H = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad \widehat{A}_i := G_i A_i H = \begin{bmatrix} \widehat{A}_{i11} & 0 \\ \widehat{A}_{i21} & I_{n-r} \end{bmatrix}, \qquad \widehat{P}_i := G_i^{-T} P_i H = \begin{bmatrix} \widehat{P}_{i11} & 0 \\ \widehat{P}_{i21} & \widehat{P}_{i22} \end{bmatrix}, \tag{3.25}$$

where $\widehat{A}_{i11} = A_{i11} - A_{i12}A_{i22}^{-1}A_{i21}$, $\widehat{A}_{i21} = A_{i22}^{-1}A_{i21}$, $\widehat{P}_{i11} = P_{i11}$, $\widehat{P}_{i21} = A_{i12}^T P_{i11} + A_{i22}^T P_{i21}$, and $\widehat{P}_{i21} = A_{i22}^T P_{i22}$. According to (3.25), denote

$$\begin{gathered} \widehat{A}_{di} := G_i A_{di} H = \begin{bmatrix} \widehat{A}_{di11} & \widehat{A}_{di12} \\ \widehat{A}_{di21} & \widehat{A}_{di22} \end{bmatrix}, \qquad \widehat{Q}_{il} := H^T Q_{il} H = \begin{bmatrix} \widehat{Q}_{il11} & \widehat{Q}_{il12} \\ \widehat{Q}_{il21} & \widehat{Q}_{il22} \end{bmatrix}, \\ \widehat{Z}_{iv} := G_i^{-T} Z_{iv} G_i^{-1} = \begin{bmatrix} \widehat{Z}_{iv11} & \widehat{Z}_{iv12} \\ \widehat{Z}_{iv21} & \widehat{Z}_{iv22} \end{bmatrix}, \qquad \widehat{M}_{iv} := H^T M_{iv} G_i^{-1} = \begin{bmatrix} \widehat{M}_{iv11} & \widehat{M}_{iv12} \\ \widehat{M}_{iv21} & \widehat{M}_{iv22} \end{bmatrix}, \\ \widehat{N}_{iv} := H^T N_{iv} G_i^{-1} = \begin{bmatrix} \widehat{N}_{iv11} & \widehat{N}_{iv12} \\ \widehat{N}_{iv21} & \widehat{N}_{iv22} \end{bmatrix}, \qquad \widehat{S}_{iv} := H^T S_{iv} G_i^{-1} = \begin{bmatrix} \widehat{S}_{iv11} & \widehat{S}_{iv12} \\ \widehat{S}_{iv21} & \widehat{S}_{iv22} \end{bmatrix}, \\ l = 1,2,3, \quad v = 1,2 \end{gathered} \tag{3.26}$$

and let

$$\xi(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} := H^{-1} x(t) = x(t), \tag{3.27}$$

where $\xi_1(t) \in \mathbf{R}^r$ and $\xi_2(t) \in \mathbf{R}^{n-r}$. Then, for any $\sigma(t) = i$, $i \in \mathcal{I}$, system (2.5) is a restricted system equivalent (r.s.e.) to

$$\begin{aligned} \dot{\xi}_1(t) &= \widehat{A}_{i11}\xi_1(t) + \widehat{A}_{di11}\xi_1(t-d(t)) + \widehat{A}_{di12}\xi_2(t-d(t)), \\ -\xi_2(t) &= \widehat{A}_{i21}\xi_1(t) + \widehat{A}_{di21}\xi_1(t-d(t)) + \widehat{A}_{di22}\xi_2(t-d(t)). \end{aligned} \tag{3.28}$$

By (3.2) and Schur complement, we have

$$\begin{bmatrix} \Phi_{i11} & \Phi_{i12} \\ * & \Phi_{i22} \end{bmatrix} < 0. \tag{3.29}$$

Pre- and postmultiplying this inequality by $\operatorname{diag}\{H^T, H^T\}$ and $\operatorname{diag}\{H, H\}$, respectively, noting the expressions in (3.25) and (3.26), and using Schur complement, we have

$$\begin{bmatrix} \widehat{P}_{i22}^T + \widehat{P}_{i22} + \sum_{l=1}^{3}\widehat{Q}_{il22} & \widehat{P}_{i22}^T\widehat{A}_{di22}^T \\ * & -(1-\mu)e^{-\alpha d_2}\widehat{Q}_{i322} \end{bmatrix} < 0. \tag{3.30}$$

Pre- and postmultiplying this inequality by $[-\widehat{A}_{di22}^T \;\; I]$ and its transpose, respectively, and noting $\widehat{Q}_{i122} > 0$, $\widehat{Q}_{i222} > 0$, and $\mu \geq 0$, we obtain

$$\left(e^{(1/2)\alpha d_2}\widehat{A}_{di22}\right)^T \widehat{Q}_{i322}\left(e^{(1/2)\alpha d_2}\widehat{A}_{di22}\right) - \widehat{Q}_{i322} < 0. \tag{3.31}$$

Then, according to Lemma 5 in Kharitonov et al. [37], we can deduce that there exist constants $\hbar_i > 1$ and $\eta_i \in (0,1)$ such that

$$\left\| \left(e^{(1/2)\alpha d_2}\widehat{A}_{di22}\right)^l \right\| \leq \hbar_i e^{-\eta_i l}, \quad l = 0, 1, \ldots. \tag{3.32}$$

Define

$$\begin{gathered} t^0 = t, \quad t^j = t^{j-1} - d\left(t^{j-1}\right), \quad j = 1, 2, \ldots, \\ \left\|\widehat{A}_{21}\right\| = \max_{\forall i \in \mathcal{I}}\left\|\widehat{A}_{i21}\right\|, \quad \left\|\widehat{A}_{d21}\right\| = \max_{\forall i \in \mathcal{I}}\left\|\widehat{A}_{di21}\right\|, \quad \left\|\widehat{A}_{d22}\right\| = \max_{\forall i \in \mathcal{I}}\left\|\widehat{A}_{di22}\right\|, \quad \forall i \in \mathcal{I}. \end{gathered} \tag{3.33}$$

Now, following similar line as in Part 3 in Theorem 1 of Lin and Fei [31], it can easily be obtained that

$$\|\xi_2(t)\| \leq (\chi_1 + \chi_2 + \chi_3 + \chi_4 + \chi_5)e^{-(1/2)(\alpha-(\ln\beta)/T_a)(t-t_0)}\|x_{t_0}\|_c, \tag{3.34}$$

where

$$\chi_1 = \prod_{j=0}^{k} \hbar_{i_j} e^{-\eta_{i_j} T_{i_j}},$$

$$\chi_2 = \hbar_{i_k} \widehat{A}_{21} \sqrt{\frac{\lambda_2}{\lambda_1}} \frac{e^{\eta_{i_k}}}{e^{\eta_{i_k}} - 1},$$

$$\chi_3 = \hbar_{i_k} e^{(1/2)\alpha d_2} \widehat{A}_{d21} \sqrt{\frac{\lambda_2}{\lambda_1}} \frac{e^{\eta_{i_k}}}{e^{\eta_{i_k}} - 1}, \tag{3.35}$$

$$\chi_4 = \widehat{A}_{21} \sqrt{\frac{\lambda_2}{\lambda_1}} \sum_{p=1}^{k} \left\{ \hbar_{i_{p-1}} \left[\prod_{q=p}^{k} \hbar_{i_q} e^{-\eta_{i_q} T_{i_q}} \right] \frac{e^{\eta_{i_{p-1}}}}{e^{\eta_{i_{p-1}}} - 1} \right\},$$

$$\chi_5 = e^{(1/2)\alpha d_2} \widehat{A}_{d21} \sqrt{\frac{\lambda_2}{\lambda_1}} \sum_{p=1}^{k} \left\{ \hbar_{i_{p-1}} \left[\prod_{q=p}^{k} \hbar_{i_q} e^{-\eta_{i_q} T_{i_q}} \right] \frac{e^{\eta_{i_{p-1}}}}{e^{\eta_{i_{p-1}}} - 1} \right\}.$$

$T_{i_k}, T_{i_{k-1}}, \ldots, T_{i_0}$ are positive finite integers, respectively, satisfying

$$\begin{aligned}
&t^{T_{i_k}} \in (t_{k-1}, t_k], \qquad t^{T_{i_k}} \longrightarrow t_k,\\
&t^{T_{i_k}+T_{i_{k-1}}} \in (t_{k-2}, t_{k-1}], \qquad t^{T_{i_k}+T_{i_{k-1}}} \longrightarrow t_{k-1},\\
&\qquad \vdots\\
&t^{T_{i_k}+\cdots+T_{i_0}} \in (-d_2, t_0], \qquad t^{T_{i_k}+\cdots+T_{i_0}} \longrightarrow t_0.
\end{aligned} \tag{3.36}$$

Combining (3.27), (3.23) and (3.34) yields that system (2.5) is exponentially stable for any switching sequence $\mathcal{S}$ with average dwell time $T_a \geq T_a^* = \ln\beta/\alpha$. This completes the proof. □

Remark 3.2. Theorem 3.1 provides a sufficient condition of the exponential admissibility for the switched singular time-delay system (2.5). Note that due to the existence of algebraic constraints in system states, the stability analysis of switched singular time-delay systems is much more complicated than that for switched state-space time-delay systems [21–23, 25, 38]. Note also that the condition established in Theorem 3.1 is not only delay-range-dependent but also decay-rate-dependent. The delay-range-dependence makes the result less conservative, while the decay-rate-dependence enables one to control the transient process of differential and algebraic subsystems with a unified performance specification.

Remark 3.3. Different from the integral inequality method used in our previous work [31], the free-weighting matrix method [39] is adopted when deriving Theorem 3.1, and thus no three-product terms, for example, $A_i^T Z_{iv} A_i$, $A_{di}^T Z_{iv} A_{di}$, and so forth, are involved, which greatly facilitates the SF and SOF controllers design, as seen in Section 4.

Remark 3.4. If $\beta = 1$ in $T_a \geq T_a^* = (\ln\beta)/\alpha$, which leads to $P_{i11} \equiv P_{j11}$, $Q_{il} \equiv Q_{jl}$, $Z_{iv} \equiv Z_{jv}$, $l = 1, 2, 3$, $v = 1, 2$, for all $i, j \in \mathcal{I}$, and $T_a^* = 0$, then system (2.5) possesses a common Lyapunov function, and the switching signals can be arbitrary.

Now, the following theorem presents a sufficient condition on exponential admissibility with a weighted H_∞ performance of the switched singular time-delay system (2.1) with $u(t) = 0$.

Theorem 3.5. *For prescribed scalars $\alpha > 0$, $\gamma > 0$, $0 \le d_1 \le d_2$, and $0 < \mu < 1$, if for each $i \in \mathcal{I}$, there exist matrices $Q_{il} > 0$, $l = 1,2,3$, $Z_{iv} > 0$, M_{iv}, N_{iv}, S_{iv}, $v = 1,2$, and P_i with the form of* (3.1) *such that*

$$\Phi_i = \begin{bmatrix} \widetilde{\Phi}_{i11} & \widetilde{\Phi}_{i12} & \Phi_{i13} & -S_{i1}E & \widetilde{\Phi}_{i15} & C_i^T & c_1 N_{i1} & c_{12} S_{i1} & c_{12} M_{i1} \\ * & \widetilde{\Phi}_{i22} & \Phi_{i23} & -S_{i2}E & \widetilde{\Phi}_{i25} & C_{di}^T & c_1 N_{i2} & c_{12} S_{i2} & c_{12} M_{i2} \\ * & * & \Phi_{i33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{i44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \widetilde{\Phi}_{i55} & D_{wi}^T & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -c_1 Z_{i1} & 0 & 0 \\ * & * & * & * & * & * & * & -c_{12} Z_{i2} & 0 \\ * & * & * & * & * & * & * & * & -c_{12} Z_{i2} \end{bmatrix} < 0, \tag{3.37}$$

where

$$\begin{aligned} &\widetilde{\Phi}_{i11} = \Phi_{i11} + A_i^T U_i A_i, \qquad \widetilde{\Phi}_{i12} = \Phi_{i12} + A_i^T U_i A_{di}, \qquad \widetilde{\Phi}_{i15} = P_i^T B_{wi} + A_i^T U_i B_{wi}, \\ &\widetilde{\Phi}_{i22} = \Phi_{i22} + A_{di}^T U_i A_{di}, \qquad \widetilde{\Phi}_{i25} = A_{di}^T U_i B_{wi}, \qquad \widetilde{\Phi}_{i55} = -\gamma^2 I + B_{wi}^T U_i B_{wi}, \end{aligned} \tag{3.38}$$

and Φ_{i11}, Φ_{i12}, Φ_{i13}, Φ_{i22}, Φ_{i23}, Φ_{i33}, Φ_{i44}, and U_i are defined in (3.2). *Then, system* (2.1) *with $u(t) = 0$ is exponentially admissible with a weighted H_∞ performance γ for any switching sequence $\mathcal{S}$ with average dwell time $T_a \ge T_a^* = (\ln \beta)/\alpha$, where $\beta \ge 1$ satisfying* (3.4).

Proof. Choose the piecewise Lyapunov function defined by (3.8). Since (3.37) implies (3.2), system (2.1) with $u(t) = 0$ and $w(t) = 0$ is exponentially admissible by Theorem 3.1. On the other hand, similar to the proof of Theorem 3.1, from (3.37), we have that for $t \in [t_k, t_{k+1})$,

$$\dot{V}_{i_k}(x_t) + \alpha V_{i_k}(x_t) + \Gamma(t) \le 0, \tag{3.39}$$

where $\Gamma(t) = z^T(t) z(t) - \gamma^2 w^T(t) w(t)$. This implies that

$$V_{i_k}(x_t) \le e^{-\alpha(t-t_k)} V_{i_k}(x_{t_k}) - \int_{t_k}^{t} e^{-\alpha(t-s)} \Gamma(s) ds. \tag{3.40}$$

By induction, we have

$$\begin{aligned} V_{i_k}(x_t) &\leq \beta e^{-\alpha(t-t_k)} V_{i_{k-1}}(x_{t_k}) - \int_{t_k}^{t} e^{-\alpha(t-s)}\Gamma(s)ds \\ &\vdots \\ &\leq \beta^k e^{-\alpha t} V_{i_0}(x_0) - \int_{t_k}^{t} e^{-\alpha(t-s)}\Gamma(s)ds - \sum_{p=1}^{k-p} \beta^{k-p} \int_{t_p}^{t_{p+1}} e^{-\alpha(t-s)}\Gamma(s)ds \\ &= e^{-\alpha t + N_\alpha(0,t)\ln\beta} V_{i_0}(x_0) - \int_0^t e^{-\alpha(t-s)+N_\alpha(s,t)\ln\beta}\Gamma(s)ds. \end{aligned} \tag{3.41}$$

Under zero initial condition, (3.41) gives

$$0 \leq -\int_0^t e^{-\alpha(t-s)+N_\alpha(s,t)\ln\beta}\Gamma(s)ds. \tag{3.42}$$

Multiplying both sides of (3.42) by $e^{-N_\alpha(0,t)\ln\beta}$ yields

$$\int_0^t e^{-\alpha(t-s)-N_\alpha(0,s)\ln\beta} z^T(s)z(s)ds \leq \gamma^2 \int_0^t e^{-\alpha(t-s)-N_\alpha(0,s)\ln\beta} w^T(s)w(s)ds. \tag{3.43}$$

Noting that $N_\alpha(0,s) \leq s/T_a$ and $T_a \geq T_a^* = (\ln\beta)/\alpha$, we get $N_\alpha(0,s)\ln\beta \leq \alpha s$. Then, it follows from (3.43) that $\int_0^t e^{-\alpha(t-s)-\alpha s} z^T(s)z(s)ds \leq \gamma^2 \int_0^t e^{-\alpha(t-s)} w^T(s)w(s)ds$. Integrating both sides of this inequality from $t = 0$ to ∞ leads to inequality (2.7). This completes the proof of Theorem 3.5. □

Remark 3.6. Note that when $\beta = 1$, which is a trivial case, system (2.1) with $u(t) = 0$ achieves the normal H_∞ performance γ under arbitrary switching.

4. Controller Design

In this section, based on the results of the previous section, we are to deal with the design problems of both SF and SOF controllers for the switched singular time-delay system (2.1).

4.1. SF Controller Design

Applying the SF controller (2.8) to system (2.1) gives the following closed-loop system:

$$\begin{aligned} E\dot{x}(t) &= \overline{A}_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d(t)) + B_{w\sigma(t)}w(t), \\ z(t) &= \overline{C}_{\sigma(t)}x(t) + C_{d\sigma(t)}x(t-d(t)) + D_{w\sigma(t)}w(t), \\ x(\theta) &= \phi(\theta), \quad \theta \in [-d_2, 0], \end{aligned} \tag{4.1}$$

where

$$\overline{A}_{\sigma(t)} = A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)}, \qquad \overline{C}_{\sigma(t)} = C_{\sigma(t)} + D_{\sigma(t)}K_{\sigma(t)}. \tag{4.2}$$

The following theorem presents a sufficient condition for solvability of the SF controller design problem for system (2.1).

Theorem 4.1. *For prescribed scalars $\alpha > 0$, $\gamma > 0$, $0 \leq d_1 \leq d_2$, and $0 < \mu < 1$, if for each $i \in \mathcal{I}$, and given scalars ϵ_{if}, $f = 1, 2, \ldots, 6$, $\epsilon_{i7} > 0$, and $\epsilon_{i8} > 0$, there exist matrices $R_{il} > 0$, $l = 1, 2, 3$, $Z_{iv} > 0$, T_i, and X_i of the following form:*

$$X_i = \begin{bmatrix} X_{i11} & 0 \\ X_{i21} & X_{i22} \end{bmatrix}, \tag{4.3}$$

with $X_{i11} \in \mathbf{R}^r$, $X_{i11} > 0$, and X_{i22} being invertible, such that

$$\begin{bmatrix}
\Psi_{i11} & \Psi_{i12} & \Psi_{i13} & \Psi_{i14} & B_{wi} & \Psi_{i16} & \Psi_{i17} & c_{12}\epsilon_{i5}I & c_{12}\epsilon_{i3}I & \Psi_{i110} & \Psi_{i111} & \Xi_i \\
* & \Psi_{i22} & \Psi_{i23} & \Psi_{i24} & 0 & \Psi_{i26} & \Psi_{i27} & c_{12}\epsilon_{i6}I & c_{12}\epsilon_{i4}I & \Psi_{i210} & \Psi_{i211} & 0 \\
* & * & \Psi_{i33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Psi_{i44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\gamma^2 I & D_{wi}^T & 0 & 0 & 0 & d_1 B_{wi}^T & d_{12} B_{wi}^T & 0 \\
* & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -c_1 Z_{i1} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -c_{12} Z_{i2} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -c_{12} Z_{i2} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & \Psi_{i1010} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & \Psi_{i1111} & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\Gamma_i
\end{bmatrix} < 0, \tag{4.4}$$

where

$$\Psi_{i11} = \text{Sym}\left\{A_i X_i + B_i T_i + \epsilon_{i1} E X_i\right\} + \alpha X_i^T E^T,$$

$$\Psi_{i12} = A_{di} R_{i3} + \epsilon_{i2} X_i^T E^T + \epsilon_{i5} E R_{i3} - \epsilon_{i3} E R_{i3},$$

$$\Psi_{i13} = \epsilon_{i3} E R_{i1} - \epsilon_{i1} E R_{i1}, \qquad \Psi_{i14} = -\epsilon_{i5} E R_{i2},$$

$$\Psi_{i16} = T_i^T D_i^T + X_i^T C_i^T, \qquad \Psi_{i17} = c_1 \epsilon_{i1} I,$$

$$\Psi_{i110} = d_1 T_i^T B_i^T + d_1 X_i^T A_i^T, \qquad \Psi_{i111} = d_{12} T_i^T B_i^T + d_{12} X_i^T A_i^T,$$

$$\Psi_{i22} = -(1-\mu) e^{-\alpha d_2} R_{i3} + \text{Sym}\left\{\epsilon_{i6} E R_{i3} - \epsilon_{i4} E R_{i3}\right\},$$

$$\Psi_{i23} = \epsilon_{i4} E R_{i1} - \epsilon_{i2} E R_{i1}, \qquad \Psi_{i24} = -\epsilon_{i6} E R_{i2},$$
$$\Psi_{i26} = R_{i3} C_{di}^T, \qquad \Psi_{i27} = c_1 \epsilon_{i2} I, \qquad \Psi_{i210} = d_1 R_{i3} A_{di}^T,$$
$$\Psi_{i211} = d_{12} R_{i3} A_{di}^T, \qquad \Psi_{i33} = -e^{-\alpha d_1} R_{i1}, \qquad \Psi_{i44} = -e^{-\alpha d_2} R_{i2},$$
$$\Psi_{i1010} = -2 d_1 \epsilon_{i7} I + d_1 \epsilon_{i7}^2 Z_{i1}, \qquad \Psi_{i1111} = -2 d_{12} \epsilon_{i8} I + d_{12} \epsilon_{i8}^2 Z_{i2},$$
$$\Xi_i = \begin{bmatrix} X_i^T & X_i^T & X_i^T \end{bmatrix}, \qquad \Gamma_i = \operatorname{diag}\{R_{i1}, R_{i2}, R_{i3}\}. \tag{4.5}$$

Then, there exists an SF controller (2.8) *such that the closed-loop system* (4.1) *with* $d(t)$ *satisfying* (2.2) *is exponentially admissible with a weighted* H_∞ *performance* γ *for any switching sequence* $\mathcal{S}$ *with average dwell time* $T_a \geq T_a^* = \ln \beta / \alpha$*, where* $\beta \geq 1$ *satisfies*

$$X_{i11} \geq \beta^{-1} X_{j11}, \quad R_{il} \geq \beta^{-1} R_{jl}, \quad Z_{iv} \leq \beta Z_{jv}, \quad l = 1,2,3, \ v = 1,2, \ \forall i, j \in \mathcal{I}. \tag{4.6}$$

Moreover, the feedback gain of the controller is

$$K_i = T_i X_i^{-1}, \quad i \in \mathcal{I}. \tag{4.7}$$

Proof. According to Theorem 3.5, the closed-loop system (4.1) is exponentially admissible with a weighted H_∞ performance γ if for each $i \in \mathcal{I}$, there exist matrices $Q_{il} > 0$, $l = 1,2,3$, $Z_{iv} > 0$, M_{iv}, N_{iv}, S_{iv}, $v = 1,2$, and P_i with the form of (3.1) such that inequality (3.37) holds with A_i and C_i instead of $\overline{A}_i$ and $\overline{C}_i$, respectively. By Schur complement, (3.37) is equivalent to

$$\begin{bmatrix}
\Phi'_{i11} & \Phi_{i12} & \Phi_{i13} & -S_{i1}E & P_i^T B_{wi} & \overline{C}_i^T & c_1 N_{i1} & c_{12} S_{i1} & c_{12} M_{i1} & d_1 \overline{A}_i^T & d_{12} \overline{A}_i^T \\
* & \Phi_{i22} & \Phi_{i23} & -S_{i2}E & 0 & C_{di}^T & c_1 N_{i2} & c_{12} S_{i2} & c_{12} M_{i2} & d_1 A_{di}^T & d_{12} A_{di}^T \\
* & * & \Phi_{i33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \Phi_{i44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\gamma^2 I & D_{wi}^T & 0 & 0 & 0 & d_1 B_{wi}^T & d_{12} B_{wi}^T \\
* & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -c_1 Z_{i1} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -c_{12} Z_{i2} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -c_{12} Z_{i2} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -d_1 Z_{i1}^{-1} & 0 \\
* & * & * & * & * & * & * & * & * & * & -d_{12} Z_{i2}^{-1}
\end{bmatrix} < 0, \tag{4.8}$$

where Φ_{i12}, Φ_{i13}, Φ_{i22}, Φ_{i23}, Φ_{i33}, and Φ_{i44} are defined in (3.2), and

$$\Phi'_{i11} = \operatorname{Sym}\left\{ P_i^T \overline{A}_i + N_{i1} E \right\} + \sum_{l=1}^{3} Q_{il} + \alpha E^T P_i. \tag{4.9}$$

Since $P_{i11} > 0$ and P_{i22} is invertible, then P_i is invertible. Let

$$X_i = P_i^{-1}, \qquad R_{i1} = Q_{i1}^{-1}, \qquad R_{i2} = Q_{i2}^{-1}, \qquad R_{i3} = Q_{i3}^{-1}. \tag{4.10}$$

By (3.1), X_i has the form of (4.3). Pre- and postmultiplying (4.8) by $\mathrm{diag}\{X_i^T, R_{i3}, R_{i1}, R_{i2}, I, I, I, I, I, I, I\}$ and its transpose, respectively, and noting (4.10), we obtain

$$\begin{bmatrix} \Phi''_{i11} & \Phi''_{i12} & \Phi''_{i13} & \Phi''_{i14} & B_{wi} & \Phi''_{i16} & \Phi''_{i17} & \Phi''_{i18} & \Phi''_{i19} & \Phi''_{i110} & \Phi''_{i111} \\ * & \Phi''_{i22} & \Phi''_{i23} & \Phi''_{i24} & 0 & \Phi''_{i26} & \Phi''_{i27} & \Phi''_{i28} & \Phi''_{i29} & \Psi_{i210} & \Psi_{i211} \\ * & * & \Psi_{i33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{i44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & D_{wi}^T & 0 & 0 & 0 & d_1 B_{wi}^T & d_{12} B_{wi}^T \\ * & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -c_1 Z_{i1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -c_{12} Z_{i2} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -c_{12} Z_{i2} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -d_1 Z_{i1}^{-1} & 0 \\ * & * & * & * & * & * & * & * & * & * & -d_{12} Z_{i2}^{-1} \end{bmatrix} < 0, \tag{4.11}$$

where

$$\begin{gathered} \Phi''_{i11} = \mathrm{Sym}\left\{\overline{A}_i X_i + X_i^T N_{i1} E X_i\right\} + X_i^T \sum_{l=1}^{3} Q_{il} X_i + \alpha X_i^T E^T, \\ \Phi''_{i12} = A_{di} R_{i3} + X_i^T (N_{i2} E)^T R_{i3} + X_i^T S_{i1} E R_{i3} - X_i^T M_{i1} E R_{i3}, \\ \Phi''_{i13} = X_i^T (M_{i1} E - N_{i1} E) R_{i1}, \qquad \Phi''_{i14} = -X_i^T S_{i1} E R_{i2}, \\ \Phi''_{i16} = X_i^T \overline{C}_i^T, \qquad \Phi''_{i17} = c_1 X_i^T N_{i1}, \qquad \Phi''_{i18} = c_{12} X_i^T S_{i1}, \\ \Phi''_{i19} = c_{12} X_i^T M_{i1}, \qquad \Phi''_{i110} = d_1 X_i^T \overline{A}_i^T, \qquad \Phi''_{i111} = d_{12} X_i^T \overline{A}_i^T, \\ \Phi''_{i22} = -(1-\mu) e^{-\alpha d_2} R_{i3} + \mathrm{Sym}\{R_{i3} S_{i2} E R_{i3} - R_{i3} M_{i2} E R_{i3}\}, \\ \Phi''_{i23} = R_{i3} (M_{i2} E - N_{i2} E) R_{i1}, \qquad \Phi''_{i24} = -R_{i3} S_{i2} E R_{i2}, \\ \Phi''_{i26} = R_{i3} C_{di}^T, \qquad \Phi''_{i27} = c_1 R_{i3} N_{i2}, \qquad \Phi''_{i28} = c_{12} R_{i3} S_{i2}, \qquad \Phi''_{i29} = c_{12} R_{i3} M_{i2}. \end{gathered} \tag{4.12}$$

Now, introducing change of variables

$$\begin{gathered} N_{i1} = \epsilon_{i1} X_i^{-T}, \qquad N_{i2} = \epsilon_{i2} R_{i3}^{-1}, \qquad M_{i1} = \epsilon_{i3} X_i^{-T}, \qquad M_{i2} = \epsilon_{i4} R_{i3}^{-1}, \\ S_{i1} = \epsilon_{i5} X_i^{-T}, \qquad S_{i2} = \epsilon_{i6} R_{i3}^{-1}, \qquad T_i = K_i X_i, \end{gathered} \tag{4.13}$$

where $\epsilon_{if}, f = 1, 2, \ldots, 6$ are scalars, noting the fact that

$$-Z_{i1}^{-1} \le -2\epsilon_{i7} I + \epsilon_{i7}^2 Z_{i1}, \qquad -Z_{i2}^{-1} \le -2\epsilon_{i8} I + \epsilon_{i8}^2 Z_{i2}, \tag{4.14}$$

where ϵ_{i7} and ϵ_{i8} are positive scalars, and using Schur complement on (4.11), we can easy obtain (4.4). In addition, by (3.4) and (4.10), it is easily to verify that the condition (4.6) is equivalent to (3.4). This completes the proof. □

Remark 4.2. Scalars ϵ_{ih}, $h = 1, 2, \dots, 8$, $i \in \mathcal{I}$, in Theorem 4.1 are tuning parameters which need to be specified first; such tuning parameters are frequently encountered when dealing with the SF control problem of singular time-delay systems; see, for example, Ma et al. [27], Shu and Lam [40], and Wu et al. [38]. A simple way to choose these tuning parameters is using the trial-and-error method. In fact, (4.4) for fixed ϵ_{ih}, is bilinear matrix inequality (BMI) regarding these tuning parameters. Therefore, if one can accept more computation burden, better results can be obtained by directly applying some existing optimization algorithms, such as the program fminsearch in the optimization toolbox of MATLAB, the branch-and-band algorithm [41], and the branch-and-cut algorithm [42].

4.2. SOF Controller Design

Connecting the SOF controller (2.9) to system (2.1) yields the closed-loop system

$$\begin{aligned} E\dot{x}(t) &= \overline{A}'_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d(t)) + B_{w\sigma(t)}w(t), \\ z(t) &= \overline{C}'_{\sigma(t)}x(t) + C_{d\sigma(t)}x(t-d(t)) + D_{w\sigma(t)}w(t), \end{aligned} \tag{4.15}$$

where

$$\overline{A}'_{\sigma(t)} = A_{\sigma(t)} + B_{\sigma(t)}F_{\sigma(t)}L_{\sigma(t)}, \qquad \overline{C}'_{\sigma(t)} = C_{\sigma(t)} + D_{\sigma(t)}F_{\sigma(t)}L_{\sigma(t)}. \tag{4.16}$$

The following theorem presents a sufficient condition for solvability of the SOF controller design problem for system (2.1).

Theorem 4.3. *For prescribed scalars $\alpha > 0$, $\gamma > 0$, $0 \le d_1 \le d_2$, and $0 < \mu < 1$, if for each $i \in \mathcal{I}$, and a given matrix J_i, there exist matrices $Q_{il} > 0$, $l = 1, 2, 3$, $Z_{iv} > 0$, and P_i of the form* (3.2) *such that*

$$\begin{bmatrix} \Lambda_{i11} & \Lambda_{i12} & \Lambda_{i13} & \Lambda_{i14} & -S_{i1}E & J_i^T B_{wi} & \overline{C}_i'^T & c_1 N_{i1} & c_{12} S_{i1} & c_{12} M_{i1} \\ * & \Lambda_{i22} & \Lambda_{i23} & 0 & 0 & J_i^T B_{wi} & 0 & 0 & 0 & 0 \\ * & * & \Lambda_{i33} & \Lambda_{i34} & -S_{i2}E & 0 & C_{di}^T & c_1 N_{i2} & c_{12} S_{i2} & c_{12} M_{i2} \\ * & * & * & \Lambda_{i44} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Lambda_{i55} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & D_{wi}^T & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -c_1 Z_{i1} & 0 & 0 \\ * & * & * & * & * & * & * & * & -c_{12} Z_{i2} & 0 \\ * & * & * & * & * & * & * & * & * & -c_{12} Z_{i2} \end{bmatrix} < 0, \tag{4.17}$$

where c_1, c_{12}, *and* U_i *are defined in* (3.2), *and*

$$\begin{gathered}
\Lambda_{i11} = \operatorname{Sym}\left\{J_i\overline{A}_i' + N_{i1}E\right\} + \sum_{l=1}^{3} Q_{il} + \alpha E^T P_i, \\
\Lambda_{i12} = -J_i + \overline{A}_i'^T J_i^T + P_i^T, \qquad \Lambda_{i13} = J_i A_{di} + E^T N_{i2}^T + S_{i1}E - M_{i1}E, \\
\Lambda_{i14} = M_{i1}E - N_{i1}E, \qquad \Lambda_{i22} = -J_i^T - J_i + U_i, \qquad \Lambda_{i23} = J_i A_{di}, \\
\Lambda_{i33} = -(1-\mu)e^{-\alpha d_2} Q_{i3} + \operatorname{Sym}\{S_{i2}E - M_{i2}E\}, \\
\Lambda_{i34} = M_{i2}E - N_{i2}E, \qquad \Lambda_{i44} = -e^{-\alpha d_1} Q_{i1}, \qquad \Lambda_{i55} = -e^{-\alpha d_2} Q_{i2}.
\end{gathered} \tag{4.18}$$

Then, there exists an SOF controller (2.9) *such that the closed-loop system* (4.15) *with* $d(t)$ *satisfying* (2.2) *is exponentially admissible with a weighted* H_∞ *performance* γ *for any switching sequence* $\mathcal{S}$ *with average dwell time* $T_a \geq T_a^* = \ln\beta/\alpha$, *where* $\beta \geq 1$ *satisfying* (3.4).

Proof. From Theorem 3.5, we know that system (4.15) is exponentially admissible with a weighted H_∞ performance γ for any switching sequence $\mathcal{S}$ with average dwell time $T_a \geq T_a^* = (\ln\beta)/\alpha$, where $\beta \geq 1$ satisfying (3.4), if for each $i \in \mathcal{I}$, there exist matrices $Q_{il} > 0$, $l = 1,2,3$, $Z_{iv} > 0$, M_{iv}, N_{iv}, S_{iv}, $v = 1,2$, and P_i with the form of (3.1) such that the inequality (4.10) with A_i and C_i instead of $\overline{A}_i'$ and $\overline{C}_i'$, respectively, holds. By decomposing Φ_i in (4.10), we obtain that for each $i \in \mathcal{I}$,

$$\Phi_i = \Pi(i)\Lambda_i\Pi_i^T < 0, \tag{4.19}$$

where Λ_i is exactly the left half of the inequality (4.17), and

$$\Pi(i) = \begin{bmatrix}
I & \overline{A}_i'^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & A_{di}^T & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\
0 & B_{wi}^T & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I
\end{bmatrix}. \tag{4.20}$$

Hence, the condition (4.17) implies $\Phi_i < 0$. This completes the proof. □

Remark 4.4. Note that there exist product terms between the Lyapunov and system matrices in inequality (3.37) of Theorem 3.5, which will bring some difficulties in solving the SOF controller design problem. To resolve this problem, in the proof of Theorem 4.3, we have made a decoupling between the Lyapunov and system matrices by introducing a slack matrix variable J_i and then obtained a new inequality (4.17). It should be pointed that in Haidar et al. [32], a sufficient condition for solvability of the SOF controller design problem for the deterministic singular time-delay system has been proposed. However, the controller gain

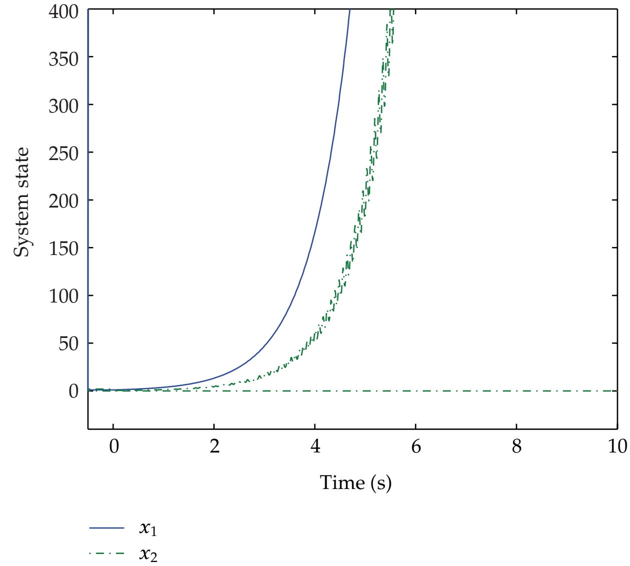

Figure 1: The state trajectories of the open-loop subsystem 1.

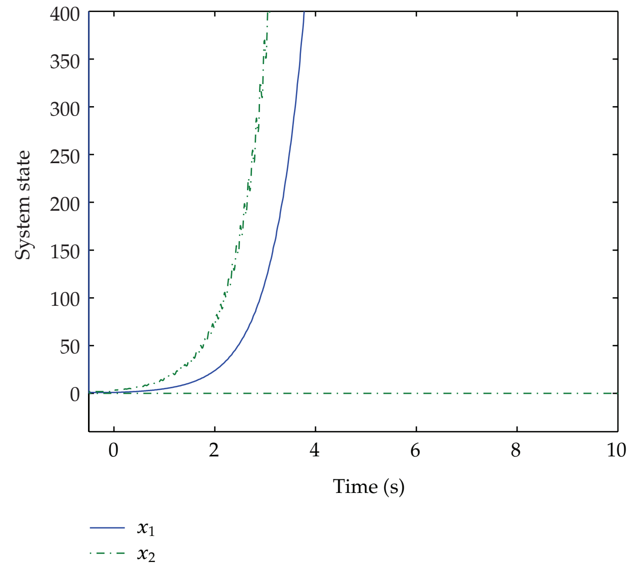

Figure 2: The state trajectories of the open-loop subsystem 2.

was computed by using an iterative LMI algorithm, which was complex. Although the new inequality (4.17) may be conservative mainly due to the introduction of matrix variable J_i, the introduced decoupling technique enables us to obtain a more easily tractable condition for the synthesis of SOF controller.

Remark 4.5. Matrices J_i, $i \in \mathcal{I}$, in Theorem 4.3 can be specified by the algorithm stated in Remark 3.6.

Remark 4.6. In this paper, we have only discussed a special case of the derivative matrix E having no switching modes. If E also has switching modes, then E is changed to E_i, $i \in \mathcal{I}$. In this case, the transformation matrices P and Q should become P_i and Q_i, and we have $P_i E_i Q_i = \text{diag}\{I_{r_i}, 0\}$. Accordingly, the state of the transformed system becomes $\tilde{x}(t) = [\tilde{x}_{i1}^T(t) \ \ \tilde{x}_{i2}^T(t)]^T$ with $\tilde{x}_{i1}^T(t) \in \mathbf{R}^{r_i}$, which means that there does not exist one common state space coordinate basis for all subsystems, and thus it is complicated to discuss the transformed system. Hence, some assumptions for the matrices E_i (e.g., E_i, $i \in \mathcal{I}$, have the same right zero subspace [43]) should be given so that the matrices Q_i remain the same; in this case, the method presented here is also valid. However, the general case of E with switching modes is an interesting problem for future investigation via other methods.

5. Numerical Examples

In this section, we present two illustrative examples to demonstrate the applicability and effectiveness of the proposed approach.

Example 5.1. Consider the switched system (2.5) with $I = 2$ (i.e., there are two subsystems) and the related parameters are given as follows:

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_1 = \begin{bmatrix} 0.73 & 0 \\ 0 & -1 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} -1.1 & 1 \\ 0 & 0.5 \end{bmatrix}, \tag{5.1}$$
$$A_2 = \begin{bmatrix} 0.4 & 0 \\ -0.1 & -1 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} -1 & 0.1 \\ 0 & 0.1 \end{bmatrix}$$

and $d_1 = 0.1$, $d_2 = 0.3$, $\mu = 0.4$, and $\alpha = 0.5$. It can be checked that the previous two subsystems are both stable independently. Consider the quadratic approach (see Remark 3.3, $\beta = 1$, and we know that it requires a common Lyapunov functional for all subsystems); by simulation, it can be found that there is no feasible solution to this case, that is to say, there is no common Lyapunov functional for all subsystems. Now, we consider the average dwell time scheme, and set $\beta = 1.25$, and solving the LMIs (3.2) gives the following solutions:

$$P_1 = 10^3 \times \begin{bmatrix} 0.0354 & 0 \\ 0.0256 & 1.3047 \end{bmatrix}, \qquad Q_{11} = \begin{bmatrix} 1.2978 & -1.6836 \\ -1.6836 & 78.9994 \end{bmatrix},$$
$$Q_{12} = \begin{bmatrix} 0.4821 & -1.6749 \\ -1.6749 & 78.5452 \end{bmatrix}, \qquad Q_{13} = 10^3 \times \begin{bmatrix} 0.0002 & 0.0093 \\ 0.0093 & 1.8512 \end{bmatrix},$$

$$Z_{11} = \begin{bmatrix} 128.9500 & 2.7682 \\ 2.7682 & 308.4441 \end{bmatrix}, \qquad Z_{12} = \begin{bmatrix} 128.1143 & 4.4005 \\ 4.4005 & 323.5358 \end{bmatrix},$$

$$M_{11} = \begin{bmatrix} -81.4620 & -2.8201 \\ 11.2329 & 0.2679 \end{bmatrix}, \qquad M_{12} = \begin{bmatrix} 59.2544 & 2.0497 \\ -62.6780 & -1.9627 \end{bmatrix},$$

$$N_{11} = \begin{bmatrix} -82.1186 & -1.7810 \\ 11.3273 & 0.1165 \end{bmatrix}, \qquad N_{12} = \begin{bmatrix} 58.7098 & 1.2698 \\ -62.8565 & -1.1401 \end{bmatrix},$$

$$S_{11} = \begin{bmatrix} -0.1540 & 0.0183 \\ 0.0297 & 0.2129 \end{bmatrix}, \qquad S_{12} = \begin{bmatrix} -0.2125 & -0.0163 \\ -0.0270 & -0.3559 \end{bmatrix},$$

$$P_{2} = \begin{bmatrix} 43.6700 & 0 \\ -66.4319 & 953.1992 \end{bmatrix}, \qquad Q_{21} = \begin{bmatrix} 1.3435 & -0.9553 \\ -0.9553 & 76.7387 \end{bmatrix},$$

$$Q_{22} = \begin{bmatrix} 0.4965 & -1.3638 \\ -1.3638 & 76.0801 \end{bmatrix}, \qquad Q_{23} = 10^{3} \times \begin{bmatrix} 0.0002 & 0.0078 \\ 0.0078 & 1.5081 \end{bmatrix},$$

$$Z_{21} = \begin{bmatrix} 116.4480 & 2.3702 \\ 2.3702 & 308.2446 \end{bmatrix}, \qquad Z_{22} = \begin{bmatrix} 110.6046 & 3.6134 \\ 3.6134 & 323.0948 \end{bmatrix},$$

$$M_{21} = \begin{bmatrix} -73.9202 & -2.4255 \\ 7.4257 & 0.2466 \end{bmatrix}, \qquad M_{22} = \begin{bmatrix} 52.5267 & 1.7299 \\ -2.7859 & -0.1158 \end{bmatrix},$$

$$N_{21} = \begin{bmatrix} -74.6177 & -1.5240 \\ 7.4918 & 0.1543 \end{bmatrix}, \qquad N_{22} = \begin{bmatrix} 51.9573 & 1.0641 \\ -2.7465 & -0.0661 \end{bmatrix},$$

$$S_{21} = \begin{bmatrix} -0.1625 & -0.0042 \\ 0.0150 & 0.0002 \end{bmatrix}, \qquad S_{22} = \begin{bmatrix} -0.2236 & -0.0085 \\ 0.0150 & 0.0022 \end{bmatrix}, \tag{5.2}$$

which means that the aforementioned switched system is exponentially admissible. Moreover, by further analysis, we find that the allowable minimum of β is $\beta_{\min} = 1.046$ when $\alpha = 0.5$ is fixed; in this case, $T_a^* = (\ln \beta_{\min})/\alpha = 0.0899$. By the previous analysis, we know that the average dwell time approach proposed in this paper is less conservative than the quadratic approach.

Example 5.2. Consider the switched system (2.1) with $I = 2$ and

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0.9 & 0 \\ 1 & -5 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0.5 & 0.1 \\ 1 & 0.1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}, \qquad B_{w1} = \begin{bmatrix} 0.5 \\ 0.03 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \qquad C_{d1} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \qquad D_1 = 1.1, \qquad D_{w1} = 0.15$$

$$A_2 = \begin{bmatrix} 0.5 & 0.1 \\ 5 & -2 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.2 & 0.5 \\ 1.5 & 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -4 \\ -1 \end{bmatrix}, \quad B_{w2} = \begin{bmatrix} 0.3 \\ 0.03 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \quad C_{d2} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \quad D_2 = 1, \quad D_{w2} = 0.1, \tag{5.3}$$

and $d(t) = 0.3 + 0.2\sin(1.5t)$. A straightforward calculation gives $d_1 = 0.1$, $d_2 = 0.5$, and $\mu = 0.3$. By simulation, it can be checked that the previous two subsystems with $u(t) = 0$ are both unstable, and the state responses of the corresponding open-loop systems are shown in Figures 1 and 2, respectively, with the initial condition given by $\phi(t) = [1 \;\; 2]^T$, $t \in [-0.5, 0]$. In view of this, our goal is to design an SF control $u(t)$ in the form of (2.8) and an SOF control $u(t)$ in the form of (2.9), such that the closed-loop system is exponentially admissible with a weighted H_∞ performance $\gamma = 1.5$.

For SF control law, set $\alpha = 0.4$, $\beta = 1.05$ (thus $T_a \ge T_a^* = (\ln \beta)/\alpha = 0.122$), and choose $\epsilon_{11} = 0.2$, $\epsilon_{12} = 0.1$, $\epsilon_{13} = 0.1$, $\epsilon_{14} = 0.02$, $\epsilon_{15} = 0.004$, $\epsilon_{16} = 0.03$, $\epsilon_{17} = 1.9$, $\epsilon_{18} = 1$, $\epsilon_{21} = 0.06$, $\epsilon_{22} = 0.1$, $\epsilon_{23} = 0.14$, $\epsilon_{24} = 0.17$, $\epsilon_{25} = 0.1$, $\epsilon_{26} = 0.1$, $\epsilon_{27} = 0.4$, and $\epsilon_{28} = 0.1$. Solving the LMIs (4.4), we obtain the following solutions:

$$\begin{gathered} X_1 = \begin{bmatrix} 0.0930 & 0 \\ -0.0297 & 0.2059 \end{bmatrix}, \quad R_{11} = \begin{bmatrix} 1.0444 & 0.0008 \\ 0.0008 & 191.0844 \end{bmatrix}, \quad R_{12} = \begin{bmatrix} 0.4461 & 0.0009 \\ 0.0009 & 200.5541 \end{bmatrix}, \\ R_{13} = \begin{bmatrix} 0.0889 & -0.1241 \\ -0.1241 & 0.8811 \end{bmatrix}, \quad Z_{11} = \begin{bmatrix} 0.8979 & 0.0010 \\ 0.0010 & 0.8617 \end{bmatrix}, \quad Z_{12} = \begin{bmatrix} 1.7609 & 0.0194 \\ 0.0194 & 1.4794 \end{bmatrix}, \\ X_2 = \begin{bmatrix} 0.0932 & 0 \\ 0.0208 & 0.0847 \end{bmatrix}, \quad R_{21} = \begin{bmatrix} 1.0397 & 0.0008 \\ 0.0008 & 191.0844 \end{bmatrix}, \quad R_{22} = \begin{bmatrix} 0.4442 & 0.0009 \\ 0.0009 & 200.5541 \end{bmatrix}, \\ R_{23} = \begin{bmatrix} 0.0900 & -0.1233 \\ -0.1233 & 0.8771 \end{bmatrix}, \quad Z_{21} = \begin{bmatrix} 0.9017 & 0.0011 \\ 0.0011 & 0.8640 \end{bmatrix}, \quad Z_{22} = \begin{bmatrix} 1.7882 & 0.0220 \\ 0.0220 & 1.4824 \end{bmatrix}, \\ T_1 = \begin{bmatrix} -0.2000 & 0.0046 \end{bmatrix}, \quad T_2 = \begin{bmatrix} -1.6983 & -0.0889 \end{bmatrix}. \end{gathered} \tag{5.4}$$

Therefore, from (4.7), the gain matrices of an SF controller can be obtained as

$$K_1 = \begin{bmatrix} -2.1437 & 0.0221 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 2.4825 & 0.1243 \end{bmatrix}. \tag{5.5}$$

For SOF control law, let α, β be the same as in the SF control case, and choose

$$J_1 = \mathrm{diag}\{1.08, 3.07\}, \quad J_2 = \mathrm{diag}\{3.95, 1.58\}. \tag{5.6}$$

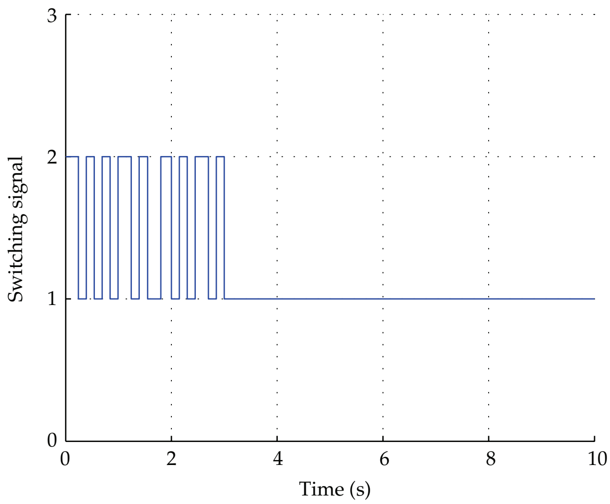

Figure 3: Switching signal with the average dwell time $T_a > 0.13$.

By solving the LMIs (4.17), we obtain the following solutions:

$$P_1 = \begin{bmatrix} 0.1155 & 0 \\ -5.8842 & 17.7617 \end{bmatrix}, \quad Q_{11} = \begin{bmatrix} 3.3116 & 0.5239 \\ 0.5239 & 2.1174 \end{bmatrix}, \quad Q_{12} = \begin{bmatrix} 1.5055 & 0.5231 \\ 0.5231 & 2.1194 \end{bmatrix},$$

$$Q_{13} = \begin{bmatrix} 1.1241 & 1.3009 \\ 1.3009 & 3.5238 \end{bmatrix}, \quad Z_{11} = \begin{bmatrix} 5.9711 & -0.2713 \\ -0.2713 & 10.1082 \end{bmatrix}, \quad Z_{12} = \begin{bmatrix} 4.0344 & -0.1822 \\ -0.1822 & 7.5545 \end{bmatrix},$$

$$M_{11} = \begin{bmatrix} -13.1122 & 0.5931 \\ 0.1486 & -0.0065 \end{bmatrix}, \quad M_{12} = \begin{bmatrix} 13.8147 & -0.6248 \\ -0.0015 & 0.0001 \end{bmatrix}, \quad N_{11} = \begin{bmatrix} -15.3140 & 0.6970 \\ 0.1666 & -0.0070 \end{bmatrix},$$

$$N_{12} = \begin{bmatrix} 13.1090 & -0.5965 \\ -0.0014 & 0.0001 \end{bmatrix}, \quad S_{11} = \begin{bmatrix} -0.0092 & 0.0004 \\ -0.0028 & 0.0006 \end{bmatrix}, \quad S_{12} = \begin{bmatrix} -1.1016 & 0.0500 \\ 0.0002 & 0.0001 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0.1153 & 0 \\ -10.1494 & 4.3434 \end{bmatrix}, \quad Q_{21} = \begin{bmatrix} 3.3132 & 0.5242 \\ 0.5242 & 2.1163 \end{bmatrix}, \quad Q_{22} = \begin{bmatrix} 1.5061 & 0.5231 \\ 0.5231 & 2.1182 \end{bmatrix},$$

$$Q_{23} = \begin{bmatrix} 1.1243 & 1.3007 \\ 1.3007 & 3.5222 \end{bmatrix}, \quad Z_{21} = \begin{bmatrix} 5.9744 & -0.2714 \\ -0.2714 & 10.1049 \end{bmatrix}, \quad Z_{22} = \begin{bmatrix} 4.0385 & -0.1824 \\ -0.1824 & 7.5509 \end{bmatrix},$$

$$M_{21} = \begin{bmatrix} -13.2335 & 0.5985 \\ 0.0221 & -0.0010 \end{bmatrix}, \quad M_{22} = \begin{bmatrix} 13.9337 & -0.6301 \\ -0.0086 & 0.0004 \end{bmatrix}, \quad N_{21} = \begin{bmatrix} -15.4420 & 0.7025 \\ 0.0237 & -0.0011 \end{bmatrix},$$

$$N_{22} = \begin{bmatrix} 13.2333 & -0.6018 \\ -0.0102 & 0.0003 \end{bmatrix}, \quad S_{21} = \begin{bmatrix} -0.0038 & 0.0002 \\ -0.0007 & 0.0000 \end{bmatrix}, \quad S_{22} = \begin{bmatrix} -1.1076 & 0.0501 \\ -0.0001 & -0.0000 \end{bmatrix},$$

$$K_1 = 15.1634, \qquad F_2 = 1.6543. \tag{5.7}$$

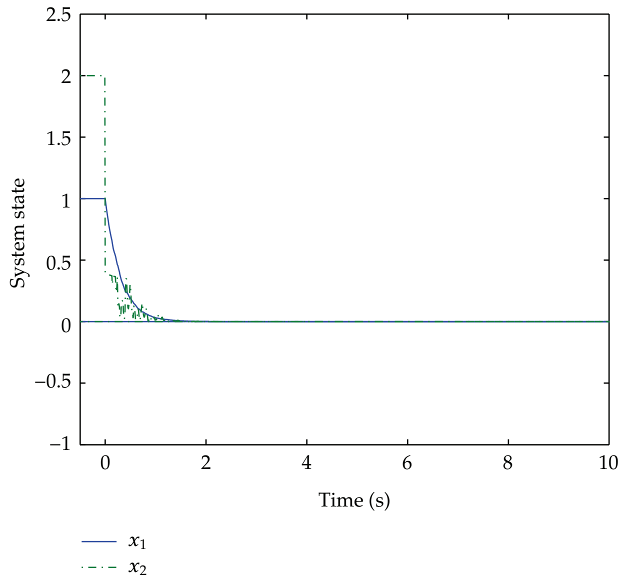

Figure 4: The state trajectories of the closed-loop system under SF control.

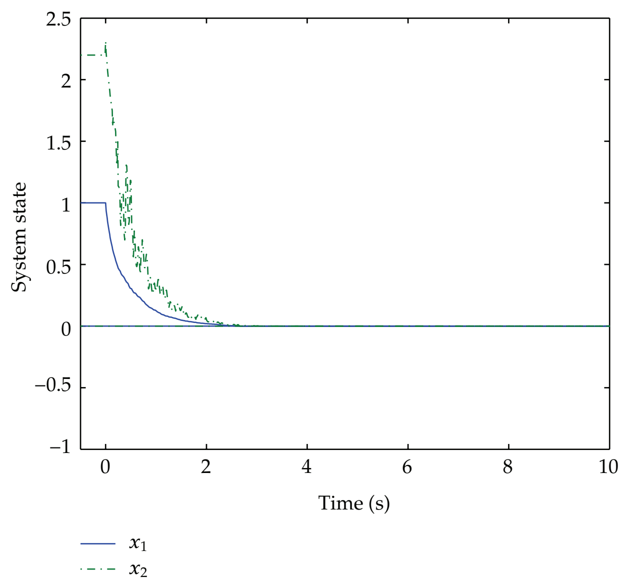

Figure 5: The state trajectories of the closed-loop system under SOF control.

To show the effectiveness of the designed SF and SOF controllers, giving a random switching signal with the average dwell time $T_a \geq 0.13$ as shown in Figure 3, we get the state responses using the SF and SOF controllers for the system as shown in Figures 4 and 5, respectively, for the given initial condition $\phi(t) = [1 \ \ 2]^T$, $t \in [-0.5, 0]$. It is obvious that the designed controllers are feasible and ensure the stability of the closed-loop systems despite the interval time-varying delays.

6. Conclusions

In this paper, the problems of exponential admissibility and H_∞ control for a class of continuous-time switched singular systems with interval time-varying delay have been investigated. A class of switching signals specified by the average dwell time has been identified for the unforced systems to be exponentially admissible with a weighted H_∞ performance. The state feedback and static output feedback controllers have been designed, and their corresponding solvability conditions have been established by using the LMI technique. Simulation results have demonstrated the effectiveness of the proposed design method.

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