Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 474983, 9 pages
doi:10.1155/2012/474983

Research Article

On Fixed Point Theorems in Intuitionistic Fuzzy Metric Spaces

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Received 11 November 2011; Accepted 5 December 2011

Academic Editor: Nazim I. Mahmudov

The author extends two fixed point theorems (due to Gregori, Sapena, and Žikić, resp.) in fuzzy metric spaces to intuitionistic fuzzy metric spaces.

1. Introduction

In this paper, we pay our attention to the fixed point theory on intuitionistic fuzzy metric spaces. Since Zadeh [1] introduced the theory of fuzzy sets, many authors have studied the character of fuzzy metric spaces in different ways [2–5]. Among others, fixed point theorem was an important subject. Gregori and Sapena [6] investigated fixed point theorems for fuzzy contractive mappings defined on fuzzy metric spaces. Recently, Žikić [7] proved a fixed point theorem for mappings on fuzzy metric space which improved the result of Gregori and Sapena. As further development, Atanassov [8] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets, and later there has been much progress in the study of intuitionistic fuzzy sets [9, 10]. Using the idea of intuitionistic sets, Park [11] defined the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space. Recently, several authors studied the structure of intuitionistic fuzzy metric spaces and fixed point theorems for the mappings defined on intuitionistic fuzzy metric spaces. We refer the reader to [11–13] for further details. In this paper, we will prove the following two fixed point theorems.

The first theorem extends Gregori-Sapena's fixed point theorem [6] in fuzzy metric spaces to complete intuitionistic fuzzy metric spaces. As preparation, we recall the definition of s-increasing sequence [6]. A sequence $\{t_n\}$ of positive real numbers is said to be an s-increasing sequence if there exists $m_0 \in \mathbb{N}$ such that $t_m + 1 \leq t_{m+1}$, for all $m \geq m_0$.

Theorem 1.1. *Let $(X, M, N, *, \Diamond)$ be a complete intuitionistic fuzzy metric space such that for every s-increasing sequence $\{t_n\}$ and arbitrary $x, y \in X$,*

$$\lim_{n \to \infty} \prod_{i=n}^{\infty} M(x, y, t_i) = 1, \qquad \lim_{n \to \infty} \prod_{i=n}^{\infty} N(x, y, t_i) = 0 \tag{1.1}$$

hold.

Let $k \in (0,1)$ and $T : X \to X$ be a mapping satisfying $M(Tx, Ty, kt) \geq M(x, y, t)$ and $N(Tx, Ty, kt) \leq N(x, y, t)$ for all $x, y \in X$. Then T has a unique fixed point.

The second theorem extends Žikić's fixed point theorem [7] in fuzzy metric space to intuitionistic fuzzy metric space.

Theorem 1.2. *Let $(X, M, N, *, \Diamond)$ be a complete intuitionistic fuzzy metric space such that for some $\sigma_0 \in (0,1)$ and $x_0 \in X$,*

$$\lim_{n \to \infty} \prod_{i=n}^{\infty} M\left(x_0, Tx_0, \frac{1}{\sigma_0^i}\right) = 1, \qquad \lim_{n \to \infty} \prod_{i=n}^{\infty} N\left(x_0, Tx_0, \frac{1}{\sigma_0^i}\right) = 0 \tag{1.2}$$

hold.

Let $k \in (0,1)$ and $T : X \to X$ be a mapping satisfying $M(Tx, Ty, kt) \geq M(x, y, t)$ and $N(Tx, Ty, kt) \leq N(x, y, t)$ for all $x, y \in X$. Then T has a unique fixed point.

2. Basic Notions and Preliminary Results

For the sake of completeness, in this section we will recall some definitions and preliminaries on intuitionistic fuzzy metric spaces.

Definition 2.1 (see [14]). Let X be a nonempty fixed set. An *intuitionistic fuzzy set A* is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}, \tag{2.1}$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and the degree of nonmembership of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

For developing intuitionistic fuzzy topological spaces, in [10], Çoker introduced the intuitionistic fuzzy sets $0_\sim$ and $1_\sim$ in X as follows.

Definition 2.2 (see [10]). $0_\sim = \{\langle x, 0, 1\rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle : x \in X\}$.

By Definition 2.2, Çoker defined the notion of intuitionistic fuzzy topological spaces.

Definition 2.3 (see [10]). An *intuitionistic fuzzy topology* on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms:

(T1) $0_{\sim}, 1_{\sim} \in \tau$;

(T2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;

(T3) $\bigcup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an *intuitionistic fuzzy topological space*.

Definition 2.4 (see [15]). A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a *continuous t-norm* (triangular norm) if $*$ satisfies the following conditions:

(a) $*$ is associative and commutative;

(b) $*$ is continuous;

(c) $a * 1 = a$ for all $a \in [0,1]$;

(d) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0,1]$.

By this definition, it is easy to see that $1 * 1 = 1$. According to condition (a), the following product is well defined: $M(x_1, y_1, t_1) * M(x_2, y_2, t_2) * \cdots * M(x_n, y_n, t_n)$, and we will denote it by $\prod_{i=1}^{i=n} M(x_i, y_i, t_i)$.

Definition 2.5 (see [15]). A binary operation $\Diamond : [0,1] \times [0,1] \to [0,1]$ is a *continuous t-conorm* (triangular conorm) if $\Diamond$ satisfies the following conditions:

(e) $\Diamond$ is associative and commutative;

(f) $\Diamond$ is continuous;

(g) $a \Diamond 0 = a$ for all $a \in [0,1]$;

(h) $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0,1]$.

By this definition, it is easy to see that $0 \Diamond 0 = 0$. According to condition (e), the following product is well defined: $N(x_1, y_1, t_1) \Diamond N(x_2, y_2, t_2) \Diamond \cdots \Diamond N(x_n, y_n, t_n)$, and we also denote this product by $\prod_{i=1}^{i=n} N(x_i, y_i, t_i)$.

Remark 2.6. The origin of concepts of t-norms and related t-conorms was in the theory of statistical metric spaces in the work of Menger [5]. These concepts are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. Basic examples of t-norms are $a * b = ab$ and $a * b = \min\{a, b\}$, and basic examples of t-conorms are $a \Diamond b = \max\{a, b\}$ and $a \Diamond b = \min\{1, a + b\}$.

Definition 2.7 (see [13]). A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an *intuitionistic fuzzy metric space* if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm, and M, N are fuzzy sets on $X \times X \times [0, \infty)$ satisfying the following conditions:

(IFm 1) $M(x, y, t) + N(x, y, t) \leq 1$;

(IFm 2) $M(x, y, 0) = 0$;

(IFm 3) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$;

(IFm 4) $M(x, y, t) = M(y, x, t)$;

(IFm 5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$, $s, t > 0$;

(IFm 6) $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left continuous;

(IFm 7) $\lim_{t \to \infty} M(x, y, t) = 1$ for all $x, y \in X$;

(IFm 8) $N(x, y, 0) = 1$;

(IFm 9) $N(x, y, t) = 0$ for all $t > 0$ if and only if $x = y$;

(IFm 10) $N(x, y, t) = N(y, x, t)$;

(IFm 11) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$, $s, t > 0$;

(IFm 12) $N(x, y, \cdot) : [0, \infty) \to [0, 1]$ is right continuous;

(IFm 13) $\lim_{t \to \infty} N(x, y, t) = 0$ for all $x, y \in X$.

We denote by (M, N) the intuitionistic fuzzy metric on X. In intuitionistic fuzzy metric space X, it is easy to see $M(x, y, \cdot)$ is nondecreasing and $N(x, y, \cdot)$ is nonincreasing for all $x, y \in X$. We also note that the successive product $\prod$ with respect to $M(x, y, t)$ is in the sense of $*$ and the successive product $\prod$ with respect to $N(x, y, t)$ is in the sense of $\lozenge$ throughout this paper.

Definition 2.8. Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space. Then

(I) a sequence $\{x_n\}$ in X is *Cauchy sequence* if and only if for each $t > 0$ and $p > 0$,

$$\lim_{n \to \infty} M(x_n, x_{n+p}, t) = 1, \qquad \lim_{n \to \infty} N(x_n, x_{n+p}, t) = 0, \tag{2.2}$$

(II) a sequence $\{x_n\}$ in X is *convergent to* $x \in X$ if and only if for each $t > 0$,

$$\lim_{n \to \infty} M(x_n, x, t) = 1, \qquad \lim_{n \to \infty} N(x_n, x, t) = 0. \tag{2.3}$$

Definition 2.9. An intuitionistic fuzzy metric space is said to be *complete* if and only if every Cauchy sequence is convergent.

3. Proof of Theorem 1.1

In this section, we will give the proof of Theorem 1.1 of the present paper.

Proof. Select an arbitrary point $x \in X$. Let $x_n = T^n(x), n \in \mathbb{N}$. We have

$$\begin{aligned} M(x_1, x_2, t) &= M\left(T(x), T^2(x), t\right) \geq M\left(x, T(x), \frac{t}{k}\right) = M\left(x, x_1, \frac{t}{k}\right), \\ N(x_1, x_2, t) &= N\left(T(x), T^2(x), t\right) \leq N\left(x, T(x), \frac{t}{k}\right) = N\left(x, x_1, \frac{t}{k}\right). \end{aligned} \tag{3.1}$$

By induction it follows that $M(x_n, x_{n+1}, t) \geq M(x, x_1, t/k^n)$ and $N(x_n, x_{n+1}, t) \leq N(x, x_1, t/k^n)$.

Let $t > 0$. For $m, n \in \mathbb{N}$, without loss of generality, we suppose $n < m$; if we choose $s_i > 0$, $i = n, \dots, m-1$, satisfying $s_n + s_{n+1} + \cdots + s_{m-1} \le 1$, then we have

$$\begin{aligned} M(x_n, x_m, t) &\ge M(x_n, x_{n+1}, s_n t) * \cdots * M(x_{m-1}, x_m, s_{m-1} t) \\ &\ge M\left(x, x_1, \frac{s_n t}{k^n}\right) * \cdots * M\left(x, x_1, \frac{s_{m-1} t}{k^{m-1}}\right), \\ N(x_n, x_m, t) &\le N(x_n, x_{n+1}, s_n t) \Diamond \cdots \Diamond N(x_{m-1}, x_m, s_{m-1} t) \\ &\le N\left(x, x_1, \frac{s_n t}{k^n}\right) \Diamond \cdots \Diamond N\left(x, x_1, \frac{s_{m-1} t}{k^{m-1}}\right). \end{aligned} \tag{3.2}$$

In particular, since $\sum_{n=1}^{\infty} 1/n(n+1) = 1$, taking $s_i = 1/i(i+1)$, $i = n, \dots, m-1$, one achieves

$$M(x_n, x_m, t) \ge M\left(x, x_1, \frac{t}{n(n+1)k^n}\right) * \cdots * M\left(x, x_1, \frac{t}{(m-1)mk^{m-1}}\right), \tag{3.3}$$

$$N(x_n, x_m, t) \le N\left(x, x_1, \frac{t}{n(n+1)k^n}\right) \Diamond \cdots \Diamond N\left(x, x_1, \frac{t}{(m-1)mk^{m-1}}\right). \tag{3.4}$$

We define $t_n = t/n(n+1)k^n$. It is preliminary to show that $(t_{n+1} - t_n) \to \infty$, as $n \to \infty$, so $\{t_n\}$ is an s-increasing sequence, and hence we get

$$\lim_{m\to\infty} \prod_{n=m}^{\infty} M\left(x, x_1, \frac{t}{n(n+1)k^n}\right) = 1, \qquad \lim_{m\to\infty} \prod_{n=\mathrm{m}}^{\infty} N\left(x, x_1, \frac{t}{n(n+1)k^n}\right) = 0. \tag{3.5}$$

The combination of (3.3), (3.4), and (3.5) implies $\lim_{n\to\infty} M(x_n, x_m, t) = 1$ and $\lim_{n\to\infty} N(x_n, x_m, t) = 0$ for $m > n$. Hence $\{x_n\}$ is a Cauchy sequence. Since X is complete, there is $y \in X$ such that $\lim_{n\to\infty} x_n = y$. We claim y is a fixed point of T. In fact, it is easy to see

$$\begin{aligned} M(T(y), y, t) &\ge \left\{\lim_{n\to\infty} M\left(T(y), T(x_n), \frac{t}{2}\right)\right\} * \left\{\lim_{n\to\infty} M\left(x_{n+1}, y, \frac{t}{2}\right)\right\} \\ &\ge \left\{\lim_{n\to\infty} M\left(y, x_n, \frac{t}{2k}\right)\right\} * \left\{\lim_{n\to\infty} M\left(x_{n+1}, y, \frac{t}{2}\right)\right\} \\ &= 1 * 1, \\ N(T(y), y, t) &\le \left\{\lim_{n\to\infty} N\left(T(y), T(x_n), \frac{t}{2}\right)\right\} \Diamond \left\{\lim_{n\to\infty} N\left(x_{n+1}, y, \frac{t}{2}\right)\right\} \\ &\le \left\{\lim_{n\to\infty} N\left(y, x_n, \frac{t}{2k}\right)\right\} \Diamond \left\{\lim_{n\to\infty} N\left(x_{n+1}, y, \frac{t}{2}\right)\right\} \\ &= 0 \Diamond 0. \end{aligned} \tag{3.6}$$

Thus $M(T(y), y, t) = 1$ and $N(T(y), y, t) = 0$, and we obtain $T(y) = y$. In the sequel, we show the uniqueness of the fixed point. We assume $T(z) = z$ for some $z \in X$. We have

$$\begin{aligned}
1 &\geq M(y,z,t) = M(Ty,Tz,t) \\
&\geq M\left(y,z,\frac{t}{k}\right) = M\left(T(y),T(z),\frac{t}{k}\right) \\
&\geq M\left(y,z,\frac{t}{k^2}\right) = M\left(T(y),T(z),\frac{t}{k^2}\right) \\
&\cdots \\
&\geq \lim_{n\to\infty} M\left(y,z,\frac{t}{k^n}\right) \\
&= 1, \\
0 &\leq N(y,z,t) = N(Ty,Tz,t) \\
&\leq N\left(y,z,\frac{t}{k}\right) = N\left(T(y),T(z),\frac{t}{k}\right) \\
&\leq N\left(y,z,\frac{t}{k^2}\right) = N\left(T(y),T(z),\frac{t}{k^2}\right) \\
&\cdots \\
&\leq \lim_{n\to\infty} N\left(y,z,\frac{t}{k^n}\right) \\
&= 0.
\end{aligned} \tag{3.7}$$

Thus we get $M(y, z, t) = 1$ and $N(y, z, t) = 0$, and hence $y = z$. The proof is complete. □

4. Proof of Theorem 1.2

In this section, we will give the proof of Theorem 1.2 by three lemmas.

Lemma 4.1. *For any monotonely nondecreasing function $F : (0,\infty) \to [0,1]$, the following implication holds:*

$$\lim_{n\to\infty} \prod_{i=n}^{\infty} F\left(\sigma_0^i\right) = 0 \Longrightarrow \lim_{n\to\infty} \prod_{i=n}^{\infty} F\left(\sigma^i\right) = 0 \tag{4.1}$$

for all $\sigma \in (0,1)$, where the infinite product $\prod$ is in the sense of $\diamondsuit$.

Proof

Case 1 ($\sigma < \sigma_0$). For $i \in \mathbb{N}$, $\sigma^i < \sigma_0^i$, and since F is nondecreasing, $F(\sigma^i) \leq F(\sigma_0^i)$ hold. And hence $\prod_{i=n}^{\infty} F(\sigma^i) \leq \prod_{i=n}^{\infty} F(\sigma_0^i)$, $n \in \mathbb{N}$. So implication (4.1) holds.

Case 2 ($\sigma \geq \sigma_0$). If $\sigma = \sqrt{\sigma_0}$, it follows

$$\begin{aligned}\prod_{i=2m}^{\infty} F\left(\sigma^i\right) &= \left[\prod_{i=m}^{\infty} F\left(\sigma^{2i}\right)\right] \Diamond \left[\prod_{i=m}^{\infty} F\left(\sigma^{2i+1}\right)\right] \\ &\leq \left[\prod_{i=m}^{\infty} F\left(\sigma_0^i\right)\right] \Diamond \left[\prod_{i=m}^{\infty} F\left(\sigma_0^i\right)\right].\end{aligned} \tag{4.2}$$

Then we have $\lim_{m\to\infty}\prod_{i=2m}^{\infty} F(\sigma^i) \leq 0\Diamond 0 = 0$. And $\lim_{m\to\infty}\prod_{i=2m+1}^{\infty} F(\sigma^i) \leq \lim_{m\to\infty}\prod_{i=2m+2}^{\infty} F(\sigma^i) = 0$. Thus it follows that $\lim_{m\to\infty}\prod_{i=m}^{\infty} F(\sigma^i) = 0$ for $\sigma = \sqrt{\sigma_0}$. Since F is nondecreasing, it is easy to show $\lim_{m\to\infty}\prod_{i=m}^{\infty} F(\sigma^i) = 0$ for $\sigma < \sqrt{\sigma_0}$.

For an arbitrary $\sigma > \sigma_0$, there exists $m \in \mathbb{N}$ such that $\sigma < \sigma_0^{[(1/2)^m]}$, and we can repeat the above process m-times to get $\lim_{m\to\infty}\prod_{i=m}^{\infty} F(\sigma^i) = 0$.

Lemma 4.2. *For any monotonely nonincreasing function $G : (0,\infty) \to [0,1]$, the following implication holds:*

$$\lim_{n\to\infty}\prod_{i=n}^{\infty} G\left(\sigma_0^i\right) = 1 \Longrightarrow \lim_{n\to\infty}\prod_{i=n}^{\infty} G\left(\sigma^i\right) = 1 \tag{4.3}$$

for all $\sigma \in (0,1)$, where the infinite product $\prod$ is in the sense of $*$.

Proof. One can take a similar procedure as in the proof of Lemma 4.1 to complete the proof of this lemma. For simplicity, we omit the detailed argument. We refer the reader to [7] for further details. □

Lemma 4.3. *We define $x_n = T^n(x_0)(n \in \mathbb{N})$. Then $\{x_n\}$ is a Cauchy sequence.*

Proof. We assume $F(x) = N(x_0, T(x_0), 1/x)$ and $G(x) = M(x_0, T(x_0), 1/x)$ for $x > 0$. Then $F(x)$ ($G(x)$) is nondecreasing (nonincreasing) mapping from $(0,\infty)$ into $[0,1]$. Taking $1 > \sigma > k$, by Lemmas 4.1 and 4.2, we have

$$\lim_{n\to\infty}\prod_{i=n}^{\infty} M\left(x_0, T(x_0), \frac{1}{(k/\sigma)^i}\right) = 1, \qquad \lim_{n\to\infty}\prod_{i=n}^{\infty} N\left(x_0, T(x_0), \frac{1}{(k/\sigma)^i}\right) = 0. \tag{4.4}$$

Since $\sigma < 1$, $\sum_{n=1}^{\infty}\sigma^n < \infty$, for any $\varepsilon_0 > 0$ there exists n_0 such that $\sum_{n=n_0}^{\infty}\sigma^n < \varepsilon_0$. For the above $\varepsilon_0 > 0$, if $m > n > n_0$ and $t > \varepsilon_0$,

$$\begin{aligned}M(x_n, x_m, t) \geq M(x_n, x_m, \varepsilon_0) &\geq \prod_{i=n}^{m-1} M\left(x_i, x_{i-1}, \sigma^i\right) \\ &\geq \prod_{i=n}^{m-1} M\left(x_0, Tx_0, \frac{\sigma^i}{k^i}\right) \\ &= \prod_{i=n}^{m-1} M\left(x_0, Tx_0, \frac{1}{(k/\sigma)^i}\right),\end{aligned}$$

$$
\begin{aligned}
N(x_n, x_m, t) \le N(x_n, x_m, \varepsilon_0) &\le \prod_{i=n}^{m-1} N\left(x_i, x_{i-1}, \sigma^i\right) \\
&\le \prod_{i=n}^{m-1} N\left(x_0, Tx_0, \frac{\sigma^i}{k^i}\right) \\
&= \prod_{i=n}^{m-1} N\left(x_0, Tx_0, \frac{1}{(k/\sigma)^i}\right)
\end{aligned}
\tag{4.5}
$$

hold.

And according to (4.4), we have $\lim_{n\to\infty} M(x_n, x_m, t) = 1$ and $\lim_{n\to\infty} N(x_n, x_m, t) = 0$ for $m > n$. So $\{x_n\}$ is Cauchy sequence.

Since X is complete, there exists some $y \in X$ such that $\lim_{n\to\infty} x_n = y$. One can prove y is the unique fixed point of T by repeating the same process as in the proof of Theorem 1.1. Thus, we complete the proof of Theorem 1.2. □

Acknowledgments

The author is grateful to Professor Nazim I. Mahmudov and the anonymous referees for their helpful comments and constructive suggestions. This work was supported partly by the National Natural Science Foundation of China (nos. 11026105 and 11026106), the Scientific Research Fund of Zhejiang Province Education Department (no. Y201119292), and the Foundation for Development of Mathematics Subject of HDU (ZX100204004-10).

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