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Research Article

Uniformly Strong Persistence for a Delayed Predator-Prey Model

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An asymptotically periodic predator-prey model with time delay is investigated. Some sufficient conditions for the uniformly strong persistence of the system are obtained. Our result is an important complementarity to the earlier results.

1. Introduction

The dynamical behavior including boundedness, stability, permanence, and existence of periodic solutions of predator-prey systems has attracted a great deal of attention and many excellent results have already been derived. For example, Gyllenberg et al. [1] studied limit cycles of a competitor-competitor-mutualist Lotka-Volterra model. Mukherjee [2] made a discussion on the uniform persistence in a generalized prey-predator system with parasitic infection. Aggelis et al. [3] considered the coexistence of both prey and predator populations of a prey-predator model. Agiza et al. [4] investigated the chaotic phenomena of a discrete prey-predator model with Holling type II. Sen et al. [5] analyzed the bifurcation behavior of a ratio-dependent prey-predator model with the Allee effect. Zhang and Luo [6] gave a theoretical study on the existence of multiple positive periodic solutions for a delayed predator-prey system with stage structure for the predator. Nindjin and Aziz-Alaoui [7] focused on the persistence and global stability in a delayed Leslie-Gower-type three species food chain. Ko and Ryu [8] discussed the coexistence states of a nonlinear Lotka-Volterra-type predator-prey model with cross-diffusion. Fazly and Hesaaraki [9] dealt with periodic solutions of a predator-prey system with monotone functional responses. One can see [10–19] and so forth for more related studies. However, the research work on asymptotically periodic predator-prey model is very few at present.

The so-called asymptotically periodic function is that a function $\overline{a}(t)$ can be expressed by the form $\overline{a}(t) = a(t)+\widetilde{a}(t)$, where $a(t)$ is a periodic function and $\widetilde{a}(t)$ satisfies $\lim_{t\to+\infty}\widetilde{a}(t) = 0$.

In 2006, Kar and Batabyal [20] investigated the stability and bifurcation of the following predator-prey model with time delay

$$\begin{aligned}\frac{dx}{dt} &= x\left[r - \frac{r}{K}x - \frac{\alpha_1 y}{a_1 + x} - \frac{\alpha_2 z}{a_2 + x}\right],\\ \frac{dy}{dt} &= y\left[-d_1 + \frac{\beta_1\alpha_1 x(t-\tau)}{a_1 + x(t-\tau)} - \gamma y\right],\\ \frac{dz}{dt} &= z\left[-d_2 + \frac{\beta_2\alpha_2 x(t-\tau)}{a_2 + x(t-\tau)} - \delta z\right],\end{aligned} \tag{1.1}$$

with initial conditions $x(0) \ge 0,\ y(0) \ge 0,\ z(0) \ge 0$, where $z(t)$ denotes the densities of prey; $y(t)$ and $z(t)$ denote the densities of two predators, respectively, at time t; γ and δ denote the intraspecific competition coefficients of the predators; β_1 and β_2 denote the conversion of biomass constant; d_1 and d_2 are the death rate of first and second predator species, respectively; α_1 is the maximum values of per capita reduction rate of x due to y and α_2 is the maximum values of per capita reduction rate of x due to z; a_1 and a_2 are half saturation constants. τ is time delay in the prey species. All the parameters are positive constants. For details, one can see [20].

It will be pointed out that all biological and environment parameters in model (1.1) are constants in time. However, any biological or environmental parameters are naturally subject to fluctuation in time. Thus the effects of a periodically varying environment are important for evolutionary theory as the selective forces on systems in a fluctuating environment differ from those in a stable environment. Therefore, the assumptions of periodicity of the parameters are a way of incorporating the periodicity the environment (such as seasonal effects of weather, food supplies, and mating habits). Inspired by above considerations and considering the asymptotically periodic function, in this paper, we will modify system (1.1) as follows:

$$\begin{aligned}\frac{dx}{dt} &= x\left[r(t) + \widetilde{r}(t) - \frac{r(t)+\widetilde{r}(t)}{K(t)+\widetilde{K}(t)}x - \frac{(\alpha_1(t)+\widetilde{\alpha}_1(t))y}{a_1(t)+\widetilde{a}_1(t)+x} - \frac{(\alpha_2(t)+\widetilde{\alpha}_2(t))z}{a_2(t)+\widetilde{a}_2(t)+x}\right],\\ \frac{dy}{dt} &= y\left[-\left(d_1(t)+\widetilde{d}_1(t)\right) + \frac{\left(\beta_1(t)+\widetilde{\beta}_1(t)\right)(\alpha_1(t)+\widetilde{\alpha}_1(t))x(t-\tau)}{a_1(t)+\widetilde{a}_1(t)+x(t-\tau)} - \left(\gamma(t)+\widetilde{\gamma}(t)\right)y\right],\\ \frac{dz}{dt} &= z\left[-\left(d_2(t)+\widetilde{d}_2(t)\right) + \frac{\left(\beta_2(t)+\widetilde{\beta}_2(t)\right)(\alpha_2(t)+\widetilde{\alpha}_2(t))x(t-\tau)}{a_2(t)+\widetilde{a}_2(t)+x(t-\tau)} - \left(\delta(t)+\widetilde{\delta}(t)\right)z\right],\end{aligned} \tag{1.2}$$

with initial conditions $x(0) \ge 0,\ y(0) \ge 0,\ z(0) \ge 0$.

The principle object of this paper is to explore the uniformly strong persistence of system (1.2). There are very few papers which deal with this topic, see [10, 21].

In order to obtain our results, we always assume that system (1.2) satisfies (H1) $\alpha_i(t)$, $\beta_i(t)$, $a_i(t)$, $d_i(t)(i = 1,2)$, $r(t)$, $\gamma(t)$, $\delta(t)$, $K(t)$ are continuous, nonnegative periodic functions; $\tilde{\alpha}_i(t)$, $\tilde{\beta}_i(t)$, $\tilde{a}_i(t)$, $\tilde{d}_i(t)(i = 1,2)$, $\tilde{R}(t)$, $\tilde{\gamma}(t)$, $\tilde{\delta}(t)$, $\tilde{K}(t)$ are continuous, nonnegative asymptotically items of asymptotically periodic functions.

2. Uniformly Strong Persistence

In this section, we will present some result about the uniformly strong persistence of system (1.2). For convenience and simplicity in the following discussion, we introduce the notations, definition, and Lemmas. Let

$$0 < f^l = \lim_{t\to+\infty} \inf f(t) \le \lim_{t\to+\infty} \sup f(t) = f^u < +\infty. \tag{2.1}$$

In view of the definitions of lower limit and upper limit, it follows that for any $\varepsilon > 0$, there exists $T > 0$ such that

$$f^l - \varepsilon \le f(t) \le f^u + \varepsilon, \quad \text{for } t \ge T. \tag{2.2}$$

Definition 2.1. The system (1.2) is said to be strong persistence, if every solution $x(t)$ of system (1.2) satisfied

$$0 < \lim_{t\to+\infty} \inf x(t) \le \lim_{t\to+\infty} \sup x(t) \le \delta < +\infty. \tag{2.3}$$

Lemma 2.2. *Both the positive and nonnegative cones of R^2 are invariant with respect to system* (1.2).

It follows from Lemma 2.2 that any solution of system (1.2) with a nonnegative initial condition remains nonnegative.

Lemma 2.3 (see [10]). *If $a > 0$, $b > 0$, and $\dot{x}(t) \ge (\le) x(t)(b - ax^{\alpha}(t))$, where α is a positive constant, when $t \ge 0$ and $x(0) > 0$, we have*

$$x(t) \ge (\le) \left(\frac{b}{a}\right)^{1/\alpha} \left[1 + \left(\frac{bx^{-\alpha}(0)}{a} - 1\right)e^{-b\alpha t}\right]^{-1/\alpha}. \tag{2.4}$$

In the following, we will be ready to state our result.

Theorem 2.4. *Let P_1, P_2, P_3, and Q_1 be defined by* (2.7), (2.10), (2.13), *and* (2.16), *respectively. Assume that conditions* (H1) *and*

(H2) $\alpha_i^u \beta_i^u > d_i^l (i = 1,2)$, $r^l a_1^l a_2^l > a_1^u a_2^l P_2 + a_1^l a_2^u P_3$,

(H3) $\alpha_1^l \beta_1^l Q_1 > d_1^u (a_1^u + P_1)$, $\alpha_2^l \beta_2^l Q_1 > d_2^u (a_2^u + P_1)$

hold, then system (1.2) *is uniformly strong persistence.*

Proof. It follows from (2.2) that for any $\varepsilon > 0$, there exists $T_1 > 0$ such that for $t \geq T_1$,

$$\begin{gathered} r^l - \varepsilon \leq r(t) \leq r^u + \varepsilon, \quad -\varepsilon < \widetilde{r}(t) < \varepsilon, \\ K^l - \varepsilon \leq K(t) \leq K^u + \varepsilon, \quad -\varepsilon < \widetilde{K}(t) < \varepsilon, \\ a_1^l - \varepsilon \leq a_1(t) \leq a_1^u + \varepsilon, \quad -\varepsilon < \widetilde{a}_1(t) < \varepsilon, \\ a_2^l - \varepsilon \leq a_2(t) \leq a_2^u + \varepsilon, \quad -\varepsilon < \widetilde{a}_2(t) < \varepsilon, \\ \alpha_1^l - \varepsilon \leq \alpha_1(t) \leq \alpha_1^u + \varepsilon, \quad -\varepsilon < \widetilde{\alpha}_1(t) < \varepsilon, \\ \alpha_2^l - \varepsilon \leq \alpha_2(t) \leq \alpha_2^u + \varepsilon, \quad -\varepsilon < \widetilde{\alpha}_2(t) < \varepsilon. \end{gathered} \tag{2.5}$$

Substitute (2.5) into the first equation of system (1.2), then we have

$$\begin{aligned} \frac{dx}{dt} &= x\left[r(t) + \widetilde{r}(t) - \frac{r(t) + \widetilde{r}(t)}{K(t) + \widetilde{K}(t)}x - \frac{(\alpha_1(t) + \widetilde{\alpha}_1(t))y}{a_1(t) + \widetilde{a}_1(t) + x} - \frac{(\alpha_2(t) + \widetilde{\alpha}_2(t))z}{a_2(t) + \widetilde{a}_2(t) + x}\right] \\ &\leq x\left[r(t) + \widetilde{r}(t) - \frac{r(t) + \widetilde{r}(t)}{K(t) + \widetilde{K}(t)}x\right] \leq x(t)\left[(r^u + 2\varepsilon) - \frac{r^l - 2\varepsilon}{K^u + 2\varepsilon}x(t)\right]. \end{aligned} \tag{2.6}$$

By Lemma 2.3, we get

$$\lim_{t \to +\infty} \sup x(t) \leq \frac{r^u K^u}{r^l} := P_1. \tag{2.7}$$

Then for any $\varepsilon > 0$, there exists $T_2 > T_1 > 0$ such that

$$x(t) \leq P_1 + \varepsilon, \quad t \geq T_2. \tag{2.8}$$

Similarly, from (2.2) and the second equation of system (1.2), we obtain that for any $\varepsilon > 0$, there exists $T_3 > T_2 > 0$ such that

$$\begin{aligned} \dot{y}(t) &= y\left[-\left(d_1(t) + \widetilde{d}_1(t)\right) + \frac{\left(\beta_1(t) + \widetilde{\beta}_1(t)\right)(\alpha_1(t) + \widetilde{\alpha}_1(t))x(t-\tau)}{a_1(t) + \widetilde{a}_1(t) + x(t-\tau)} - (\gamma(t) + \widetilde{\gamma}(t))y\right] \\ &\leq y(t)\left[-\left(d_1^l - 2\varepsilon\right) - \left(\gamma^l - 2\varepsilon\right)y(t) + (\beta_1^u + 2\varepsilon)(\alpha_1^u + 2\varepsilon)\right]. \end{aligned} \tag{2.9}$$

In view of Lemma 2.3, we derive

$$\lim_{t \to +\infty} \sup y(t) \leq \frac{\alpha_1^u \beta_1^u - d_1^l}{\gamma^l} := P_2. \tag{2.10}$$

Then for any $\varepsilon > 0$, there exists $T_4 > T_3 > 0$ such that

$$y(t) \le P_2 + \varepsilon, \quad t \ge T_4. \tag{2.11}$$

From (2.2) and the third equation of system (1.2), we obtain that for any $\varepsilon > 0$, there exists $T_5 > T_4 > 0$ such that

$$\begin{aligned}
\dot{z}(t) &= z\left[-\left(d_2(t) + \tilde{d}_2(t)\right) + \frac{\left(\beta_2(t) + \tilde{\beta}_2(t)\right)(\alpha_2(t) + \tilde{\alpha}_2(t))x(t-\tau)}{a_2(t) + \tilde{a}_2(t) + x(t-\tau)} - \left(\delta(t) + \tilde{\delta}(t)\right)z\right] \\
&\le z(t)\left[-\left(d_2^l - 2\varepsilon\right) - \left(\delta^l - 2\varepsilon\right)z(t) + \left(\beta_2^u + 2\varepsilon\right)\left(\alpha_2^u + 2\varepsilon\right)\right].
\end{aligned} \tag{2.12}$$

In view of Lemma 2.3, we derive

$$\limsup_{t\to+\infty} z(t) \le \frac{\alpha_2^u \beta_2^u - d_2^l}{\delta^l} := P_3. \tag{2.13}$$

Then for any $\varepsilon > 0$, there exists $T_6 > T_5 > 0$ such that

$$z(t) \le P_3 + \varepsilon, \quad t \ge T_6. \tag{2.14}$$

According (2.8), (2.11), (2.14) and the first equation of system (1.2), we obtain that for any $\varepsilon > 0$, there exists $T_7 > T_6 > 0$ such that

$$\begin{aligned}
\frac{dx}{dt} &= x\left[r(t) + \tilde{r}(t) - \frac{r(t) + \tilde{r}(t)}{K(t) + \tilde{K}(t)}x - \frac{(\alpha_1(t) + \tilde{\alpha}_1(t))y}{a_1(t) + \tilde{a}_1(t) + x} - \frac{(\alpha_2(t) + \tilde{\alpha}_2(t))z}{a_2(t) + \tilde{a}_2(t) + x}\right] \\
&\ge x(t)\left[\left(r^l - 2\varepsilon\right) - \frac{r^u + 2\varepsilon}{K^l - 2\varepsilon}x(t) - \frac{\left(\alpha_1^u + 2\varepsilon\right)(P_2 + \varepsilon)}{a_1^l - 2\varepsilon} - \frac{\left(\alpha_2^u + 2\varepsilon\right)(P_3 + \varepsilon)}{a_2^l - 2\varepsilon}\right].
\end{aligned} \tag{2.15}$$

Using Lemma 2.3 again, we have

$$\liminf_{t\to+\infty} x(t) \ge \frac{K^l\left(r^l a_1^l a_2^l - a_1^u a_2^l P_2 - a_1^l a_2^u P_3\right)}{a_1^l a_2^l r^u} := Q_1. \tag{2.16}$$

Thus for any $\varepsilon > 0$, there exists $T_8 > T_7 > 0$ such that

$$x(t) \ge Q_1 - \varepsilon. \tag{2.17}$$

According (2.8), (2.11), (2.14) and the second equation of system (1.2), we obtain that for any $\varepsilon > 0$, there exists $T_9 > T_8 > 0$ such that

$$\begin{aligned}\dot{y}(t) &= y\left[-\left(d_1(t)+\widetilde{d}_1(t)\right)+\frac{\left(\beta_1(t)+\widetilde{\beta}_1(t)\right)(\alpha_1(t)+\widetilde{\alpha}_1(t))x(t-\tau)}{a_1(t)+\widetilde{a}_1(t)+x(t-\tau)}-(\gamma(t)+\widetilde{\gamma}(t))y\right] \\ &\geq y(t)\left[-(d_1^u+2\varepsilon)-(\gamma^u+2\varepsilon)y(t)+\frac{\left(\beta_1^l-2\varepsilon\right)\left(\alpha_1^l-2\varepsilon\right)(Q_1-\varepsilon)}{a_1^u+2\varepsilon+P_1+\varepsilon}\right].\end{aligned} \tag{2.18}$$

Using Lemma 2.3 again, we have

$$\lim_{t\to+\infty}\inf y(t) \geq \frac{\beta_1^l\alpha_1^l Q_1 - d_1^u(a_1^u+P_1)}{\gamma^u(a_1^u+P_1)} := Q_2. \tag{2.19}$$

Thus for any $\varepsilon > 0$, there exists $T_{10} > T_9 > 0$ such that

$$y(t) \geq Q_2 - \varepsilon. \tag{2.20}$$

According (2.8), (2.11), (2.14) and the third equation of system (1.2), we obtain that for any $\varepsilon > 0$, there exists $T_{11} > T_{10} > 0$ such that

$$\begin{aligned}\dot{z}(t) &= z\left[-\left(d_2(t)+\widetilde{d}_2(t)\right)+\frac{\left(\beta_2(t)+\widetilde{\beta}_2(t)\right)(\alpha_2(t)+\widetilde{\alpha}_2(t))x(t-\tau)}{a_2(t)+\widetilde{a}_2(t)+x(t-\tau)}-\left(\delta(t)+\widetilde{\delta}(t)\right)z\right] \\ &\geq y(t)\left[-(d_2^u+2\varepsilon)-(\delta^u+2\varepsilon)z(t)+\frac{\left(\beta_2^l-2\varepsilon\right)\left(\alpha_2^l-2\varepsilon\right)(Q_1-\varepsilon)}{a_2^u+2\varepsilon+P_1+\varepsilon}\right].\end{aligned} \tag{2.21}$$

Using Lemma 2.3 again, we have

$$\lim_{t\to+\infty}\inf z(t) \geq \frac{\beta_2^l\alpha_2^l Q_1 - d_2^u(a_2^u+P_1)}{\delta^u(a_2^u+P_1)} := Q_3. \tag{2.22}$$

Thus the proof of Theorem 2.4 is complete. □

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