*Research Article*

# Cluster Anti-Synchronization of Complex Networks with Nonidentical Dynamical Nodes

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This paper investigates a new cluster antisynchronization scheme in the time-varying delays coupled complex dynamical networks with nonidentical nodes. Based on the community structure of the networks, the controllers are designed differently between the nodes in one community that have direct connections to the nodes in other communities and the nodes without direct connections with the nodes in other communities strategy; some sufficient criteria are derived to ensure cluster anti-synchronization of the network model. Particularly, the weight configuration matrix is not assumed to be irreducible. The numerical simulations are performed to verify the effectiveness of the theoretical results.

## 1. Introduction

During the past decade, the research on synchronization and dynamical behavior analysis of complex network systems has become a new and important direction in this field [1–6]; many control approaches and many different synchronization phenomena have been developed, such as impulsive control, pinning control and complete synchronization, phase synchronization, cluster synchronization, mixed synchronization, and generalized synchronization, which have been investigated since ten years ago in [7, 8] and references therein.

Cluster synchronization means that nodes in the same group synchronize with each other, but there is no synchronization between nodes in different groups [9, 10]. Wang et al. [11] investigated the cluster synchronization of the dynamical networks with community structure and nonidentical nodes and with identical local dynamics for all individual nodes in each community, which were considered by using some feedback control schemes. Wu and Lu [12] investigated cluster synchronization in the adaptive complex dynamical networks

with nonidentical nodes by a local control method and a novel adaptive strategy for the coupling strengths of the networks. Qin and Chen [13] investigated the stability of selected cluster synchronization in coupled Josephson equations. Ma et al. [14] showed that the arbitrarily selected cluster synchronization manifolds could be stabilized by constructing a special coupled matrix for connected chaotic networks. Wu et al. [15] investigated the anti-synchronization (AS) problem of two general complex dynamical networks with nondelayed and delayed coupling using pinning adaptive control method. Based on Lyapunov stability theory and Barbalat lemma, a sufficient condition is derived to guarantee the AS between two networks with nondelayed and delayed coupling. However, there is few theoretical result on the cluster anti-synchronization of linearly coupled complex networks with time-varying delays coupling.

Motivated by the aforementioned discussions, this paper aims to analyze the cluster anti-synchronization problem for the time-varying delays coupled complex dynamical networks. The main contributions of this paper are threefold: (1) the local dynamics in each community are identical, but those of different communities are nonidentical. (2) For achieving the synchronization, based on the community structure of the networks, the controllers are designed differently between the nodes in one community which have direct connections to the nodes in other communities and the nodes without direct connections with the nodes in other communities strategy; some sufficient criteria are derived to ensure cluster anti-synchronization of the network model. (3) According to Lyapunov stability theory, the sufficient conditions for achieving cluster anti-synchronization are obtained analytically. Compared with some similar designs, our controllers are very simple.

The paper is organized as follows: the network model is introduced followed by some definitions, lemmas, and hypotheses in Section 2. The cluster anti-synchronization of the complex coupled networks is discussed in Section 3. Simulations are obtained in Section 4. Finally, in Section 5 the various conclusions are discussed.

## 2. Model and Preliminaries

The network with time-varying delays coupling can be described by

$$
c\dot{x}_i(t) = f_{\phi_i}(t, x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma_1 x_j(t) + c\sum_{j=1}^{N} b_{ij}\Gamma_2 x_j\big(t - \eta_{\phi_i}(t)\big), \quad i = 1, 2, \ldots, N, \tag{2.1}
$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ is the state vector of node $i$, $c > 0$ is the coupling strength, and $f_{\phi_i} : R^n \to R^n$ describes the local dynamics of nodes in the $\phi_i$ th community. For any pair of nodes $i$ and $j$, if $\phi_i \neq \phi_j$, that is, nodes $i$ and $j$ belong to different communities, then $f_{\phi_i} \neq f_{\phi_j}$, $\eta_{\phi_i}(t)$ is time-varying delay. $\Gamma_1 \in R^{n \times n}$ and $\Gamma_2 \in R^{n \times n}$ are inner-coupling matrices, for simplicity; we assume that $\Gamma_1$, $\Gamma_2$ are diagonal matrices with positive diagonal elements, $\Gamma_1 = \mathrm{diag}(\rho_1, \rho_2, \ldots, \rho_n)$ with $\rho_i \geq 0$, $\Gamma_2 = \mathrm{diag}(\theta_1, \theta_2, \ldots, \theta_n)$ with $\theta_i \geq 0$. $A = (a_{ij})_{N \times N}$, $B = (b_{ij})_{N \times N}$ are the weight configuration matrices. If there is a connection from node $i$ to node $j$ ($j \neq i$), then $a_{ij} = a_{ji} > 0$, $b_{ij} = b_{ji} > 0$; otherwise $a_{ij} = a_{ji} = 0$, $b_{ij} = b_{ji} = 0$, and the diagonal elements of matrix $A$, $B$ are defined as

$$
a_{ii} = -\sum_{j=1, j \neq i}^{N} a_{ji}, \quad b_{ii} = -\sum_{j=1, j \neq i}^{N} b_{ji}, \quad i = 1, 2, \ldots, N. \tag{2.2}
$$

Particularly, the weight configuration matrices are not assumed to be irreducible.

When the control inputs $v_i(t)$, $u_i(t) \in R^n$ $(i = 1, 2, \ldots, N)$ are introduced, the controlled dynamical network with respect to network (2.1) can be written as

$$
\begin{aligned}
\dot{x}_i(t) &= f_{\phi_i}(t, x_i(t)) + c \sum_{j=1}^{N} a_{ij} \Gamma_1 x_j(t) \\
&\quad + c \sum_{j=1}^{N} b_{ij} \Gamma_2 x_j(t - \eta_{\phi_i}(t)) + u_i(t), \quad \phi_i(t) \in \bar{J}_{\phi_i}, \ i = 1, 2, \ldots, N, \\
\dot{x}_i(t) &= f_{\phi_i}(t, x_i(t)) + c \sum_{j=1}^{N} a_{ij} \Gamma_1 x_j(t) \\
&\quad + c \sum_{j=1}^{N} b_{ij} \Gamma_2 x_j(t - \eta_{\phi_i}(t)) + v_i(t), \quad \phi_i(t) \in J_{\phi_i} - \bar{J}_{\phi_i}, \ i = 1, 2, \ldots, N,
\end{aligned}
\tag{2.3}
$$

where $J_{\phi_i}$ denotes all the nodes in the $\phi_i$ th community and $\bar{J}_{\phi_i}$ represents the nodes in the $\phi_i$ th community, which have direct links with the nodes in other communities.

In this paper, let $\{C_1, C_2, \ldots, C_m\}$ denote $m$ $(2 \leq m \leq N)$ communities of the networks and $\bigcup_{i=1}^{m} C_i = \{1, 2, \ldots N\}$. If node $i$ belongs to the $j$th community, then we denote $\phi_i = j$. We employ $f_i(\cdot)$ to represent the local dynamics of all nodes in the $i$th community. The local dynamics of individual nodes in different communities are assumed to be nonidentical, that is, if $\phi_i \neq \phi_j$, then $f_{\phi_i} \neq f_{\phi_j}$. Let $s_{\phi_i}(t)$ be a solution of an isolated node in the $\phi_i$ th community, that is, $\dot{s}_i(t) = f_{\phi_i}(t, s_i(t))$, where $\lim_{t \to \infty} \|s_i(t) - s_j(t)\| \neq 0$ $(i \neq j)$ and the set $S = \{s_1(t), s_2(t), \ldots, s_m(t)\}$ is used as the cluster anti-synchronization manifold for network (2.3). Cluster anti-synchronization can be realized if and only if the manifold S is stable, where $s_k(t)$ may be an equilibrium point, a periodic orbit, or even a chaotic orbit.

*Definition 2.1* (see [15]). The dynamical network (2.1) is said to achieve cluster anti-synchronization (CAS) if

$$
c \lim_{t \to \infty} \|e_i(t)\| = \lim_{t \to \infty} \|x_i(t) + s_{\phi_i}(t)\| = 0, \quad i = 1, 2, \ldots, N.
\tag{2.4}
$$

**Lemma 2.2.** *For any two vectors $x$ and $y$, a matrix $Q > 0$ with compatible dimensions, one has*

$$
2x^T y \leq x^T Q x + y^T Q^{-1} y.
\tag{2.5}
$$

*Assumption 2.3* (see [15]). For any $x = (x_1, x_2, \ldots, x_n)^T \in R^n$, $y = (y_1, y_2, \ldots, y_n)^T \in R^n$, there exists a positive constant $L$ such that, $(y - x)^T (f(t, y) - f(t, x)) \leq L(y - x)^T \Gamma (y - x)$, where $\Gamma$ is a positive definite matrix. Here $x$ and $y$ are time-varying vectors.

*Assumption 2.4.* $f(x, t)$ is an odd function of $x$, that is, $f(-x, t) = -f(x, t)$ for arbitrary $x \in R^n$.

*Assumption 2.5.* $\eta_{\phi_i}(t)$ is a differential function with $0 \leq \dot{\eta}_{\phi_i}(t) \leq \varepsilon \leq 1$. Clearly, this assumption is certainly ensured if the delay $\eta_{\phi_i}(t)$ is constant.

## 3. Main Results

In this section, a control scheme is developed to synchronize a delayed complex network with nonidentical nodes to any smooth dynamics $s_{\phi_i}(t)$. Let synchronization errors $e_i(t) = x_i(t) + s_{\phi_i}(t)$ for $i = 1, 2, \ldots, N$, according to system (2.1); the error dynamical system can be derived as

$$
\begin{aligned}
\dot{e}_i(t) &= \tilde{f}_{\phi_i}(t, x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma_1 e_j(t) + c\sum_{j=1}^{N} b_{ij}\Gamma_2 e_j(t - \eta_{\phi_i}(t)) - c\sum_{i=1}^{N} a_{ij}\Gamma_1 s_{\phi_i}(t) \\
&\quad - c\sum_{i=1}^{N} b_{ij}\Gamma_2 s_{\phi_i}(t - \eta_{\phi_i}(t)) + u_i(t), \quad \phi_i(t) \in \overline{J}_{\phi_i}, \ i = 1, 2, \ldots, N,
\end{aligned}
$$

$$
\begin{aligned}
\dot{e}_i(t) &= \tilde{f}_{\phi_i}(t, x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma_1 e_j(t) \\
&\quad + c\sum_{j=1}^{N} b_{ij}\Gamma_2 e_j(t - \eta_{\phi_i}(t)) + v_i(t), \quad \phi_i(t) \in J_{\phi_i} - \overline{J}_{\phi_i}, \ i = 1, 2, \ldots, N,
\end{aligned}
\tag{3.1}
$$

where $\tilde{f}_{\phi_i}(t, e_i(t)) = f_{\phi_i}(t, x_i(t)) + f_{\phi_i}(t, s_{\phi_i}(t))$, for $i = 1, 2, \ldots, N$.

According to the diffusive coupling condition (2.2) of the matrix $A$, $B$, we have

$$
c\sum_{i=1}^{N} a_{ij}\Gamma_1 s_{\phi_i}(t) + c\sum_{i=1}^{N} b_{ij}\Gamma_2 s_{\phi_i}(t - \eta_{\phi_i}(t)) = 0, \quad i \in J_{\phi_i} - \overline{J}_{\phi_i}.
\tag{3.2}
$$

On the basis of this property, for achieving cluster anti-synchronization, we design the controllers as follows:

$$
u_i(t) = c\sum_{i=1}^{N} a_{ij}\Gamma_1 s_{\phi_i}(t) + c\sum_{i=1}^{N} b_{ij}\Gamma_2 s_{\phi_i}(t - \eta_{\phi_i}(t)) - d_i e_i(t), \quad i \in \overline{J}_{\phi_i},
$$

$$
v_i(t) = -d_i e_i(t), \quad i \in J_{\phi_i} - \overline{J}_{\phi_i},
\tag{3.3}
$$

where $\dot{d}_i = k_i e_i^T(t) e_i(t)$, $d_i$ are the feedback strength and $k_i$ are arbitrary positive constants.

It is easy to see that the synchronization of the controlled complex network (2.1) is achieved if the zero solution of the error system (3.1) is globally asymptotically stable, which is ensured by the following theorem.

Let $\tilde{e}_j(t) = (e_{1j}(t), e_{2j}(t), \ldots, e_{Nj}(t))^T$, $D = \text{diag}(d_1, \ldots, d_N)^T$; then we have the following result.

**Theorem 3.1.** *Suppose Assumptions 2.3–2.5 hold. Consider the network* (2.1) *via control law* (3.3). *If the following conditions hold:*

$$
\lambda_{\max}(\Gamma)L + c\lambda_{\max}(Q) + \frac{1}{2}c\lambda_{\max}\left(PP^T\right) + \frac{c}{2(1 - \varepsilon)} < d,
\tag{3.4}
$$

*then the system* (2.3) *is cluster anti-synchronization, where d is sufficiently large positive constant to be determined.*

*Proof.* From Assumptions 2.3 and 2.4, we get

$$(y + x)^T (f(t, y) + f(t, x)) = (y - (-x))^T (f(t, y) - f(t, -x)) \leq L(y + x)^T \Gamma(y + x) \quad (3.5)$$

for $x, y \in R^n$.

Construct the following Lyapunov functional:

$$V(t) = \frac{1}{2} \sum_{i=1}^{N} e_i^T(t) e_i(t) + \frac{c}{2(1 - \varepsilon)} \sum_{i=1}^{N} \int_{t - \eta_{\phi_i}(t)}^{t} e_i^T(\xi) e_i(\xi) d\xi + \frac{1}{2} \sum_{i=1}^{N} \frac{(d_i - d)^2}{k_i}. \quad (3.6)$$

Calculating the derivative of $V(t)$, we have

$$\dot{V}(t) = \sum_{i=1}^{N} e_i^T(t) \left[ \tilde{f}_{\phi_i}(t, e_i(t)) + c \sum_{j=1}^{N} a_{ij} \Gamma_1 e_j(t) + c \sum_{j=1}^{N} b_{ij} \Gamma_2 e_j(t - \eta_{\phi_i}(t)) - d_i e_i(t) \right]$$

$$+ \frac{c}{2(1 - \varepsilon)} \sum_{i=1}^{N} e_i^T(t) e_i(t) - c \left( \frac{1 - \dot{\eta}_{\phi_i}(t)}{2(1 - \varepsilon)} \right) \sum_{i=1}^{N} e_i^T(t - \eta_{\phi_i}(t)) e_i(t - \eta_{\phi_i}(t)) \quad (3.7)$$

$$+ \sum_{i=1}^{N} (d_i - d) e_i^T(t) e_i(t).$$

By the Assumptions 2.3–2.5, we have

$$\dot{V}(t) \leq \sum_{i=1}^{N} L e_i^T(t) \Gamma e_i(t) + c \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} e_i^T(t) \Gamma_1 e_j(t) + c \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij} e_i^T(t) \Gamma_2 e_j(t - \eta_{\phi_i}(t))$$

$$+ \frac{c}{2(1 - \varepsilon)} \sum_{i=1}^{N} e_i^T(t) e_i(t) - \frac{c}{2} \sum_{i=1}^{N} e_i^T(t - \eta_{\phi_i}(t)) e_i(t - \eta_{\phi_i}(t)) - \sum_{i=1}^{N} d e_i^T(t) e_i(t)$$

$$\leq \sum_{i=1}^{N} L e_i^T(t) \Gamma e_i(t) + c e^T(A \otimes \Gamma_1) e + c e^T(B \otimes \Gamma_2) e(t - \eta_{\phi_i}(t)) + \frac{c}{2(1 - \varepsilon)} \sum_{i=1}^{N} e_i^T(t) e_i(t)$$

$$- \frac{c}{2} e^T(t - \eta_{\phi_i}(t)) e(t - \eta_{\phi_i}(t)) - d e^T(t) e(t).$$

$$(3.8)$$

Let $e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T \in R^{nN}, Q = (A \otimes \Gamma_1), P = (B \otimes \Gamma_2)$, where $\otimes$ represents the Kronecker product. Then

$$\dot{V}(t) \leq \lambda_{\max}(\Gamma) L e^T(t) e(t) + c e^T(t) Q e(t) + c e^T(t) P e(t - \eta_{\phi_i}(t)) + \frac{c}{2(1 - \varepsilon)} e^T(t) e(t)$$

$$- \frac{c}{2} e^T(t - \eta_{\phi_i}(t)) e(t - \eta_{\phi_i}(t)) - d e^T(t) e(t).$$

$$(3.9)$$

(a)



(b)



(c)

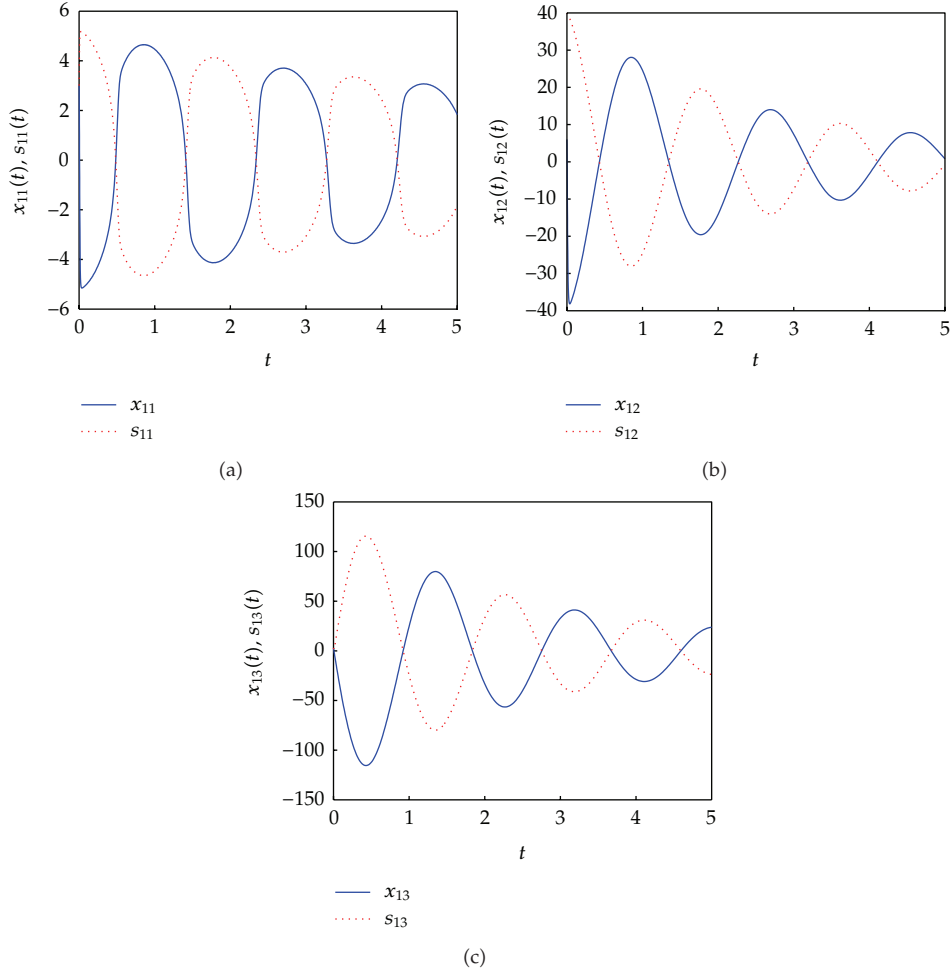**Figure 1:** The trajectories of nodes in the first community.

By Lemma 2.2, we have

$$\dot{V}(t) \le \lambda_{\max}(\Gamma) L e^T(t)e(t) + ce^T(t)Qe(t) + \frac{c}{2}e^T(t)PP^T e(t) + \frac{c}{2(1-\varepsilon)}e^T(t)e(t) - de^T(t)e(t)$$

$$\le \left( \lambda_{\max}(\Gamma) L + c\lambda_{\max}(Q) + \frac{1}{2}c\lambda_{\max}\left(PP^T\right) + \frac{c}{2(1-\varepsilon)} - d \right)e^T(t)e(t).$$

$$(3.10)$$

Therefore, if we have $\lambda_{\max}(\Gamma)L + c\lambda_{\max}(Q) + (1/2)c\lambda_{\max}(PP^T) + c/2(1-\varepsilon) < d$, then

$$\dot{V}(t) \le -e^T(t)e(t).$$

$$(3.11)$$

Choose $\lambda_{\max}(\Gamma)L + c\lambda_{\max}(Q) + (1/2)c\lambda_{\max}(PP^T) + c/2(1-\varepsilon) + 1 < d$. Thus, one obtains $\dot{V}(t) \le -e^T(t)e(t) \le 0$. It is obvious that the largest invariant set contained in set $E = \{\dot{V}(t) = 0\} = \{e_i(t) = 0, i = 1, 2, \ldots, N\}$ is $\tilde{Q} = \{e_i(t) = 0, d_i(t) = d, i = 1, 2, \ldots, N\}$.
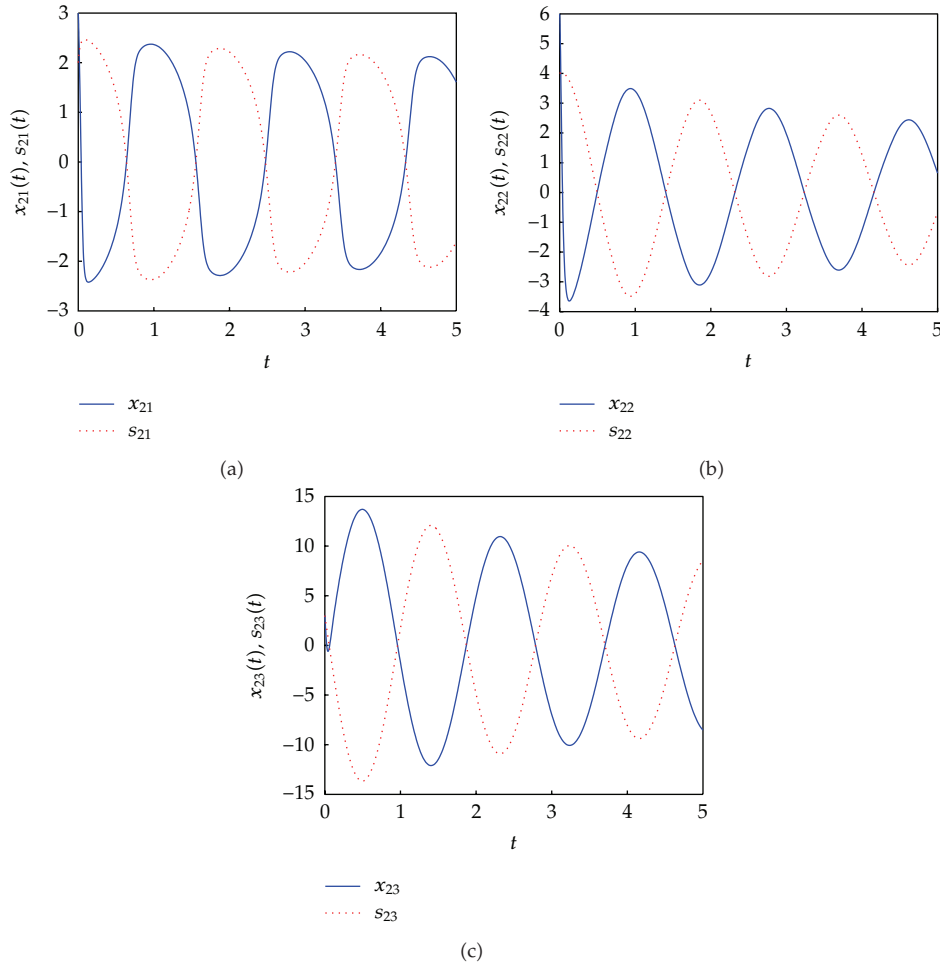
(a)

(b)

(c)

**Figure 2:** The trajectories of nodes in the second community.

Based on LaSalle invariance principle, starting with any initial values of the error dynamical system, the trajectory asymptotically converges to the largest invariant $\widetilde{Q}$ which implies that $\lim_{t\to\infty}\|e_i(t)\| = 0$ for $i = 1, 2, \ldots, N$. Therefore, cluster anti-synchronization in the network (2.3) is achieved under the controllers (3.3). This completes the proof. □

**Corollary 3.2.** *When $A = 0$, network* (2.1) *is translated into*

$$\dot{x}_i(t) = f_{\phi_i}(t, x_i(t)) + c\sum_{j=1}^{N} b_{ij}\Gamma_2 x_j\left(t - \eta_{\phi_i}(t)\right), \quad i = 1, 2, \ldots, N. \tag{3.12}$$

*We design the controllers, as follows, then the complex networks can also achieve synchronization, where*

$$u_i(t) = c\sum_{i=1}^{N} b_{ij}\Gamma_2 s_{\phi_i}\left(t - \eta_{\phi_i}(t)\right) - d_i e_i(t), \quad i \in \overline{J}_{\phi_i},$$

$$v_i(t) = -d_i e_i(t), \quad i \in J_{\phi_i} - \overline{J}_{\phi_i}. \tag{3.13}$$
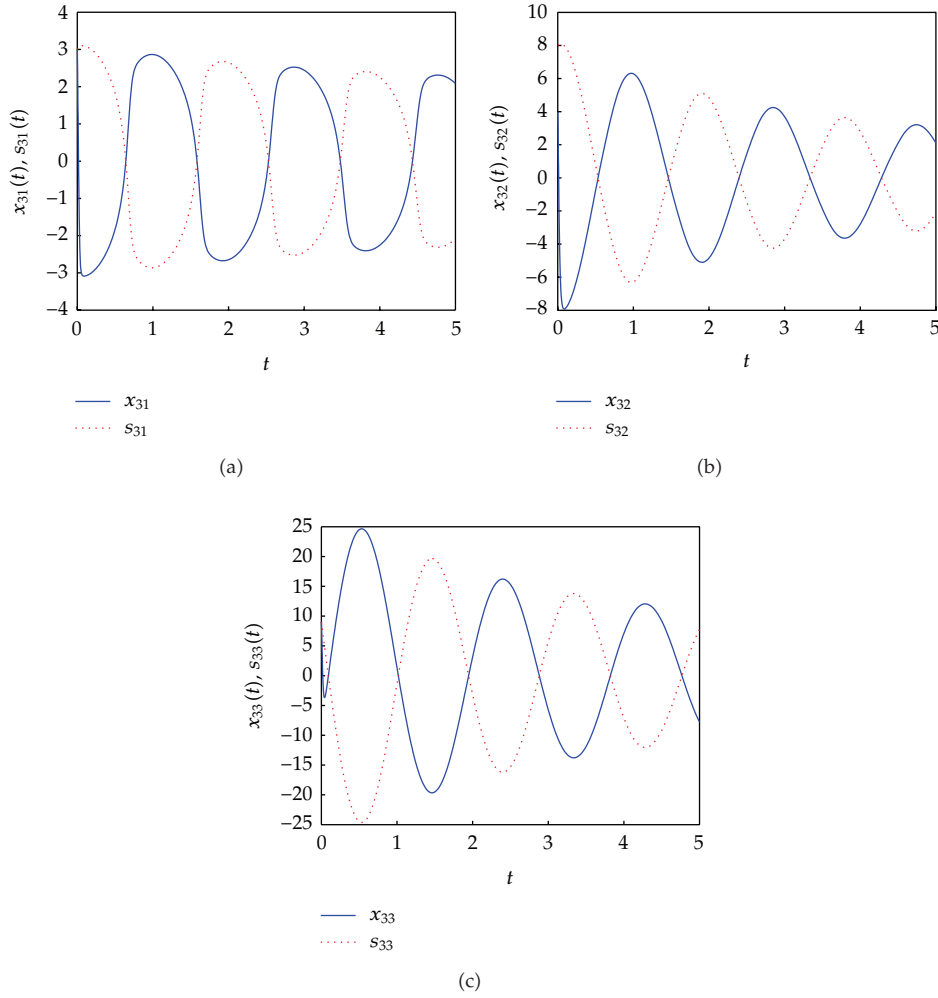
(a)



(b)



(c)

**Figure 3:** The trajectories of nodes in the third community.

**Corollary 3.3.** *When $B = 0$, network (2.1) is translated into*

$$\dot{x}_i(t) = f_{\phi_i}(t, x_i(t)) + c \sum_{j=1}^{N} a_{ij} \Gamma_1 x_j(t), \quad i = 1, 2, \ldots, N. \tag{3.14}$$

*We design the controllers, as follows, then the complex networks can also achieve synchronization, where*

$$u_i(t) = c \sum_{i=1}^{N} a_{ij} \Gamma_1 s_{\phi_i}(t) - d_i e_i(t), \quad i \in \overline{J}_{\phi_i},$$

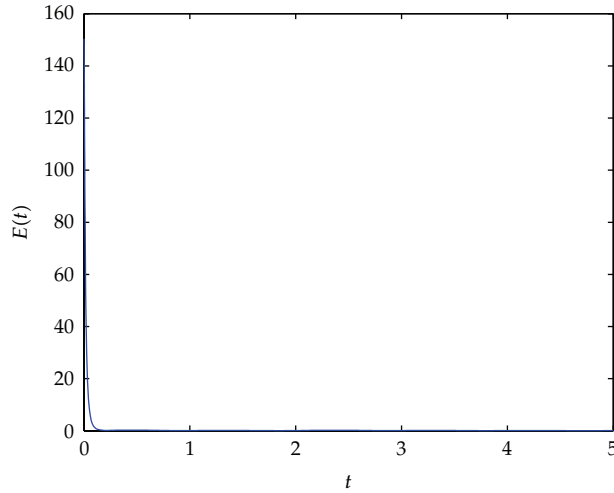$$v_i(t) = -d_i e_i(t), \quad i \in J_{\phi_i} - \overline{J}_{\phi_i}. \tag{3.15}$$

**Figure 4:** Time evolution of the synchronization error $E(t)$.
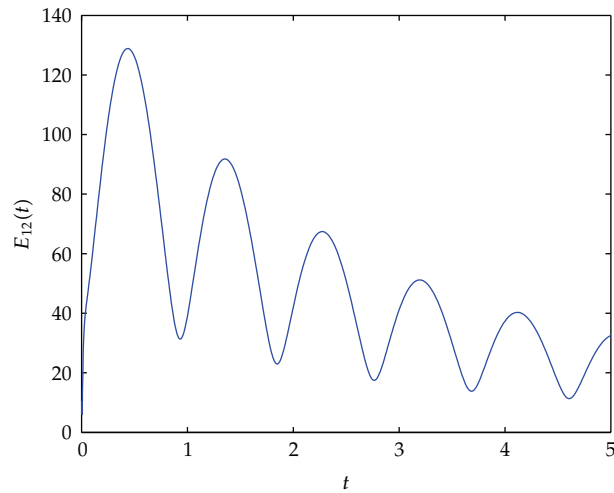


**Figure 5:** Time evolution of the synchronization error $E_{12}(t)$.

## 4. Illustrative Examples

In this section, several numerical examples are provided to illustrate the proposed synchronization methods. The nodes dynamics are the following well-known modified Chua's circuit [16] with different system parameters. Considering the following network:

$$\dot{x}_i(t) = f_{\phi_i}(t, x_i(t)) + c \sum_{j=1}^{N} a_{ij} \Gamma_1 x_j(t)$$

$$+ c \sum_{j=1}^{N} b_{ij} \Gamma_2 x_j\big(t - \eta_{\phi_i}(t)\big) + u_i(t), \quad \phi_i(t) \in \overline{J}_{\phi_i}, \ i = 1, 2, \ldots, N,$$
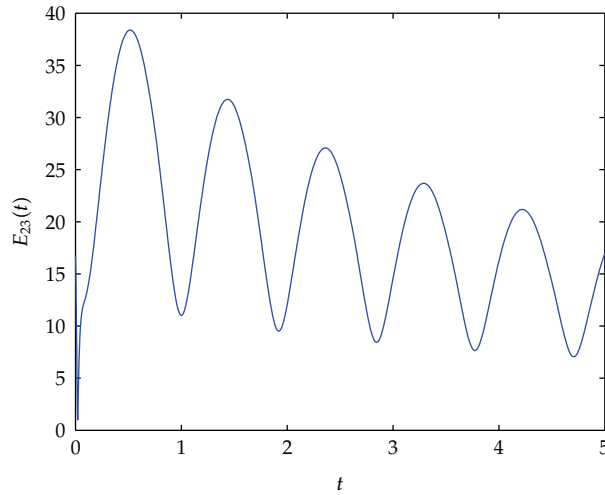
**Figure 6:** Time evolution of the synchronization error $E_{23}(t)$.

$$\dot{x}_i(t) = f_{\phi_i}(t, x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma_1 x_j(t)$$

$$+ c\sum_{j=1}^{N} b_{ij}\Gamma_2 x_j(t - \eta_{\phi_i}(t)) + v_i(t), \quad \phi_i(t) \in J_{\phi_i} - \bar{J}_{\phi_i}, \ i = 1, 2, \ldots, N,$$

(4.1)

where $x_i(t) = (x_{1i}(t), x_{2i}(t), x_{3i}(t))^T$, $f_1(t, x_i(t)) = D_1 x_i(t) + h_1(x_i(t))$, $f_2(t, x_i(t)) = D_2 x_i(t) + h_2(x_i(t))$, $f_3(t, x_i(t)) = D_3 x_i(t) + h_3(x_i(t))$, $h_1(x_i) = h_2(x_i) = h_3(x_i) = ((-20/7)x_{i1}^3, 0, 0)^T$, $\Gamma_1 = \Gamma_2 = \text{Diag}\{1, 1, 1\}$,

$$\eta_1(t) = \frac{0.1e^t}{1 + e^t}, \qquad \eta_2(t) = \frac{2e^t}{1 + e^t}, \qquad \eta_3(t) = \frac{0.8e^t}{1 + e^t},$$

$$D_1 = \begin{bmatrix} -\dfrac{10}{7} & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -12 & 0 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} -\dfrac{10}{7} & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -13 & 0 \end{bmatrix}, \qquad D_3 = \begin{bmatrix} -\dfrac{10}{7} & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14 & 0 \end{bmatrix},$$

(4.2)

$$c = 0.01, \qquad d = 10, \qquad A = B = \begin{bmatrix} -2 & 1 & 1 & -2 & 1 & 1 \\ -1 & -3 & 0 & 1 & 2 & 1 \\ 1 & 0 & -3 & 1 & 1 & 0 \\ 1 & 1 & 1 & -4 & 1 & 0 \\ 1 & 1 & 1 & 0 & -4 & 1 \\ 1 & 1 & 0 & 0 & 0 & -2 \end{bmatrix}.$$

Simulation results are given in Figures 1, 2, and 3. Cluster anti-synchronization is achieved by the controller (3.3).
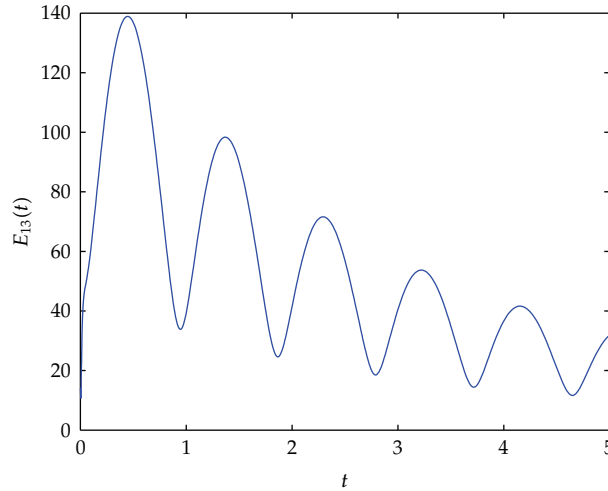
**Figure 7:** Time evolution of the synchronization error $E_{13}(t)$.

The following quantities are utilized to measure the process of cluster anti-synchronization

$$
\begin{aligned}
E(t) &= \sum_{i=1}^{N} \left\| x_i(t) + s_{\phi_i}(t) \right\|, \\
E_{12}(t) &= \left\| x_u(t) - x_v(t) \right\|, \quad u \in C_1, \ v \in C_2, \\
E_{13}(t) &= \left\| x_u(t) - x_v(t) \right\|, \quad u \in C_1, \ v \in C_3, \\
E_{23}(t) &= \left\| x_u(t) - x_v(t) \right\|, \quad u \in C_2, \ v \in C_3,
\end{aligned}
\tag{4.3}
$$

where $E(t)$ is the error of cluster synchronization for this controlled network (2.2); $E_{12}(t)$, $E_{13}(t)$, and $E_{23}(t)$ are the errors between two communities; cluster anti-synchronization is achieved if the synchronization error $E(t)$ converges to zero and $E_{12}(t)$, $E_{13}(t)$, and $E_{23}(t)$ do not as $t \to \infty$. Simulation results are given in Figures 4, 5, 6, and 7.

## 5. Conclusions

The cluster anti-synchronization in community networks has been studied in this paper, based on the community structure of the networks. Particularly, weight configuration matrix is not assumed to be irreducible. Some simple and useful criteria are derived by constructing an effective control scheme. The synchronization criteria are independent of time delay. Finally, the developed techniques are applied in three complex community networks. The numerical simulations are performed to verify the effectiveness of the theoretical results.

## Acknowledgments

# References

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