Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 340450, 7 pages
doi:10.1155/2012/340450

Research Article

A Note on Practical Stability of Nonlinear Vibration Systems with Impulsive Effects

Qishui Zhong,[1] Hongcai Li,[2] Hui Liu,[2] and Juebang Yu[3]

[1] *Institute of Astronautics & Aeronautics, University of Electronic Science and Technology of China, Chengdu 610054, China*
[2] *National Key Lab of Vehicular Transmission, Beijing Institute of Technology, Beijing 100081, China*
[3] *College of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu 610054, China*

Correspondence should be addressed to Qishui Zhong, zhongqs@uestc.edu.cn

Received 23 March 2012; Accepted 26 July 2012

Academic Editor: Junjie Wei

This paper addresses the issue of vibration characteristics of nonlinear systems with impulsive effects. By utilizing a T-S fuzzy model to represent a nonlinear system, a general strict practical stability criterion is derived for nonlinear impulsive systems.

1. Introduction

In recent years, nonlinear vibration and its control have been widely studied due to undesirable or harmful behaviors under many circumstances [1–6]. Furthermore, during working condition, plates usually work with disturbances which can be described with impulsive effects, and impulsive differential equations usually used to dealt with this kind of problem. A number of papers have deal with the theory of impulsive differential equations [7–10] and its applications to many kind of systems [11, 12]. Impulsive control has been demonstrated to be an effective and attractive control method to stabilize linear and nonlinear systems [11, 13], especially to stabilize various chaotic vibration systems (e.g., see [14–16]).

Furthermore, a highly nonlinear system can usually be represented by the T-S fuzzy model [17]. The T-S fuzzy model is described by fuzzy IF-THEN rules where the consequent parts represent local linear models for nonlinear systems. Then we can use linear theory to analyze chaotic systems by means of T-S fuzzy models at a certain domain [18, 19].

In the study of the Lyapunov stability, an interesting set of problems deals with bringing sets close to a certain state, rather than the state $x = 0$ [20]. The desired state of system may be mathematically unstable and yet the system may oscillate sufficiently near this state that its performance is acceptable. Many problems fall into this category including the travel of a space vehicle between two points, an aircraft or a missile which may oscillate around a mathematically unstable course yet its performance may be acceptable, and the problem in a chemical process of keeping the temperature within certain bounds. Such considerations led to the notion of practical stability which is neither weaker nor stronger than the Lyapunov stability [21, 22]. In a practical control problem, one aims at controlling a system into a certain region of interest instead of an exact point. If one wants to control a system to an exact point, the expense may be prohibitively high in some cases.

In this paper, we study the practical stability for the nonlinear impulsive system based on T-S fuzzy model. After a general strictly practical stability criterion is derived, some simple and easily verified sufficient conditions are given to stabilize the nonlinear vibration system.

Notation: $A > 0$ (< 0) means A is a symmetrical positive (negative) definite matrix. R^{+}, R_{+}, and N stand for, respectively, the set of all positive real numbers, nonnegative real numbers, and the set of natural numbers. $\|x\|$ denotes the Euclidian norm of vector x, and $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ mean the minimal and maximal eigenvalues of matrix A, respectively.

2. Problem Formulation

Consider the following nonlinear system

$$\dot{x}(t) = f(x(t)), \tag{2.1}$$

where $x(t) \in R^{n}$ is the state variable, $f \in C[R^{n}, R^{n}]$. We can construct the fuzzy model for (2.1) as follows: rule (i): if $z_1(t)$ is $M_{i1}, \ldots$, and $z_p(t)$ is M_{ip}, THEN $x(t) = A_i x(t), i = 1, 2, \ldots, r$, in which $z_i(t), \ldots, z_p(t)$ are the premise variables, each M_{ij} $(j = 1, 2, \ldots, p)$ is a fuzzy set, and $A_i \in R^{n\times n}$ is a constant matrix. With a center-average defuzzifier, the overall fuzzy system is represented as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(t) A_i x(t), \tag{2.2}$$

where r is the number of fuzzy implications, $h_i(t) = w_i(t)/(\sum_{i=1}^{r} w_i(t))$, $w_i(t) = \prod_{l=1}^{p} M_{il}(z_l(t))$, and $M_{il}(z_l(t))$ is the grade of membership of $z_l(t)$ in M_{il}. Of course, $h_i(t) \geq 0$ and $\sum_{i=1}^{r} h_i(t) = 1$. Note that system (2.2) can locally represent system (2.1).

Disturbances, acting on system (2.1), are given by a sequence $\{t_k, I_k(x(t_k))\}$, where $0 < t_1 < t_2 < \cdots < t_k < \cdots$, $t_k \to \infty$ as $k \to \infty$, and $I_k \in C[R^{n}, R^{n}]$ denotes the incremental change of the state at time t_k. Thus, the following impulsive differential equation is obtained

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(t) A_i x(t), \quad \forall t \geq 0,\ t \neq t_k, \\ \Delta x(t) &= I_k(x(t)), \quad t = t_k,\ k \in N, \end{aligned} \tag{2.3}$$

in which $\Delta x(t) = x(t_k^{+}) - x(t_k), x(t_k^{+}) = \lim_{t\to t_k^{+}} x(t_k), k \in N$.

Assume that $I_k(0) \equiv 0$ for all k so that the trivial solution of (2.3) exists. Denote $S(\rho) = \{x \in R^n : \|x\| < \rho\}$.

We will introduce the following classes of function spaces, definitions, and theorems for future use:

$$K = \{a \in C[R_+, R_+] : a \text{ is strictly increasing and } a(0) = 0\}.$$
$$C_K = \{a \in C[R_+^2, R_+] : a(t, u) \in K \text{ for each } t \in R_+\}.$$

Definition 2.1. Let $V : R_+ \times R^n \to R_+$, then V is said to belong to class v_0 if:

(1) V is continuous in $(t_{k-1}, t_k] \times R^n$ and for each $x \in R^n, k = 1, 2, \ldots,$

$$\lim_{(t,y)\to(t_k^+,x)} V(t,y)\lim = V(t_k^+, x) \tag{2.4}$$

exits;

(2) V is locally Lipschitzian in x.

Definition 2.2. For $(t, x) \in (t_{k-1}, t_k] \times R^n$, we define the derivatives of $V(t, x)$ as

$$D^+V(t,x) = \lim_{s\to 0^+} \sup \frac{1}{s}\left[V(t+s, x+sf(t,x)) - V(t,x)\right], \tag{2.5}$$

$$D^-V(t,x) = \lim_{s\to 0^-} \inf \frac{1}{s}\left[V(t+s, x+sf(t,x)) - V(t,x)\right]. \tag{2.6}$$

Definition 2.3. The trivial solution of the system (2.1) is said to be

(1) practically stable, if given (λ, A) with $0 < \lambda < A$, one has $\|x_0\| < \lambda$ implies $\|x(t)\| < A, t \ge t_0$ for some $t_0 \in R_+$;

(2) strictly stable, if (1) holds and for every $\mu \le \lambda$ there exists $B < \mu$ such that $\|x_0\| > \mu$ implies $\|x(t)\| > B, t \ge t_0$.

Theorem 2.4 (see [23]). *Suppose that*

(1) $0 < \lambda < A < \rho$;

(2) *there exists* $V_1 : R_+ \times S(\rho) \to R_+, V_1 \in v_0, a_1 \in C_K, b_1 \in K$ *such that* $a_1(t_0, \lambda) \le b_1(A)$ *for some* $t_0 \in R_+$, *and for* $(t, x) \in R_+ \times S(\rho)$,

$$\begin{aligned} &b_1(\|x\|) \le V_1(t,x) \le a_1(t_0, \|x\|), \\ &D^+V_1(t,x) \le 0, \quad t \ne t_k, \\ &V_1(t^+, x + I_k(x)) \le V_1(t,x), \quad t = t_k, \end{aligned} \tag{2.7}$$

(3) *there exists* $V_2 : R_+ \times S(\rho) \rightarrow R_+, V_2 \in v_0, a_2, b_2 \in K$ *such that for* $(t, x) \in R_+ \times S(\rho)$,

$$\begin{aligned} &b_2(\|x\|) \leq V_2(t,x) \leq a_2(\|x\|) \\ &D_- V_2(t,x) \geq 0, \quad t \neq t_k, \\ &V_2(t^+, x + I_k(x)) \geq V_2(t,x), \quad t = t_k. \end{aligned} \tag{2.8}$$

Then, the trivial solution of (2.1) *is strictly practically stable.*

3. Main Results

In this section, we will give some strict practical stability criteria of nonlinear vibration system (2.3).

Theorem 3.1. *The trivial solution of system* (2.3) *is strictly practically stable if there exists a matrix* $P > 0$, $\alpha \leq 0$ *and the following conditions hold:*

$$PA_i + A_i^T P \leq \alpha I, \quad i = 1, \ldots, r, \tag{3.1}$$

$$\|x(t_k) + I_k(x(t_k))\|^2 \leq \frac{\lambda_{\min}(P)}{\lambda_{\max}(P)} \|x(t_k)\|^2, \quad k \in N. \tag{3.2}$$

Proof. Consider the Lyapunov function $V_1(x(t)) = (1/2)x^T(t)Px(t)$ and $V_2(x(t)) = \exp(-x^T(t)Px(t))$. It yields that

$$\begin{aligned} &\frac{1}{2}\lambda_{\min}(P)\|x(t)\|^2 \leq V_1(x(t)) \leq \frac{1}{2}\lambda_{\max}(P)\|x(t)\|^2, \\ &\exp\left(-\lambda_{\min}(P)\|x(t)\|^2\right) \geq V_2(x(t)) \geq \exp\left(-\lambda_{\max}(P)\|x(t)\|^2\right). \end{aligned} \tag{3.3}$$

When $t \neq t_k$, since $PA_i + A_i^T P \leq \alpha I$, $i = 1, 2, \ldots, r$ and $\alpha \leq 0$, the derivatives of them are

$$\begin{aligned} D^+ V_1(x(t)) &= \frac{1}{2}\dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\ &= \frac{1}{2}\sum_{i=1}^{r} h_i(t)x^T(t)\left\{PA_i + A_i^T P\right\}x(t) \\ &\leq \frac{\alpha}{2}\|x(t)\|^2 \\ &\leq 0, \quad t \in (t_{k-1}, t_k),\ k \in N,\ x \in S(\rho), \end{aligned} \tag{3.4}$$

$$\begin{aligned} D_- V_2(x(t)) &= -\exp\left(-x^T(t)Px(t)\right)\left(\dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t)\right) \\ &= -\exp\left(-x^T(t)Px(t)\right)\left(\sum_{i=1}^{r} h_i(t)x^T(t)\left\{PA_i + A_i^T P\right\}x(t)\right) \end{aligned}$$

$$\begin{aligned}&\geq -\alpha \exp\left(-x^T(t)Px(t)\right)\|x(t)\|^2\\&\geq 0, \quad t\in(t_{k-1},t_k),\ k\in N,\ x\in S(\rho).\end{aligned} \tag{3.5}$$

Let $a_1(x) = x^T(t)Px(t)$, $b_1(x) = (1/4)V_1(x(t))$, $a_2(x) = 2\exp(-x^T(t)Px(t))$, and $b_2(x) = \exp(-x^T(t)Px(t))$, we can easily obtain

$$\begin{aligned}&b_1(\|x\|)\leq V_1(x(t))\leq a_1(t_0,\|x\|),\\&b_2(\|x\|)\leq V_2(x(t))\leq a_2(\|x\|).\end{aligned} \tag{3.6}$$

When $t = t_k$, according to (3.2) and (3.6), we have

$$\begin{aligned}V_1(x(t)+I_k(x(t))) &= \frac{1}{2}(x(t)+I_k(x(t)))^T P(x(t)+I_k(x(t)))\\&\leq \frac{1}{2}\lambda_{\max}(P)\|(x(t)+I_k(x(t)))\|^2\\&\leq \frac{1}{2}\lambda_{\min}(P)\|(x(t))\|^2\\&\leq V_1(x(t)),\end{aligned} \tag{3.7}$$

$$\begin{aligned}V_2(x(t)+I_k(x(t))) &= \exp\Big((-(x(t)+I_k(x(t)))^T P(x(t)+I_k(x(t)))\Big)\\&\geq \exp\left(-\lambda_{\max}(P)\|(x(t)+I_k(x(t)))\|\right)^2\\&\geq \exp\left(-\lambda_{\min}(P)\|(x(t))\|^2\right)\\&\geq V_2(x(t)).\end{aligned} \tag{3.8}$$

Then, with Theorem 2.4, the trivial solution of system (2.3) is strictly practical stable. □

Remark 3.2. If system (2.3) is practically stabilizable, we can design a general linear or nonlinear impulsive control law $\{t_k, I_k(x(t_k))\}$ for system (2.1), which can make the system oscillate sufficiently near the aimed state and the performance is considered acceptable, that is, the vibration of system (2.1) is depressed in the sense of practical stability under the impulsive control.

If $I_k(x(t_k)) = C_k x(t_k), k\in N$, where each $C_k\in R^{n\times n}$ is constant matrix, then system (2.3) is rewritten by

$$\begin{aligned}&\dot{x}(t) = \sum_{i=1}^{r} h_i(t)A_i x(t), \quad \forall t\geq 0,\ t\neq t_k,\\&\Delta x(t) = C_k x(t), \quad t = t_k,\ k\in N.\end{aligned} \tag{3.9}$$

From (3.2), it is easily obtained that

$$\|I + C_k\|^2 \leq \frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}, \quad k \in N. \tag{3.10}$$

Then, we have the following corollary.

Corollary 3.3. *The trivial solution of system* (3.9) *is strictly practically stable if there exist a matrix* $P > 0$, $\alpha \leq 0$ *and the conditions* (3.1) *and* (3.10) *hold.*

4. Conclusions

In this paper, some strict practical stability criteria have been put forward for nonlinear impulsive systems based on their T-S models. The reported results are helpful to consider the vibration characteristics of nonlinear systems with impulsive disturbances and also to control the vibration of nonlinear system via the impulsive control law.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grants no. 50905018 and 51075033, the Foundation of Science and Technology on Vehicle Transmission Laboratory, the Fundamental Research Funds for the Central Universities under Grant no. ZYGX2009J088, and Sichuan Science & Technology Plan under Grant no. 2011JY0001.

References

[1] B. Lehman, J. Bentsman, S. Verduyn Lunel, and E. I. Verriest, "Vibrational control of nonlinear time lag systems with bounded delay: averaging theory, stabilizability, and transient behavior," *IEEE Transactions on Automatic Control*, vol. 39, no. 5, pp. 898–912, 1994.

[2] A. E. Alshorbagy, M. A. Eltaher, and F. F. Mahmoud, "Free vibration characteristics of a functionally graded beam by finite element method," *Applied Mathematical Modelling*, vol. 35, no. 1, pp. 412–425, 2011.

[3] B. Farshi and A. Assadi, "Development of a chaotic nonlinear tuned mass damper for optimal vibration response," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 11, pp. 4514–4523, 2011.

[4] B. K. Eshmatov, "Nonlinear vibrations and dynamic stability of viscoelastic orthotropic rectangular plates," *Journal of Sound and Vibration*, vol. 300, no. 3–5, pp. 709–726, 2007.

[5] X. Zhou, Y. Wu, Y. Li, and Z. Wei, "Hopf bifurcation analysis of the Liu system," *Chaos, Solitons & Fractals*, vol. 36, no. 5, pp. 1385–1391, 2008.

[6] X. Zhou, Y. Wu, Y. Li, and H. Xue, "Adaptive control and synchronization of a novel hyperchaotic system with uncertain parameters," *Applied Mathematics and Computation*, vol. 203, no. 1, pp. 80–85, 2008.

[7] V. Lakshmikantham, D. D. Bainov, and P. S. Simeonov, *Theory of Impulsive Differential Equations*, Series in Modern Applied Mathematics, World Scientific, Singapore, 1989.

[8] Q. Zhong, J. Bao, Y. Yu, and X. Liao, "Exponential stabilization for discrete Takagi-Sugeno fuzzy systems via impulsive control," *Chaos, Solitons & Fractals*, vol. 41, no. 4, pp. 2123–2127, 2009.

[9] X. H. Tang, Z. He, and J. S. Yu, "Stability theorem for delay differential equations with impulses," *Applied Mathematics and Computation*, vol. 131, no. 2-3, pp. 373–381, 2002.

[10] T. Yang, “Impulsive control,” *IEEE Transactions on Automatic Control*, vol. 44, no. 5, pp. 1081–1083, 1999.
[11] Q. Zhong, J. Bao, Y. Yu, and X. Liao, “Impulsive control for T-S fuzzy model-based chaotic systems,” *Mathematics and Computers in Simulation*, vol. 79, no. 3, pp. 409–415, 2008.
[12] M. De La Sen and N. Luo, “A note on the stability of linear time-delay systems with impulsive inputs,” *IEEE Transactions on Circuits and Systems. I*, vol. 50, no. 1, pp. 149–152, 2003.
[13] X. Liu and G. Ballinger, “Uniform asymptotic stability of impulsive delay differential equations,” *Computers & Mathematics with Applications*, vol. 41, no. 7-8, pp. 903–915, 2001.
[14] X. Liu, “Impulsive stabilization and control of chaotic system,” *Nonlinear Analysis: Theory, Methods & Applications*, vol. 47, no. 2, pp. 1081–1092, 2001.
[15] J. Sun, “Impulsive control of a new chaotic system,” *Mathematics and Computers in Simulation*, vol. 64, no. 6, pp. 669–677, 2004.
[16] Q. S. Zhong, J. F. Bao, Y. B. Yu, and X. F. Liao, “Impulsive control for fractional-order chaotic systems,” *Chinese Physics Letters*, vol. 25, no. 8, pp. 2812–2815, 2008.
[17] T. Takagi and M. Sugeno, “Fuzzy identification of systems and its applications to modeling and control,” *IEEE Transactions on Systems, Man and Cybernetics*, vol. 15, no. 1, pp. 116–132, 1985.
[18] Y. W. Wang, Z. H. Guan, and H. O. Wang, “Impulsive synchronization for Takagi-Sugeno fuzzy model and its application to continuous chaotic system,” *Physics Letters, Section A*, vol. 339, no. 3–5, pp. 325–332, 2005.
[19] Y.-W. Wang, Z.-H. Guan, H. O. Wang, and J.-W. Xiao, “Impulsive control for T-S fuzzy system and its application to chaotic systems,” *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering*, vol. 16, no. 8, pp. 2417–2423, 2006.
[20] I. M. Stamova, “Vector Lyapunov functions for practical stability of nonlinear impulsive functional differential equations,” *Journal of Mathematical Analysis and Applications*, vol. 325, no. 1, pp. 612–623, 2007.
[21] V. Lakshmikantham, S. Leela, and A. A. Martynyuk, *Practical Stability of Nonlinear Systems*, World Scientific, Teaneck, NJ, USA, 1990.
[22] T. Yang, J. A. K. Suykens, and L. O. Chua, “Impulsive control of nonautonomous chaotic systems using practical stabilization,” *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering*, vol. 8, no. 7, pp. 1557–1564, 1998.
[23] S. Li, X. Song, and A. Li, “Strict practical stability of nonlinear impulsive systems by employing two Lyapunov-like functions,” *Nonlinear Analysis: Real World Applications*, vol. 9, no. 5, pp. 2262–2269, 2008.