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Research Article

The Global Existence of Nonlinear Evolutionary Equation with Small Delay

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We investigate the global existence of the delayed nonlinear evolutionary equation $\partial_t u + Au = f(u(t), u(t-\tau))$. Our work space is the fractional powers space X^α. Under the fundamental theorem on sectorial operators, we make use of the fixed-point principle to prove the local existence and uniqueness theorem. Then, the global existence is obtained by Gronwall's inequality.

1. Introduction

On the existence for solutions of evolutionary equations, there are many works and methods [1–7]. For example, the fixed principle [1, 3–5, 7] and Galerkin approximations [2, 6]. They are very classical methods to prove existence and uniqueness. Generally speaking, there are four solution concepts. That is, weak solution, mild solution, strong solution, and classical solution. We can obtain different types for different conditions. For instance [1], consider the following inhomogeneous initial value problem:

$$\begin{gathered} u'(t) + Au = f(t), \quad t > 0, \\ u(0) = x \in X, \end{gathered} \tag{1.1}$$

where X is Banach space. If the nonlinearity $f \in L^1(0, T; X)$, the initial value problem has a unique mild solution. If the nonlinearity f is differentiable a.e. on $[0, T]$ and $f' \in L^1(0, T; X)$, then for every $x \in D(A)$ the initial value problem has a unique strong solution. Furthermore, if the nonlinearity $f \in L^1(0, T; X)$ is locally Hölder continuous, then the initial value problem has a unique classical solution.

In the article [5], the author considered scalar reaction-diffusion equations with small delay

$$u'(t) - \Delta u = f(u(t), u(t-\tau)). \tag{1.2}$$

There the nonlinearity is assumed to be locally lipschitz and to satisfy the one-sided growth estimates

$$\begin{aligned} f(u,v) &\leq (u+1)\gamma(v), \quad u \geq 0, \\ f(u,v) &\geq -(|u|+1)\gamma(v), \quad u \leq 0, \end{aligned} \tag{1.3}$$

for some continuous γ. To prove existence, he treated the equation stepwise as a nonautonomous undelayed parabolic partial differential equation on the time intervals $[(j-1)\tau, j\tau]$ by regarding the delayed values as fixed. His strategy was to mimic the results of Henry [3, Theorem 3.3.3 and Corollary 3.3.5], but with his assumption of Hölder continuity in replaced by p-integrability. Many authors had investigated the nondelayed one in [8–10].

In this paper, we consider the following nonlinear evolutionary equation with small delay:

$$\begin{gathered} u'(t) + Au = f(u(t), u(t-\tau)), \quad t > 0, \\ u|_{[-\tau,0]} = \varphi(t). \end{gathered} \tag{1.4}$$

Under the hypothesis of (A1), (A2), and (A3) (see Section 2), we firstly make use of the fixed principle to prove the local existence and uniqueness theorem. Then we obtain the global existence and uniqueness by Gronwall inequality. In the whole paper, our work space is fractional powers space X^α. Its definition can be referred to [1, 3, 4].

2. Preliminaries

In this section, we will give some basic notions and facts. Firstly, basic assumptions are listed.

(A1) Let A be a positive, sectorial operator on a Banach Space X. e^{-At} is an analytic semigroup generated by $-A$. Fractional powers operator A^α is well defined. Fractional powers space $X^\alpha = D(A^\alpha)$ with the graph norm $\|u\|_\alpha = \|A^\alpha u\|_X$. For simplicity, we will denote $\|\cdot\|_X$ as $\|\cdot\|$.

(A2) For some $0 < \alpha < 1$, the nonlinearity $f : X^\alpha \times X^\alpha \to X$ is locally Lipschitz in (u, v). More precisely, there exists a neighborhood U such that for $u_i, v_i \in U$ and some constant L

$$\left\| f(u_1, v_1) - f(u_2, v_2) \right\| \leq L(\|u_1 - u_2\|_\alpha + \|v_1 - v_2\|_\alpha). \tag{2.1}$$

(A3) The initial value $\varphi(t)$ is Hölder continuous from $[-\tau, 0]$ to X^α.

Definition 2.1. Let I be an interval. A function u is called a (classical) solution of (1.4) in the space X^α provided that $u : I \to X^\alpha$ is continuously differentiable on I with $\partial_t u \in C(I, X)$ and satisfies (1.4) everywhere in I.

Obviously the (classical) solution of (1.4) can be expressed by the variation of constant formula

$$u(t) = e^{-At}x + \int_0^t e^{-A(t-s)} f(u(s), u(s-\tau))ds, \quad \text{for } t \geq 0, \tag{2.2}$$

where we let $\varphi(0) = x$. Next we come to the main theorem on analytic semigroup which is extremely important in the study of the dynamics of nonlinear evolutionary equations [4].

Theorem 2.2 (fundamental theorem on sectorial operators). *Let A be a positive, sectorial operator on a Banach Space X and e^{-At} be the analytic semigroup generated by $-A$. Then the following statements hold.*

(i) *For any $\alpha \geq 0$, there is a constant $C_\alpha > 0$ such that for all $t > 0$*

$$\left\| A^\alpha e^{-At} \right\|_{L(X)} \leq C_\alpha t^{-\alpha} e^{-at}, \quad (a > 0). \tag{2.3}$$

(ii) *For $0 < \alpha \leq 1$, there is a constant $C_\alpha > 0$ such that for $t \geq 0$ and $x \in D(A^\alpha)$*

$$\left\| e^{-At}x - x \right\| \leq C_\alpha t^\alpha \| A^\alpha x \|. \tag{2.4}$$

(iii) *For every $\alpha \geq 0$, there is a constant $C_\alpha > 0$ such that for all $t > 0$ and $x \in X$*

$$\left\| \left(e^{-A(t+h)} - e^{-At} \right) x \right\|_\alpha \leq C_\alpha |h| t^{-(1+\alpha)} \|x\|. \tag{2.5}$$

Lemma 2.3 (Gronwall's equality, [2–4]). *Let $v(t) \geq 0$ and be continuous on $[t_0, T]$. If there exists positive constants a, b, α $(\alpha < 1)$ such that for $t \in [t_0, T]$*

$$v(t) \leq a + b \int_{t_0}^t (t-s)^{\alpha-1} v(s) d, \tag{2.6}$$

then there exists positive constant M such that for $t \in [t_0, T]$

$$v(t) \leq Ma. \tag{2.7}$$

3. Main Results

Theorem 3.1. *Suppose (A1), (A2), and (A3) hold. Then there exists a sufficiently small $T > 0$ such that* (1.4) *has a unique solution on $[-\tau, T]$.*

Proof. For convenience, we still denote $\varphi(0) = x$. Select $\delta > 0$ and construct set

$$V = \{(u, v) \mid \|u - x\|_\alpha \le \delta,\ \|v - x\|_\alpha \le \delta\}. \tag{3.1}$$

Let $B = \|f(x, x)\|$, choose sufficient small $T < \tau$ such that

$$\left\| \left(e^{-At} - I\right) x \right\|_\alpha \le \frac{\delta}{2}, \quad 0 \le t < T, \tag{3.2}$$

$$C_\alpha (B + 2L\delta) \int_0^T u^{-\alpha} e^{-au} du \le \frac{\delta}{2}. \tag{3.3}$$

Let Y be the Banach space $C([-\tau, T]; X)$ with the usual supremum norm which we denote by $\|\cdot\|_Y$. Let S be the nonempty closed and bounded subset of Y defined by

$$S = \left\{ y : y \in Y,\ \left\| y(t) - A^\alpha x \right\| \le \delta \right\}. \tag{3.4}$$

On S we define a mapping F by

$$Fy(t) = \begin{cases} e^{-tA} A^\alpha x + \displaystyle\int_0^t A^\alpha e^{-(t-s)A} f\left(A^{-\alpha} y(s), A^{-\alpha} y(s - \tau)\right) ds, & 0 < t < T, \\ A^\alpha \varphi(t), & -\tau \le t \le 0. \end{cases} \tag{3.5}$$

Next we will utilize the contraction mapping theorem to prove the existence of fixed point. In order to complete this work, we need to verify that F maps S into itself and F is a contraction mapping on S with the contraction constant $\le 1/2$.

It is easy to see from (3.4) and (3.5) that for $-\tau \le t \le 0$, $F : S \to S$. For $0 < t < T$, considering (2.1), (2.3), (3.2), and (3.3), we obtain

$$\begin{aligned} \left\| Fy(t) - A^\alpha x \right\| &\le \left\| e^{-tA} A^\alpha x - A^\alpha x \right\| + \int_0^t \left\| A^\alpha e^{-(t-s)A} f\left(A^{-\alpha} y(s), A^{-\alpha} y(s - \tau)\right) \right\| ds \\ &\le \left\| e^{-tA} A^\alpha x - A^\alpha x \right\| + \int_0^t \left\| A^\alpha e^{-(t-s)A} f\left(A^{-\alpha} y(s), A^{-\alpha} y(s - \tau)\right) - f(x, x) \right\| ds \\ &\quad + \int_0^t \left\| A^\alpha e^{-(t-s)A} f(x, x) \right\| ds \\ &\le \frac{\delta}{2} + C_\alpha (2L\delta + B) \int_0^t (t - s)^{-\alpha} e^{-a(t-s)} ds \\ &\le \frac{\delta}{2} + C_\alpha (2L\delta + B) \int_0^T u^{-\alpha} e^{-au} du \\ &\le \delta. \end{aligned} \tag{3.6}$$

Therefore $F : S \to S$. Furthermore if $y_1, y_2 \in S$ then from (3.3) and (3.5)

$$
\begin{aligned}
&\|Fy_1(t) - Fy_2(t)\| \\
&\quad \le \int_0^t \left\| A^{\alpha} e^{-(t-s)A} \left[f\left(A^{-\alpha} y_1(s), A^{-\alpha} y_1(s-\tau)\right) - f\left(A^{-\alpha} y_2(s), A^{-\alpha} y_2(s-\tau)\right) \right] \right\| ds \\
&\quad \le \int_0^t \left\| A^{\alpha} e^{-(t-s)A} \right\| \left\| \left[f\left(A^{-\alpha} y_1(s), A^{-\alpha} y_1(s-\tau)\right) - f\left(A^{-\alpha} y_2(s), A^{-\alpha} y_2(s-\tau)\right) \right] \right\| ds \\
&\quad \le \int_0^t C_{\alpha} (t-s)^{-\alpha} e^{-a(t-s)} L \| y_1(s) - y_2(s) \| ds \\
&\quad \le C_{\alpha} L \int_0^T u^{-\alpha} e^{-au} ds \cdot \| (y_1 - y_2) \|_Y \\
&\quad \le \frac{1}{2} \| (y_1 - y_2) \|_Y,
\end{aligned}
\tag{3.7}
$$

which implies that

$$
\|Fy_1(t) - Fy_2(t)\|_Y \le \frac{1}{2} \| (y_1 - y_2) \|_Y. \tag{3.8}
$$

By the contraction mapping theorem the mapping F has a unique fixed point $y \in S$. This fixed point satisfies the following:

$$
y(t) = e^{-tA} A^{\alpha} x + \int_0^t A^{\alpha} e^{-(t-s)A} f\left(A^{-\alpha} y(s), A^{-\alpha} y(s-\tau)\right) ds, \quad 0 < t < T, \tag{3.9}
$$

$$
y(t) = A^{\alpha} \varphi(t), \quad -\tau \le t \le 0. \tag{3.10}
$$

From (2.1) and the continuity of y it follows that $t \to f(A^{-\alpha} y(t), A^{-\alpha} y(t-\tau))$ is continuous on $[0, T]$ and a fortiori bounded on this interval. Let

$$
\left\| f\left(A^{-\alpha} y(t), A^{-\alpha} y(t-\tau)\right) \right\| \le N. \tag{3.11}
$$

Next we want to show that $t \to f(A^{-\alpha} y(t), A^{-\alpha} y(t-\tau))$ is locally Hölder continuous on $(0, T)$. To this end, we show first that the solution y of (3.9) is locally Hölder continuous on $(0, T)$.

Select $[t_0, t_1] \subset (0, T)$, $t_0 \le t < t + h \le t_1$ such that

$$\begin{aligned}
\|y(t+h) - y(t)\| \le{}& \left\|\left(e^{-hA} - I\right) A^{\alpha} e^{-tA} x\right\| \\
&+ \int_0^t \left\|\left(e^{-hA} - I\right) A^{\alpha} e^{-(t-s)A} f\left(A^{-\alpha} y(s), A^{-\alpha} y(s-\tau)\right)\right\| ds \\
&+ \int_t^{t+h} \left\|A^{\alpha} e^{-(t+h-s)A} f\left(A^{-\alpha} y(s), A^{-\alpha} y(s-\tau)\right)\right\| ds \\
={}& I_1 + I_2 + I_3.
\end{aligned} \tag{3.12}$$

Considering (2.3) and (2.4), we select $\beta \in (0, 1-\alpha)$ such that

$$\begin{aligned}
I_1 &\le C_\beta h^\beta \left\|A^{\alpha+\beta} e^{-tA} x\right\| \\
&\le C_\beta h^\beta C_{\alpha+\beta} t^{-(\alpha+\beta)} \|x\| \\
&\le M_1 h^\beta, \\
I_2 &\le N C_\beta h^\beta \int_0^t \left\|A^{\alpha+\beta} e^{-(t-s)A}\right\| ds \\
&\le N C h^\beta \int_0^t (t-s)^{-(\alpha+\beta)} ds \\
&\le M_2 h^\beta, \\
I_3 &\le N C_\alpha \int_t^{t+h} (t+h-s)^{-\alpha} ds \\
&= \frac{N C_\alpha}{1-\alpha} h^{1-\alpha} \\
&\le M_3 h^\beta.
\end{aligned} \tag{3.13}$$

Synthesizing (3.12) and (3.13), we get

$$\|y(t+h) - y(t)\| \le C h^\beta, \quad t \in [t_0, t_1] \subset (0, T). \tag{3.14}$$

So we proved the solution y of (3.9) is locally Hölder continuous on $(0, T)$. Furthermore, in view of (2.1) we have

$$\begin{aligned}
&\left\|f\left[A^{-\alpha} y(t+h), A^{-\alpha} y(t+h-\tau)\right] - f\left[A^{-\alpha} y(t), A^{-\alpha} y(t-\tau)\right]\right\| \\
&\quad \le L\left(\|y(t+h) - y(t)\| + \|y(t+h-\tau) - y(t-\tau)\|\right) \\
&\quad \le L C h^\beta + \left\|A^{\alpha} \varphi(t+h-\tau) - A^{\alpha} \varphi(t-\tau)\right\| \\
&\quad \le M h^\gamma.
\end{aligned} \tag{3.15}$$

Let y be the solution of (3.9) and (3.10) and $\tilde{f}(t) = f(A^{-\alpha}y(t), A^{-\alpha}y(t-\tau))$. In view of locally Hölder continuous on $(0,T)$ of $\tilde{f}(t)$, consider the inhomogeneous initial value problem

$$\begin{gathered} u'(t) + Au = \tilde{f}(t), \quad 0 < t < T, \\ u(0) = x. \end{gathered} \tag{3.16}$$

By Corollary 4.3.3 in [1], this problem has a unique solution and the solution is given by

$$u(t) = e^{-tA}x + \int_0^t e^{-(t-s)A} f\left(A^{-\alpha}y(s), A^{-\alpha}y(s-\tau)\right) ds. \tag{3.17}$$

Each term of (3.17) is in $D(A)$ and a fortiori in $D(A^\alpha)$. Operating on both sides of (3.17) with A^α we find

$$A^\alpha u(t) = e^{-tA}A^\alpha x + \int_0^t A^\alpha e^{-(t-s)A} f\left(A^{-\alpha}y(s), A^{-\alpha}y(s-\tau)\right) ds. \tag{3.18}$$

By (3.9) the right-hand side of (3.18) equals $y(t)$ and therefore $u(t) = A^{-\alpha}y(t)$. So for $0 < t < T$, by (3.17) we have

$$u(t) = e^{-tA}x + \int_0^t e^{-(t-s)A} f(u(s), u(s-\tau)) ds. \tag{3.19}$$

So u is a $u \in C^1(0,T;X)$ solution of (1.4). The uniqueness of u follows readily from the uniqueness of the solutions of (3.9) and (3.16), and the proof is complete. □

Before giving our global existence theorem, we should first prove extended theorem of solution.

Theorem 3.2 (extended theorem). *Assume that (A1), (A2), and (A3) hold. And also assume that for every closed bounded set $B \subset U$, the image $f(B)$ is bounded in X. If u is a solution of* (1.1) *on $[-\tau, T_{\max})$, then either $T_{\max} = +\infty$ or there exists a sequence $t_n \to T_{\max}$ as $n \to +\infty$ such that $u(t_n) \to \partial U$. (If U is unbounded, the point at infinity is included in ∂U.)*

Proof. Suppose $T_{\max} < +\infty$, there exists a closed bounded B subset of U and $\tau_0 < T_{\max}$ such that for $\tau_0 \le t < T_{\max} u(t) \in B$. We prove there exists $x^* \in B$ such that

$$\lim_{t \to T_{\max}^-} u(t) = x^* \tag{3.20}$$

in X^α, which implies the solution may be extended beyond time $T_{\max}$.

Now let

$$C = \sup\left\{ \left\| f(u,v) \right\|, (u,v) \in B \right\}. \tag{3.21}$$

We show firstly that $\|u(t)\|_\beta$ remains bounded as $t \to T_{\max}^-$ for any $\beta \in [\alpha, 1)$.

Observe that if $\alpha \le \beta < 1$, $\tau_0 \le t < T_{\max}$, in view of (2.3) and (3.19) we have

$$\begin{aligned}
\|u(t)\|_\beta &\le \left\|A^{\beta-\alpha}e^{-tA}\right\|\|x\|_\alpha + \int_0^t \left\|A^\beta e^{-(t-s)A}\right\|\|f(u(s),u(s-\tau))\|ds \\
&\le C_{\beta-\alpha}t^{-(\beta-\alpha)}\|x\|_\alpha + C_\beta C\int_0^t (t-s)^{-\beta}ds \\
&= C_{\beta-\alpha}t^{-(\beta-\alpha)}\|x\|_\alpha + \frac{C_\beta C}{1-\beta}t^{1-\beta} \\
&\le M, \quad 0 < \tau_0 \le t < T_{\max}.
\end{aligned} \tag{3.22}$$

Secondly, suppose $\tau_0 \le t_1 < t < T_{\max}$, so

$$u(t) - u(t_1) = \left(e^{-(t-t_1)A} - I\right)u(t_1) + \int_{t_1}^t e^{-(t-s)A}f(u(s),u(s-\tau))ds. \tag{3.23}$$

From (2.3) and (2.4) we get

$$\begin{aligned}
\|u(t)-u(t_1)\|_\alpha &\le \left\|\left(e^{-(t-t_1)A} - I\right)A^\alpha u(t_1)\right\| + C\int_{t_1}^t \left\|A^\alpha e^{-(t-s)A}\right\|ds \\
&\le C_{\beta-\alpha}(t-t_1)^{\beta-\alpha}\left\|A^{\beta-\alpha+\alpha}u(t_1)\right\| + CC_\alpha\int_{t_1}^t (t-s)^{-\alpha}ds \\
&= C_{\beta-\alpha}(t-t_1)^{\beta-\alpha}\|u(t_1)\|_\beta + \frac{CC_\alpha}{1-\alpha}(t-t_1)^{1-\alpha}ds \\
&\le C_0(t-t_1)^{\beta-\alpha}.
\end{aligned} \tag{3.24}$$

Thus (3.20) holds, and the proof is completed. □

Theorem 3.3 (global existence and uniqueness). *Assume that (A1), (A2), and (A3) hold. And for all $(u,v) \in X^\alpha \times X^\alpha$, f satisfies*

$$\|f(u,v)\| \le L(\|u\|_\alpha + \|v\|_\alpha). \tag{3.25}$$

Then, the unique solution of (1.4) exists for all $t \ge -\tau$.

Proof. We need to verify that $\|u(t)\|_\alpha$ is bounded when $t \to T_{\max}^-$. As for $0 \le t < T_{\max}$

$$u(t) = e^{-tA}x + \int_0^t e^{-(t-s)A}f(u(s),u(s-\tau))ds. \tag{3.26}$$

Considering (3.25), we can obtain

$$\begin{aligned}
\|u(t)\|_\alpha &= \|A^\alpha u(t)\| \\
&\leq \left\|e^{-tA}A^\alpha x\right\| + \int_0^t \left\|A^\alpha e^{-(t-s)A}\right\| L(\|u(s)\|_\alpha + \|u(s-\tau)\|_\alpha)ds \\
&\leq C_1\|x\|_\alpha + LC_\alpha \int_0^t (t-s)^{-\alpha}\|u(s)\|_\alpha ds \\
&\quad + LC_\alpha \int_0^t (t-s)^{-\alpha}\|u(s-\tau)\|_\alpha ds,
\end{aligned} \tag{3.27}$$

For

$$\int_0^t (t-s)^{-\alpha}u(s-\tau)ds \overset{s-\tau=w}{=} \int_{-\tau}^{t-\tau} (t-\tau-w)^{-\alpha}u(w)dw. \tag{3.28}$$

Case 1. If $T_{\max} \leq \tau$. Because $u(t) = \varphi(t)$ for $-\tau \leq t \leq 0$ and φ is Hölder continuous from $[-\tau, 0]$ to X^α. Let

$$M = \max_{t\in[-\tau,0]} \left\|\varphi(t)\right\|, \tag{3.29}$$

$$\begin{aligned}
\int_0^t (t-s)^{-\alpha}\|u(s-\tau)\|_\alpha ds &= \int_{-\tau}^{t-\tau} (t-\tau-w)^{-\alpha}\left\|\varphi(w)\right\|_\alpha dw \\
&\leq M\int_{-\tau}^{0} (t-\tau-w)^{-\alpha}dw \leq \frac{M}{1-\alpha}t^{1-\alpha} \\
&\leq M_1, \quad t \in [0, T_{\max}).
\end{aligned} \tag{3.30}$$

From (3.27), we immediately get

$$\|u(t)\|_\alpha \leq a + b\int_0^t (t-s)^{-\alpha}\|u(s)\|_\alpha ds. \tag{3.31}$$

From Lemma 2.3, that is, Gronwall's inequality, we find $\|u(t)\|_\alpha \leq C$.

Case 2. If $T_{\max} > \tau$, still let

$$M = \max_{t\in[-\tau,0]} \left\|\varphi(t)\right\|, \tag{3.32}$$

because

$$\begin{aligned}
\int_0^t (t-s)^{-\alpha}\|u(s-\tau)\|_\alpha ds &= \int_{-\tau}^{t-\tau} (t-\tau-w)^{-\alpha}\|u(w)\|_\alpha dw \\
&= \int_{-\tau}^{0} (t-\tau-w)^{-\alpha}\|\varphi(w)\|_\alpha dw + \int_0^{t-\tau} (t-\tau-w)^{-\alpha}\|u(w)\|_\alpha dw \\
&\le M\int_{-\tau}^{0} (t-\tau-w)^{-\alpha} dw + \int_0^t (t-s)^{-\alpha}\|u(s)\|_\alpha ds \\
&\le M_0 + \int_0^t (t-s)^{-\alpha}\|u(s)\|_\alpha ds.
\end{aligned} \tag{3.33}$$

From (3.27) again, we obtain

$$\|u(t)\|_\alpha \le a_0 + b_0\int_0^t (t-s)^{-\alpha}\|u(s)\|_\alpha ds. \tag{3.34}$$

By Gronwall's inequality again, we get $\|u(t)\|_\alpha \le C$. This completes the proof of this theorem. □

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