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Research Article

Lag Synchronization of Hyperchaotic Systems via Intermittent Control

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Different from the most existing results, in this paper an intermittent control scheme is designed to achieve lag synchronization of coupled hyperchaotic systems. Several sufficient conditions ensuring lag synchronization are proposed by rigorous theoretical analysis with the help of the Lyapunov stability theory. Numerical simulations are also presented to show the effectiveness and feasibility of the theoretical results.

1. Introduction

Chaos is a highly interesting nonlinear phenomenon that has been much investigated for its great theoretical challenge and potential applications to many fields. Since the seminal works of Pecora and Carroll [1], the idea of synchronization of chaotic systems has received a great deal of interest among researchers from various fields. Over the past decades, several different regimes of chaos synchronization have been investigated, for example, complete synchronization [1, 2], generalized synchronization [3], projective synchronization [4], phase synchronization [5], lag synchronization [6], and anticipating synchronization [7].

On the other hand, it has been shown that the complete synchronization of chaos is practically impossible for the finite speed of signals. Chaotic lag synchronization appears as a coincidence of shift-in-time states of interactive systems. It is just synchronization lag that makes lag synchronization practically available. For instance, in the telephone communication system, the voice one hears on the receive side at time t is often the voice from the transmitter side at time $t - \tau$ [8]. So, in many cases, it is more reasonable to require the slave system to synchronize the master system with a time-delay τ. Moreover, projective synchronization of chaotic systems has attracted increasing attention due to its potential applications in secure communication and control processing.

There are many different methods including continuous control and discontinuous control that have been proposed for stabilizing and synchronizing chaotic systems, such as state feedback control [9, 10], adaptive control [11], switching control [12, 13], impulsive control [14–18], and intermittent control [19–22]. Recently, intermittent control of nonlinear system has drawn increasing interests in process control, ecosystem management, synchronizing chaotic systems, and communication, and so on. As a special form of switching control, intermittent control is also divided into two classes: state-dependent switching rule and time-switching rule. The former implies that the control operation is activated only when the states come into the certain region which is often before given, while the latter activates the control only in some finite time intervals; the system evolves freely when the time goes out of those intervals. Therefore, these intermittent control systems are open looped. Compared with continuous control method, and intermittent control method is advantageous for its efficiency.

Motivated by the above discussions, in this paper, we first investigate lag synchronization of hyperchaotic systems by periodically intermittent control. By using the Lyapunov stability theory and the intermittent control technique, the intermittent controllers and the corresponding parameter update rules are designed to obtain lag synchronization of hyperchaotic systems. Numerical simulations are reported to show their good agreement with the theoretical results.

The rest of the paper is organized as follows. In Section 2, model description is given. We will study the lag synchronization of coupled hyperchaotic systems, respectively. Accordingly, we obtain the control laws for both regimes based on the rigorous theoretical analysis. In Section 3, numerical examples are given to show the theoretical results, which is followed by the conclusions in Section 4.

2. Problem Formulations

Now we consider a novel hyperchaotic system described as [23]

$$\begin{aligned}\dot{x}_1 &= a(x_2 - x_1),\\ \dot{x}_2 &= bx_1 - x_2 - x_1x_3 + x_4,\\ \dot{x}_3 &= cx_3 + x_1x_2,\\ \dot{x}_4 &= d_1x_1 + d_2x_2,\end{aligned} \tag{2.1}$$

where x_1, x_2, x_3, and x_4 are state variables, and a, b, c, d_1, and d_2 are real parameters. It is shown that this system is hyperchaotic when the parameters are chosen as $a = 10$, $b = 28$, $c = -8/3$, and $(d_1, d_2) = (-9.3, 1)$, as shown in Figure 1.

We divide the system (2.1) into two parts, that is, linear part and nonlinear part. Then, we rewrite (2.1) as follows:

$$\dot{x}(t) = Ax(t) + f(x(t)), \tag{2.2}$$

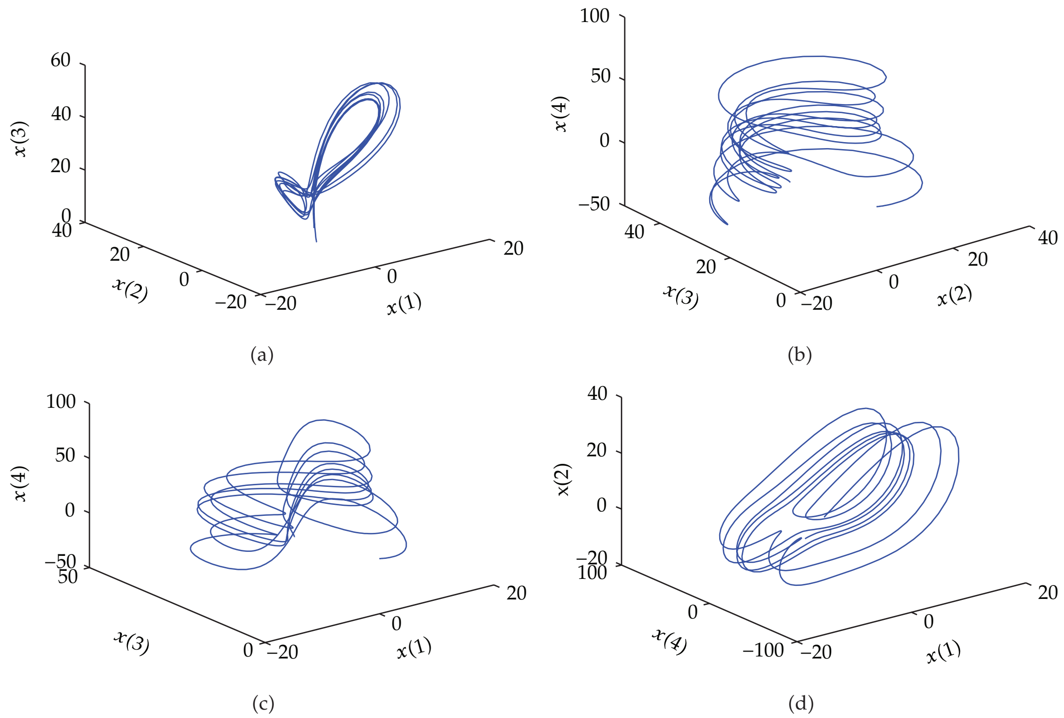

Figure 1: Several projections of hyperchaotic attractors.

where

$$x(t) = (x_1(t), x_2(t), x_3(t), x_4(t))^T, \qquad A = \begin{pmatrix} -a & a & 0 & 0 \\ b & -1 & 0 & 1 \\ 0 & 0 & c & 0 \\ d_1 & d_2 & 0 & 0 \end{pmatrix}, \qquad f(x(t)) = \begin{pmatrix} 0 \\ -x_1(t)x_3(t) \\ x_1(t)x_2(t) \\ 0 \end{pmatrix}. \tag{2.3}$$

In what follows, the coupled response system with feedback control is given by

$$\dot{y}(t) = Ay(t) + f(y(t)) + u(t), \tag{2.4}$$

where $y(t) = (y_1(t), y_2(t), y_3(t), y_4(t))^T$ is the response state. $u(t)$ is the intermittent control gain defined by

$$u(t) = \begin{cases} k(y(t) - x(t-\tau)), & nT \le t \le nT + \sigma T, \\ 0, & nT + \sigma T \le t \le (n+1)T, \end{cases} \tag{2.5}$$

where $\tau > 0$ is the propagation delay, k denotes control strength, $0 < \sigma < 1$ denotes switching rate and T denotes control period. Let $e(t) = y(t) - x(t-\tau)$ be the lag synchronization error between the systems (2.2) and (2.4), then yields the error system

$$\begin{aligned} \dot{e}(t) &= \dot{y}(t) - \dot{x}(t-\tau) \\ &= Ae(t) + f(y) - f(x(t-\tau)) + u(t). \end{aligned} \tag{2.6}$$

Under the control of the form (2.5), the lag synchronization error (2.6) can be rewritten as

$$\begin{aligned} \dot{e}(t) &= Ae(t) + f(y) - f(x(t-\tau)) + ke(t), \quad nT \leq t \leq nT + \sigma T, \\ \dot{e}(t) &= Ae(t) + f(y) - f(x(t-\tau)), \quad nT + \sigma T \leq t \leq (n+1)T. \end{aligned} \tag{2.7}$$

We now state our main results.

Theorem 2.1. *Suppose that there exist positive constants* $g_1 > 0$, $g_2 > 0$ *such that*

(1) $2kI + A + A^T + 2L_{\max}I + g_1 I \leq 0$;

(2) $A + A^T + 2L_{\max}I - g_2 I \leq 0$;

(3) $g_1\sigma - g_2(1-\sigma) > 0$.

Then, the lag synchronization error system (2.7) is globally exponentially stable, and moreover,

$$\|e(t)\| = \|e(t_0)\| \exp\left(-\frac{1}{2}(g_1\sigma - g_2(1-\sigma))(t-\sigma T)\right). \tag{2.8}$$

This implies that the two systems (2.2) and (2.4) are globally exponentially lag synchronized.

Proof. Consider the following Lyapunov function:

$$V(e(t)) = e^T(t)e(t) \tag{2.9}$$

which implies that $V(e(t)) = \|e(t)\|^2$.

When $nT \leq t \leq nT + \sigma T$, the derivative of (2.9) with respect to time t along the trajectories of the first subsystem of the system (2.7) is calculated and estimated as follows:

$$\begin{aligned}
\dot{V}(e(t)) &= e^T(t)\dot{e}(t) + e(t)\dot{e}^T(t) \\
&= e^T(t)\left[Ae(t) + f(y) - f(x(t-\tau)) + ke(t)\right] \\
&\quad + \left[Ae(t) + f(y) - f(x(t-\tau)) + ke(t)\right]^T e(t) \\
&= e^T(t)\left[2kI + A + A^T\right]e(t) + x_3(t-\tau)e_1(t)e_2(t) - x_2(t-\tau)e_1(t)e_3(t) \\
&= e^T(t)\left[2kI + A + A^T\right]e(t) + L_{\max}(|e_1(t)||e_2(t)| + |e_1(t)||e_3(t)|) \\
&\leq e^T(t)\left[2kI + A + A^T + 2L_{\max}I + g_1 I\right]e(t) - g_1 e^T(t)e(t) \\
&\leq -g_1 V(e(t)).
\end{aligned} \tag{2.10}$$

Thus, we have

$$\dot{V}(e(t)) \leq -g_1 V(e(t)), \quad nT \leq t \leq nT + \sigma T. \tag{2.11}$$

And then

$$V(e(t)) \leq V(e(nT)) \exp\left(-g_1(t - nT)\right). \tag{2.12}$$

Similarly, when $nT + \sigma T \leq t \leq (n+1)T$, we have

$$\begin{aligned}
\dot{V}(e(t)) &= e^T(t)\dot{e}(t) + e(t)\dot{e}^T(t) \\
&= e^T(t)\left[Ae(t) + f(y) - f(x(t-\tau))\right] \\
&\quad + \left[Ae(t) + f(y) - f(x(t-\tau))\right]^T e(t) \\
&= e^T(t)\left[A + A^T\right]e(t) + x_3(t-\tau)e_1(t)e_2(t) - x_2(t-\tau)e_1(t)e_3(t) \\
&= e^T(t)\left[A + A^T\right]e(t) + L_{\max}(|e_1(t)||e_2(t)| + |e_1(t)||e_3(t)|) \\
&\leq e^T(t)\left[A + A^T + 2L_{\max}I - g_2 I\right]e(t) + g_2 e^T(t)e(t) \\
&\leq g_2 V(e(t)).
\end{aligned} \tag{2.13}$$

Therefore, we derive that when $nT + \sigma T \leq t \leq (n+1)T$,

$$\begin{gathered}
\dot{V}(e(t)) \leq g_2 V(e(t)), \\
V(e(t)) \leq V(e(nT + \sigma T)) \exp\left(g_2(t - nT - \sigma T)\right).
\end{gathered} \tag{2.14}$$

Hence,

$$\begin{aligned} V(e(t)) &\le V(e(nT))\exp\left(-g_1(t-nT)\right), \quad nT \le t \le nT+\sigma T, \\ V(e(t)) &\le V(e(nT+\sigma T))\exp\left(g_2(t-nT-\sigma T)\right), \quad nT+\sigma T \le t \le (n+1)T. \end{aligned} \tag{2.15}$$

Following the same line of argument of the proof of Theorem 1 of [20], we can get

$$V(e(t)) \le V(e(t_0))\exp\left(-\left(g_1\sigma - g_2(1-\sigma)\right)(t-\sigma T)\right). \tag{2.16}$$

Therefore, for any $t \ge 0$,

$$\|e(t)\|^2 = V(e(t)) \le V(e(t_0))\exp\left(-\left(g_1\sigma - g_2(1-\sigma)\right)(t-\sigma T)\right). \tag{2.17}$$

This implies that the origin of system (2.7) is globally exponentially stable and the following estimate holds:

$$\|e(t)\| = \|e(t_0)\|\exp\left(-\frac{1}{2}\left(g_1\sigma - g_2(1-\sigma)\right)(t-\sigma T)\right), \quad t \ge 0. \tag{2.18}$$

Hence, the two systems (2.2) and (2.4) are globally exponentially lag synchronized. The proof is thus completed. □

Remark 2.2. Let λ be the largest eigenvalue of $A + A^T$. If we replace the first two conditions in Theorem 2.1 by the scalar equalities $g_1^* = -2k - \lambda - 2L_{\max}$, $g_2^* = \lambda + 2L_{\max}$, where $g_1^* \ge g_1$ and $g_2^* \le g_2$, then, Theorem 2.1 also holds. Furthermore, Theorem 2.1 will reduce the following corollary.

Corollary 2.3. *If there exist constants k, $0 < \sigma < 1$, such that $g_1^*\sigma - g_2^*(1-\sigma) > 0$, where $g_1^* = -2k - \lambda - 2L_{\max}$, $g_2^* = \lambda + 2L_{\max}$, the lag synchronization error system* (2.7) *is globally exponentially stable and the lag synchronization between the systems* (2.2) *and* (2.4) *is achieved.*

3. Numerical Examples

In this section, we will present some numerical simulations for lag synchronization of the hyperchaotic system to verify and illustrate the effectiveness of the theoretical analysis in Section 3. In all these simulations, the constants are set to be $a = 10$, $b = 28$, $c = -8/3$, and $(d_1, d_2) = (-9.3, 1)$. From Corollary 2.3, one observes that the control strength k can be estimated as follows:

$$k < -\frac{(\lambda + 2L_{\max})}{2\sigma} < 0. \tag{3.1}$$

From (3.1), one then estimates the feasible region D of control parameters (k, σ), $D = \{(k,\sigma) \mid -(\lambda + 2L_{\max})/2\sigma < 0, 0 < \sigma < 1\}$.

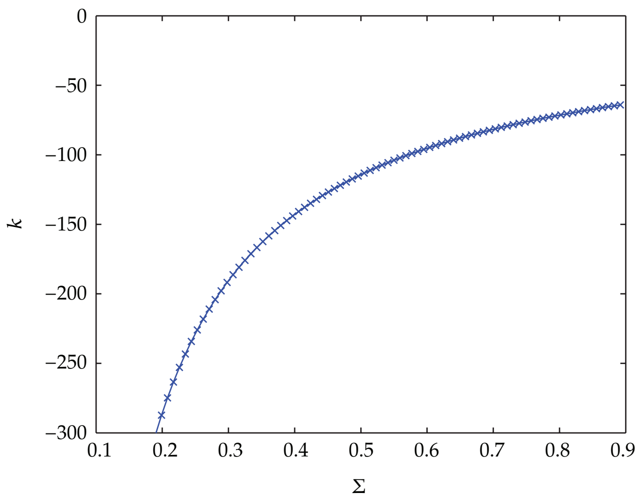

Figure 2: The feasible region D (the region below the curve) of the control parameters (k, σ).

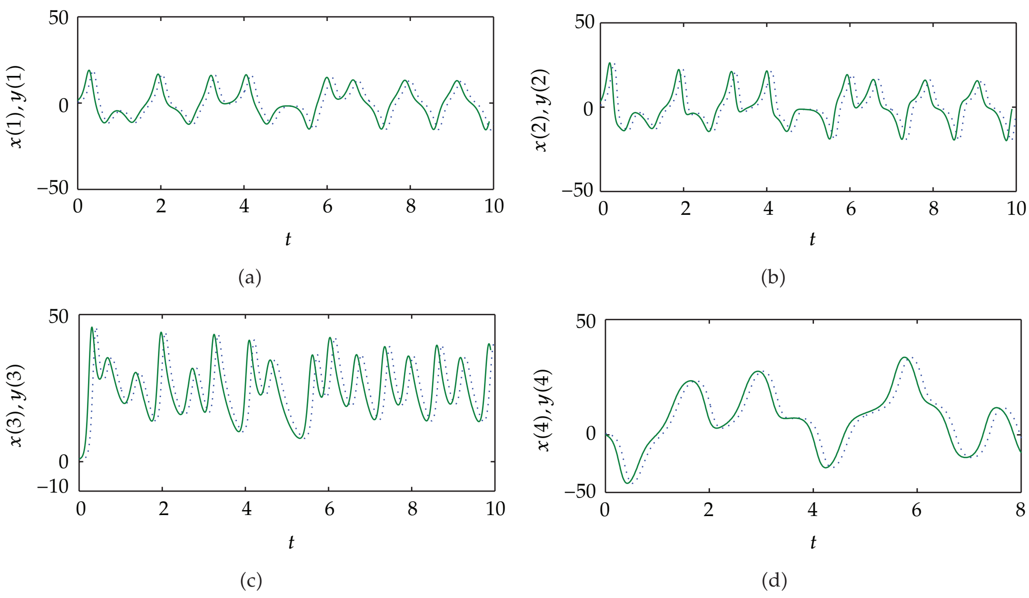

Figure 3: Dynamical behaviors of the hyperchaotic system with $\tau = 0.02$.

Based on the bound of the hyperchaotic attractor, we can choose $L_{\max} = 52$. Since the eigenvalues of the matrix $A+A^T$ are -33.5370, -5.3334, 1.2020, and 10.3350, $\lambda = \lambda_{\max}(A+A^T) = 10.3350$.

Therefore, the feasible region of control parameters (k, σ) is

$$D = \{(k, \sigma) \mid k < -57.1675/\sigma < 0, 0 < \sigma < 1\} \tag{3.2}$$

as shown in Figure 2.

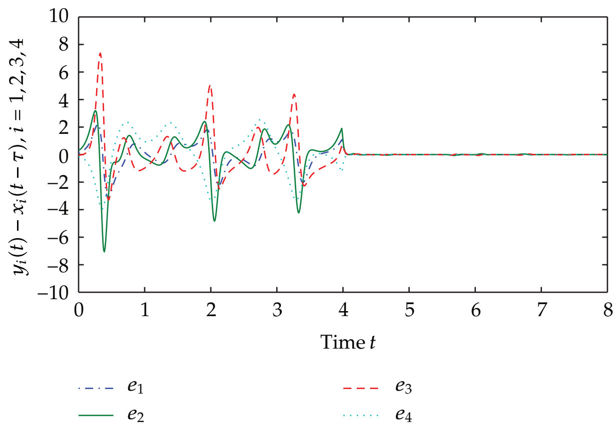

Figure 4: Lag synchronization error of the hyperchaotic system with $\tau = 0.02$.

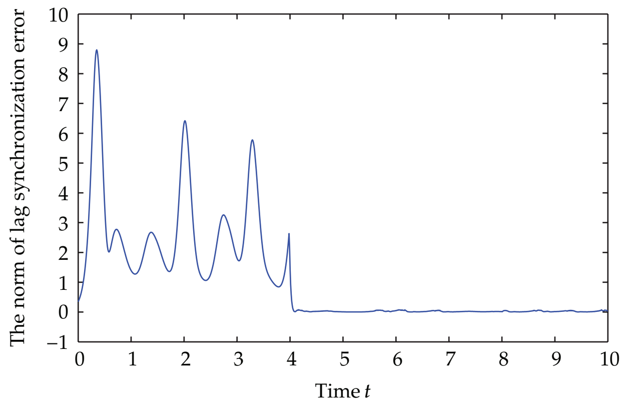

Figure 5: The norm of lag synchronization error of the hyperchaotic system with $\tau = 0.02$.

To satisfy the conditions in Theorem 2.1, we set $T = 4$, $\sigma = 0.8$, $\tau = 0.02$, and $k = -60$. Figure 3 shows dynamical behaviors of the hyperchaotic system (2.2) and its nonlinear observer (2.4). Figure 4 shows the time response curves of the lag synchronization error in the case of $\tau = 0.02$. Figure 5 shows the norm of lag synchronization error of the hyperchaotic system. From Figures 4 and 5, one can see that the state variables of the hyperchaotic systems achieve lag synchronization, which shows the correctness and effectiveness of our method via intermittent control.

4. Conclusions

In this paper, we have formulated the lag synchronization problem for hyperchaotic systems by means of periodically intermittent control and design a general periodically intermittent controller for hyperchaotic systems. Lag synchronization criteria are established based on the Lyapunov stability theory and linear matrix inequality techniques. Numerical simulations have showed the validity of theoretical result.

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