

Research Article

LMI-Based Approach for Exponential Robust Stability of High-Order Hopfield Neural Networks with Time-Varying Delays

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This paper studies the problems of global exponential robust stability of high-order hopfield neural networks with time-varying delays. By employing a new Lyapunov-Krasovskii functional and linear matrix inequality, some criteria of global exponential robust stability for the high-order neural networks are established, which are easily verifiable and have a wider adaptive.

1. Introduction

Hopfield neural networks (HNNs) with time delays and their various generalization have been successfully employed in many areas such as pattern recognition, associate memory, and combinatorial optimization (see [1–11]). Recently, the dynamics of high-order Hopfield neural networks (HOHNNs) with time delays have been considerable attention (see [12–18]), due to the neural networks have stronger approximation properties, faster convergence rate, greater storage capacity, and higher fault tolerance than lower order neural networks (see [12]). So, the stability of HOHNNs with time delays should be a focused topic of theoretical as wells as practical importance. This paper studies the problems of global exponential robust stability of high-order hopfield neural networks with time-varying delays. This paper is also an extension of our previous work [19]. By employing a new Lyapunov-Krasovskii functional and linear matrix inequality, some criteria of global exponential robust stability for the high-order neural networks are established, which are easily verifiable and have a wider adaptive.

2. Model Description and Preliminaries

We consider the following HOHNNs with time-varying delays:

$$\begin{aligned} \frac{du_i(t)}{dt} = & -a_i u_i(t) + \sum_{j=1}^n W_{ij} f_j(u_j(t)) + \sum_{j=1}^n T_{ij} g_j(u_j(t - \tau(t))) \\ & + \sum_{j=1}^n \sum_{l=1}^n T_{ijl} g_j(u_j(t - \tau(t))) g_l(u_l(t - \tau(t))) + V_i, \end{aligned} \quad (2.1)$$

$$u_i(t_0 + \theta) = \zeta_i(\theta), \quad -\tau_0 \leq \theta \leq 0, \quad i = 1, \dots, n,$$

$$0 \leq \tau(t) \leq \tau_0, \quad \underline{a}_i \leq a_i, \quad \underline{W}_{ij} \leq W_{ij} \leq \overline{W}_{ij}, \quad \underline{T}_{ij} \leq T_{ij} \leq \overline{T}_{ij},$$

$$\underline{T}_{ijl} \leq T_{ijl} \leq \overline{T}_{ijl}, \quad \underline{V}_i \leq V_i \leq \overline{V}_i, \quad i, j, l = 1, \dots, n,$$

where $u(t) = (u_1(t), \dots, u_n(t))^T \in R^n$, $a_i > 0$, W_{ij} , f_j , g_j , V_i have the same meanings as those in [13], T_{ij} , T_{ijl} are the first- and second-order synaptic weights of the system (2.1) (see [12]).

In this paper, the superscript “ T ” presents the transpose.

We assume throughout that the neuron activation functions $f_j(u_j)$, $g_j(u_j)$, $j = 1, \dots, n$, satisfy the following conditions:

$$\begin{aligned} (H_1) : |g_j(u_j)| \leq M_j, \quad 0 \leq \frac{|f_j(u_j) - f_j(v_j)|}{|u_j - v_j|} \leq l_{0j}, \quad 0 \leq \frac{|g_j(u_j) - g_j(v_j)|}{|u_j - v_j|} \leq l_{1j}, \\ \forall u_j \neq v_j, u_j, v_j, l_{0j}, l_{1j} \in R. \end{aligned} \quad (2.2)$$

From (2.2), we know

$$\begin{aligned} |f(u^1) - f(u^2)| & \leq |L_0(u^1 - u^2)|, \\ |g(u^1) - g(u^2)| & \leq |L_1(u^1 - u^2)|, \quad \forall u^1, u^2 \in R^n, \end{aligned} \quad (2.3)$$

where $u^i = [u_{i1}, \dots, u_{in}]^T \in R^n$, $f(u^i) = [f_1(u_{i1}), f_2(u_{i2}), \dots, f_n(u_{in})]^T$, $g(u^i) = [g_1(u_{i1}), g_2(u_{i2}), \dots, g_n(u_{in})]^T$, $i = 1, 2$. $L_0 = \text{diag}(l_{01}, l_{02}, \dots, l_{0n})$, and $L_1 = \text{diag}(l_{11}, l_{12}, \dots, l_{1n})$.

If there is an equilibrium point $u^* = [u_1^*, \dots, u_n^*]^T$ of system (2.1), we can rewrite system (2.1) as the following equivalent form:

$$\begin{aligned} \frac{d(u_i(t) - u_i^*)}{dt} = & -a_i(u_i(t) - u_i^*) + \sum_{j=1}^n W_{ij} (f_j(u_j(t)) - f_j(u_j^*)) \\ & + \sum_{j=1}^n \left(T_{ij} + \sum_{l=1}^n (T_{ijl} + T_{ilj}) \zeta_l \right) (g_j(u_j(t - \tau(t))) - g_j(u_j^*)) \end{aligned}$$

$$\begin{aligned}
u_i(t_0 + \theta) &= \xi_i(\theta), \quad -\tau_0 \leq \theta \leq 0, \quad i = 1, \dots, n, \\
0 \leq \tau(t) &\leq \tau_0, \quad \underline{a}_i \leq a_i, \quad \underline{W}_{ij} \leq W_{ij} \leq \overline{W}_{ij}, \quad \underline{T}_{ij} \leq T_{ij} \leq \overline{T}_{ij}, \\
\underline{T}_{ijl} &\leq T_{ijl} \leq \overline{T}_{ijl}, \quad i, j, l = 1, \dots, n,
\end{aligned} \tag{2.4}$$

where $\zeta_l = (1/2)(g_l(u_l(t - \tau(t))) + g_l(u_l^*))$ and $|\zeta_l| \leq M_l$.

We easily obtain that system (2.4) is equivalent to system (2.1) (see [13]).

The notations in this paper are quite standard:

(i)

$$\|u(t)\| = \left(\sum_{i=1}^n |u_i(t)|^2 \right)^{1/2}, \tag{2.5}$$

where $|u_i(t)|$ denotes Euclid's norm.

- (ii) $A = (a_{ij})_{n \times n} > 0 (< 0)$: a positive (negative) definite matrix, that is, $x^T A x > 0 (< 0)$ for any $x \in R^n$.
- (iii) $A = (a_{ij})_{n \times n} \geq 0$: a semipositive definite matrix, that is, $x^T A x \geq 0$ for any $x \in R^n$.
- (iv) $A \geq B$ (resp., $A > B$): this means $A - B$ is a semi-positive definite matrix (resp., positive definite).
- (v) I : identity matrix with compatible dimension.
- (vi) $I^\circ = (1, 1, \dots, 1)^T$.
- (vii) $\lambda_{\max}(A)$ (resp., $\lambda_{\min}(A)$) means the largest (resp., smallest) eigenvalue of the matrix A .
- (viii) $C([- \tau_0, 0], R^n)$ denotes a set of continuous functions.

Let $u(t, \xi) = [u_1(t, \xi), \dots, u_n(t, \xi)]^T$ denote the solution $u(t, \xi)$ to system (2.1) from the initial data $u(t_0 + \theta, \xi) = \xi(\theta)$ on $-\tau_0 \leq \theta \leq 0$ in $\xi(\theta) \in C([- \tau_0, 0], R^n)$.

Definition 2.1. The equilibrium point u^* of system (2.1) is called globally exponentially robustly stable on norm $\|\cdot\|$, if for any $\xi(\theta) \in C([- \tau_0, 0], R^n)$ there exist scalars, $J > 0$ and $\alpha > 0$ such that the solution $u(t, \xi)$ to system (2.1) with the initial condition $u(t_0 + \theta, \xi) = \xi(\theta)$ on $-\tau_0 \leq \theta \leq 0$ satisfies

$$\|u(t, \xi) - u^*\|^2 \leq J e^{-\alpha t} \sup_{-\tau_0 \leq \theta \leq 0} \|\xi(\theta) - u^*\|^2. \tag{2.6}$$

Lemma 2.2 (see [20]). *The LMI*

$$\begin{bmatrix} Q(t) & S(t) \\ S^T(t) & R(t) \end{bmatrix} > 0, \tag{2.7}$$

where $Q(t) = Q^T(t)$, $R(t) = R^T(t)$, and $S(t)$ depend on t , is equivalent to any one of the following conditions:

$$\begin{aligned} (K_1), R(t) > 0, \quad Q(t) - S(t)R^{-1}(t)S^T(t) > 0; \\ (K_2), Q(t) > 0, \quad R(t) - S^T(t)Q^{-1}(t)S(t) > 0. \end{aligned} \quad (2.8)$$

Lemma 2.3 (see [20]). Let $x \in R^n$, $y \in R^n$, and $\varepsilon > 0$. Then, one has

$$x^T y + y^T x \leq \varepsilon x^T x + \varepsilon^{-1} y^T y. \quad (2.9)$$

3. Main Results

Let

$$\begin{aligned} M &= [M_1, M_2, \dots, M_n]^T, \quad \Gamma = \text{diag}(M, M, \dots, M), \quad \zeta = [\zeta_1, \zeta_2, \dots, \zeta_n]^T, \\ \Pi &= \text{diag}(\zeta, \zeta, \dots, \zeta), \quad \underline{A} = \text{diag}(\underline{a}_i), \quad W = (W_{ij})_{n \times n'}, \quad W^+ = (W_{ij}^+)_{n \times n'}, \\ \bar{W} &= \left(\max\{|\underline{W}_{ij}|, |\bar{W}_{ij}|\} \right)_{n \times n'}, \quad \bar{W}^T = \left(\max\{|\underline{W}_{ji}|, |\bar{W}_{ji}|\} \right)_{n \times n'}, \\ T &= (T_{ij})_{n \times n'}, \quad T^+ = (T_{ij}^+)_{n \times n'}, \\ \bar{T} &= \left(\max\{|\underline{T}_{ij}|, |\bar{T}_{ij}|\} \right)_{n \times n'}, \quad \bar{T}^T = \left(\max\{|\underline{T}_{ji}|, |\bar{T}_{ji}|\} \right)_{n \times n'}, \\ T_i &= (T_{ij})_{n \times n'}, \quad T_i^T = (T_{ij}^T)_{n \times n'}, \quad T_i^+ = (T_{ij}^+)_{n \times n'}, \\ \bar{T}_i &= \left(\max\{|\underline{T}_{ijl}|, |\bar{T}_{ijl}|\} \right)_{n \times n'}, \quad \bar{T}_i^T = \left(\max\{|\underline{T}_{ilj}|, |\bar{T}_{ilj}|\} \right)_{n \times n'}, \\ T_H &= \left(T_1 + T_1^T, T_2 + T_2^T, \dots, T_n + T_n^T \right)_{n^2 \times n'}^T, \\ T_H^+ &= \left(T_1^+ + (T_1^T)^+, T_2^+ + (T_2^T)^+, \dots, T_n^+ + (T_n^T)^+ \right)_{n^2 \times n'}^T, \\ \bar{T}_H &= \left(\bar{T}_1 + \bar{T}_1^T, \bar{T}_2 + \bar{T}_2^T, \dots, \bar{T}_n + \bar{T}_n^T \right)_{n^2 \times n'}^T, \\ W_\Pi &= T + \Pi^T T_H, \quad \bar{W}_\Pi = \bar{T} + \Pi^T \bar{T}_H. \end{aligned} \quad (3.1)$$

For the purpose of simplicity, we rewrite the system (2.4) as the following vector form:

$$\begin{aligned} \frac{dy}{dt} &= -Ay(t) + WF_0(y(t)) + \left(T + \Pi^T T_H \right) F_1(y(t - \tau(t))) \\ y(t_0 + \theta) &= \varphi(\theta), \quad -\tau_0 \leq \theta \leq 0, \quad 0 \leq \tau(t) \leq \tau_0, \\ \underline{a}_i &\leq a_i, \quad \underline{W}_{ij} \leq W_{ij} \leq \bar{W}_{ij}, \quad \underline{T}_{ij} \leq T_{ij} \leq \bar{T}_{ij}, \\ \underline{T}_{ijl} &\leq T_{ijl} \leq \bar{T}_{ijl}, \quad i, j, l = 1, \dots, n, \end{aligned} \quad (3.2)$$

where $y = y(t) = u(t, \xi(\theta)) - u^*$ for any $\xi(\theta) \in C([- \tau_0, 0], R^n)$, $F_0(y(t)) = f(y(t) + u^*) - f(u^*)$, $f(y(t) + u^*) = [f_1(y_1(t) + u_1^*), f_2(y_2(t) + u_2^*), \dots, f_n(y_n(t) + u_n^*)]^T$, $F_1(y(t - \tau(t))) = g(y(t - \tau(t)) + u^*) - g(u^*)$, $g(y(t - \tau(t)) + u^*) = [g_1(y_1(t - \tau(t)) + u_1^*), g_2(y_2(t - \tau(t)) + u_2^*), \dots, g_n(y_n(t - \tau(t)) + u_n^*)]^T$, $\varphi(\theta) = \xi(\theta) - u^*$, $\xi(\theta) = (\xi_1(\theta), \dots, \xi_n(\theta))^T$.

Theorem 3.1. *Given a positive definite matrix $Q = L_1^T L_1 > 0$. The equilibrium of system (2.1) is globally exponentially robustly stable on norm $\| \cdot \|$ for any $\tau(t)$ satisfying $\dot{\tau}(t) \leq \eta < 1$, if system (2.1) satisfies (H_1) and*

$$(H_2) : C = A - W^+ L_0 - W_\Gamma^+ L_1 \tag{3.3}$$

is M-matrix,

where $W_\Gamma^+ = T^+ + \Gamma^T T_H^+$, and

$$(H_3) : \begin{bmatrix} \Lambda & L_0 & \beta \bar{W}^T & \beta \bar{W}_\Pi^T \\ L_0^T & -I & 0 & 0 \\ \beta \bar{W} & 0 & -I & 0 \\ \beta \bar{W}_\Pi & 0 & 0 & -(1 - \eta)I \end{bmatrix} \leq 0, \tag{3.4}$$

where $\bar{W}_\Pi = \bar{T} + \Pi^T \bar{T}_H$, and $\Lambda = \beta(\alpha I - 2\underline{A}) + e^{\alpha \tau_0} Q$, $\beta > 0$.

Proof of Theorem 3.1. Let

$$h_i(u_i, V_i) = a_i u_i - \sum_{j=1}^n W_{ij} f_j(u_j) - \sum_{j=1}^n T_{ij} g_j(u_j) - \sum_{j=1}^n \sum_{l=1}^n T_{ijl} g_j(u_j) g_l(u_l) + V_i = 0, \quad i = 1, \dots, n. \tag{3.5}$$

It is obvious that the solutions to (3.5) are the equilibrium points of system (2.1).

Let us define homotopic mapping as follows:

$$H(u, \lambda) = (H_1(u_1, \lambda), \dots, H_n(u_n, \lambda))^T, \tag{3.6}$$

where

$$H_i(u_i, \lambda) = \lambda h_i(u_i, V_i) + (1 - \lambda) u_i, \quad \lambda \in [0, 1]. \tag{3.7}$$

□

By homotopy invariance theorem (see, [21]), topological degree theory (see, [22]), (H_1) - (H_2) , and the proof which is similar to Theorem 3.1 in [23], we can conclude that (3.5) has at least one solution. That is, system (2.1) has at least an equilibrium point.

3.1. Part 2- Globally Exponentially Stable

Define a Lyapunov-Krasovskii functional candidate by

$$V(y(t)) = \beta e^{\alpha t} y^T(t) y(t) + e^{\alpha \tau_0} \int_{t-\tau(t)}^t e^{\alpha \theta} y^T(\theta) Q y(\theta) d\theta > 0, \quad (3.8)$$

where $Q = Q^T > 0$.

From system (3.2), its *Dini* derivative can be calculated as

$$\begin{aligned} D^+V(y(t)) &= e^{\alpha t} \left[\alpha \beta y^T(t) y(t) + y^T(t) \left(-\beta A^T - \beta A \right) y(t) \right. \\ &\quad + \beta \left(F_0^T(y(t)) W^T y(t) + y^T(t) W F_0(y(t)) \right) \\ &\quad + \beta F_1^T(y(t-\tau(t))) \left(T + \Pi^T T_H \right)^T y(t) \\ &\quad \left. + y^T(t) \left(T + \Pi^T T_H \right) F_1(y(t-\tau(t))) \right] \\ &\quad + e^{\alpha \tau_0} \left[e^{\alpha t} y^T(t) Q y(t) - e^{\alpha(t-\tau(t))} y^T(t-\tau(t)) Q y(t-\tau(t)) (1-\dot{\tau}(t)) \right]. \end{aligned} \quad (3.9)$$

Since $0 \leq \tau(t) \leq \tau_0$, $\dot{\tau}(t) \leq \eta < 1$, and $Q = L_1^T L_1 > 0$, we have

$$-e^{\alpha \tau_0} e^{\alpha(t-\tau(t))} y^T(t-\tau(t)) Q y(t-\tau(t)) (1-\dot{\tau}(t)) \leq -e^{\alpha t} y^T(t-\tau(t)) Q y(t-\tau(t)) (1-\eta), \quad (3.10)$$

From Lemma 2.3 and (2.3), we have

$$\begin{aligned} &e^{\alpha t} \beta \left(F_0^T(y(t)) W^T y(t) + y^T(t) W F_0(y(t)) \right) \\ &\leq e^{\alpha t} \beta \left(\beta y^T(t) W W^T y(t) + \frac{1}{\beta} F_0^T(y(t)) F_0(y(t)) \right) \\ &\leq e^{\alpha t} y^T(t) \left(\beta^2 W W^T + L_0^T L_0 \right) y(t) \leq e^{\alpha t} y^T(t) \left(\beta^2 \bar{W} \bar{W}^T + L_0^T L_0 \right) y(t), \\ &e^{\alpha t} \beta \left(F_1^T(y(t-\tau(t))) \left(T + \Pi^T T_H \right)^T y(t) + y^T(t) \left(T + \Pi^T T_H \right) F_1(y(t-\tau(t))) \right) \\ &\leq e^{\alpha t} \beta \left(\left(\frac{1-\eta}{\beta} \right)^{-1} y^T(t) W_\Pi W_\Pi^T y(t) + \frac{1-\eta}{\beta} F_1^T(y(t-\tau(t))) F_1(y(t-\tau(t))) \right) \\ &\leq e^{\alpha t} \left(\frac{\beta^2}{1-\eta} y^T(t) W_\Pi W_\Pi^T y(t) + (1-\eta) \left(y^T(t-\tau(t)) Q y(t-\tau(t)) \right) \right) \\ &\leq e^{\alpha t} \left(\frac{\beta^2}{1-\eta} y^T(t) \bar{W}_\Pi \bar{W}_\Pi^T y(t) + (1-\eta) \left(y^T(t-\tau(t)) Q y(t-\tau(t)) \right) \right). \end{aligned} \quad (3.11)$$

In view of $Q > 0$, $\dot{\tau}(t) \leq \eta < 1$, (3.8)–(3.11), and (H_3) , it follows from Lemma 2.2 that

$$\begin{aligned} D^+V(\mathbf{y}(t)) &\leq e^{\alpha t} \mathbf{y}^T(t) \left(\beta(\alpha I - 2A) + e^{\alpha \tau_0} Q \right. \\ &\quad \left. + \beta^2 \left(\overline{W} \overline{W}^T + \frac{1}{1-\eta} \overline{W}_\Pi \overline{W}_\Pi^T \right) + L_0^T L_0 \right) \mathbf{y}(t) \leq 0. \end{aligned} \quad (3.12)$$

From (3.12), we have

$$V(\mathbf{y}(t)) \leq V(\mathbf{y}(t_0)). \quad (3.13)$$

From (3.9) and (3.13), we can know that the solution $\mathbf{y}(t)$ to system (3.2) satisfies

$$\begin{aligned} e^{\alpha t} \beta \|\mathbf{y}(t)\|^2 &= e^{\alpha t} \left(\beta \mathbf{y}^T(t) \mathbf{y}(t) \right) \leq V(\mathbf{y}(t)) \leq V(\mathbf{y}(t_0)) \\ &\leq \left(\beta + \frac{1}{\alpha} (e^{\alpha \tau_0} - 1) \lambda_{\max}(Q) \right) \sup_{-\tau_0 \leq \theta \leq 0} \|\varphi(\theta)\|. \end{aligned} \quad (3.14)$$

So,

$$\|u(t, \xi) - u^*\|^2 \leq J e^{-\alpha t} \sup_{-\tau_0 \leq \theta \leq 0} \|\xi(\theta) - u^*\|^2, \quad (3.15)$$

where $J = 1 + 1/\alpha \beta (e^{\alpha \tau_0} - 1) \lambda_{\max}(Q)$, for any $\xi(\theta) \in C([- \tau_0, 0], R^n)$.

If there exists another equilibrium $u^{**} = [u_1^{**}, u_2^{**}, \dots, u_n^{**}]^T$ of system (2.1), we have $|u_i^* - u_i^{**}| \leq |u_i(t, \xi) - u_i^*| + |u_i(t, \xi) - u_i^{**}| \rightarrow 0$, $t \rightarrow \infty$, $i = 1, \dots, n$.

From the above proof, the system (2.1) has a unique equilibrium point u^* , which is globally exponentially robustly stable. Theorem 3.1 is proved.

Remark 3.2. When $a_i = \underline{a}_i > 0$, $\underline{b}_{ij} = b_{ij} = \overline{b}_{ij}$, $\underline{w}_{ij} = w_{ij} = \overline{w}_{ij}$, $\underline{w}_{ijl}^{(k)} = w_{ijl}^{(k)} = \overline{w}_{ijl}^{(k)}$, $\underline{V}_i = V_i = \overline{V}_i$, the system (2.1) becomes to system in [12, 14]. So, the system in [12, 14] is special case of system (2.1).

4. Conclusion

We have investigated global exponential robust stability of high-order hopfield neural networks with time-varying delays. By employing a new Lyapunov-Krasovskii functional and linear matrix inequality, some criteria of global exponential robust stability for the neural networks are established, which are easily verifiable. The systems found in the literature are special cases of the system (2.1). So the problem addressed in this paper should be a focused topic of theoretical as well as practical importance.

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