Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2012, Article ID 171406, 12 pages
doi:10.1155/2012/171406

Research Article

The Research of the Fractal Nature between Costs and Efficacy in the Brain Vascular Disease

Shuo Sun and Xiaocui Xing

Department of Mathematics, Tianjin Polytechnic University, Tianjin 300387, China

Correspondence should be addressed to Shuo Sun, sunshuo@tjpu.edu.cn

Received 16 February 2012; Accepted 25 February 2012

Academic Editor: Rudong Chen

Copyright © 2012 S. Sun and X. Xing. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Three hundred patients were randomly selected as the research object, of which 150 were on Chinese medical treatment; rehabilitation of 150 people was recorded for each patient before and after treatment by the three outcome measures (Fugle-Meyer baseline, NHISS baseline, and baseline BI), coupled with the treatment process spent in a variety of costs (mainly medicine costs, medicine,laboratory test, treatment, bed, care, diagnosis and examination fees, inspection fees). By combining the meaning of data with its practicality we get the definition of the efficacy. Via using the softwares of Excel, Matlab, and Eviews for data processing and fitting, it can be found that there exsists the fractal nature between efficacy and cost of treatment during the cerebral diseases. Then combined with the fractal theory, the application of chaotic time series, and two Fractal Indexes, the largest Lyapunov exponent and correlation dimension were extracted under two conditions of Chinese medical treatment and rehabilitation, and in the comparison of significance the brain vascular disease in traditional Chinese medicine treatment and rehabilitation was found. There were significant difference in fractal indicators of the time series of effective unit cost. At the same time, there were similar significant differences in the three outcome measures. This paper studied the fractal nature of cerebrovascular disease between the efficacy and cost and draw some fractal relationships and conclusions, so as to find better medical treatment to provide a theoretical basis for the hope of the treatment of cerebral vascular disease to provide some valuable reference.

1. Introduction and Preliminary Knowledge

The relationship between the cost of treatment and the curative effect is a problem that concern the patient and the hospital both; we select 300 patients randomly as the research object in a hospital, of which 150 were Chinese medical treatment; rehabilitation of 150 people was recorded for each patient before and after treatment by the three outcome measures (Fugle-Meyer baseline, NHISS baseline and baseline BI) by the three outcome measures (Fugle-Meyer baseline, NHISS baseline, and baseline BI) coupled with the treatment process spent in a variety of costs (mainly medicine costs, medicine,laboratory test, treatment, bed, care, Then combined with the fractal theory, the application of chaotic time series, two Fractal Indexes, the largest Lyapunov exponent and correlation dimension were extracted from under two conditions of Chinese medical treatment and rehabilitation, and in the comparison of significance, the brain Vascular disease in traditional Chinese medicine treatment and rehabilitation was found. There were significant difference in fractal indicators of the time series of unit cost effective. At the same time, there were similar significant differences in the three outcome measures. And then we got the fractal nature of cerebrovascular disease between the efficacy and cost and draw some fractal relationships and conclusions, so as to find better medical treatment to provide a theoretical basis for the hope of the treatment of cerebral vascular disease to provide some valuable reference.

1.1. Lyapunov Exponent

The basic characteristics of chaotic system, that is the system of the initial value, is extreme sensitivity; two initial value with a little difference produced a track, separated with the passage of time as the index way. Lyapunov exponent [1–3] is the volume quantitative description of the amount of this phenomenon.

Lyapunov exponent is an important quantitative index of a measurement system of dynamic characteristics; it represents the system in the phase space of convergence divergence between adjacent orbit or the average index rate, depicting the dissipative system of space phase volume contraction in the process of the geometric feature of change.

Usually all of the Lyapunov exponent spectrum ranging is in size for $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$; in the Lyapunov exponent spectrum, the smallest Lyapunov exponent decided the track of the speed of the contract, and the biggest Lyapunov exponent decided the track of the covers of whole spread of the speed of the attractor. And there generally is a sense in which the sum for all of the index $\sum_{i=1}^{n} \lambda_i$, is characterized the average speed of the rail line spread. Because the time series is based on the largest Lyapunov exponent λ_1, so we only need to find out λ_1 to quantitative characterization of phase space trajectories of to quantitatively characterize the divergence problems of two adjacent trajectories of phase space. We use λ_1 as the description of movement randomness or uncertainty of the quantitative. So,

$$\lambda_1 + \lambda_2 + \cdots + \lambda_k = \lim_{n \to \infty} \frac{1}{t_n - t_0} \sum_{i=0}^{n-1} \ln \frac{A'_k(t_i)}{A_k(t_i)}, \quad k = 1, 2, \ldots, m. \tag{1.1}$$

can be simplified into

$$\lambda_1 = \frac{1}{t_m - t_0} \sum_{k=1}^{M} \log_2 \frac{L(t_k)}{L(t_{k-1})}. \tag{1.2}$$

In which, $L(t_k)$ express t_k embedded space between the original point of the nearest point and the original point of the Euclidean distance, that is to say, given the embedded space time of an original point, using exhaustive method to find the point in the Euclidean distance of the nearest, the distance is $L(t_0)$, in the next moment t_1. The scales will evolve into $L(t_1)$, continue until all the data is received, then find out the estimated value of point λ_1.

(1) If $\lambda_1 < 0$, it means that adjacent points ultimately to be close to a point, corresponding to the stability of the fixed point and periodic motion.

(2) If $\lambda_1 > 0$, it means that adjacent point is ultimately to be separated; this corresponds to the track of the local instability, namely, in the system in phase space; regardless of the initial two track spacing being so small, the difference will cannot be predicted, because it increase at the rate of exponential with time evolving, if the track has the overall stability factors (such as global bounded, dissipation, presence of capture area), then chaotic attractors can be attained by folding and formation of chaotic attractors.

(3) The greater the index is, the more obvious the chaos characteristics is, the higher the degree of chaos is.

At present, the common calculation chaotic sequence of the largest Lyapunov exponent, Wolf method, and a small amount of data that is most widely used are the most common, and are given below in a one-dimensional data extracted from the Wolf Lyapunov index method:

(1) Applicate the time sequence to select delay time τ, according to the observation data of the total number of samples N, and structure a new sequence of n dimensional phase space; the Phase number is $m = N - (n-1)\tau$. Let chaos time sequence be $\{x_1, x_2, \ldots, x_N\}$, and use delay coordinates to phase space reconstruction

$$x = \left\{x_i \mid x_i = \left[x_i, x_{i+\tau}, x_{i+2\tau}, \ldots, x_{i+(m-1)\tau}\right]T, i = 1, 2, \ldots, m\right\}. \tag{1.3}$$

(2) In the initial phase point X_0 for basis points, In point set $\{X_i\}$ of the remaining phase point selection and X_0 nearest point X_i to the endpoint, constitute an initial vector, $X_0 \sim X_j$; Euclidean distance can be denoted by $L(t_0)$.

(3) Take time step as $k, t_1 = t_0 + k$, along the rail line initial vector forward evolution, get new vector corresponding basis points and endpoints of the new vector can be denoted as $L(t_1)$; in the corresponding period, the exponential growth rate of the system line index is recorded as

$$\lambda_1 = \frac{1}{k} \log 2 \left[\frac{L(t_1)}{L(t_0)}\right]. \tag{1.4}$$

(4) This continues until all phase, then the exponential growth rate average value of the largest Lyapunov exponent estimation.

(5) In order to increase the embedding dimension N, repeat (2), (3), (4) processes until the Lyapunov index estimation value with n changes and becomes more smoothly; so far, the obtained results are that for the largest Lyapunov exponent value.

1.2. Correlation Dimension

Correlation dimension [4] is also called relevant dimension, describes chaotic freedom information, and is a measure chaotic dynamics strange attractor in a way. In nonlinear dynamic system, high-dimension mathematical model is hard to set up and can use the time series fractal dimension to the system dynamics properties that were studied. Using time series fractal dimension figure can distinguish nonlinear system from the uncertain state to a fork or the dimension of the nonlinear system phase space may very high, even infinite, and dimension of the attractor is generally less than the dimension of the phase space.

We start from an interval of a single-variable time series $x_1, x_2, x_3 \ldots$ and structure a group of n-dimensional vector structure supporting an embedding space. As long as the embedding dimension is high enough (typically requires $n \geq 2D + 1$, D as attractor dimension), it can in the sense of topological equivalent restore the original dynamics state. For n-dimensional reconstruction of chaotic dynamical systems, there is strange attractor from point $y_j = (x_j, x_{j+\tau}, x_{j+2\tau}, \ldots, x_{j+(n-1)\tau})$. In the constructed vector y_j, first we need to define the distance between them. Because if they satisfy the axiom of distance that can be defined, it may be the largest component difference of the two vectors that act as the distance that is,

$$\left|y_i - y_j\right| = \max_{1\leq k\leq n}\left|y_{i_k} - y_{j_k}\right|. \tag{1.5}$$

For regulations, any distance less than a given value vector is called the associated vector. Set in the phase space reconstruction an N point (i.e., vector); calculation of which is associated with the vector pairs, which in all possible N^2 pairs is in proportion to be called the correlation integral:

$$C_n(r) = \frac{1}{N^2}\sum_{i,j=1}^{N}\theta\left(r - \left|y_i - y_j\right|\right). \tag{1.6}$$

Among them, θ acts as the Heaviside unit function:

$$\theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases} \tag{1.7}$$

As we know, correlation integral $C_n(r)$ in $r \to 0$ and r has the following relationships:

$$\lim_{r\to 0} C(r) \propto r^D. \tag{1.8}$$

Among them, D is called correlation dimension, proper selection of r; the D was able to describe the chaotic attractor of the self-similar structure. Approximate numerical calculation formula is

$$D_{GP} = \frac{\ln C_n(r)}{\ln r}. \tag{1.9}$$

In actual calculation, usually given some specific value of r, if the r value is too small, it has been lower than the ambient noise and the measurement error caused by the vector difference, from the formula calculated not from correlation dimension but the embedding dimension. At the time, usually for n small increase, make D unchanged, namely, double logarithmic relationship in the $\ln C_n(r) \sim \ln r$ straight section. Remove slope of 0 or ∞ lines, which is the best fitting straight line, then the slope of the line is D.

In 1983, Grassberger and Procaccia, according to the embedding theorem and phase space reconstruction thought, put forward, from time series of direct calculation of the correlation dimension algorithm, the *G-P* algorithm.

The step of *G-P* algorithm [5]

(1) Using time series $x_1, x_2, \ldots, x_{n-1}, x_n, \ldots$, give a smaller value corresponding to a reconstructed phase space (selected τ and $m = m_0$):

$$Y(t_i) = [x(t_i), x(t_i+\tau), x(t_i+2\tau), \ldots, x(t_i+(m-1)\tau)] \quad (i = 1, 2, \ldots). \tag{1.10}$$

(2) From (1.6), Calculate Correlation function

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i,j=1}^{N} \theta\left(r - \left|Y(t_i) - Y(t_j)\right|\right),$$
$$\theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases} \tag{1.11}$$

(3) For a proper range of r, attract the son dimension d and $C(r)$ should satisfy the cumulative distribution function of the linear relationship between the logarithm, that is, $d(m) = \ln C(r)/\ln r$. Thus by fitting for m_0, it corresponds to the estimate of correlation dimensions of $d(m_0)$.

(4) Embedding dimension $m_1 > m_0$, repeat steps 2 and 3, until the corresponding dimension estimation value of $d(m)$ is no longer with the increase of m in a certain error range unchanged, then the d is obtained as attractor correlation dimension.

1.3. Phase Space Reconstruction

Chaotic time series and chaotic signal refers to a chaotic system that is sampled by the observation of a single variable time series. Chaotic time series analysis is based on the theory of the phase space reconstruction, and the existing observation sequence has access to the embedding dimension, Lyapunov index, attractor dimension invariant calculation method of the original attractor.

On the time delay and embedding dimension selection problem, this paper uses the autocorrelation coefficient $C(\tau) = (\sum_{i=1}^{n-r}(x_{i-r} - \overline{x})(x_i - \overline{x}))/\sum_{i=1}^{n-r}(x_i - \overline{x})^2$ first down to the initial value of $1/E$ for optimal delay time interval τ. On the subject of unit cost effectiveness,

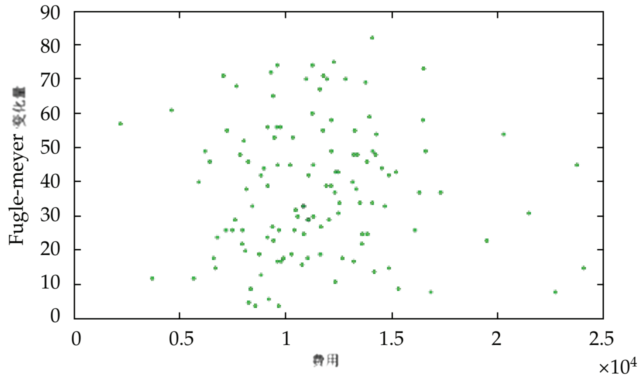

Figure 1: The scatter diagram of Fugle-Meyer baseline and total costs.

we use time series analysis; we found after the commissioning of the calculation results, $\tau = 2$. Select the appropriate value of τ, given the value of m, and draw the curve of $\ln(C(\tau))$-$\ln \tau$. Using linear regression method for processing, do not consider the τ minimum of noise and 2 maximum $\ln(C(\tau))$-$\ln \tau$ saturated zone. The middle line part of the slope is the correlation dimension D. Usually in order to get the appropriate m, we can appropriately increase it, when m increases, the general D also increases accordingly. When m increases to certain value $m_{\min}$, D is approximation to the limit. So $m_{\min}$ is accommodating the strange attractor minimum of phase space reconstruction dimension, namely, the embedding dimension m.

2. The Research of Chaos Time Sequence in Cerebral Vascular Disease Curative Effect and Cost Method

For the research of efficacy and cost of cerebral vascular disease, first, divide the Chinese medicine group and the rehabilitation group during treatment of recurrence, lost, stop, drop, and delete data, and treatment process in the fifth evaluation of the numerical and baseline numerical difference from baseline variation. The cost of total demand is the total cost; total cost divided by the actual length of stay is the average cost per day. Then, in order to reflect the effect of different duration of lower level, also eliminate flat affect. We define the efficacy for fifth evaluation of numerical and baseline numerical difference divided by the respective baseline levels, and a group of doctor of traditional Chinese medicine, and rehabilitation group in each baseline indicator of efficacy, and the same effect cost average consolidation. Secondly, in order to study the treatment of cerebral vascular disease in the course of efficacy and costs of the relationship, we will calculate the average cost divided by the corresponding effect to receive the unit cost effectiveness. Finally, we use Excel, Eviews, Matlab, and other software to do research on the efficacy and costs of the fractal nature.

2.1. Relationship between Baseline Variation and Total Cost

By using the Matlab to make the scatter diagram of total costs associated with the three baseline indicators of changes (only in the case of Fugle-Meyer saw in Figure 1), we can see that, there is no clear relationship between the total cost and the baseline change trends.

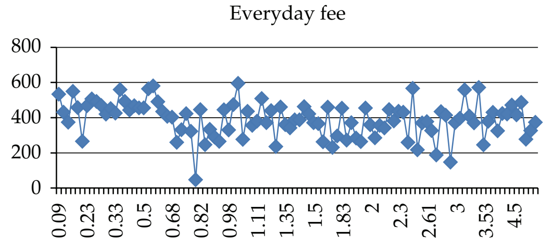

Figure 2: The line graph of Fugle-Meyer baseline effect and the average cost in Chinese medicine group.

Table 1: Efficacy and cost effectiveness of the fractal dimension of unit table.

	Chinese medicine group	The rehabilitation group
Fugle-Meyer Baseline	−1.0720549	−0.9573777263
NIHSS Baseline	−1.000721214	−0.8529432974
BI Baseline	−1.022924504	−0.942960312
Average value	−1.0319	−0.91776

2.2. Relationship between the Curative Effect and the Average Cost

The total cost divided by the actual length of stay is the average cost per day (y). Again the baseline variation divided by their respective baseline levels of worth is the normalized data without the unit influence—curative effect (x), and then make a line chart (Only with the Chinese group Fugle-Meyer baseline as an example) between y and x, we can get the Figure 2.

From the line graph, we can see that there is no deterministic function between efficacy and cost. Volatility is not regular, periodic unsteady, similar cloud boundary, peaks of the profile, the coastline, Brown movement, and so on. However, when we use a magnifying glass to see the data, no matter how magnifying multiple changes are, the graphics are similar. For these data, using chaos and fractal to describe their nature sometimes is more accurately than classical Euclidean geometry of plane body function to describe.

2.3. Research of the Fractal Nature between Efficacy and Average Costs

We now define unit cost effectiveness for the average expenses divided by the corresponding effect, applying Eviews software to calculate the fitting equation of logarithm after the efficacy (x) and after taking logarithm unit cost effectiveness (y). As can be seen in double logarithmic coordinates, efficacy and effectiveness units costs approximate linear nature and fitting test results are compared with the ideal; therefore, we believe that it have fractal properties [5–8] between the efficacy and unit cost effectiveness, its linear slope of fractal dimension (Table 1).

From this we can see that there are fractal properties between Curative effect and the unit cost effect, and there exist significant differences Chinese medicine treatment and rehabilitation of treatment between the fractal dimension. Chinese medicine group is the baseline fractal dimension of the average of −1.0319, and the rehabilitation group is the baseline fractal dimension of the average of −0.91776.

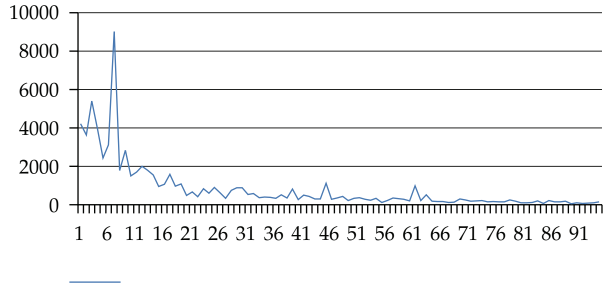

Figure 3: The time series of rehabilitation group Fugle-Meyer baseline unit cost effectiveness.

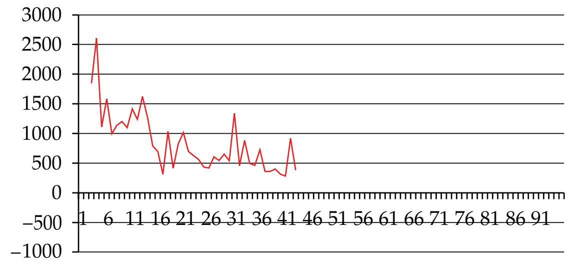

Figure 4: The time series of rehabilitation group NIHSS baseline unit cost effectiveness.

2.4. Judgment of Lyapunov Index for the Fractal Nature of Unit Cost Effectiveness

On the cerebrovascular disease unit cost effectiveness study, we assume that all subjects except the effects do not exist other differences, then throughout the treatment process, it can be seen that the same subjects' unit cost effectiveness varies with the time continuously after sorting all the experimenters' unit cost effectiveness. Therefore, it can be applied to chaotic time series on the group of doctor of traditional Chinese medicine and rehabilitation group efficacy and costs of the fractal nature of the discussion in order to find out the unit cost of the fractal dimension and the efficacy of different treatment methods on the unit cost effectiveness of the difference, so as to find a better medical treatment and provide a theoretical basis.

According to the chaotic time series theory, for the group of doctor of traditional Chinese medicine and rehabilitation group efficacy and cost composition system, one of the three indicators of efficacy and costs of evolution is determined by the interaction of the other component, and the information of the relative components is hidden in a component in the process of development. We are now through the study of unit cost effectiveness time series evolution development process (In the rehabilitation group an example is shown in Figures 3, 4, and 5) to explore the efficacy and cost between the fractal properties.

2.5. Comparison of Correlation Dimension for Different Treatment under the Unit Cost Effectiveness Fractal Properties

Using Matlab software to research programming for unit cost effectiveness time series fractal properties, after several debugging, we know that, embedding dimension is 2 or 4, the

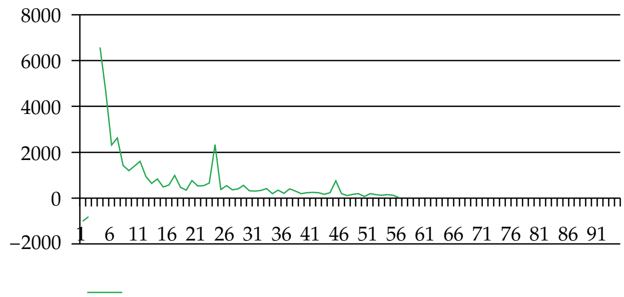

Figure 5: The time series of rehabilitation group BI baseline unit cost effectiveness.

Table 2: Lyapunov time series index table of Chinese medicine group for unit cost effectiveness.

Chinese medicine group	Fugle-Meyer baseline	NIHSS baseline	BI baseline
Embedding dimension	2	2	4
Lyapunov index	0.0615	0.1362	0.0442

Table 3: Lyapunov time series index table of the rehabilitation group for unit cost effectiveness.

The rehabilitation group	Fugle-Meyer baseline	NIHSS baseline	BI baseline
Embedding dimension	2	2	4
Lyapunov index	0.1372	0.6615	0.3019

phase space reconstruction is fitting better, respectively, for the Chinese medicine group and rehabilitation group, all indexes unit curative effect of time sequence Lyapunov cost index. so we see, Lyapunov index values are positive, so there is a fractal feature (Tables 2 and 3).

Applicate the *G-P* algorithm and use the Matlab software to extract traditional Chinese medicine and rehabilitation group unit cost effectiveness time series correlation dimension; we see there exist significant differences between Chinese medicine and rehabilitation group of unit cost effectiveness time series correlation dimension fitting line and the fitting line slope is the fractal dimension of size in the map; we know from the map, the fractal dimension is negative, and with different values of the larger changes step size (Figures 6–11).

3. Conclusion

In the process of cerebrovascular disease treated with traditional Chinese medicine and rehabilitation, there exited significant difference between efficacy and cost of the fractal properties presence.

For the study on the fractal property of the group of Chinese Medicine, we see that Fugle-Meyer baseline unit cost effectiveness on time series embedding dimension for $m = 2$, Lyapunov index is 0.0615; NIHSS baseline unit cost effectiveness on time series embedding dimension for $m = 2$, Lyapunov index is 0.1362; BI baseline unit cost effectiveness on time series embedding dimension for $m = 4$, Lyapunov index is 0.0442.

For the study on the fractal property of the rehabilitation group, we see that Fugle-Meyer baseline unit cost effectiveness on time series embedding dimension for $m = 2$, Lyapunov index is 0.1372; NIHSS baseline unit cost effectiveness on time series embedding

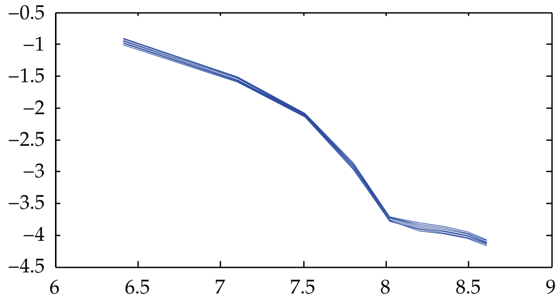

Figure 6: The fitting chart of Chinese medicine group for Fugle-Meyer baseline fractal $m = 2$.

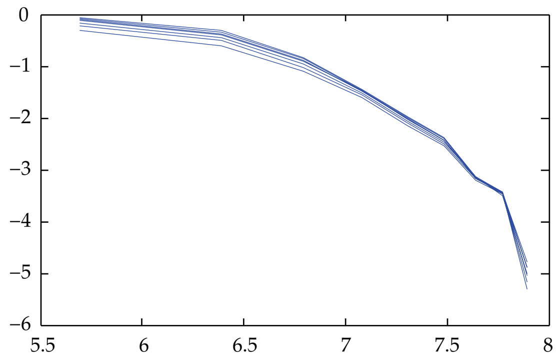

Figure 7: The fitting chart of Chinese medicine group for NIHSS baseline fractal $m = 2$.

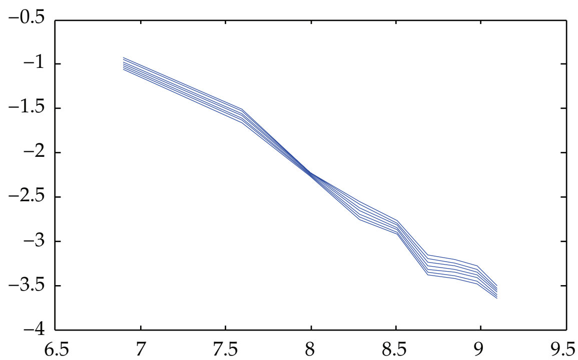

Figure 8: The fitting chart of Chinese medicine group for BI baseline fractal $m = 4$.

dimension for $m = 2$, Lyapunov index is 0.6615; BI baseline unit cost effectiveness on time series embedding dimension for $m = 4$, Lyapunov index is 0.3019.

By the graph line we fit out the fractal dimension, and the fractal dimension of either Chinese or rehabilitation group is a number between −2 to 0, which changes significantly with different step values. The selection of embedding dimension has been a rule between the three indicators in the reconstructed phase space, that either Chinese or rehabilitation group, whose Fugle-Meyer baseline unit cost effectiveness of time series and NHISS baseline unit cost effectiveness all of time series to the optimal selective embedding dimension is 2, while BI baseline unit cost effectiveness of time sequence to the optimal selective embedding

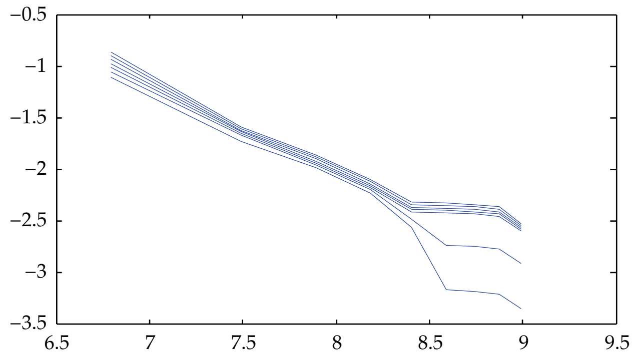

Figure 9: The fitting chart of rehabilitation group for Fugle-Meyer baseline fractal $m = 2$.

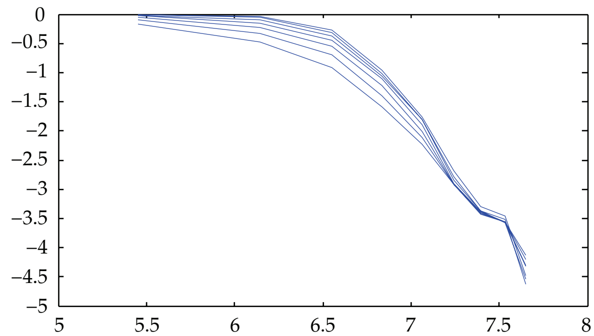

Figure 10: The fitting chart of rehabilitation group for NIHSS baseline fractal $m = 2$.

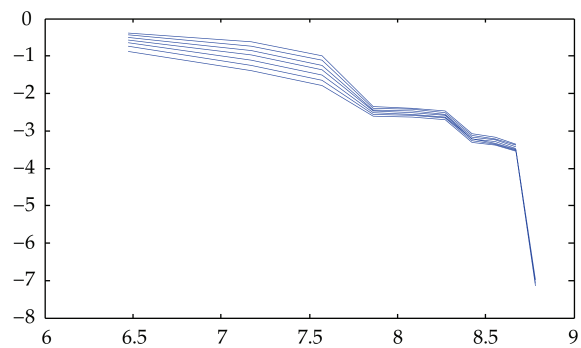

Figure 11: The fitting chart of rehabilitation group for BI baseline fractal $m = 4$.

dimension is 4. With different treatment methods, there exits similar significant difference between various primary outcomes.

References

[1] X. Li, T. Kang, H. Quan, and X. Tian, "Research based on the correlation dimension and maximum Lyapunov exponent motion HRV signal [J]," *Biomedical Engineering Research*, vol. 28, no. 3, pp. 188–192, 2009.

[2] J. Xia and D. Xiao, "Chaotic time series prediction model based on the Lyapunov index and CBP [J]," *Statistics and Decision*, vol. 19, no. 2, article 210044, 2007.
[3] B. Song and K. Zhang, Statistical theory and application for Chaotic time series and fractal.
[4] M. Zhon, Y. Long, Q. Xie, and X. Li, "Signal analysis based on the the fractal box counting dimension and multifractal blasting seismic wave," *Vibration and Shock*, vol. 29, no. 1, pp. 7–11, 2010.
[5] L. Ni, Z. Ni, H. Wu, and H. Ye, "Attribute selection method based on the fractal dimension and the ant colony algorithm," *Pattern Recognition and Artificial Intelligence*, vol. 22, no. 2, pp. 293–298, 2009.
[6] J. Lv, J. Lu et al., *Chaotic Time Series Analysis and Its Applications*, Wuhan University press.
[7] Z. Zhang, "Application of fractal theory in the field of biomedical," *Shanghai Medical University Department of Pathology of Tumors*, vol. 16, no. 2, p. 3, 1996.
[8] S. S. Cross and D. W. K. Cotton, "The fractal dimension may be a useful morphometric discriminant in histopathology," *Journal of Pathology*, vol. 166, no. 4, pp. 409–411, 1992.