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Research Article

Some Properties of Certain Integral Operators on New Subclasses of Analytic Functions with Complex Order

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We define new subclasses of p-valent meromorphic functions with complex order. We prove some properties for certain integral operators on these subclasses.

1. Introduction

Let $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc in the complex plane $\mathbb{C}$, $\mathbb{U}^* = \mathbb{U} \setminus \{0\}$, the punctured open unit disk. Let Σ_p denote the class of meromorphic functions of the form

$$f(z) = \frac{1}{z^p} + \sum_{n=0}^{\infty} a_n z^n, \quad (p \in \mathbb{N}), \tag{1.1}$$

which are analytic and p-valent in $\mathbb{U}^*$.

For $p = 1$, we obtain the class of meromorphic functions Σ.

We say that a function $f \in \Sigma_p$ is the meromorphic p-valent starlike of order α $(0 \leq \alpha < p)$ and belongs to the class $f \in \Sigma_p^*(\alpha)$, if it satisfies the inequality

$$-\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha. \tag{1.2}$$

A function $f \in \Sigma_p$ is the meromorphic p-valent convex function of order α $(0 \leq \alpha < p)$, if f satisfies the following inequality:

$$-\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, \tag{1.3}$$

and we denote this class by $\Sigma K_p(\alpha)$.

Many important properties and characteristics of various interesting subclasses of the class Σ_p of meromorphically p-valent functions were investigated extensively by (among others) Uralegaddi and Somanatha [1, 2], Liu and Srivastava [3, 4], Mogra [5, 6], Srivastava et a1. [7], Aouf et al. [8, 9], Joshi and Srivastava [10], Owa et a1. [11], and Kulkarni et al. [12].

Now, for $f \in \Sigma_p$, we define the following new subclasses.

Definition 1.1. Let a function $f \in \Sigma_p$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Sigma_{p\mathcal{M}}(\beta, b)$ if it satisfies the inequality

$$\Re\left\{p - \frac{1}{b}\left(\frac{zf'(z)}{f(z)} + p\right)\right\} < \beta, \tag{1.4}$$

where

$$p \in \mathbb{N}, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta > p. \tag{1.5}$$

Definition 1.2. Let a function $f \in \Sigma_p$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Sigma_{p\mathcal{N}}(\beta, b)$ if it satisfies the inequality

$$\Re\left\{p - \frac{1}{b}\left(\frac{zf''(z)}{f'(z)} + p + 1\right)\right\} < \beta, \tag{1.6}$$

where

$$p \in \mathbb{N}, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta > p. \tag{1.7}$$

Definition 1.3. Let a function $f \in \Sigma_p$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Sigma F_p(\beta, b)$ if it satisfies the inequality

$$\Re\left\{p - \frac{1}{b}\left(\frac{z(zf''(z) + (p+2)f'(z))}{zf'(z) + (p+1)f(z)} + p\right)\right\} < \beta, \tag{1.8}$$

where

$$p \in \mathbb{N}, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta > p. \tag{1.9}$$

For $p = 1$ in Definitions 1.1, 1.2, and 1.3, we obtain the following new subclasses of meromorphic functions Σ.

Definition 1.4. Let a function $f \in \Sigma$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Sigma_{\mathcal{M}}(\beta, b)$ if it satisfies the inequality

$$\Re\left\{1 - \frac{1}{b}\left(\frac{zf'(z)}{f(z)} + 1\right)\right\} < \beta, \tag{1.10}$$

where

$$b \in \mathbb{C} \setminus \{0\}, \quad \beta > 1. \tag{1.11}$$

Definition 1.5. Let a function $f \in \Sigma$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Sigma_{\mathcal{N}}(\beta, b)$ if it satisfies the inequality

$$\Re\left\{1 - \frac{1}{b}\left(\frac{zf''(z)}{f'(z)} + 2\right)\right\} < \beta, \tag{1.12}$$

where

$$b \in \mathbb{C} \setminus \{0\}, \quad \beta > 1. \tag{1.13}$$

For $b = 1$ in Definition 1.5, we obtain $\Sigma_N(\beta)$, the class of meromorphic function, introduced and studied by Wang et al. [13–15] (see [16–18]).

Definition 1.6. Let a function $f \in \Sigma$ be analytic in $\mathbb{U}^*$. Then f is in the class $\Sigma F(\beta, b)$ if it satisfies the inequality

$$\Re\left\{1 - \frac{1}{b}\left(\frac{z(zf''(z) + 3f'(z))}{zf'(z) + 2f(z)} + 1\right)\right\} < \beta, \tag{1.14}$$

where

$$b \in \mathbb{C} \setminus \{0\}, \quad \beta > 1. \tag{1.15}$$

Most recently, Mohammed and Darus [19] introduced the following two general integral operators of p-valent meromorphic functions Σ_p:

$$\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z) = \frac{1}{z^{p+1}} \int_0^z (u^p f_1(u))^{\gamma_1} \cdots (u^p f_n(u))^{\gamma_n} du; \tag{1.16}$$

$$\mathcal{J}_{p,\gamma_1,\ldots,\gamma_n}(z) = \frac{1}{z^{p+1}} \int_0^z \left(\frac{-u^{p+1}}{p} f_1'(u)\right)^{\gamma_1} \cdots \left(\frac{-u^{p+1}}{p} f_n'(u)\right)^{\gamma_n} du, \tag{1.17}$$

where

$$n, p \in \mathbb{N}, \quad j \in \{1, 2, 3, \ldots, n\}, \quad \gamma_j > 0. \tag{1.18}$$

For $p = 1$ in (1.16) and (1.17), respectively, we obtain the general integral operators $\mathcal{F}_{1,\gamma_1,\ldots,\gamma_n}(z) = \mathcal{H}(z)$ and $\mathcal{Q}_{1,\gamma_1,\ldots,\gamma_n}(z) = \mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$, introduced by the authors in [16, 20].

2. Main Results

In this section, considering the above new subclasses we obtain for the integral operators $F_{1,\gamma_1,\ldots,\gamma_n}(z)$ and $\mathcal{Q}_{1,\gamma_1,\ldots,\gamma_n}(z)$ some sufficient conditions for a family of functions f_i to be in the the above new subclasses.

Theorem 2.1. *Let $f_j \in \Sigma_p$. If $f_j \in \Sigma_{p\mathcal{M}}(\beta_j, b)$, then*

$$\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z) \in \Sigma F_p(\mu, b), \tag{2.1}$$

where

$$\begin{gathered} n, p \in \mathbb{N}, \quad j \in \{1, \ldots, n\}, \quad \gamma_j > 0, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta_j > p, \\ \mu = p + \sum_{j=1}^{n} \gamma_j (\beta_j - p). \end{gathered} \tag{2.2}$$

Proof. A differentiation of $\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)$ which is defined in (1.16), we get

$$\begin{aligned} z^{p+1}\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z) + (p+1)z^p\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z) &= \left(z^p f_1(z)\right)^{\gamma_1} \cdots \left(z^p f_n(z)\right)^{\gamma_n}, \\ z^{p+1}\mathcal{F}''_{p,\gamma_1,\ldots,\gamma_n}(z) + 2(p+1)z^p\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z) &+ p(p+1)z^{p-1}\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z) \\ = \sum_{j=1}^{n} \gamma_j \left(\frac{z^p f'_j(z) + p z^{p-1} f_j(z)}{z^p f_j(z)} \right) &\left[\left(z^p f_1(z)\right)^{\gamma_1} \cdots \left(z^p f_n(z)\right)^{\gamma_n} \right], \end{aligned} \tag{2.3}$$

Then from (2.3), we obtain

$$\begin{aligned} &\frac{z^{p+1}\mathcal{F}''_{p,\gamma_1,\ldots,\gamma_n}(z) + 2(p+1)z^p\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z) + p(p+1)z^{p-1}\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)}{z^{p+1}\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z) + (p+1)z^p\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)} \\ &= \sum_{j=1}^{n} \gamma_j \left(\frac{f'_j(z)}{f_j(z)} + \frac{p}{z} \right). \end{aligned} \tag{2.4}$$

By multiplying (2.4) with z yield,

$$\begin{aligned}&\frac{z^{p+1}\mathcal{F}''_{p,\gamma_1,\ldots,\gamma_n}(z)+2(p+1)z^{p}\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)+p(p+1)z^{p-1}\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)}{z^{p}\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)z^{p-1}\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)}\\&\quad=\sum_{j=1}^{n}\gamma_j\left(\frac{zf_j'(z)}{f_j(z)}+p\right).\end{aligned}\tag{2.5}$$

That is equivalent to

$$\begin{aligned}&\frac{z^{p+1}\mathcal{F}''_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+2)z^{p}\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)}{z^{p}\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)z^{p-1}\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)}+p=\sum_{j=1}^{n}\gamma_j\left(\frac{zf_j'(z)}{f_j(z)}+p\right);\\&\frac{z\left(z\mathcal{F}''_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+2)\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)}+p=\sum_{j=1}^{n}\gamma_j\left(\frac{zf_j'(z)}{f_j(z)}+p\right).\end{aligned}\tag{2.6}$$

Equivalently, the above can be written as

$$\begin{aligned}&p-\frac{1}{b}\left(\frac{z\left(z\mathcal{F}''_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+2)\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)}+p\right)\\&\quad=\sum_{j=1}^{n}\gamma_j\left\{p-\frac{1}{b}\left(\frac{zf_j'(z)}{f_j(z)}+p\right)\right\}+p-p\sum_{j=1}^{n}\gamma_j.\end{aligned}\tag{2.7}$$

Taking the real part of both terms of (2.7), we have

$$\begin{aligned}&\Re\left\{p-\frac{1}{b}\left(\frac{z\left(z\mathcal{F}''_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+2)\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)}+p\right)\right\}\\&\quad=\sum_{j=1}^{n}\gamma_j\Re\left\{p-\frac{1}{b}\left(\frac{zf_j'(z)}{f_j(z)}+p\right)\right\}+p-p\sum_{j=1}^{n}\gamma_j.\end{aligned}\tag{2.8}$$

Sine $f_j\in\Sigma_{p\mathcal{M}}(\beta_j,b)$, we get

$$\Re\left\{p-\frac{1}{b}\left(\frac{z\left(z\mathcal{F}''_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+2)\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)}+p\right)\right\}<\sum_{j=1}^{n}\gamma_j\beta_j+p-p\sum_{j=1}^{n}\gamma_j.\tag{2.9}$$

That is,

$$\Re\left\{p-\frac{1}{b}\left(\frac{z\left(z\mathcal{F}''_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+2)\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{F}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)}+p\right)\right\}<p+\sum_{j=1}^{n}\gamma_j(\beta_j-p). \tag{2.10}$$

Then

$$\mathcal{F}_{p,\gamma_1,\ldots,\gamma_n}(z)\in\Sigma F_p(\mu,b),\qquad \mu=p+\sum_{j=1}^{n}\gamma_j(\beta_j-p). \tag{2.11}$$

This completes the proof. □

Theorem 2.2. *Let $f_j\in\Sigma_p$. If $f_j\in\Sigma_{p\mathcal{N}}(\beta_j,b)$, then*

$$\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)\in\Sigma F_p(\mu,b), \tag{2.12}$$

where

$$\begin{gathered} n,p\in\mathbb{N},\quad j\in\{1,\ldots,n\},\quad \gamma_j>0,\quad b\in\mathbb{C}\setminus\{0\},\quad \beta_j>p,\\ \mu=p+\sum_{j=1}^{n}\gamma_j(\beta_j-p). \end{gathered} \tag{2.13}$$

Proof. A differentiation of $\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)$, which is defined in (1.17), we get

$$\begin{aligned} &z^{p+1}\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)z^p\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)=\left(\frac{-z^{p+1}}{p}f_1'(z)\right)^{\gamma_1}\cdots\left(\frac{-z^{p+1}}{p}f_n'(z)\right)^{\gamma_n},\\ &z^{p+1}\mathcal{Q}''_{p,\gamma_1,\ldots,\gamma_n}(z)+2(p+1)z^p\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)+p(p+1)z^{p-1}\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)\\ &\quad=\sum_{j=1}^{n}\gamma_j\left(\frac{z^{p+1}f_j''(z)+(p+1)z^pf_j'(z)}{z^{p+1}f_j'(z)}\right)\left[\left(\frac{-z^{p+1}}{p}f_1'(z)\right)^{\gamma_1}\cdots\left(\frac{-z^{p+1}}{p}f_n'(z)\right)^{\gamma_n}\right]. \end{aligned} \tag{2.14}$$

Then from (2.14), we obtain

$$\begin{aligned} &\frac{z^{p+1}\mathcal{Q}''_{p,\gamma_1,\ldots,\gamma_n}(z)+2(p+1)z^p\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)+p(p+1)z^{p-1}\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)}{z^p\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)z^{p-1}\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)}\\ &\quad=\sum_{j=1}^{n}\gamma_j\left(\frac{zf_j''(z)}{f_j'(z)}+p+1\right). \end{aligned} \tag{2.15}$$

That is equivalent to

$$\frac{z\left(z\mathcal{Q}''_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+2)\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)}+p=\sum_{j=1}^{n}\gamma_j\left(\frac{zf''_j(z)}{f'_j(z)}+p+1\right). \tag{2.16}$$

Equivalently, the above can be written as

$$\begin{aligned}&p-\frac{1}{b}\left(\frac{z\left(z\mathcal{Q}''_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+2)\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)}+p\right)\\&\quad=\sum_{j=1}^{n}\gamma_j\left\{p-\frac{1}{b}\left(\frac{zf''_j(z)}{f'_j(z)}+p+1\right)\right\}+p-p\sum_{j=1}^{n}\gamma_j.\end{aligned} \tag{2.17}$$

Taking the real part of both terms of (2.17), we have

$$\begin{aligned}&\Re\left\{p-\frac{1}{b}\left(\frac{z\left(z\mathcal{Q}''_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+2)\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)}+p\right)\right\}\\&\quad=\sum_{j=1}^{n}\gamma_j\Re\left\{p-\frac{1}{b}\left(\frac{zf''_j(z)}{f'_j(z)}+p+1\right)\right\}+p-p\sum_{j=1}^{n}\gamma_j.\end{aligned} \tag{2.18}$$

Sine $f_j \in \Sigma_{p,\mathcal{N}}(\beta_j, b)$, we get

$$\Re\left\{p-\frac{1}{b}\left(\frac{z\left(z\mathcal{Q}''_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+2)\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)}+p\right)\right\}<\sum_{j=1}^{n}\gamma_j\beta_j+p-p\sum_{j=1}^{n}\gamma_j. \tag{2.19}$$

That is,

$$\Re\left\{p-\frac{1}{b}\left(\frac{z\left(z\mathcal{Q}''_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+2)\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{Q}'_{p,\gamma_1,\ldots,\gamma_n}(z)+(p+1)\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)}+p\right)\right\}<p+\sum_{j=1}^{n}\gamma_j(\beta_j-p). \tag{2.20}$$

Then

$$\mathcal{Q}_{p,\gamma_1,\ldots,\gamma_n}(z)\in\Sigma F_p(\mu,b),\qquad \mu=p+\sum_{j=1}^{n}\gamma_j(\beta_j-p). \tag{2.21}$$

This completes the proof. □

Putting $p = 1$ in Theorem 2.1, we have the following.

Theorem 2.3. *Let* $f_j \in \Sigma$. *If* $f_j \in \Sigma_{\mathcal{M}}(\beta_j, b)$, *then*

$$\mathcal{H}_n(z) \in \Sigma F(\mu, b), \tag{2.22}$$

where

$$n \in \mathbb{N}, \quad j \in \{1, \dots, n\}, \quad \gamma_j > 0, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta_j > 1,$$
$$\mu = 1 + \sum_{j=1}^{n} \gamma_j (\beta_j - 1). \tag{2.23}$$

Putting $p = 1$ in Theorem 2.2, we have the following.

Theorem 2.4. *Let* $f_j \in \Sigma$. *If* $f_j \in \Sigma_{\mathcal{N}}(\beta_j, b)$, *then*

$$\mathcal{H}_{\gamma_1, \dots, \gamma_n}(z) \in \Sigma F(\mu, b), \tag{2.24}$$

where

$$n \in \mathbb{N}, \quad j \in \{1, \dots, n\}, \quad \gamma_j > 0, \quad b \in \mathbb{C} \setminus \{0\}, \quad \beta_j > 1,$$
$$\mu = 1 + \sum_{j=1}^{n} \gamma_j (\beta_j - 1). \tag{2.25}$$

for other work that we can look at regarding integral operators see [17, 21, 22].

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