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Research Article

Dynamics of Fuzzy BAM Neural Networks with Distributed Delays and Diffusion

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Constructing a new Lyapunov functional and employing inequality technique, the existence, uniqueness, and global exponential stability of the periodic oscillatory solution are investigated for a class of fuzzy bidirectional associative memory (BAM) neural networks with distributed delays and diffusion. We obtained some sufficient conditions ensuring the existence, uniqueness, and global exponential stability of the periodic solution. The results remove the usual assumption that the activation functions are differentiable. An example is provided to show the effectiveness of our results.

1. Introduction

The bidirectional associative memory (BAM) neural network was first introduced by Kosko [1, 2]. These models generalize the single-layer autoassociative Hebbian correlator to a two layer pattern-matched heteroassociative circuits. BAM neural network is composed of neurons arranged in two-layers, the X-layer and the Y-layer. The BAM neural network has been used in many fields such as image processing, pattern recognition, and automatic control [3]. Recently, the stability and the periodic oscillatory solutions of BAM neural networks have been studied (see, e.g., [1–25]). In 2002, Cao and Wang [7] derived some sufficient conditions for the global exponential stability and existence of periodic oscillatory solution of BAM neural networks with delays. Some authors [16, 19, 25] studied the BAM neural networks with distributed delays, which are more appropriate when neural networks have a multitude of parallel pathways with a variety of axon sizes and lengths.

However, strictly speaking, diffusion effects cannot be avoided in the neural networks when electrons are moving in asymmetric electromagnetic fields. So we must consider that activations vary in space as well as in time. Song et al. [25] have considered the stability of BAM neural networks with diffusion effects, which are expressed by partial differential equations. In this paper, we would like to integrate fuzzy operations into BAM neural networks. Speaking of fuzzy operations, T. Yang and L. B. Yang [26] first introduced fuzzy cellular neural networks (FCNNs) combining those operations with cellular neural networks. So far, researchers have founded that FCNNs are useful in image processing, and some results have been reported on stability and periodicity of FCNNs [26–32]. However, to the best of our knowledge, few authors consider the global exponential stability and existence of periodic solutions for fuzzy BAM neural networks with distributed delays and diffusion terms. In [33], Li studied the global exponential stabilities of both the equilibrium point and the periodic solution for a class of BAM fuzzy neural networks with delays and reaction-diffusion terms. Motivated by the above discussion, in this paper, by constructing a suitable Lyapunov functional and employing inequality technique, we will derive some sufficient conditions of the global exponential stability and existence of periodic solutions for fuzzy BAM neural networks with distributed delays and diffusion terms.

2. System Description and Preliminaries

In this paper, we consider the globally exponentially stable and periodic fuzzy BAM neural networks with distributed delays and diffusion terms described by partial differential equations with delays:

$$\begin{aligned}
\frac{\partial u_i(t,x)}{\partial t} &= \sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i}{\partial x_k}\right) - a_i u_i(t,x) + \sum_{j=1}^{m} c_{ji}\int_{-\infty}^{t} K_{ji}(t-s) f_j(v_j(s,x))ds \\
&\quad + \bigwedge_{j=1}^{m}\alpha_{ji}\int_{-\infty}^{t} K_{ji}(t-s) f_j(v_j(s,x))ds + \bigvee_{j=1}^{m}\beta_{ji}\int_{-\infty}^{t} K_{ji}(t-s) f_j(v_j(s,x))ds \\
&\quad + \bigwedge_{j=1}^{m} T_{ji}\omega_j + \bigvee_{j=1}^{m} H_{ji}\omega_j + I_i(t), \\
\frac{\partial v_j(t,x)}{\partial t} &= \sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D^*_{jk}\frac{\partial v_j}{\partial x_k}\right) - b_j v_j(t,x) + \sum_{i=1}^{n} d_{ij}\int_{-\infty}^{t} N_{ij}(t-s) g_i(u_i(s,x))ds \\
&\quad + \bigwedge_{i=1}^{n} p_{ij}\int_{-\infty}^{t} N_{ij}(t-s) g_i(u_i(s,x))ds + \bigvee_{i=1}^{n} q_{ij}\int_{-\infty}^{t} N_{ij}(t-s) g_i(u_i(s,x))ds \\
&\quad + \bigwedge_{i=1}^{n} S_{ij}\omega_i + \bigvee_{i=1}^{n} L_{ij}\omega_i + J_j(t),
\end{aligned} \tag{2.1}$$

where n and m correspond to the number of neurons in X-layer and Y-layer, respectively. For $i = 1,2,\ldots,n, j = 1,2,\ldots,m, x = (x_1,\ x_2,\ldots,x_l)^T \in \Omega \subset R^l$, Ω is a bounded compact set with smooth boundary $\partial\Omega$ and mess $(\Omega) > 0$ in space R^l. $u = (u_1,u_2,\ldots,u_n)^T \in R^n, v = (v_1,v_2,\ldots,v_m)^T \in R^m$. $u_i\ (t,x)$ and $v_j(t,x)$ are the state of the ith neuron and the jth neurons at time t and in space x, respectively. $a_i > 0,\ b_j > 0$, and they denote the rate with which the ith neuron and jth neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs; c_{ji} and d_{ij} are constants, denoting the connection weights. $\alpha_{ji}, \beta_{ji}, T_{ji}$, and H_{ji} are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template in X-layer, respectively; p_{ij}, q_{ij}, S_{ij}, and L_{ij} are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template in Y-layer, respectively; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operations, respectively; ω_j, ω_i denote external input of the ith neurons in X-layer and external input of the jth neurons in Y-layer, respectively; $I_i(t)$ and $J_j(t)$ denote the external inputs on the ith neurons in X-layer and the jth neurons in Y-layer at time t, respectively; $I_i : R^+ \to R$ and $J_j : R^+ \to R$ are continuously periodic functions with periodic ω., that is, $I_i(t+\omega) = I_i(t), J_j(t+\omega) = J_j(t)$. $K_{ji}(\cdot)$ and $N_{ij}(\cdot)$ are delay kernels functions. $g_i(\cdot)$ and $f_j(\cdot)$ are signal transmission functions of ith neurons and jth neurons at time t and in space x. Smooth functions $D_{ik}(t,x,u) \geq 0$ and $D^*_{jk}(t,x,v) \geq 0$ correspond to the transmission diffusion operators along the ith neurons and the jth neurons, respectively.

The boundary conditions and the initial conditions are given by

$$\begin{aligned} \frac{\partial u_i}{\partial n} &:= \left(\frac{\partial u_i}{\partial x_1}, \frac{\partial u_i}{\partial x_2}, \ldots, \frac{\partial u_i}{\partial x_l}\right)^T = 0, \quad i = 1,2,\ldots,n, \\ \frac{\partial v_j}{\partial n} &:= \left(\frac{\partial v_j}{\partial x_1}, \frac{\partial v_j}{\partial x_2}, \ldots, \frac{\partial v_j}{\partial x_l}\right)^T = 0, \quad j = 1,2,\ldots,m, \end{aligned} \tag{2.2}$$

$$\begin{aligned} u_i(s,x) &= \phi_{ui}(s,x), \quad s \in (-\infty, 0],\ i = 1,2,\ldots,n, \\ v_j(s,x) &= \varphi_{vj}(s,x), \quad s \in (-\infty, 0],\ i = 1,2,\ldots,m, \end{aligned} \tag{2.3}$$

where $\phi_{ui}(s,x)$ and $\varphi_{vj}(s,x)$ ($i = 1,2,\ldots,n; j = 1,2,\ldots,m.$) are continuous bounded functions defined on $(-\infty,0] \times \Omega$, respectively.

Throughout the paper, we give the following assumptions.

(A1) The signal transmission functions $f_j(\cdot)$, $g_i(\cdot)$ ($i = 1,2,\ldots,n, j = 1, 2,\ldots,m$) are Lipschtiz continuous on R with Lipschtiz constants μ_j and ν_i, namely, for any $x, y \in R$,

$$|f_j(x) - f_j(y)| \leq \mu_j |x - y|, \qquad |g_i(x) - g_i(y)| \leq \nu_i |x - y|, \qquad f_j(0) = g_i(0) = 0. \tag{2.4}$$

(A2) The delay kernels $K_{ji}, N_{ij} : [0,\infty) \to [0,\infty)(i = 1,2,\ldots,n; j = 1,2,\ldots,m)$ are nonnegative continuous functions that satisfy the following conditions:

(i) $\int_0^\infty K_{ji}(s)ds = \int_0^\infty N_{ij}(s)ds = 1$,

(ii) $\int_0^\infty sK_{ji}(s)ds < \infty$, $\int_0^\infty sN_{ij}(s)ds < \infty$,

(iii) there exists a positive constant η such that

$$\int_0^\infty s e^{\eta s} K_{ji}(s)ds < \infty, \qquad \int_0^\infty s e^{\eta s} N_{ij}(s)ds < \infty. \tag{2.5}$$

To be convenient, we introduce some notations. Let $u_i = u_i(t,x), v_j = v_j(t,x)$. Let $u^* = (u_1^*, u_2^*, \ldots, u_n^*)^T$ and $v^* = (v_1^*, v_2^*, \ldots, v_m^*)$ be the equilibrium of system (2.1). We denote

$$
\begin{aligned}
\|u(t,x) - u^*\| &= \left[\int_\Omega \sum_{i=1}^{n} |u_i - u_i^*|^2 dx\right]^{1/2}, \\
\|\phi_u(s,x) - u^*\| &= \sup_{-\infty \le s \le 0} \left[\int_\Omega \sum_{i=1}^{n} |\phi_{ui}(s,x) - u_i^*|^2 dx\right]^{1/2}, \\
\|v(t,x) - v^*\| &= \left[\int_\Omega \sum_{j=1}^{m} \left|v_j - v_j^*\right|^2 dx\right]^{1/2}, \\
\|\phi_v(s,x) - v^*\| &= \sup_{-\infty \le s \le 0} \left[\int_\Omega \sum_{j=1}^{m} \left|\varphi_{vj}(s,x) - v_j^*\right|^2 dx\right]^{1/2}.
\end{aligned}
\tag{2.6}
$$

Definition 2.1. The equilibrium $u^* = (u_1^*,\ u_2^*, \ldots, u_n^*)^T$, $v^* = (v_1^*,\ v_2^*, \ldots, v_m^*)^T$ of the delay fuzzy BAM neural networks (2.1) is said to be globally exponentially stable, if there exist positive constants $M \ge 1$, $\lambda > 0$ such that

$$
\|u(t,x) - u^*\| + \|v(t,x) - v^*\| \le M\left(\|\phi_u(s,x) - u^*\| + \|\varphi_v(s,x) - v^*\|\right)e^{-\lambda t} \tag{2.7}
$$

for every solution $u(t,x), v(t,x)$ of the delay fuzzy BAM neural networks (2.1) with the initial conditions (2.2) and (2.3) for all $t > 0$.

Definition 2.2. If $f(t) : R \to R$ *is a* continuous function, then the upper right derivative of $f(t)$ *is* defined as

$$
D^+ f(t) = \lim_{h \to 0^+} \sup \frac{1}{h}\left(f(t+h) - f(t)\right). \tag{2.8}
$$

Lemma 2.3 (see [26]). *Suppose x and y are two states of system* (2.1), *then one has*

$$
\begin{aligned}
\left|\bigwedge_{j=1}^{n} \alpha_{ij} g_j(x) - \bigwedge_{j=1}^{n} \alpha_{ij} g_j(y)\right| &\le \sum_{j=1}^{n} |\alpha_{ij}| |g_j(x) - g_j(y)|, \\
\left|\bigvee_{j=1}^{n} \beta_{ij} g_j(x) - \bigvee_{j=1}^{n} \beta_{ij} g_j(y)\right| &\le \sum_{j=1}^{n} |\beta_{ij}| |g_j(x) - g_j(y)|.
\end{aligned}
\tag{2.9}
$$

The remainder of this paper is organized as follows. In Section 3, we will study global exponential stability of fuzzy BAM neural networks (2.1). In Section 4, we present the existence of periodic solution for fuzzy BAM neural networks (2.1). In Section 5, an example will be given to illustrate effectiveness of our results obtained. We will give a general conclusion in Section 6.

3. Global Exponential Stability

In this section, we will discuss the global exponential stability of fuzzy BAM neural networks (2.1) by constructing suitable functional.

Theorem 3.1. *Under assumptions (A1) and (A2), if there exist $\delta_i > 0, \delta_{n+j} > 0$ such that*

$$\delta_i\left[-2a_i + \sum_{j=1}^{m}\left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right)\mu_j\right] + \nu_i\sum_{j=1}^{m}\delta_{n+j}\left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right) < 0,$$
$$\delta_{n+j}\left[-2b_j + \sum_{i=1}^{n}\left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right)\nu_i\right] + \mu_j\sum_{i=1}^{n}\delta_i\left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right) < 0, \tag{3.1}$$

where $i = 1,2,\ldots,n, j = 1,2,\ldots,m$ then the equilibrium u^*, v^* of system (2.1) *is the globally exponentially stable.*

Proof. By using (3.1), we can choose a small number $\lambda > 0$ such that

$$\delta_i\left[\lambda - 4a_i + 2\sum_{j=1}^{m}\left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right)\mu_j\right] + 2\nu_i\sum_{j=1}^{m}\delta_{n+j}\left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right)\int_0^{\infty} N_{ij}(s)e^{\lambda s}ds < 0,$$
$$\delta_{n+j}\left[\lambda - 4b_j + 2\sum_{i=1}^{n}\left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right)\nu_i\right] + 2\mu_j\sum_{i=1}^{n}\delta_i\left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right)\int_0^{\infty} K_{ji}(s)e^{\lambda s}ds < 0. \tag{3.2}$$

It is well known that the bounded functions always guarantee the existence of an equilibrium point for system (2.1). The uniqueness of the equilibrium for system (2.1) will follow from the global exponential stability to be established below.

Suppose $(u_1(t,x),\ldots,u_n(t,x),v_1(t,x),\ldots,v_m(t,x))^T$ is any solution of system (2.1). Rewrite (2.1) as follows

$$\frac{\partial(u_i - u_i^*)}{\partial t} = \sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial(u_i - u_i^*)}{\partial x_k}\right) - a_i(u_i - u_i^*)$$
$$+ \sum_{j=1}^{m}c_{ji}\int_{-\infty}^{t} K_{ji}(t-s)\left(f_j(v_j) - f_j\left(v_j^*\right)\right)ds$$

$$
\begin{aligned}
&+\bigwedge_{j=1}^{m}\alpha_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j(v_j)ds-\bigwedge_{j=1}^{m}\alpha_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j\left(v_j^*\right)ds\\
&+\bigvee_{j=1}^{m}\beta_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j(v_j)ds-\bigvee_{j=1}^{m}\beta_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j\left(v_j^*\right)ds,
\end{aligned}
\tag{3.3}
$$

$$
\begin{aligned}
\frac{\partial\left(v_j-v_j^*\right)}{\partial t}&=\sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{jk}^*\frac{\partial\left(v_j-v_j^*\right)}{\partial x_k}\right)-b_j\left(v_j-v_j^*\right)\\
&+\sum_{i=1}^{n}d_{ij}\int_{-\infty}^{t}N_{ij}(t-s)\left(g_i(u_i)-g_i(u_i^*)\right)ds\\
&+\bigwedge_{i=1}^{n}p_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_i(u_i)ds-\bigwedge_{i=1}^{n}p_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_i(u_i^*)ds\\
&+\bigvee_{j=1}^{m}q_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_i(u_i)ds-\bigvee_{j=1}^{m}q_{ij}\int_{-\infty}^{t}N_{ij}(t-s)g_i(u_i^*)ds.
\end{aligned}
\tag{3.4}
$$

Multiply both sides of (3.3) by $u_i-u_i^*$ and integrate, we have

$$
\begin{aligned}
\frac{1}{2}\frac{d}{dt}\int_{\Omega}(u_i-u_i^*)^2dx&=\int_{\Omega}(u_i-u_i^*)\left[\sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial(u_i-u_i^*)}{\partial x_k}\right)\right]dx-\int_{\Omega}a_i(u_i-u_i^*)^2dx\\
&+\int_{\Omega}(u_i-u_i^*)\left[\sum_{j=1}^{m}c_{ji}\int_{-\infty}^{t}K_{ji}(t-s)\left(f_j(v_j)-f_j\left(v_j^*\right)\right)ds\right]dx\\
&+\int_{\Omega}(u_i-u_i^*)\left[\bigwedge_{j=1}^{m}\alpha_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j(v_j)ds\right]dx\\
&-\int_{\Omega}(u_i-u_i^*)\left[\bigwedge_{j=1}^{m}\alpha_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j\left(v_j^*\right)ds\right]dx\\
&+\int_{\Omega}(u_i-u_i^*)\left[\bigvee_{j=1}^{m}\beta_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j(v_j)ds\right]dx\\
&-\int_{\Omega}(u_i-u_i^*)\left[\bigvee_{j=1}^{m}\beta_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j\left(v_j^*\right)ds\right]dx.
\end{aligned}
\tag{3.5}
$$

Applying the boundary condition (2.2) and the Gauss formula, we get

$$\begin{aligned}
&\int_{\Omega}(u_i-u_i^*)\left[\sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial(u_i-u_i^*)}{\partial x_k}\right)\right]dx\\
&\quad=\int_{\Omega}(u_i-u_i^*)\left[\sum_{k=1}^{l}\nabla\left(D_{ik}\frac{\partial(u_i-u_i^*)}{\partial x_k}\right)_{k=1}^{l}\right]dx\\
&\quad=\int_{\Omega}\nabla\cdot\left((u_i-u_i^*)D_{ik}\frac{\partial(u_i-u_i^*)}{\partial x_k}\right)_{k=1}^{l}dx-\int_{\Omega}\left(D_{ik}\frac{\partial(u_i-u_i^*)}{\partial x_k}\right)_{k=1}^{l}\nabla(u_i-u_i^*)dx\\
&\quad=\int_{\partial\Omega}\left((u_i-u_i^*)D_{ik}\frac{\partial(u_i-u_i^*)}{\partial x_k}\right)_{k=1}^{l}ds-\sum_{k=1}^{l}\int_{\Omega}D_{ik}\left(\frac{\partial(u_i-u_i^*)}{\partial x_k}\right)^2dx\\
&\quad=-\sum_{k=1}^{l}\int_{\Omega}D_{ik}\left(\frac{\partial(u_i-u_i^*)}{\partial x_k}\right)^2dx\le 0
\end{aligned}\tag{3.6}$$

in which $\nabla=(\partial/(\partial x_1),\partial/(\partial x_2),\ldots,\partial/(\partial x_l))^T$ is the gradient operator and

$$\left(D_{ik}\frac{\partial(u_i-u_i^*)}{\partial x_k}\right)_{k=1}^{l}=\left(D_{i1}\frac{\partial(u_i-u_i^*)}{\partial x_i},\ D_{i1}\frac{\partial(u_i-u_i^*)}{\partial x_2},\ldots,\ D_{i1}\frac{\partial(u_i-u_i^*)}{\partial x_l}\right)^T.\tag{3.7}$$

From (3.5), (3.6), hypothesis (A1), Lemma 2.3, and the Holder inequality, we have

$$\begin{aligned}
&\frac{d\|u_i-u_i^*\|^2}{dt}\\
&\quad\le -2a_i\int_{\Omega}(u_i-u_i^*)^2dx+2\int_{\Omega}(u_i-u_i^*)\\
&\qquad\times\left[\sum_{j=1}^{m}c_{ji}\int_{-\infty}^{t}K_{ji}(t-s)\left(f_j(v_j)-f_j\left(v_j^*\right)\right)ds\right]dx\\
&\qquad+\int_{\Omega}2(u_i-u_i^*)\left[\bigwedge_{j=1}^{m}\alpha_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j(v_j)ds-\bigwedge_{j=1}^{m}\alpha_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j\left(v_j^*\right)ds\right]dx\\
&\qquad+\int_{\Omega}2(u_i-u_i^*)\left[\bigvee_{j=1}^{m}\beta_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j(v_j)ds-\bigvee_{j=1}^{m}\beta_{ji}\int_{-\infty}^{t}K_{ji}(t-s)f_j\left(v_j^*\right)ds\right]dx
\end{aligned}$$

$$\le -2a_i\int_{\Omega}(u_i-u_i^*)^2dx+2\int_{\Omega}(u_i-u_i^*)\left[\sum_{j=1}^{m}\mu_j|c_{ji}|\int_{-\infty}^{t}K_{ji}(t-s)\left|v_j-v_j^*\right|ds\right]dx$$
$$+2\int_{\Omega}(u_i-u_i^*)\left[\sum_{j=1}^{m}\mu_j(|\alpha_{ji}|+|\beta_{ji}|)\int_{-\infty}^{t}K_{ji}(t-s)\left|v_j-v_j^*\right|ds\right]dx$$
$$\le -2a_i(u_i-u_i^*)^2+2\sum_{j=1}^{m}(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\int_{-\infty}^{t}K_{ji}(t-s)|u_i-u_i^*|\mu_j\left|v_j-v_j^*\right|ds. \tag{3.8}$$

Multiply both sides of (3.4) by $v_i-v_i^*$, similarly, we get

$$\frac{d\left|v_j-v_j^*\right|^2}{dt}\le -2b_j\left(v_j-v_j^*\right)^2+2\sum_{i=1}^{n}(|d_{ij}|+|p_{ij}|+|q_{ij}|)\int_{-\infty}^{t}N_{ij}(t-s)\left|v_j-v_j^*\right|\nu_i|u_i-u_i^*|ds. \tag{3.9}$$

We construct a Lyapunov functional

$$\begin{aligned} V(t)=\int_{\Omega}\sum_{i=1}^{n}\delta_i\Bigg[&|u_i-u_i^*|^2e^{\lambda t}+2\sum_{j=1}^{m}(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\mu_j\int_0^{\infty}K_{ji}(s)\\ &\times\int_{t-s}^{t}e^{\lambda(\zeta+s)}\left|v_j(\zeta,x)-v_j^*\right|^2d\zeta\,ds\Bigg]dx\\ +\int_{\Omega}\sum_{j=1}^{m}\delta_{n+j}\Bigg[&\left|v_j-v_j^*\right|^2e^{\lambda t}+2\sum_{i=1}^{n}(|d_{ij}|+|p_{ij}|+|q_{ij}|)\nu_i\int_0^{\infty}N_{ij}(s)\\ &\times\int_{t-s}^{t}e^{\lambda(\zeta+s)}|u_i(\zeta,x)-u_i^*|^2d\zeta\,ds\Bigg]dx. \end{aligned} \tag{3.10}$$

Calculating the upper right derivative $D^+V(t)$ of $V(t)$ along the solution of (3.3) and (3.4), from (3.8) and (3.9), we get

$$\begin{aligned} D^+V(t)=\int_{\Omega}\sum_{i=1}^{n}\delta_i\Bigg[&2|u_i-u_i^*|\frac{\partial|u_i-u_i^*|}{\partial t}e^{\lambda t}+\lambda e^{\lambda t}|u_i-u_i^*|^2\\ &+2e^{\lambda t}\sum_{j=1}^{m}(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\mu_j\\ &\times\int_0^{\infty}K_{ji}(s)e^{\lambda s}\left|v_j(t,x)-v_j^*\right|^2ds-2e^{\lambda t}\sum_{j=1}^{m}(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\mu_j \end{aligned}$$

$$\times \int_0^\infty K_{ji}(s) \times \left|v_j(t-s,x) - v_j^*\right|^2 ds \Bigg] dx$$

$$+ \int_\Omega \sum_{j=1}^m \delta_{n+j} \Bigg[2\left|v_j - v_j^*\right| \frac{\partial\left|v_j - v_j^*\right|}{\partial t} e^{\lambda t} + \lambda e^{\lambda t}\left|v_j - v_j^*\right|^2$$

$$+ 2e^{\lambda t} \sum_{i=1}^n \left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right) \nu_i \int_0^\infty N_{ij}(s) e^{\lambda s} |u_i(t,x) - u_i^*|^2 ds$$

$$-2e^{\lambda t} \sum_{i=1}^n \left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right) \nu_i \int_0^\infty N_{ij}(s) |u_i(t-s,x) - u_i^*|^2 ds \Bigg] dx$$

$$\leq \int_\Omega \sum_{i=1}^n \delta_i \Bigg[2e^{\lambda t} \Bigg(-2a_i(u_i - u_i^*)^2 + 2\sum_{j=1}^m \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right)$$

$$\times \int_{-\infty}^t K_{ji}(t-s) |u_i - u_i^*| \times \mu_j \left|v_j - v_j^*\right| ds \Bigg) + \lambda e^{\lambda t} |u_i - u_i^*|^2$$

$$+ 2e^{\lambda t} \sum_{j=1}^m \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right) \mu_j \int_0^\infty K_{ji}(s) e^{\lambda s} \left|v_j(t,x) - v_j^*\right|^2 ds$$

$$- 2e^{\lambda t} \sum_{j=1}^m \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right) \mu_j \int_0^\infty K_{ji}(s)$$

$$\times \left|v_j(t-s,x) - v_j^*\right|^2 ds \Bigg] dx$$

$$+ \int_\Omega \sum_{j=1}^m \delta_{n+j} \Bigg[2e^{\lambda t} \Bigg(-2b_j\left(v_j - v_j^*\right)^2$$

$$+ 2\sum_{i=1}^n \left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right)$$

$$\times \int_{-\infty}^t N_{ij}(t-s) \left|v_j - v_j^*\right| \nu_i |u_i - u_i^*| ds \Bigg)$$

$$+ \lambda e^{\lambda t} \left|v_j - v_j^*\right|^2 + 2e^{\lambda t} \sum_{i=1}^n \left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right) \nu_i$$

$$\times \int_0^\infty N_{ij}(s) e^{\lambda s} |u_i(t,x) - u_i^*|^2 ds$$

$$- 2e^{\lambda t} \sum_{i=1}^n \left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right) \nu_i$$

$$\times \int_0^\infty N_{ij}(s) |u_i(t-s,x) - u_i^*|^2 ds \Bigg] dx$$

$$\begin{aligned}
&\le \int_\Omega e^{\lambda t}\sum_{i=1}^{n}\delta_i\Bigg\{\Bigg[-4a_i|u_i-u_i^*|^2+\lambda|u_i-u_i^*|^2+4\sum_{j=1}^{m}(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\\
&\qquad\times\int_{-\infty}^{t}K_{ji}(t-s)|u_i-u_i^*|\mu_j\left|v_j-v_j^*\right|ds\Bigg]\\
&\qquad+2\sum_{j=1}^{m}(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\mu_j\int_0^\infty K_{ji}(s)e^{\lambda s}\left|v_j(t,x)-v_j^*\right|^2ds\\
&\qquad-2\sum_{j=1}^{m}(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\mu_j\int_0^\infty K_{ji}(s)\left|v_j(t-s,x)-v_j^*\right|^2ds\Bigg\}dx\\
&+\int_\Omega e^{\lambda t}\sum_{j=1}^{m}\delta_{n+j}\Bigg\{\Bigg[-4b_j\left|v_j-v_j^*\right|^2+\lambda\left|v_j-v_j^*\right|^2+4\sum_{i=1}^{n}(|d_{ij}|+|p_{ij}|+|q_{ij}|)\\
&\qquad\times\int_{-\infty}^{t}N_{ij}(t-s)\left|v_j-v_j^*\right|\nu_i|u_i-u_i^*|ds\Bigg]\\
&\qquad+2\sum_{i=1}^{n}(|d_{ij}|+|p_{ij}|+|q_{ij}|)\times\nu_i\int_0^\infty N_{ij}(s)e^{\lambda s}|u_i(t,x)-u_i^*|^2ds\\
&\qquad-2\sum_{i=1}^{n}(|d_{ij}|+|p_{ij}|+|q_{ij}|)\nu_i\\
&\qquad\times\int_0^\infty N_{ij}(s)|u_i(t-s,x)-u_i^*|^2ds\Bigg\}dx.
\end{aligned}\tag{3.11}$$

Estimating the right of (3.11) by elemental inequality $2ab \le a^2+b^2$, we obtain that

$$\begin{aligned}
D^+V &\le \int_\Omega e^{\lambda t}\sum_{i=1}^{n}\delta_i\Bigg\{\Bigg[-4a_i|u_i-u_i^*|^2+\lambda|u_i-u_i^*|^2+2\sum_{j=1}^{m}(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\times\mu_j|u_i-u_i^*|^2\Bigg]\\
&\qquad+2\sum_{j=1}^{m}(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|)\mu_j\int_0^\infty K_{ji}(s)e^{\lambda s}\left|v_j(t,x)-v_j^*\right|^2ds\Bigg\}dx\\
&+\int_\Omega e^{\lambda t}\sum_{j=1}^{m}\delta_{n+j}\Bigg\{\Bigg[-4b_j\left|v_j-v_j^*\right|^2+\lambda\left|v_j-v_j^*\right|^2\\
&\qquad+2\sum_{i=1}^{n}(|d_{ij}|+|p_{ij}|+|q_{ij}|)\nu_i\left|v_j-v_j^*\right|^2\Bigg]
\end{aligned}$$

$$
\begin{aligned}
&\qquad\qquad +2\sum_{i=1}^{n}\left(|d_{ij}|+|p_{ij}|+|q_{ij}|\right)\nu_i\int_0^{\infty}N_{ij}(s)e^{\lambda s}\left|u_i(t,x)-u_i^*\right|^2 ds\Bigg\}dx \\
&= \int_{\Omega}e^{\lambda t}\sum_{i=1}^{n}\Bigg\{\delta_i\left[\lambda-4a_i+2\sum_{j=1}^{m}\left(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|\right)\mu_j\right] \\
&\qquad +2\nu_i\sum_{j=1}^{m}\delta_{n+j}\left(|d_{ij}|+|p_{ij}|+|q_{ij}|\right)\int_0^{\infty}N_{ij}(s)e^{\lambda s}ds\Bigg\}\left|u_i-u_i^*\right|^2 dx \\
&\quad +\int_{\Omega}e^{\lambda t}\sum_{j=1}^{m}\Bigg\{\delta_{n+j}\left[\lambda-4b_j+2\sum_{i=1}^{n}\left(|d_{ij}|+|p_{ij}|+|q_{ij}|\right)\nu_i\right] \\
&\qquad +2\mu_j\sum_{i=1}^{n}\delta_i\left(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|\right)\int_0^{\infty}K_{ji}(s)e^{\lambda s}ds\Bigg\}\left|v_j-v_j^*\right|^2 dx \le 0.
\end{aligned}
\tag{3.12}
$$

Therefore,

$$
V(t)\le V(0),\qquad t\ge 0. \tag{3.13}
$$

Noting that

$$
e^{\lambda t}\min_{1\le i\le n+m}(\delta_i)\left(\|u(t,x)-u^*\|+\|v(t,x)-v^*\|\right)\le V(t),\qquad t\ge 0. \tag{3.14}
$$

On the other hand, we have

$$
\begin{aligned}
V(0) &= \int_{\Omega}\sum_{i=1}^{n}\delta_i\Bigg[\left|u_i(0,x)-u_i^*\right|^2+2\sum_{j=1}^{m}\left(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|\right)\mu_j\int_0^{\infty}K_{ji}(s) \\
&\qquad\times\int_{-s}^{0}e^{\lambda(\zeta+s)}\left|v_j(\zeta,x)-v_j^*\right|^2 d\zeta\, ds\Bigg]dx \\
&\quad +\int_{\Omega}\sum_{j=1}^{m}\delta_{n+j}\Bigg[\left|v_j(0,x)-v_j^*\right|^2+2\sum_{i=1}^{n}\left(|d_{ij}|+|p_{ij}|+|q_{ij}|\right)\nu_i\int_0^{\infty}N_{ij}(s) \\
&\qquad\times\int_{-s}^{0}e^{\lambda(\zeta+s)}\left|u_i(\zeta,x)-u_i^*\right|^2 d\zeta\, ds\Bigg]dx \\
&\le \left[\max_{1\le i\le n}(\delta_i)+2\max_{1\le j\le m}(\delta_{n+j})\sum_{i=1}^{n}\left(|d_{ij}|+|p_{ij}|+|q_{ij}|\right)\nu_i\int_0^{\infty}N_{ij}(s)se^{\lambda s}\right]\|\phi_u(s,x)-u^*\| \\
&\quad +\left[\max_{1\le j\le m}(\delta_{n+j})+2\max_{1\le i\le n}(\delta_i)\sum_{j=1}^{m}\left(|c_{ji}|+|\alpha_{ji}|+|\beta_{ji}|\right)\mu_j\int_0^{\infty}K_{ji}(s)se^{\lambda s}\right]\|\varphi_v(s,x)-v^*\|.
\end{aligned}
\tag{3.15}
$$

Let

$$M = \max\left\{ \max_{1\le i\le n}(\delta_i) + 2\max_{1\le j\le m}(\delta_{n+j}) \sum_{i=1}^{n}\left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right)\nu_i \int_0^{\infty} N_{ij}(s) s e^{\lambda s}, \right.$$
$$\left. \max_{1\le j\le m}(\delta_{n+j}) + 2\max_{1\le i\le n}(\delta_i) \sum_{j=1}^{m}\left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right)\mu_j \int_0^{\infty} K_{ji}(s) s e^{\lambda s} \right\}, \tag{3.16}$$

then $M \ge 1$ and

$$\|u(t,x) - u^*\| + \|v(t,x) - v^*\| \le M\left(\|\phi_u(s,x) - u^*\| + \|\varphi_v(s,x) - v^*\|\right)e^{-\lambda t}. \tag{3.17}$$

This implies that the equilibrium point of system (2.1) is globally exponentially stable. The proof is completed. □

Corollary 3.2. *Suppose (A1) and (A2) hold. Then, the equilibrium u^*, v^* of system* (2.1) *is the globally exponentially stable, if the following conditions are satisfied:*

$$-2a_i + \sum_{j=1}^{m}\left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right)\mu_j + \nu_i \sum_{j=1}^{m}\left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right) < 0,$$
$$-2b_j + \sum_{i=1}^{n}\left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right)\nu_i + \mu_j \sum_{i=1}^{n}\left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right) < 0. \tag{3.18}$$

4. Periodic Oscillatory Solutions of Fuzzy BAM Neural Networks

In this section, we consider the existence and uniqueness of periodic oscillatory solutions for system (2.1).

Theorem 4.1. *Suppose that assumption (A1) and (A2) hold. Then, there exists exactly an ω-periodic solution of system* (2.1)*, with the initial values* (2.2) *and* (2.3)*, and all other solutions converge exponentially to it as $t \to \infty$. If there exists $\delta_i \ge 0, \delta_{n+j} > 0$ such that*

$$\delta_i\left[-2a_i + \sum_{j=1}^{m}\left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right)\mu_j\right] + \nu_i \sum_{j=1}^{m}\delta_{n+j}\left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right) < 0,$$
$$\delta_{n+j}\left[-2b_j + \sum_{i=1}^{n}\left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right)\nu_i\right] + \mu_j \sum_{i=1}^{n}\delta_i\left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right) < 0, \tag{4.1}$$

where $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$.

Proof. Let

$$C = \left\{\Phi \mid \Phi = (\phi_u, \varphi_v)^T = (\phi_{u1},\ldots, \phi_{un}, \varphi_{v1},\ldots,\varphi_{vm})^T, \Phi : (-\infty,0)\times(-\infty,0) \longrightarrow R^{n+m}\right\}. \tag{4.2}$$

For any $\Phi \in C$, we define

$$\|\Phi\| = \sup_{-\infty \le s \le 0} \left[\int_\Omega \sum_{i=1}^{n} |\phi_{ui}|^2 dx \right]^{1/2} + \sup_{-\infty \le s \le 0} \left[\int_\Omega \sum_{j=1}^{m} |\varphi_{vi}|^2 dx \right]^{1/2}. \tag{4.3}$$

Then, C is the Banach space of continuous functions.

For any $(\phi_u, \varphi_v)^T, (\phi'_u, \varphi'_v)^T \in C$, we denote the solutions of system (2.1) through $(0,0)^T, (\phi_u, \varphi_v)^T$, and $(0,0)^T, (\phi'_u, \varphi'_v)^T$ as

$$\begin{aligned} u(t, \phi_u, x) &= \left(u_1(t, \phi_u, x), \dots, u_n(t, \phi_u, x)\right)^T, \\ v(t,\ \varphi_v,\ x) &= \left(v_1(t, \varphi_v, x), \dots, v_m(t, \varphi_v, x)\right)^T, \\ u(t, \phi'_u, x) &= \left(u_1(t, \phi'_u, x), \dots, u_n(t, \phi'_u, x)\right)^T, \\ v(t, \varphi'_v, x) &= \left(v_1(t, \varphi'_v,\ x), \dots, v_m(t, \varphi'_v, x)\right)^T, \end{aligned} \tag{4.4}$$

respectively. And define that

$$u_t(\phi_u, x) = u(t+\theta, \phi_u, x), \qquad v_t(\varphi_v, x) = v(t+\theta, \varphi_v, x), \quad \theta \in (-\infty, 0], \quad t \ge 0, \tag{4.5}$$

then $(u_t(\phi_u, x), v_t(\varphi_v, x)^T(\phi_u, x),\ v_t(\varphi_v, x)^T \in C$ for $t \ge 0$. Let $u_{ix} = u_i\ (t, \phi_u, x) - u_i\ (t, \phi'_u, x), v_{jx} = v_j\ (t, \varphi_v,\ x) - v_j\ (t, \varphi'_v,\ x)$. Therefore, it follows from system (2.1) that

$$\begin{aligned} \frac{\partial u_{ix}}{\partial t} = &\sum_{k=1}^{l} \frac{\partial}{\partial x_k}\left(D_{ik} \frac{\partial u_{ix}}{\partial x_k}\right) - a_i u_{ix} \\ &+ \sum_{j=1}^{m} c_{ji} \int_{-\infty}^{t} K_{ji}(t-s)\left[f_j(v_j(s, \varphi_v, x)) - f_j(v_j(s, \varphi'_v, x))\right] ds \\ &+ \bigwedge_{j=1}^{m} \alpha_{ji} \int_{-\infty}^{t} K_{ji}(t-s) f_j(v_j(s, \varphi_v, x)) ds \\ &- \bigwedge_{j=1}^{m} \alpha_{ji} \int_{-\infty}^{t} K_{ji}(t-s) f_j(v_j(s, \varphi'_v, x)) ds \\ &+ \bigvee_{j=1}^{m} \beta_{ji} \int_{-\infty}^{t} K_{ji}(t-s) f_j(v_j(s, \varphi_v, x)) ds \\ &- \bigvee_{j=1}^{m} \beta_{ji} \int_{-\infty}^{t} K_{ji}(t-s) f_j(v_j(s, \varphi'_v, x)) ds, \end{aligned}$$

$$\begin{aligned}
\frac{\partial v_{jx}}{\partial t} &= \sum_{k=1}^{l} \frac{\partial}{\partial x_k}\left(D_{jk}^{*} \frac{\partial v_{jx}}{\partial x_k}\right) - b_j v_{jx} \\
&+ \sum_{i=1}^{n} d_{ij} \int_{-\infty}^{t} N_{ij}(t-s)\left[g_i(u_i(s,\phi_u,x)) - f_j(v_j(s,\phi_u',x))\right] ds \\
&+ \bigwedge_{i=1}^{n} p_{ij} \int_{-\infty}^{t} N_{ij}(t-s) g_i(u_i(s,\phi_u,x)) ds \\
&- \bigwedge_{i=1}^{n} p_{ij} \int_{-\infty}^{t} N_{ij}(t-s) g_i(u_i(s,\phi_u',x)) ds \\
&+ \bigvee_{i=1}^{n} q_{ij} \int_{-\infty}^{t} N_{ij}(t-s) g_i(u_i(s,\phi_u,x)) ds \\
&- \bigvee_{i=1}^{n} q_{ij} \int_{-\infty}^{t} N_{ij}(t-s) g_i(u_i(s,\phi_v',x)) ds.
\end{aligned} \tag{4.6}$$

Considering the following Lyapunov functional

$$\begin{aligned}
V(t) &= \int_{\Omega} \sum_{i=1}^{n} \delta_i |u_{ix}|^2 e^{\lambda t} dx + \int_{\Omega} \sum_{j=1}^{m} \delta_{n+j} |v_{jx}|^2 e^{\lambda t} dx \\
&+ \int_{\Omega} \sum_{i=1}^{n} 2\delta_i \sum_{j=1}^{m} \left(|c_{ji}| + |\alpha_{ji}| + |\beta_{ji}|\right) \\
&\times \mu_j \int_{0}^{\infty} K_{ji}(s) \int_{t-s}^{t} e^{\lambda(\zeta+s)} \left|v_j(\zeta,\varphi_v,x) - v_j(\zeta,\varphi_v',x)\right|^2 d\zeta\, ds dx \\
&+ \int_{\Omega} \sum_{j=1}^{m} 2\delta_{n+j} \sum_{i=1}^{n} \left(|d_{ij}| + |p_{ij}| + |q_{ij}|\right) \nu_i \\
&\times \int_{0}^{\infty} N_{ij}(s) \int_{t-s}^{t} e^{\lambda(\zeta+s)} \left|u_i(\zeta,\phi_u,x) - u_i(\zeta,\phi_u',x)\right|^2 d\zeta\, ds dx.
\end{aligned} \tag{4.7}$$

By a minor modification of the proof of Theorem 3.1, we can easily obtain

$$\|u(t,\phi_u,x) - u(t,\phi_u',x)\| + \|v(t,\varphi_v,x) - v(t,\varphi_v',x)\| \le M\left(\|\phi_u - \phi_u'\| + \|\varphi_v - \varphi_v'\|\right) e^{-\lambda t}, \tag{4.8}$$

for all $t \geq 0$. We can choose a positive integer N such that

$$Me^{-\lambda N\omega} \leq \frac{1}{4} \tag{4.9}$$

and define a Poincare mapping $C \to C$ by

$$P\left((\phi_u, \varphi_v)^T\right) = (u_\omega(\phi_u), u_\omega(\varphi_v))^T. \tag{4.10}$$

It follows that

$$P^N\left((\phi_u, \varphi_v)^T\right) = (u_{N\omega}(\phi_u), u_{N\omega}(\varphi_v))^T. \tag{4.11}$$

Letting $t = n\omega$, we, from (4.8) to (4.11), obtain that

$$\left|P^N\left((\phi_u, \varphi_v)^T\right) - P^N\left((\phi'_u, \varphi'_v)^T\right)\right| \leq \frac{1}{2}\left|(\phi_u, \varphi_v)^T - (\phi'_u, \varphi'_v)^T\right|. \tag{4.12}$$

It implies that P^N is a contraction mapping. Hence, there exist a unique equilibrium point $(\phi^*_u, \varphi^*_v)^T \in C$ such that

$$P^N\left((\phi^*_u, \varphi^*_v)^T\right) = (\phi^*_u, \varphi^*_v)^T. \tag{4.13}$$

Note that

$$P^N\left(P\left((\phi^*_u, \varphi^*_v)^T\right)\right) = P\left(P^N\left((\phi^*_u, \varphi^*_v)^T\right)\right) = P\left((\phi^*_u, \varphi^*_v)^T\right). \tag{4.14}$$

Let $(u(t, \phi^*_u), v(t, \varphi^*_v))^T$ be the solution of system (2.1) through $((0,0)^T, (\phi^*_u, \varphi^*_v)^T)$, then $(u(t+\omega, \phi^*_u, x), v(t+\omega, \varphi^*_v, x))^T$ is also the solution of system (2.1). It is clear that

$$(u_{t+\omega}(\phi^*_u), v_{t+\omega}(\varphi^*_v))^T = (u_t(\phi^*_u), v_t(\varphi^*_v))^T \tag{4.15}$$

for all $t \geq 0$. It follows from (4.15) that

$$(u(t+\omega, \phi^*_u, x), v(t+\omega, \varphi^*_v, x))^T = (u(t, \phi^*_u, x), v(t, \varphi^*_v, x))^T, \quad t \geq 0, \tag{4.16}$$

which shows that $(u(t, \phi^*_u, x), v(t, \varphi^*_v, x))^T$ is exactly an ω-periodic solution of system (2.1) with the initial conditions (2.2) and (2.3) and other solutions of system (2.1) with the initial conditions (2.2) and (2.3) converge exponentially to it as $t \to \infty$. □

5. An Illustrative Example

In this section, we will give an example to illustrate feasible our result.

Example 5.1. Consider the following system

$$\begin{aligned}
\frac{\partial u_i(t,x)}{\partial t} &= \sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D_{ik}\frac{\partial u_i}{\partial x_k}\right) - a_i u_i(t,x) \\
&\quad + \sum_{j=1}^{m} c_{ji}\int_{-\infty}^{t} K_{ji}(t-s) f_j\left(v_j(s,x)\right)ds \\
&\quad + \bigwedge_{j=1}^{m} \alpha_{ji}\int_{-\infty}^{t} K_{ji}(t-s) f_j\left(v_j(s,x)\right)ds \\
&\quad + \bigvee_{j=1}^{m} \beta_{ji}\int_{-\infty}^{t} K_{ji}(t-s) f_j\left(v_j(s,x)\right)ds \\
&\quad + \bigwedge_{j=1}^{m} T_{ji}\omega_j + \bigvee_{j=1}^{m} H_{ji}\omega_j + I_i, \\
\frac{\partial v_j(t,x)}{\partial t} &= \sum_{k=1}^{l}\frac{\partial}{\partial x_k}\left(D^{*}_{jk}\frac{\partial v_j}{\partial x_k}\right) - b_j v_j(t,x) \\
&\quad + \sum_{i=1}^{n} d_{ij}\int_{-\infty}^{t} N_{ij}(t-s) g_i(u_i(s,x))ds \\
&\quad + \bigwedge_{i=1}^{n} p_{ij}\int_{-\infty}^{t} N_{ij}(t-s) g_i(u_i(s,x))ds \\
&\quad + \bigvee_{i=1}^{n} q_{ij}\int_{-\infty}^{t} N_{ij}(t-s) g_i(u_i(s,x))ds \\
&\quad + \bigwedge_{i=1}^{n} S_{ij}\omega_i + \bigvee_{i=1}^{n} L_{ij}\omega_i + J_j,
\end{aligned} \tag{5.1}$$

where

$$\begin{gathered}
K_{ji}(t) = N_{ij}(t) = te^{-t}, \quad i = 1,2,\ j = 1,2, \\
f_1(r) = f_2(r) = g_1(r) = g_2(r) = \frac{1}{2}(|r+1| - |r-1|).
\end{gathered} \tag{5.2}$$

It is obvious that $f(\cdot), g(\cdot)$ satisfy assumption (A1) and $K(\cdot), N(\cdot)$ satisfy assumption (A2), moreover $\mu_1 = \mu_2 = \nu_1 = \nu_2 = 1$. Let

$$\begin{aligned}
&c_{11} = \frac{1}{2}, \quad c_{21} = 1, \quad c_{12} = -\frac{1}{2}, \quad c_{22} = \frac{2}{3}, \\
&d_{11} = \frac{1}{3}, \quad d_{21} = \frac{1}{2}, \quad d_{12} = -\frac{1}{3}, \quad d_{22} = \frac{2}{3}, \\
&\alpha_{11} = \frac{2}{3}, \quad \alpha_{21} = \frac{1}{2}, \quad \alpha_{12} = -\frac{1}{4}, \quad \alpha_{22} = \frac{1}{3}, \\
&\beta_{11} = \frac{1}{3}, \quad \beta_{21} = \frac{1}{2}, \quad \beta_{12} = -\frac{1}{4}, \quad \beta_{22} = \frac{5}{3}, \\
&p_{11} = \frac{2}{3}, \quad p_{21} = \frac{1}{4}, \quad p_{12} = -\frac{1}{2}, \quad p_{22} = \frac{2}{3}, \\
&q_{11} = \frac{2}{3}, \quad q_{21} = \frac{1}{2}, \quad q_{12} = -\frac{1}{4}, \quad q_{22} = \frac{1}{3}, \\
&a_1 = 4, \quad a_2 = 3.5, \quad b_1 = 3, \quad b_2 = 4.5.
\end{aligned} \tag{5.3}$$

By simply calculating, we can get

$$\begin{aligned}
&-2a_1 + \sum_{j=1}^{2}\left(|c_{j1}| + |\alpha_{j1}| + |\beta_{j1}|\right)\mu_j + \nu_1\sum_{j=1}^{2}\left(|d_{1j}| + |p_{1j}| + |q_{1j}|\right) = -1.75 < 0, \\
&-2a_2 + \sum_{j=1}^{2}\left(|c_{j2}| + |\alpha_{j2}| + |\beta_{j2}|\right)\mu_j + \nu_2\sum_{j=1}^{2}\left(|d_{2j}| + |p_{2j}| + |q_{2j}|\right) = -0.75 < 0, \\
&-2b_1 + \sum_{i=1}^{2}\left(|d_{i1}| + |p_{i1}| + |q_{i1}|\right)\nu_i + \mu_1\sum_{i=1}^{2}\left(|c_{1i}| + |\alpha_{1i}| + |\beta_{1i}|\right) = -0.583 < 0, \\
&-2b_2 + \sum_{i=1}^{2}\left(|d_{i2}| + |p_{i2}| + |q_{i2}|\right)\nu_i + \mu_2\sum_{i=1}^{2}\left(|c_{2i}| + |\alpha_{2i}| + |\beta_{2i}|\right) = -1.583 < 0.
\end{aligned} \tag{5.4}$$

Since all the conditions of Corollary 3.2 are satisfied, therefore the system (5.1) has a unique equilibrium point which is globally exponentially stable.

6. Conclusion

In this paper, we have studied the exponential stability of the equilibrium point and the existence of periodic solutions for fuzzy BAM neural networks with distributed delays and diffusion. Some sufficient conditions set up here are easily verified, and the conditions which are only correlated with parameters of the system (2.1) are independent of time delays. The results obtained in this paper remove the assumption about the activation functions with differentiable and only require the activation functions are bounded and Lipschitz continuous. Thus, it allows us even more flexibility in choosing activation functions.

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