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Research Article

A Study on the Holding Capacity Safety Factors for Torpedo Anchors

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The use of powerful numerical tools based on the finite-element method has been improving the prediction of the holding capacity of fixed anchors employed by the offshore oil industry. One of the main achievements of these tools is the reduction of the uncertainty related to the holding capacity calculation of these anchors. Therefore, it is also possible to reduce the values of the associated design safety factors, which have been calibrated relying on models with higher uncertainty, without impairing the original level of structural safety. This paper presents a study on the calibration of reliability-based safety factors for the design of torpedo anchors considering the statistical model uncertainty evaluated using results from experimental tests and their correspondent finite-element-based numerical predictions. Both working stress design (WSD) and load and resistance factors design (LRFD) design methodologies are investigated. Considering the WSD design methodology, the single safety is considerably lower than the value typically employed in the design of torpedo anchors. Moreover, a LRFD design code format for torpedo anchors is more appropriate since it leads to designs having less-scattered safety levels around the target value.

1. Introduction

Torpedo anchors, as shown in Figure 1, are a cost-effective and practical solution for anchoring taut mooring lines of floating units as shown in Figure 2. This anchor has a "rocket" shape and is embedded into the seabed by free fall using its own weight as driven energy. The structure of a typical torpedo anchor usually comprises four different components [1, 2]:

(a) (b)

Figure 1: Typical torpedo anchor with four flukes: (a) conical tip and (b) top with detail of the padeye.

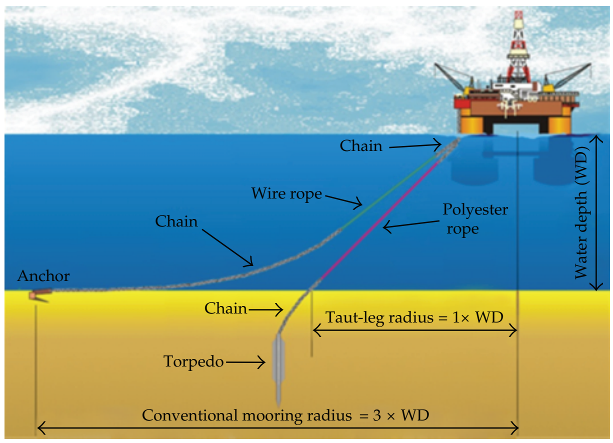

Figure 2: Torpedo anchor for mooring lines of floating units.

a padeye that connects the first chain segment of the mooring line to the anchor; a ballasted shaft of carbon steel; four flukes, which are approximately rectangular (torpedo anchors with no flukes may also be employed); a conical tip that is designed to help the embedment of the anchor. Typical torpedo anchors weight varies from 350 kN up to 980 kN.

These anchors have been first employed to hold flexible risers just after their touchdown points (TDPs) in order to avoid that the loads generated by the floating unit movements were transferred to subsea equipments, such as wet Christmas trees [1]. In 2003 a high holding capacity torpedo anchor was designed to sustain the loads imposed by the mooring lines of a floating production, storage, and offloading (FPSO) [3] and, since then, various other floating production systems have been anchored using the torpedo concept [4]. Furthermore, recently, Henriques Jr. et al. [4] presented a new torpedo anchor with a total weight of 1200 kN and larger flukes that will be employed to moor FPSOs in ultradeep waters. These characteristics, altogether, significantly increase its hold capacity when compared to the traditional anchors with 980 kN in weight.

Due to their "rocket" shape, the load position and inclination with respect to their flukes, traditional methods, such as conventional beam-column analysis procedures (otherwise known as the P-Y method) or the simple formulae proposed by API guidelines [5] for predicting the axial capacity of slender piles, often cannot be directly employed to estimate the holding capacity of these anchors.

However, numerical procedures, based on the finite element (FE) method, have recently been proposed by Brandão et al. [2] and de Sousa et al. [6]), and experimental tests performed by PETROBRAS have demonstrated their good accuracy [7]. An important outcome of these FE-based models is the lower uncertainty in their predictions when compared to traditional methods. Therefore, it is possible to reduce the design safety factors values that have been calibrated relying on prediction models with higher uncertainty, without impairing the original level of structural safety.

There is no specific guideline for the design of torpedo anchors. Therefore, it is common practice to follow the indications from API RP 2A [5] for piles foundations to support offshore structures. This guideline prescribes that the safety factor associated with these piles, according to a WSD design methodology, under operating environmental conditions and during producing operations is equal to 2.0. It seems that this relatively large safety factor is mainly related to the high prediction model uncertainty due to the large number of pile capacity prediction models available at the moment of the calibration process.

This paper presents a study on the calibration of safety factors for the ultimate limit state design of torpedo anchors. One important aspect of this study is the availability of results from holding capacity tests of six torpedo anchors installed in Campos Basin, offshore Brazil [7]. These results made it possible to assess the model uncertainty statistics associated with the FE-based model proposed by de Sousa et al. [6].

Both working stress design (WSD) and load and resistance factors design (LRFD) methodologies are investigated. Concerning the traditional WSD methodology, it is shown that, for the same actual design safety level of the traditional offshore piles, its single safety factor can be significantly lowered. However, the use of WSD design methodology results in designs with very scattered safety levels depending on the ratio of the functional and environmental load actions. Aiming at overcoming this drawback, a LRFD methodology calibration is also investigated in this paper. The results show that the structural safety levels of LRFD-based designs are more uniform than the WSD-based ones.

In the following sections, firstly, a brief review of the FE model proposed by de Sousa et al. [6] is presented followed by a comparison between the theoretical and experimental holding capacities. After that, the reliability based-design study is presented, and, finally, the main conclusions of this work are stated.

2. Finite-Element (FE) Model

2.1. Overview

Recently, de Sousa et al. [6] have proposed a FE model devoted to predict the undrained load capacity of torpedo anchors. This model employs isoparametric solid finite elements to model both the soil medium and the anchor. These elements are capable of representing the physical nonlinear behavior of the soil medium and of the anchor itself. Large deformations may also be considered. The soil-anchor interaction is ensured by surface-to-surface contact elements placed on the external surface of the anchor and the surrounding soil medium. A general overview of this model is presented in Figure 3. Next, its main aspects are briefly reviewed.

2.2. Soil Modeling

2.2.1. Constitutive Matrix

The soil is assumed to be a perfectly elastoplastic isotropic material with physical properties variable with depth. Hence, the relation between stresses and strains is given in the following form:

$$[\boldsymbol{\Delta\sigma}] = [\mathbf{B_{ep}}] \cdot [\boldsymbol{\Delta\varepsilon}]. \tag{2.1}$$

The constitutive elastoplastic matrix, $[\mathbf{B_{ep}}]$, is given by [9] as follows:

$$[\mathbf{B_{ep}}] = [\mathbf{B_{el}}] - \frac{[\mathbf{B_{el}}] \cdot \{\partial P([\boldsymbol{\sigma}], [\mathbf{m}])/\partial\boldsymbol{\sigma}\} \cdot \{\partial F([\boldsymbol{\sigma}], [\mathbf{k}])/\partial\boldsymbol{\sigma}\}^T \cdot [\mathbf{B_{el}}]}{\{\partial F([\boldsymbol{\sigma}], [\mathbf{k}])/\partial\boldsymbol{\sigma}\}^T \cdot [\mathbf{B_{el}}] \cdot \{\partial P([\boldsymbol{\sigma}], [\mathbf{m}])/\partial\boldsymbol{\sigma}\}}. \tag{2.2}$$

The elastic total stress constitutive matrix can be split into two parts, as follows [9]:

$$[\mathbf{B_{el}}] = [\mathbf{B_{eff}}] + [\mathbf{B_{pore}}]. \tag{2.3}$$

In order to represent the nonlinear material behavior of the soil, the Drucker-Prager model was chosen. This model approximates the irregular hexagon of the Mohr-Coulomb failure surface by a circle, and, consequently, the Drucker-Prager yield function is a cylindrical cone. The yield function, $F(\cdot,\cdot)$, and plastic potential function, $P(\cdot,\cdot)$, for this model are given by [10] as follows:

$$\begin{aligned} F([\boldsymbol{\sigma}], [\mathbf{k}]) &= \sqrt{J_2} + k_{\mathrm{DP}}^{(1)} \cdot I_1 + k_{\mathrm{DP}}^{(2)}, \\ P([\boldsymbol{\sigma}], [\mathbf{m}]) &= \sqrt{J_2} + m_{\mathrm{DP}}^{(1)} \cdot I_1 + m_{\mathrm{DP}}^{(2)}, \end{aligned} \tag{2.4}$$

where $k_{\mathrm{DP}}^{(1)}$, $k_{\mathrm{DP}}^{(2)}$, $m_{\mathrm{DP}}^{(1)}$, and $m_{\mathrm{DP}}^{(2)}$ are stress state parameters. These parameters depend on the adopted approximation to the Mohr-Coulomb hexagon and are functions of the internal friction angle, ϕ, the dilation angle, ψ, and the cohesion, c, of the soil. Figure 4 presents three possible approximations. In the proposed model, the circumscribed Drucker-Prager approximation is used. It is assumed that the Drucker-Prager circle touches the Mohr-Coulomb hexagon at a Lode angle of +30°, which leads to the parameters [9] as follows:

$$k_{\mathrm{DP}}^{(1)} = \frac{2 \cdot \mathrm{sen}(\phi)}{\sqrt{3} \cdot [3 + \mathrm{sen}(\phi)]}, \qquad m_{\mathrm{DP}}^{(1)} = \frac{2 \cdot \mathrm{sen}(\psi)}{\sqrt{3} \cdot [3 + \mathrm{sen}(\psi)]}, \qquad k_{\mathrm{DP}}^{(2)} = m_{\mathrm{DP}}^{(2)} = \frac{6 \cdot c \cdot \cos(\phi)}{\sqrt{3} \cdot [3 + \mathrm{sen}(\phi)]}. \tag{2.5}$$

It is worth mentioning that the rules for designing deep offshore foundations, such as the API RP 2A [5], typically assume that loading occurs rapidly enough so that no drainage

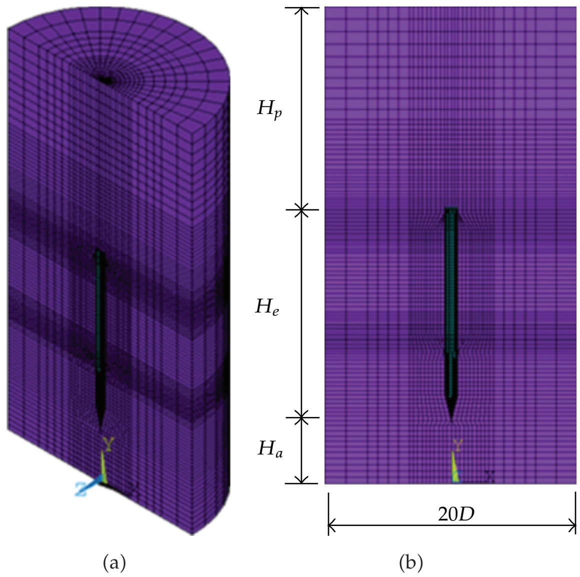

Figure 3: General view of a typical FE mesh.

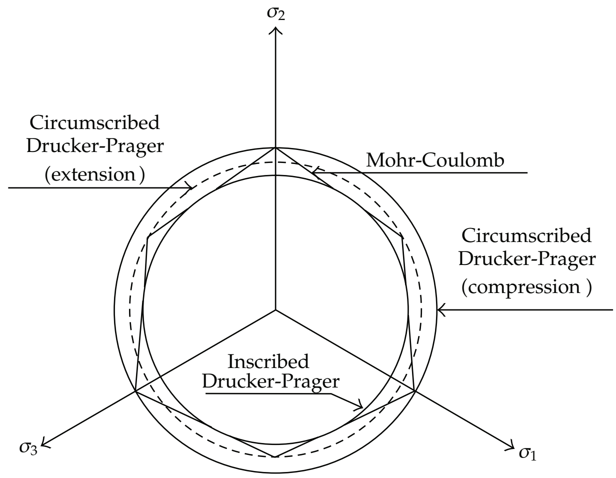

Figure 4: Drucker-Prager and Mohr-Coulomb yield surfaces in the deviatoric plane.

and hence no dissipation of excess pore pressures occur. This leads to an undrained condition and, as a consequence, the undrained shear strength of the soil, s_u, may replace the cohesion, c, in (2.5).

On the other hand, if the rate of loading is slow enough such that no excess pore pressures are developed and sufficient time has elapsed since any previous application of stresses such that all excess pore pressures have been dissipated, a drained analysis would have to be performed. Lieng et al. [11], however, indicate that the undrained loading capacity of a deep penetrating anchor, such as a torpedo anchor, is quite close to the drained capacity, and this is also assumed in this work. Nevertheless, this aspect has yet to be validated for torpedo anchors.

2.2.2. FE Mesh Characteristics

The undrained response of a torpedo anchor embedded in clay is a classical J_2 elastoplastic problem that requires no locking for incompressible materials and reasonable bending behavior in order to obtain acceptable answers. Moreover, as high plastic strains are expected in the soil, the elements may become highly distorted, and insensitivity to these distortions is demanded. Finally, coarse meshes are often required for computational efficiency, and elements capable of providing sufficient accuracy are also needed. Aiming at addressing all these requirements, a possible choice, according to Wriggers and Korelc [12], is the use of enhanced strain hexahedrical and prismatic isoparametric finite elements with three degrees of freedom at each node: translations in directions X, Y, and Z. These elements avoid both bend and volumetric locking by adding 13 internal degrees of freedom to the element [13, 14].

An outline of the main dimensions of the soil mesh is shown in Figure 3. The proposed mesh is a cylinder with a base diameter of $20D$, where D is the pile diameter. The height of the cylinder is given by the sum of the penetration of the torpedo anchor, H_p, the length of the anchor, H_e, and the distance of the tip of the torpedo anchor to the bottom of the FE mesh, H_a.

Each "slice" of the cylinder has its own physical properties, and, consequently, variable strength, density, longitudinal and transverse moduli, and Poisson coefficients may be assumed.

The vertical displacement of the cylinder is restrained at the nodes associated with its base. The displacements in X and Z directions (see Figure 3) of nodes in the outer wall of the cylinder are restrained. Once a plane of symmetry is found, the out-of-plane displacements are also restrained. In order to avoid any influence of these boundary conditions on the response of the anchor, several mesh tests were performed. In these tests, the cylinder diameter and the distance from the tip of the anchor to the bottom of the mesh (H_a) were varied. It was observed that a diameter of $20D$ for the cylinder and a distance of 5.0 m for H_a was enough to simulate an "infinite" media.

2.3. Anchor Modeling

The torpedo anchor is modeled with eight node isoparametric solid elements analogous to those used in soil representation. These elements, as pointed out before, are capable of considering both material and geometrical nonlinearities. A typical FE mesh of a torpedo anchor is shown in Figure 5.

In order to properly represent the bending of the flukes and the shaft and, consequently, avoid shear locking problems, the numerical integration of each element is performed with the enhanced strain method.

It is worth mentioning that neither the padeye at the top of the anchor nor the mooring line is represented in the proposed model. The load from the mooring line is applied at the gravity center of the padeye, where a node is placed and rigidly connected to the top of the anchor by rigid bars, as presented in Figure 6.

2.4. Anchor-Soil Interaction

The model proposed by de Sousa et al. [6] employs contact elements that allow relative large displacement and separation between the surfaces in contact. These elements are placed on the external surface of the torpedo anchor and on the surrounding soil. A contact detection

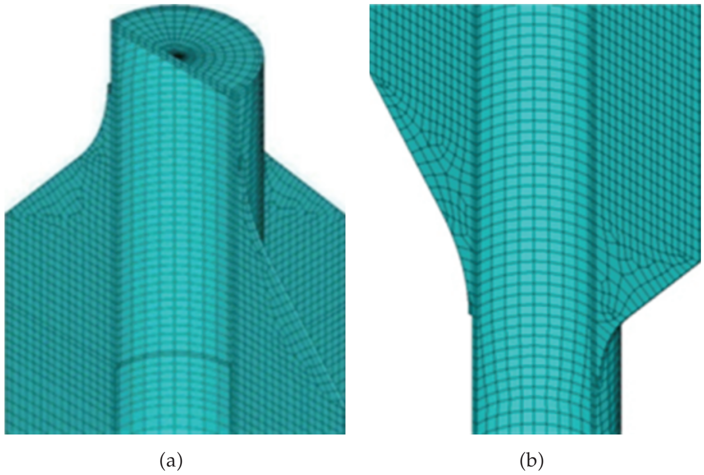

Figure 5: General view of a FE mesh for a torpedo anchor: (a) top and (b) bottom part of the flukes.

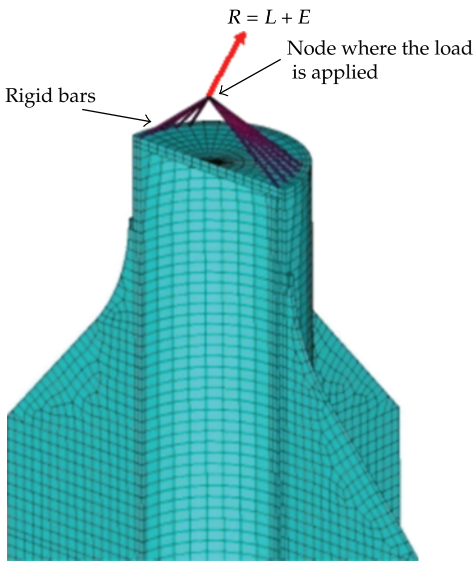

Figure 6: Load application.

algorithm based on the pinball technique and contact forces evaluated with the augmented Lagrangian method are employed in the formulation of these elements.

Relative sliding between the anchor and the soil is allowed when the shear stresses along the outer wall of the anchor exceed the values estimated with the Mohr-Coulomb friction model, which is given by [5] as follows:

$$\tau_{\max}(z) = \alpha(z) \cdot s_u(z) + K_0(z) \cdot p_o(z) \cdot \tan(\chi). \tag{2.6}$$

The friction angle between the soil and the anchor wall, χ, is stated as [5]

$$\chi = \phi - 5^{\circ}. \tag{2.7}$$

The adhesion factor, α, may be calculated by the following formula [5]:

$$\alpha(z) = \begin{cases} 0.5 \cdot \psi(z)^{-0.5}, & \psi(z) \leq 1.0, \\ 0.5 \cdot \psi(z)^{-0.25}, & \psi(z) > 1.0, \end{cases} \tag{2.8}$$

where

$$\psi(z) = \frac{s_u(z)}{p_o(z)}. \tag{2.9}$$

2.5. Solution Procedure and Implementation

The FE analysis is divided in three different steps. In the first one, the initial stress state, *that is*, the stresses in the soil prior to the imposition of any kind of structural load to the anchor, is generated. The model proposed by de Sousa et al. [6] does not simulate the anchor penetration in the soil, and it is assumed, by hypothesis, that the anchor is loaded with the soil stress state undisturbed by the installation process. The anchor is thereby supposed to be "wished in place," and this undisturbed stress state is imposed to the soil in the first step of the analysis.

However, the undrained shear strength of the soil and its stress state are deeply affected by the dynamic installation process and the setup time for reconsolidation or recovery of shear strength after installation [15]. The direct consideration of this modified stress state and undrained shear strength is quite complex and, therefore, the hypothesis of an undisturbed stress state, as employed in this work, is commonly assumed in the design of different types of offshore anchors such as suction anchors or other types of dynamic penetrating anchors [16–18]. However, the validity of this hypothesis is subject of intense debate. Lieng et al. [11], relying on FE simulations, postulate that reconsolidation after installation of a typical dynamic anchor would occur in less than six months, and, as a consequence, a holding capacity determined from undrained analyses with an adhesion factor close to 1.0 is expected to be valid. Richardson et al. [19], relying on centrifuge tests and theoretical models, indicate that 50% of the operational capacity of a dynamic anchor is attained 35 up to 350 days after its installation. According to the same authors, 90% of the operational capacity is reached 2.4 up to 24 years after the installation of the anchor. These values are extremely dependent on the horizontal coefficient of consolidation, and, therefore, authors recommend that dynamic anchors should be installed in soils that consolidation proceeds quickly. Furthermore, Richardson et al. [19] also indicate that values higher than 1.0 for the adhesion factor are reached in the long term leading to the conclusion that the undrained holding capacity of the anchor would increase with time, and this capacity would even be higher than the one estimated with an undisturbed stress state. Mirza [20] state that the time required by offshore piles to reach their full-operational capacity varies between 1 and 2 years, and Jeanjean [21] indicates that 90% of the operational capacity of suction anchors installed in soft Gulf of Mexico clays is attained in 90 days or less.

A typical torpedo anchor is firstly loaded approximately three months after its installation, and, therefore, a great part of its full operational capacity has already been reached. Moreover, the initial imposed loads are normally much lower than its actual load capacity, as its design relies on the maximum load obtained in mooring analyses that consider both intact and damaged (one mooring line broken) conditions in the mooring system and also several environmental load cases. These load cases encompass, for instance, extreme waves and currents. Hence, as the objective of the proposed FE model is to evaluate the undrained operational holding capacity of the anchor, it seems to be reasonable to assume that full reconsolidation is achieved before the maximum operational load is imposed to the anchor. Nevertheless, in order to improve the proposed design method, more experimental and also theoretical studies on this aspect have to be performed.

In the second step of the analysis, the self weight of the anchor is imposed. The analysis is restarted with the stress state from the first step, and gravity acts on the anchor. Finally, in the third step, the total load is applied with adaptable increments since, as the surrounding soil progressively fails, the stability of the solution is affected, and lower load increments are required. Typically, this last load step starts with an initial load increment of 5% of the total load applied, and as the analysis progresses, it can be reduced to 1% of the total load.

At each load step, convergence is achieved if the Euclidian norm (L2) of the residual forces and moments is less than 0.1% of the absolute value of the total forces and moments applied.

The model was implemented in a program called ESTACAS. This software generates FE meshes to be analyzed with ANSYS [22] program, where the following finite elements are employed: SOLID185 in order to model the soil and the torpedo anchor; CONTA174 and TARGE170 are used to simulate the contact between the soil and the anchor.

3. Comparison with Experimental Tests

Aiming at validating the torpedo anchor geotechnical and structural design methodology, PETROBRAS conducted six full-scale tests in Campos Basin, offshore Brazil [7]. These six full-scale tests were divided into two sets.

In the first set of tests, three torpedoes (type 1), each one weighing 350 kN, were installed at a location with a water depth of 500 m. At this location, soil was characterized as being pure clay after cone penetration tests (CPTs) had been performed. The torpedoes embedment depths varied from 8 m up to 11 m. Besides, the torpedoes were pulled out with load inclinations of about 40° [7]. The second set of tests consisted of pulling out three torpedoes (type 2) weighting 430 kN each. The water depth at this second location is 150 m, and the soil comprises both clay and sand layers. The torpedoes final embedment depths, in this case, varied from 6 m to 8 m, and the load was imposed with inclinations varying from 50° to 67°.

Table 1 presents the relation between the experimental measured pull-out loads and those estimated with the model proposed by de Sousa et al. [6].

Despite all uncertainties related to the model and the geotechnical parameters involved, Table 1 indicates that the pull-out loads estimated with the FE model agree quite well to the measured loads. Hence, these results will serve as a basis to the reliability-based design that is presented next.

4. Reliability-Based Design

Modern probabilistic design codes aim at designing new structures with a specified target probability of failure. This can be achieved by fully probabilistic or deterministic codes [23]. The latter is more common in engineering design. In these codes the target probability of failure is not stated but it is implicitly achieved by means of safety factors. These safety factors

Table 1: Relation between measured and numerical pull-out loads.

Torpedo	Experimental/numerical ratio	
	T35	T43
1	1.04	1.11
2	0.84	1.08
3	0.85	1.04

are calibrated by using standard structural reliability methods [23, 24], in which all random variables (and their uncertainties) are represented by means of probability distributions.

The two main deterministic design methodologies for ultimate limit state (ULS) are the working stress design (WSD) and load and resistance factor design (LRFD). Focusing specifically on the case of torpedo anchors for mooring systems (see Figure 6) under the WSD methodology, the design equation can be written as

$$\frac{R_k}{S_F} \geq L_k + E_k, \tag{4.1}$$

where the characteristic functional load effect, L_k, is associated with the initial mooring line pretension applied to the anchor, and the characteristic environmental load effect, E_k, corresponds to the total tension acting on the anchor, when environmental loads are applied, minus the functional one. According to this classification, one can observe that the functional load effect has smaller uncertainties than the environmental one.

In the case of the LRFD methodology, the design equation can be written as

$$R_k \geq \gamma_L L_k + \gamma_E E_k, \tag{4.2}$$

where γ_L and γ_E are the partial safety factors associated with functional and environmental load effects, respectively.

The safety factors of (4.1) and (4.2) are obtained by means of a calibration process. The main idea of the calibration process is to obtain safety factors that guarantee a certain target failure probability, pf_T, for the design. The target probability of failure may be stated as

$$\mathrm{pf}_T = \Phi(-\beta_T). \tag{4.3}$$

The calibration process is undertaken by means of reliability analyses considering various design cases with different E_k/L_k ratios and also different statistical descriptions of loadings and anchor resistances that are expected to be found in practice. Then, the safety factors can be defined, for instance, for LRFD methodology (similar procedure applies to the WSD methodology) through an optimization process to solve the following problem:

$$\min g(\gamma_L, \gamma_E) = \frac{1}{N}\sum_{i=1}^{N}\left(\beta_T - \beta_i(\gamma_L, \gamma_E)\right)^2, \tag{4.4}$$

where β_i is the reliability index of a given torpedo anchor design case that can be obtained by a reliability analysis method, for example, first-order reliability method (FORM), second-order reliability method (SORM), or Monte Carlo simulation [23, 24]. In order to pursue this kind of analysis, firstly, the limit state function must be defined. For the present study, it can be stated as

$$G(\mathbf{X}) = R - L - E, \tag{4.5}$$

where $\mathbf{X}$ is the vector containing the random variables associated with the torpedo anchor holding capacity (R), functional load (L), and environmental load (E). The variable R can be represented by

$$R = C \cdot R_M, \tag{4.6}$$

where R_M is the predicted holding capacity evaluated by a given methodology of analysis, and C is the modeling uncertainty that represents the statistical bias between the measured and predicted pile-bearing capacity. Therefore, some experimental data is necessary to statistically define this random variable. In fact, C takes into account all uncertainties regarding the soil parameters, model representation, and so forth.

In this work, all reliability analyses have been performed with Monte Carlo simulation approach which is applied to solve the following multidimensional integral:

$$\mathrm{pf} = \Phi(-\beta) = \int_{\Omega} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}, \tag{4.7}$$

where Ω is the failure domain, that is, $G(\mathbf{X}) \leq 0.0$, and $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability distribution of the random variables $\mathbf{X}$ for the case under consideration.

4.1. Safety Factor Calibration

4.1.1. Random Variables Modeling

Tests with six torpedo anchors installed in clay and clay-sand soils have recently been performed [7]. The measured torpedo anchor holding capacities has been divided by their corresponding predicted numerical estimation, R_M, according to the model presented before, to determine the model uncertainty, C, statistics. The values obtained are presented in Table 1. The mean, μ, and standard deviation, $\overline{\sigma}$, for C according to Table 1 data correspond to 0.993 and 0.118, respectively. In order to properly take into account the limited number of experiments ($N = 6$) in the reliability analysis, it is necessary to apply some statistical modeling technique for reduced sample data as explained in Ditlevsen and Madsen [25]. Assuming that the C variable has a previous lognormal distribution, as indicated in Fenton [8], this variable should be modeled by the following Student's t-distribution to account for the number N of experiments:

$$f_C(\mathbf{x}) = \frac{1}{\xi\mathbf{x}} \sqrt{\frac{d}{d+2}} \frac{\Gamma((d+1)/2)}{\Gamma(d/2)\sqrt{d\pi}} \left(1 + \frac{t(\mathbf{x})^2}{d}\right)^{-(1/2)(d+1)}; \quad d = n - 1, \tag{4.8}$$

where

$$t(\mathbf{x}) = \frac{\ln(\mathbf{x}) - \lambda}{\xi}\sqrt{\frac{d}{d+2}}, \quad n = 6,$$
$$\xi = \sqrt{\ln\left[1 + \left(\frac{\overline{\sigma}}{\mu}\right)^2\right]} = 0.1183; \qquad \lambda = \ln(\mu) - 0.5 \cdot \xi^2 = -0.0137. \tag{4.9}$$

Then, considering Table 1 data, the mean and standard deviation of this Student's t-distribution becomes

$$\mu_C = 1.003, \qquad \overline{\sigma}_C = 0.193. \tag{4.10}$$

It is important to notice that, due to the limited data sample, $\overline{\sigma}_C$ is 64% higher than the sample standard deviation, $\overline{\sigma}$. These values also show that the FE model for the torpedo-holding capacity has a lower uncertainty when compared, for instance, with prediction methods reported in an API-supported study for axial bearing capacity of single piles in clays [8]. In this latter case, the mean and standard deviation for the modeling uncertainty variable C has been, respectively, 1.04 and 0.34.

In the reliability analysis of marine structures, the functional loading is usually modeled by a normal distribution with a coefficient of variation (CoV or δ) in the order of 0.05–0.10. In this work, a CoV (δ_L) of 0.07 is assumed. The environmental load effect is usually represented by its annual extreme value distribution which, in most of the cases, corresponds to a Gumbel distribution. Considering the inherent random variability of the environmental parameters related to waves, wind and current, and also modeling uncertainties in the estimation of this complex load effect [26], it is reasonable to assume a CoV (δ_E) in the order of 0.10–0.30 for this distribution.

4.1.2. Characteristic Design Parameters

The characteristic design parameters used in (4.1) and (4.2) must be clearly defined in any calibration process or design code. Different values for these parameters are also related to different sets of safety factors.

In the present study, the characteristic value of the torpedo anchor-bearing capacity, R_k, is defined as that predicted by the FE-based model previously described [6]. The characteristic value of the functional load effect, L_k, has been assumed as its estimated mean value, that is, $L_k = \mu_L$. In the case of the environmental load effect, following the traditional design practice in offshore engineering, its characteristic value has been defined as the most probable value of the 100-yr extreme distribution. This value corresponds to that related to 99% fractile of the annual extreme cumulative distribution [27].

In the calibration process, a broader range of location water depths for the floating units has been assumed by considering the ratio L_k/E_k to be between 1.0 and 5.5. The lower ratio is associated with deeper waters in which the functional load at the anchor can be of the same order as the environmental one. The converse applies to the higher ratio. These ratios account for actual floating systems installed in water depths ranging approximately from 200 m to 2500 m [28].

4.1.3. Target Safety Index

In order to obtain the target safety index β_T for the calibration process, firstly, all cases investigated have been designed according to the API guidelines [5] considering an intact mooring system, that is, using the WSD methodology with a safety factor equals to 2.0. Secondly, the reliability index of each case has been evaluated considering the uncertainty modeling mean and standard deviation parameters as 1.04 and 0.34, respectively, taken from Fenton [8]. Finally, the target reliability index has been taken as the average of all these reliability indexes. Considering specifically cases for $\delta_E = 0.10, 0.125, \ldots, 0.30$, the target reliability index, β_T, has been found to be approximately 2.9. Figure 7 illustrates the results that have been obtained. In summary, the target reliability index assumed in this work is more or less the same that is implicitly assumed when a single pile in clay is designed to support an axial loading.

4.1.4. WSD Calibration

The optimized WSD safety factor, S_F, for the holding capacity design of torpedo anchors (see (4.1)), considering the previously discussed FE model uncertainty, has been found to be 1.45. Figure 8 shows a comparison between the target reliability index and the reliability indexes of all design cases investigated considering the optimized safety factor. When compared to the safety factor of 2.0, which is usually employed by the offshore oil industry, the modeling uncertainty reduction obtained with the FE model causes a significant reduction in the safety factor.

Comparing Figures 7 and 8, it is important to notice that a larger model uncertainty also results in more scattered reliability indexes among the design cases.

4.1.5. LRFD Calibration

As it is well known in the public literature [29], the use of a single safety factor in the WSD methodology leads to more scattered designs concerning their safety levels. The LRFD methodology, which attributes partial safety factors for different loading sources (see (4.2)), is an alternative to reduce this scatter.

In order to obtain less-scattered reliability indices for design cases considered in the calibration process, two sets of safety factors (γ_L, γ_E) have been identified. The LRFD methodology design check for the holding capacity of torpedo anchors will be read as

$$R_k \geq \max(T_{1k}, T_{2k}), \tag{4.11}$$

where

$$\begin{aligned} T_{1k} &= 1.35 \cdot L_k + 1.43 \cdot E_k, \\ T_{2k} &= 1.72 \cdot L_k + 1.27 \cdot E_k. \end{aligned} \tag{4.12}$$

The two sets of safety factors in (4.12), (1.35, 1.43) and (1.72, 1.27), have been obtained by applying the optimization process (4.4) twice: firstly, for all design cases in the range $1.0 \leq L_k/E_k \leq 2.0$ and, after that, for the other design cases in the range $2.0 < L_k/E_k \leq 5.0$.

A comparison between the target reliability index and the reliability indexes of all cases investigated considering the LRFD design, (4.4), is shown in Figure 9.

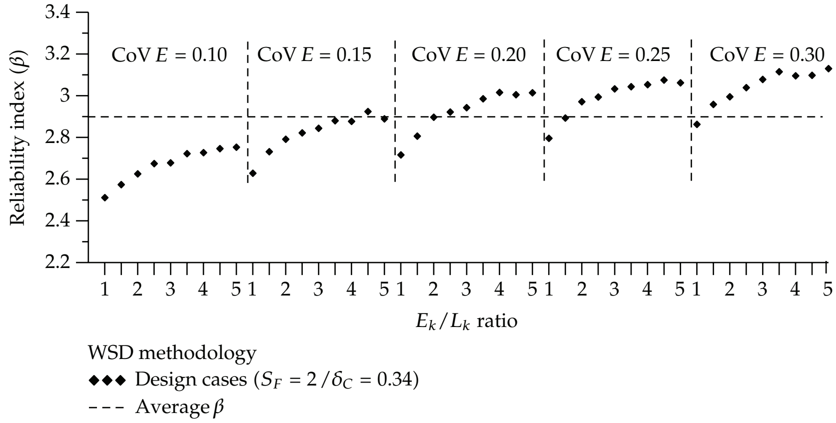

Figure 7: Reliability indexes of designs considering WSD methodology with $S_F = 2.0$ and $\delta_C = 0.34$ [8].

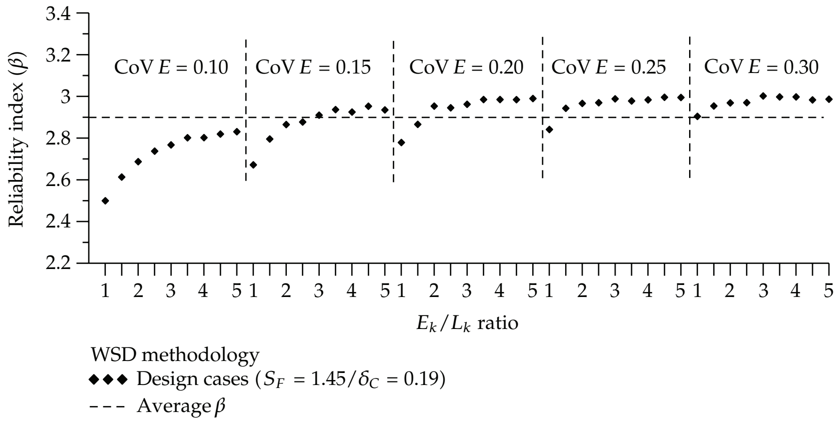

Figure 8: Reliability indexes of calibrated WSD designs considering $\delta_C = 0.19$ (experimental tests).

For both WSD and LRFD design methodologies, the average of safety index values has been the target reliability index $\beta_T = 2.9$; however, their coefficients of variation have been, respectively, 4.5% and 2.5%. This shows that LRFD methodology gives designs with safety levels that are slightly less scattered around the target value established for the calibration process.

5. Final Remarks

This paper has presented a reliability-based design study for torpedo anchors considering that their ultimate bearing capacity is predicted numerically by a nonlinear finite element model that represents the soil medium and the anchor itself. By considering the results of six full-scale experimental pull out tests, it was observed that the numerical model is practically unbiased and presents a relatively low level of statistical uncertainty. Using the model uncertainty statistics in reliability-based calibration process, it was shown that the same level of structural safety implied in the traditional design of offshore piles can be

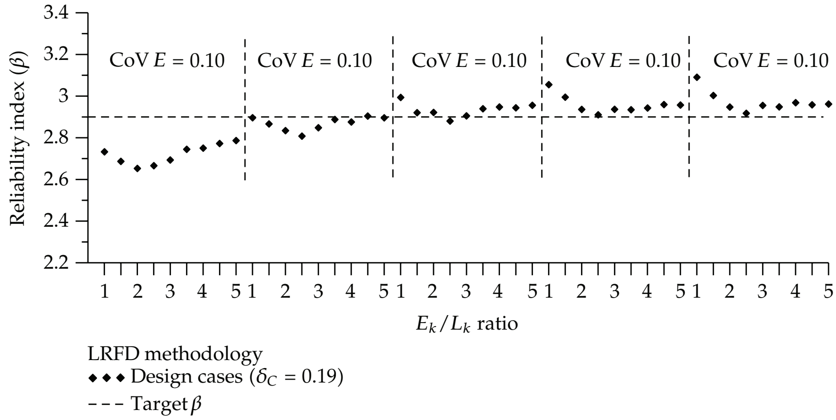

Figure 9: Reliability indexes of calibrated LRFD designs considering $\delta_C = 0.19$ (experimental tests).

achieved with the use of lower safety factors. For instance, considering the WSD design methodology, the single safety factor has a significant reduction dropping from 2.0 to 1.45.

This calibration study has also shown that a LRFD design code format for torpedo anchors is the most appropriate one since it leads to designs having less-scattered safety levels around the target value. In this case, different safety factors are proposed for the environmental and functional parcels of the total load. These factors are also lower than the one proposed by API RP 2A [5].

An important point to be noticed is that it is very likely that the safety factors presented in this paper can become even lower when more experimental tests are performed since a great part of the model uncertainty considered in the present work comes from the very limited number of experiments available. More experiments are thereby required in order to estimate these new safety factors.

Nomenclature

$\mathbf{B}_{\mathbf{el}}$:	Purely elastic total stress constitutive matrix
$\mathbf{B}_{\mathbf{eff}}$:	Effective stress constitutive matrix
$\mathbf{B}_{\mathbf{ep}}$:	Elastoplastic constitutive matrix of the soil
$\mathbf{B}_{\mathbf{pore}}$:	Constitutive matrix of the fluid
c:	Cohesion of the soil
C:	Model uncertainty
D:	Diameter of the torpedo anchor including its flukes
E:	Environmental load effect applied to the anchor
$F(\cdot,\cdot)$:	Yield function
$G(\cdot)$:	Limit state function
H_a:	Distance of the tip of the torpedo anchor to the bottom of the FE mesh
H_e:	Length of the torpedo anchor
H_p:	Penetration of the torpedo anchor into the soil (embedment depth)
I_1:	First invariant of the stress tensor
J_2:	Second invariant of the stress tensor
$\mathbf{k} = (k^{(1)}, k^{(2)})$:	Stress state parameters related to the yield function

K_0: Coefficient of lateral earth pressure
L: Functional load effect applied to the anchor
LRFD: Load and resistance factors design
$\mathbf{m} = (m^{(1)}, m^{(2)})$: Stress state parameters related to the plastic potential function
N: Total number of design cases considered in the calibration process
$P(\cdot,\cdot)$: Plastic potential function
p_o: Effective overburden pressure
pf_T: Target probability of failure
R: Holding capacity of the anchor
s_u: Undrained shear strength of the soil
S_F: Safety factor
WSD: Working stress design
$\mathbf{X}$: Random variables vector
z: Depth below mudline.

Greek

$\boldsymbol{\Delta\varepsilon}$: Incremental total strain vector
$\boldsymbol{\Delta\sigma}$: Incremental total stress vector
α: Adhesion factor
$\beta_i(\cdot,\cdot)$: Reliability index of ith case designed using the current set of safety factors $(\cdot,\cdot)$
β_T: Reliability index associated with pf_T
χ: Friction angle between the soil and the torpedo anchor wall
δ: Coefficient of variation
ϕ: Internal friction angle of the soil
$\Phi(\cdot)$: Standard normal cumulative distribution
γ_L, γ_E: Partial safety factors applied to functional and environmental load effects, respectively
μ: Mean
$\boldsymbol{\sigma}$: Stress state in the soil
$\overline{\sigma}$: Standard deviation
σ_y: Equivalent failure stress
$\tau_{\max}$: Limiting shear stress
ψ: Dilation angle of the soil.

Subscripts

C: Model uncertainty
DP: Related to the Drucker-Prager criterion
E: Environmental load effect
k: Related to characteristic (or nominal) values
L: Functional load effect
M: Related to model predictions
T: Target value.

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