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Research Article

Novel Stability Criteria of Nonlinear Uncertain Systems with Time-Varying Delay

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The problem of robust exponential stabilization for dynamical nonlinear systems with uncertainties and time-varying delay is considered in the paper. By constructing the proposed Lyapunov-Krasovskii functional approach, continuous state feedback controllers are put forward, and the criteria which guarantee the exponential stabilization of the nonlinear systems with uncertainties and time-varying delay are established in terms of solutions to the standard Riccati differential equations. Furthermore, based on the Lyapunov method and the linear matrix inequality approach, the sufficient conditions of exponential stability for a class of uncertain systems with time-varying delays and nonlinear perturbations are derived. Finally, two numerical examples are given to demonstrate the validity of the results.

1. Introduction

The stability problem and stabilization problem of time-delay systems are important problems not yet completely solved and continuously investigated by many people. For control systems with delayed state, existing stability criteria can be classified into two categories, that is, delay-independent ones and delay-dependent ones, and delay-independent ones are usually more conservative than the delay-dependent ones. On the other hand, it is unavoidable to include uncertain parameters and perturbations in practical control systems due to modeling errors, measurement errors, approximations, and so on. Therefore, the robust stabilization problem of uncertain dynamical systems with time-varying delays has attracted considerable attention of many researchers in recent years, and most of these papers have always adopted linear matrix equalities to guarantee the exponential stability of dynamical systems by employing Leibniz-Newton formula or different transformations.

For instance, by employing a descriptor model transformation and a decomposition technique of the delay term matrix, the robust stability of uncertain linear systems with a single time-varying delay and nonlinear perturbations is investigated in [1]. Without introducing any free weighting matrices, in [2], new delay-dependent stability criteria for time-delay systems have been derived by introducing a new type of Lyapunov functional which contains some triple-integral terms and fully uses the information on the lower bound of the delay. In [3], the exponential stability of linear distributed parameter systems with time-varying delays is studied through Linear Operator Inequalities which are reduced to standard Linear Matrix Inequalities. Based on the Lyapunov-Krasovskii functional approach, in [4], a stability criterion of uncertain linear systems with interval time-varying delay is derived by introducing some relaxation matrices that can be used to reduce the conservatism of the criteria. In terms of linear matrix inequality, [5] proposes a new delay-dependent stability criterion for dynamic systems with time-varying delays and nonlinear perturbations. In [6], a class of linear systems with unknown norm-bounded uncertainties and time-varying delays is investigated with Leibniz-Newton formula and linear matrix inequalities, which allow computing simultaneously the two bounds that characterize the exponential stability rate of the solution. In [7], the system is transformed into another one by the transformation of $z(t) = e^{\alpha t}x(t)$, and then delay-dependent stability criteria have been derived in terms of a matrix inequality (LMI) which can be easily solved using efficient convex optimization algorithms. In [8], by constructing a suitable augmented Lyapunov's functional and utilizing free weight matrices, the criterion for stabilization of uncertain dynamic systems with time-varying delays is established in terms of linear matrix inequalities. Using the method of the matrix equality, [9] has presented some improved stability criteria to guarantee that the uncertain systems with two successive delay components are robustly, asymptotically stable. In [10, 11], the stabilization of uncertain dynamic systems is considered. In [12], the stability analysis problem of linear neutral delay differential systems with multiple time delays is investigated. Using the Lyapunov method, some sufficient conditions are presented for the asymptotic stability of systems. In [13], the problem of robust stabilization of a class of nonlinear dynamical systems with delayed perturbations is considered. Based on the stability of the nominal systems, a new stabilizing control law for exponential stability of the system is designed using Lyapunov stability theory.

Being different from them, this paper chooses the Riccati differential equation to solve the stabilization of uncertain time-delay systems. The sufficient conditions which guarantee that the uncertain systems with time-varying delay and nonlinear perturbations are exponentially stabilizable are presented by employing Lyapunov-Krasovskii functional, and the controllers are constructed.

The rest of this paper is organized as follows. Section 2 presents notations, assumptions, and lemmas which can be used in the proof of theorems. In Section 3, the main results on the robust exponential stabilization for the uncertain systems with time-varying delay and nonlinear perturbations are given. Two numerical examples are provided to illustrate the use of our results in Section 4. Finally, the conclusion follows in Section 5.

2. Preliminaries

For convenience, we now introduce the following notations that will be employed throughout the paper. The notation R^n is used to denote the n-dimensional space. R^+ denotes the set of all real nonnegative numbers. $R^{n\times r}$ denotes the space of all $(n\times r)$ matrices. $\langle x, y\rangle$ or $x^T y$ denotes the scalar product of two vectors x and y. $\|x\|$ denotes the Euclidean vector norm of x. $\lambda(A)$ denotes the set of all eigenvalues of A. $\lambda_{\max}(A)$ is defined by $\lambda_{\max}(A) = \max\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$. $\lambda_{\min}(A)$ is defined by $\lambda_{\min}(A) = \min\{\mathrm{Re}\,\lambda : \lambda \in \lambda(A)\}$. For $h \geq 0$, $C([-h,0], R^n)$ denotes the set of all R^n-valued continuous functions mapping $[-h,0]$ into R^n. A matrix A is semipositive definite ($A \geq 0$) if $\langle Ax, x\rangle \geq 0$, for all $x \in R^n$. A is positive definite ($A > 0$) if $\langle Ax, x\rangle > 0$ for all $x \neq 0$. $A \geq B$ means $A - B \geq 0$. Also, x_t is defined by $x_t = \{x(t+s), s \in [-h,0]\}$, and $\|x_t\|$ is defined by $\|x_t\| = \sup_{s\in[-h,0]}\|x(t+s)\|$.

Now, let us consider a class of uncertain systems with time-varying delay and nonlinear perturbations of the form

$$\begin{aligned}\dot{x}(t) &= [A(t) + \Delta A(\upsilon,t)]x(t) + [A_1(t) + \Delta A_1(\xi,t)]x(t-h(t)) \\ &\quad + [B(t) + \Delta B(\varsigma,t)]u(t) + f(t, x(t), x(t-h(t)), u(t)), \\ x(t) &= \phi(t), \quad t \in [-h,0],\ h \geq 0,\end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control function, $A(t), A_1(t), B(t)$ are continuous matrixes of appropriate dimensions on R^+, and $\Delta A(\upsilon,t)$, $\Delta A_1(\xi,t)$, $\Delta B(\varsigma,t)$ represent the system uncertainties and are assumed to be continuous in all their arguments. Moreover, $(\upsilon,\xi,\varsigma) \in \psi$ is the uncertain vector, and $\psi \subset R^L$ is a compact set. In addition, perturbation $f(t,x(t),x(t-h(t)),u(t)) : R^+ \times R^n \times R^n \times R^m \to R^n$ is a given nonlinear function satisfying $f(t,0,0,0) = 0$ for all $t \geq 0$, and there exist scalars $a, b, d > 0$ such that

$$\|f(t,x(t),x(t-h(t)),u(t))\| \leq a\|x(t)\| + b\|x(t-h(t))\| + d\|u(t)\|, \tag{2.2}$$

for all $(t,x(t),x(t-h(t)),u(t)) \in R^+ \times R^n \times R^n \times R^m$. The initial function $\phi(t) \in C([-h,0],R^n)$, $h > 0$, has its norm $\|\phi\| = \sup_{s\in[-h,0]}\|\phi(s)\|$. The delay $h(t)$ is a continuous function satisfying

(H1) $0 \leq h(t) \leq h$, $\dot{h}(t) \leq \delta < 1$, for all $t \geq 0$.

The purpose of this paper is to design a state feedback controller $u(t) = K(t)x(t)$ such that the closed-loop system of (2.1)

$$\begin{aligned}\dot{x}(t) &= [A(t) + \Delta A(\upsilon,t) + B(t)K(t) + \Delta B(\varsigma,t)K(t)]x(t) \\ &\quad + [A_1(t) + \Delta A_1(\xi,t)]x(t-h(t)) + f(t,x(t),x(t-h(t)),u(t)), \\ x(t) &= \phi(t), \quad t \in [-h,0],\ h \geq 0,\end{aligned} \tag{2.3}$$

is robustly α-exponentially stable, that is, every solution $x(t,\phi)$ of the system satisfies

$$\exists N > 0,\ \alpha > 0, \quad \|x(t,\phi)\| \leq N\|\phi\|e^{-\alpha t}, \quad \forall t \in R^+, \tag{2.4}$$

for all the uncertainties $\Delta A(\upsilon,t), \Delta A_1(\xi,t), \Delta B(\varsigma,t)$.

Before proposing our theorems, we introduce for (2.1) the following standard assumptions and lemmas that will be needed for deriving the main results.

Assumption 1. For all $(x,t) \in R^n \times R$ there exist continuous matrix functions $H(\upsilon,t)$, $H_1(\xi,t)$, $E(\varsigma,t)$ of appropriate dimensions such that

$$\Delta A(\upsilon,t) = B(t)H(\upsilon,t), \qquad \Delta A_1(\xi,t) = B(t)H_1(\xi,t), \qquad \Delta B(\varsigma,t) = B(t)E(\varsigma,t). \tag{2.5}$$

Remark 2.1. Assumption 1 defines the matching condition about the uncertainties and is a rather standard assumption for robust control problem (see, e.g., [10, 14–16]).

Lemma 2.2 (see [15]). *For any real vectors a, b and any matrix $Q > 0$ with appropriate dimensions, it follows that*

$$2a^T b \leq a^T Q a + b^T Q^{-1} b. \tag{2.6}$$

Lemma 2.3 (see [11]). *Given constant symmetric matrices S_1, S_2, S_3, and $S_1 = S_1^T < 0$, $S_3 = S_3^T > 0$, then $S_1 + S_2 S_3^{-1} S_2^T < 0$ if and only if*

$$\begin{bmatrix} S_1 & S_2 \\ S_2^T & -S_3 \end{bmatrix} < 0. \tag{2.7}$$

3. Main Results

In this section, we will present our main results on the robust exponential stabilization of system (2.1).

Given positive numbers α, β, ε_i, $i = 1,2,3,4$, we set

$$\begin{gathered}
\omega = (\varepsilon_1 + \varepsilon_3 + \varepsilon_4 + \varepsilon_2 h), \qquad \overline{A}(t) = A(t) + \alpha I, \\
P_\beta(t) = P(t) + \beta I, \qquad \rho_\upsilon(t) = \max_{\upsilon} \| H(\upsilon,t) \|, \\
\rho_\xi(t) = \max_{\xi} \| H_1(\xi,t) \|, \qquad \mu(t) = \min_{\varsigma} \left[\frac{1}{2} \lambda_{\min} \left(E(\varsigma,t) + E^T(\varsigma,t) \right) \right], \qquad p = \sup_{t \in R^+} \| P(t) \|, \\
R(t) = -\frac{4}{\varepsilon_1 e^{-2\alpha h}(1-\delta)} A_1(t) A_1^T(t) - \left[\frac{1}{4(1+\mu(t))^2} + \frac{4\rho_\xi^2(t)}{\varepsilon_1 e^{-2\alpha h}(1-\delta)} + \varepsilon_3^{-1} \rho_\upsilon^2(t) - 1 \right] B(t) B^T(t) \\
- \left(\varepsilon_4^{-1} a^2 + \frac{2b^2}{\varepsilon_1 e^{-2\alpha h}(1-\delta)} + d^2 \right) I.
\end{gathered} \tag{3.1}$$

We need the following assumption.

Assumption 2. For any $t > 0$, $\mu(t) > -1$.

Theorem 3.1. *Suppose that condition (H1) and Assumptions 1-2 hold. If there exist positive numbers* α, β, ε_i, $i = 1,2,3,4$, *and a symmetric positive semidefinite matrix function* $P(t)$ *satisfying the following Riccati differential equation:*

$$\dot{P}(t) + \overline{A}^T(t)P_\beta(t) + P_\beta(t)\overline{A}(t) - P_\beta(t)R(t)P_\beta(t) + \omega I = 0, \tag{3.2}$$

then system (2.1) *is robustly* α*-exponentially stabilizable with feedback control*

$$u(t) = -\frac{1}{2(1+\mu(t))}B^T(t)P_\beta(t)x(t). \tag{3.3}$$

Moreover, the solution $x(t,\phi)$ *satisfies the condition*

$$\|x(t,\phi)\| \le \sqrt{\frac{p+\beta+h\varepsilon_1+(1/2)h^2\varepsilon_2}{\beta}}\|\phi\|e^{-\alpha t}, \quad t \ge 0. \tag{3.4}$$

Proof. Let $u(t) = K(t)x(t)$, where

$$K(t) = -\frac{1}{2(1+\mu(t))}B^T(t)P_\beta(t), \quad t \ge 0. \tag{3.5}$$

For the closed-loop system (2.3) of system (2.1), we consider the following Lyapunov-Krasovskii functional:

$$V(t,x_t) = V_1 + V_2 + V_3, \tag{3.6}$$

where

$$\begin{aligned} V_1 &= \langle P(t)x(t), x(t)\rangle + \beta\langle x(t), x(t)\rangle, \\ V_2 &= \varepsilon_1\int_{t-h(t)}^{t} e^{2\alpha(s-t)}\|x(s)\|^2 ds, \\ V_3 &= \varepsilon_2\int_{-h}^{0}\int_{t+\tau-h(t+\tau)}^{t} e^{2\alpha(s-t)}\|x(s)\|^2 ds\, d\tau. \end{aligned} \tag{3.7}$$

The time derivative of V along the trajectory of (2.3) is given by

$$\dot{V}(t,x_t) = \dot{V}_1 + \dot{V}_2 + \dot{V}_3, \tag{3.8}$$

where

$$
\begin{aligned}
\dot{V}_1 &= \langle \dot{P}(t)x(t), x(t)\rangle + 2\langle P_\beta(t)\dot{x}(t), x(t)\rangle \\
&= \left\langle \left(\dot{P}(t) + A^T(t)P_\beta(t) + P_\beta(t)A(t)\right)x(t), x(t) \right\rangle + 2\langle P_\beta(t)B(t)H(\upsilon,t)x(t), x(t)\rangle \\
&\quad + 2\langle P_\beta(t)A_1(t)x(t-h(t)), x(t)\rangle + 2\langle P_\beta(t)B(t)H_1(\xi,t)x(t-h(t)), x(t)\rangle \\
&\quad + 2\langle P_\beta(t)B(t)K(t)x(t), x(t)\rangle + 2\langle P_\beta(t)B(t)E(\varsigma,t)K(t)x(t), x(t)\rangle \\
&\quad + 2\langle P_\beta(t)f(t, x(t), x(t-h(t)), u(t)), x(t)\rangle, \\
\dot{V}_2 &= -2\alpha V_2 + \varepsilon_1\|x(t)\|^2 - \varepsilon_1 e^{-2\alpha h(t)}\|x(t-h(t))\|^2\left(1-\dot{h}(t)\right) \\
&\le -2\alpha V_2 + \varepsilon_1\|x(t)\|^2 - \varepsilon_1 e^{-2\alpha h}\|x(t-h(t))\|^2(1-\delta), \\
\dot{V}_3 &= -2\alpha V_3 + \varepsilon_2\int_{-h}^{0}\left[\|x(t)\|^2 - e^{-2\alpha(\tau-h(t+\tau))}\|x(t+\tau-h(t+\tau))\|^2\left(1-\dot{h}(t+\tau)\right)\right]d\tau \\
&\le -2\alpha V_3 + \varepsilon_2 h\|x(t)\|^2 - \varepsilon_2\int_{-h}^{0} e^{-2\alpha(\tau-h(t+\tau))}\|x(t+\tau-h(t+\tau))\|^2(1-\delta)d\tau \\
&\le -2\alpha V_3 + \varepsilon_2 h\|x(t)\|^2.
\end{aligned}
\tag{3.9}
$$

Applying Lemma 2.2 gives

$$
\begin{aligned}
&2\langle P_\beta(t)A_1(t)x(t-h(t)), x(t)\rangle \\
&\quad\le \frac{4}{\varepsilon_1(1-\delta)e^{-2\alpha h}}\left\langle P_\beta(t)A_1(t)A_1^T(t)P_\beta(t)x(t), x(t)\right\rangle + \frac{\varepsilon_1(1-\delta)e^{-2\alpha h}}{4}\langle x(t-h(t)), x(t-h(t))\rangle, \\
&2\langle P_\beta(t)B(t)H_1(\xi,t)x(t-h(t)), x(t)\rangle \\
&\quad\le \frac{\varepsilon_1(1-\delta)e^{-2\alpha h}}{4}\langle x(t-h(t)), x(t-h(t))\rangle \\
&\qquad + \frac{4}{\varepsilon_1(1-\delta)e^{-2\alpha h}}\left\langle P_\beta(t)B(t)H_1(\xi,t)H_1^T(\xi,t)B^T(t)P_\beta(t)x(t), x(t)\right\rangle \\
&\quad\le \frac{\varepsilon_1(1-\delta)e^{-2\alpha h}}{4}\langle x(t-h(t)), x(t-h(t))\rangle + \frac{4\rho_\xi^2(t)}{\varepsilon_1(1-\delta)e^{-2\alpha h}}\left\langle P_\beta(t)B(t)B^T(t)P_\beta(t)x(t), x(t)\right\rangle, \\
&2\langle P_\beta(t)B(t)H(\upsilon,t)x(t), x(t)\rangle \\
&\quad\le \varepsilon_3\langle x(t), x(t)\rangle + \varepsilon_3^{-1}\left\langle P_\beta(t)B(t)H(\upsilon,t)H^T(\upsilon,t)B^T(t)P_\beta(t)x(t), x(t)\right\rangle \\
&\quad\le \varepsilon_3\langle x(t), x(t)\rangle + \varepsilon_3^{-1}\rho_\upsilon^2(t)\left\langle P_\beta(t)B(t)B^T(t)P_\beta(t)x(t), x(t)\right\rangle.
\end{aligned}
\tag{3.10}
$$

Using (2.2) and (3.3), we get

$$
\begin{aligned}
&2\langle P_\beta(t) f(t, x(t), x(t-h(t)), u(t)), x(t)\rangle \\
&\quad \le 2\left\| f(t, x(t), x(t-h(t)), u(t))\right\| \left\| P_\beta(t)x(t)\right\| \\
&\quad \le 2a\|x(t)\| \left\| P_\beta(t)x(t)\right\| + 2b\|x(t-h(t))\| \left\| P_\beta(t)x(t)\right\| + 2d\|u(t)\| \left\| P_\beta(t)x(t)\right\| \\
&\quad \le \varepsilon_4^{-1}a^2 \left\| P_\beta(t)x(t)\right\|^2 + \varepsilon_4\|x(t)\|^2 + \frac{2b^2}{\varepsilon_1 e^{-2\alpha h}(1-\delta)}\left\| P_\beta(t)x(t)\right\|^2 \\
&\qquad + \frac{\varepsilon_1 e^{-2\alpha h}(1-\delta)}{2}\|x(t-h(t))\|^2 + d^2\left\| P_\beta(t)x(t)\right\|^2 + \|u(t)\|^2 \\
&\quad = \left(\varepsilon_4^{-1}a^2 + \frac{2b^2}{\varepsilon_1 e^{-2\alpha h}(1-\delta)} + d^2\right)\left\langle P_\beta^2(t)x(t), x(t)\right\rangle + \varepsilon_4\langle x(t), x(t)\rangle \\
&\qquad + \frac{1}{4(1+\mu(t))^2}\left\langle P_\beta(t)B(t)B^T(t)P_\beta(t)x(t), x(t)\right\rangle + \frac{\varepsilon_1 e^{-2\alpha h}(1-\delta)}{2}\|x(t-h(t))\|^2.
\end{aligned}
\tag{3.11}
$$

In addition, it is easy to obtain the following:

$$
\begin{aligned}
&2\langle P_\beta(t)B(t)K(t)x(t), x(t)\rangle + 2\langle P_\beta(t)B(t)E(\varsigma,t)K(t)x(t), x(t)\rangle \\
&\quad = -\frac{1}{1+\mu(t)}x^T(t)P_\beta(t)B(t)\left[I + \frac{1}{2}\left(E(\varsigma,t) + E^T(\varsigma,t)\right)\right]B^T(t)P_\beta(t)x(t) \\
&\quad \le -\frac{1}{1+\mu(t)}\lambda_{\min}\left[I + \frac{1}{2}\left(E(\varsigma,t) + E^T(\varsigma,t)\right)\right]\left\| B^T(t)P_\beta(t)x(t)\right\|^2 \\
&\quad = -\left\langle P_\beta(t)B(t)B^T(t)P_\beta(t)x(t), x(t)\right\rangle.
\end{aligned}
\tag{3.12}
$$

The last equality is got because of $\mu(t) = \min_\varsigma[(1/2)\lambda_{\min}(E(\varsigma,t) + E^T(\varsigma,t))]$.

Therefore, we get

$$
\begin{aligned}
&\dot V(t, x_t) + 2\alpha V(t, x_t) \\
&\quad \le \Bigg\langle \Bigg\{ \dot P(t) + \overline{A}^T(t)P_\beta(t) + P_\beta(t)\overline{A}(t) + \frac{4}{\varepsilon_1(1-\delta)e^{-2\alpha h}}P_\beta(t)A_1(t)A_1^T(t)P_\beta(t) \\
&\qquad + \left(\frac{1}{4(1+\mu(t))^2} + \frac{4\rho_\xi^2(t)}{\varepsilon_1(1-\delta)e^{-2\alpha h}} + \varepsilon_3^{-1}\rho_\upsilon^2(t) - 1\right)P_\beta(t)B(t)B^T(t)P_\beta(t) \\
&\qquad + \left(\varepsilon_4^{-1}a^2 + \frac{2b^2}{\varepsilon_1(1-\delta)e^{-2\alpha h}} + d^2\right)P_\beta^2(t) + (\varepsilon_1 + \varepsilon_3 + \varepsilon_4 + \varepsilon_2 h)I\Bigg\}x(t), x(t)\Bigg\rangle.
\end{aligned}
\tag{3.13}
$$

Then introducing (3.2) into (3.13) we can get

$$\dot{V}(t,x_t) + 2\alpha V(t,x_t) \le 0, \quad \forall t \ge 0. \tag{3.14}$$

It is obvious that

$$e^{2\alpha t}\dot{V}(t,x_t) + 2\alpha e^{2\alpha t} V(t,x_t) \le 0. \tag{3.15}$$

Integrating both sides of (3.15) from 0 to t, we get

$$V(t,x_t) \le V(0,x_0)e^{-2\alpha t}, \quad \forall t \ge 0. \tag{3.16}$$

On the other hand, from the expression of $V(t,x_t)$, it is easy to see that

$$\beta\|x(t)\|^2 \le V(t,x_t), \quad \forall t \ge 0. \tag{3.17}$$

In addition, since

$$\begin{gathered} V_1(0,x_0) \le (p+\beta)\|\phi\|^2, \qquad V_2(0,x_0) = \varepsilon_1 \int_{-h(0)}^{0} e^{2\alpha s}\|x(s)\|^2 ds \le \varepsilon_1 \int_{-h}^{0} \|\phi\|^2 ds \le \varepsilon_1 h \|\phi\|^2, \\ V_3(0,x_0) = \varepsilon_2 \int_{-h}^{0}\int_{\tau-h(\tau)}^{0} e^{2\alpha s}\|x(s)\|^2 ds\, d\tau \le \varepsilon_2 \int_{-h}^{0}\int_{\tau-h}^{0} \|\phi\|^2 ds\, d\tau \le \frac{1}{2}\varepsilon_2 h^2 \|\phi\|^2, \end{gathered} \tag{3.18}$$

we get $V(0,x_0) \le (p+\beta+h\varepsilon_1+(1/2)h^2\varepsilon_2)\|\phi\|^2$.

Hence we have

$$\|x(t,\phi)\| \le \sqrt{\frac{p+\beta+h\varepsilon_1+(1/2)h^2\varepsilon_2}{\beta}}\|\phi\| e^{-\alpha t}, \quad t \ge 0. \tag{3.19}$$

So the closed-loop system (2.3) is exponentially stable. This completes the proof. □

Remark 3.2. Note that from the proof of Theorem 3.1, condition (3.2) can be relaxed via the following matrix inequality:

$$\dot{P}(t) + \overline{A}^T(t)P_\beta(t) + P_\beta(t)\overline{A}(t) - P_\beta(t)R(t)P_\beta(t) + \omega I \le 0. \tag{3.20}$$

In addition, we consider a class of uncertain systems with time-varying delays and simple nonlinear perturbations as follows:

$$\begin{aligned} \dot{x}(t) &= [A + \Delta A(\upsilon,t)]x(t) + [A_1 + \Delta A_1(\xi,t)]x(t-h(t)) \\ &\quad + (B + \Delta B(\varsigma,t))u + f(t,x(t),x(t-h(t))), \\ x(t) &= \phi(t), \quad t \in [-h,0],\ h \ge 0, \end{aligned} \tag{3.21}$$

where A, A_1, B are constant matrices of appropriate dimensions and $\Delta A(\upsilon,t)$, $\Delta A_1(\xi,t)$, $\Delta B(\varsigma,t)$ are unknown time-varying uncertain matrices, and there exist scalars $a,b,d,l_1,l_2,l_3>0$ such that

$$\begin{gathered}\|f(t,x(t),x(t-h(t)))\|\le a\|x(t)\|+b\|x(t-h(t))\|,\quad \forall(t,x(t))\in R^{+}\times R^{n},\\ \Delta A(\upsilon,t)\Delta A^{T}(\upsilon,t)\le l_1 I,\quad \Delta A_1(\xi,t)\Delta A_1^{T}(\xi,t)\le l_2 I,\quad \|\Delta B\|\le l_3.\end{gathered}\tag{3.22}$$

Given positive numbers α, β, ε_i, $i=1,2,3,4$, we set

$$\begin{aligned}&\widehat{A}=A+\alpha I,\qquad \overline{P}_{\beta}=P+\beta I,\qquad \omega=\varepsilon_1+\varepsilon_3+\varepsilon_4+\varepsilon_2 h,\\ &\overline{R}=-\frac{4}{\varepsilon_1 e^{-2\alpha h}(1-\delta)}A_1A_1^{T}+BB^{T}\\ &\qquad-\left(\varepsilon_3^{-1}l_1+\varepsilon_4^{-1}a^2+\frac{4l_2}{\varepsilon_1(1-\delta)e^{-2\alpha h}}+\frac{2b^2}{\varepsilon_1(1-\delta)e^{-2\alpha h}}+l_3\left\|B^{T}\right\|\right)I.\end{aligned}\tag{3.23}$$

We have the following theorem.

Theorem 3.3. *Suppose that condition (H1) holds. If there exist positive numbers α, β, ε_i, $i=1,2,3,4$, and a symmetric positive define matrix X such that the following LMIs hold:*

$$\begin{pmatrix}X\widehat{A}^{T}+\widehat{A}X-\overline{R} & X\\ X & -\omega^{-1}I\end{pmatrix}\le 0,\tag{3.24}$$

$$\begin{pmatrix}-X & X\\ X & -\beta^{-1}I\end{pmatrix}<0,\tag{3.25}$$

then system (3.21) *is robustly exponentially stabilizable with feedback control*

$$u(t)=-\frac{1}{2}B^{T}X^{-1}x(t).\tag{3.26}$$

Moreover, the solution $x(t,\phi)$ satisfies the condition

$$\|x(t,\phi)\|\le\sqrt{\frac{\lambda_{\max}(P)+\beta+h\varepsilon_1+(1/2)h^2\varepsilon_2}{\beta}}\|\phi\|e^{-\alpha t},\quad t\ge 0,\tag{3.27}$$

where $P=X^{-1}-\beta I$ and $\lambda_{\max}(P)$ represents the maximum eigenvalue of P.

Proof. From (3.25), we get that P is a symmetric positive define matrix.

For system (3.21), select a Lyapunov-Krasovskii functional as

$$V(t,x_t)=V_1+V_2+V_3,\tag{3.28}$$

where

$$V_1 = \langle Px(t), x(t)\rangle + \beta\langle x(t), x(t)\rangle,$$
$$V_2 = \varepsilon_1 \int_{t-h(t)}^{t} e^{2\alpha(s-t)}\|x(s)\|^2 ds, \tag{3.29}$$
$$V_3 = \varepsilon_2 \int_{-h}^{0}\int_{t+\tau-h(t+\tau)}^{t} e^{2\alpha(s-t)}\|x(s)\|^2 ds\, d\tau.$$

The time derivative of V along trajectory of (3.21) is given by

$$\dot{V}(t, x_t) = \dot{V}_1 + \dot{V}_2 + \dot{V}_3, \tag{3.30}$$

where

$$\begin{aligned}
\dot{V}_1 &= 2\left\langle \overline{P}_\beta \dot{x}(t), x(t)\right\rangle \\
&= \left\langle \left(A^T\overline{P}_\beta + \overline{P}_\beta A\right)x(t), x(t)\right\rangle + 2\left\langle \overline{P}_\beta \Delta A(\upsilon, t)x(t), x(t)\right\rangle + 2\left\langle \overline{P}_\beta A_1 x(t-h(t)), x(t)\right\rangle \\
&\quad + 2\left\langle \overline{P}_\beta \Delta A_1(\xi, t)x(t-h(t)), x(t)\right\rangle + 2\left\langle \overline{P}_\beta f(t, x(t), x(t-h(t))), x(t)\right\rangle \\
&\quad + 2\left\langle \overline{P}_\beta BKx(t), x(t)\right\rangle + 2\left\langle \overline{P}_\beta \Delta BKx(t), x(t)\right\rangle, \\
\dot{V}_2 &= -2\alpha V_2 + \varepsilon_1\|x(t)\|^2 - \varepsilon_1 e^{-2\alpha h(t)}\|x(t-h(t))\|^2(1-\dot{h}(t)) \\
&\le -2\alpha V_2 + \varepsilon_1\|x(t)\|^2 - \varepsilon_1 e^{-2\alpha h}\|x(t-h(t))\|^2(1-\delta), \\
\dot{V}_3 &= -2\alpha V_3 + \varepsilon_2 \int_{-h}^{0}\left[\|x(t)\|^2 - e^{-2\alpha(\tau-h(t+\tau))}\|x(t+\tau-h(t+\tau))\|^2(1-\dot{h}(t+\tau))\right]d\tau \\
&\le -2\alpha V_3 + \varepsilon_2 h\|x(t)\|^2 - \varepsilon_2\int_{-h}^{0} e^{-2\alpha(\tau-h(t+\tau))}\|x(t+\tau-h(t+\tau))\|^2(1-\delta)d\tau \\
&\le -2\alpha V_3 + \varepsilon_2 h\|x(t)\|^2, \\
\overline{P}_\beta &= P + \beta I.
\end{aligned} \tag{3.31}$$

Applying Lemma 2.2 and inequality (3.22) gives

$$
\begin{aligned}
&2\left\langle \overline{P}_\beta \Delta A(\upsilon,t)x(t), x(t)\right\rangle \\
&\quad\le \varepsilon_3^{-1}\left\langle \overline{P}_\beta \Delta A(\upsilon,t)\Delta A^T(\upsilon,t)\overline{P}_\beta x(t), x(t)\right\rangle + \varepsilon_3\langle x(t), x(t)\rangle \\
&\quad\le \varepsilon_3^{-1} l_1\left\langle \overline{P}_\beta^2 x(t), x(t)\right\rangle + \varepsilon_3\langle x(t), x(t)\rangle, \\
&2\left\langle \overline{P}_\beta A_1 x(t-h(t)), x(t)\right\rangle \\
&\quad\le \frac{4}{\varepsilon_1(1-\delta)e^{-2\alpha h}}\left\langle \overline{P}_\beta A_1 A_1^T \overline{P}_\beta x(t), x(t)\right\rangle \\
&\qquad+ \frac{\varepsilon_1(1-\delta)e^{-2\alpha h}}{4}\langle x(t-h(t)), x(t-h(t))\rangle, \\
&2\left\langle \overline{P}_\beta \Delta A_1(\xi,t) x(t-h(t)), x(t)\right\rangle \\
&\quad\le \frac{4}{\varepsilon_1(1-\delta)e^{-2\alpha h}}\left\langle \overline{P}_\beta \Delta A_1(\xi,t)\Delta A_1^T(\xi,t) \overline{P}_\beta x(t), x(t)\right\rangle \\
&\qquad+ \frac{\varepsilon_1(1-\delta)e^{-2\alpha h}}{4}\langle x(t-h(t)), x(t-h(t))\rangle \\
&\quad\le \frac{4l_2}{\varepsilon_1(1-\delta)e^{-2\alpha h}}\left\langle \overline{P}_\beta^2 x(t), x(t)\right\rangle + \frac{\varepsilon_1(1-\delta)e^{-2\alpha h}}{4}\langle x(t-h(t)), x(t-h(t))\rangle, \\
&2\left\langle \overline{P}_\beta f(t, x(t), x(t-h(t))), x(t)\right\rangle \\
&\quad\le 2\| f(t, x(t), x(t-h(t)))\| \left\|\overline{P}_\beta(t) x(t)\right\| \\
&\quad\le 2a\|x(t)\|\left\|\overline{P}_\beta x(t)\right\| + 2b\|x(t-h(t))\|\left\|\overline{P}_\beta x(t)\right\| \\
&\quad\le \varepsilon_4^{-1}a^2\left\|\overline{P}_\beta x(t)\right\|^2 + \varepsilon_4\|x(t)\|^2 + \frac{2b^2}{\varepsilon_1 e^{-2\alpha h}(1-\delta)}\left\|\overline{P}_\beta x(t)\right\|^2 + \frac{\varepsilon_1 e^{-2\alpha h}(1-\delta)}{2}\|x(t-h(t))\|^2 \\
&\quad= \left(\varepsilon_4^{-1}a^2 + \frac{2b^2}{\varepsilon_1 e^{-2\alpha h}(1-\delta)}\right)\left\langle \overline{P}_\beta^2 x(t), x(t)\right\rangle + \varepsilon_4\langle x(t), x(t)\rangle + \frac{\varepsilon_1 e^{-2\alpha h}(1-\delta)}{2}\|x(t-h(t))\|^2.
\end{aligned}
\tag{3.32}
$$

Noticing

$$
\begin{aligned}
2\left\langle \overline{P}_\beta BKx(t), x(t)\right\rangle &= -\left\langle \overline{P}_\beta BB^T\overline{P}_\beta x(t), x(t)\right\rangle, \\
2\left\langle \overline{P}_\beta \Delta BKx(t), x(t)\right\rangle &= \left\langle \overline{P}_\beta \Delta BB^T\overline{P}_\beta x(t), x(t)\right\rangle \le l_3\left\|B^T\right\|\left\|\overline{P}_\beta x(t)\right\|^2,
\end{aligned}
\tag{3.33}
$$

we get

$$\begin{aligned}&\dot{V}(t,x_t)+2\alpha V(t,x_t)\\&\quad\le\Bigg\langle\Bigg\{\widehat{A}^T\overline{P}_\beta+\overline{P}_\beta\widehat{A}+\overline{P}_\beta\left(\frac{4}{\varepsilon_1(1-\delta)e^{-2\alpha h}}A_1A_1^T-BB^T\right)\overline{P}_\beta+(\varepsilon_1+\varepsilon_3+\varepsilon_4+\varepsilon_2 h)I\\&\qquad+\left(\varepsilon_3^{-1}l_1+\varepsilon_4^{-1}a^2+\frac{4l_2}{\varepsilon_1(1-\delta)e^{-2\alpha h}}+\frac{2b^2}{\varepsilon_1(1-\delta)e^{-2\alpha h}}+l_3\left\|B^T\right\|\right)\overline{P}_\beta^2\Bigg\}x(t),x(t)\Bigg\rangle.\end{aligned}\tag{3.34}$$

Hence we obtain

$$\dot{V}(t,x_t)+2\alpha V(t,x_t)\le x^T(t)\left(\widehat{A}^T\overline{P}_\beta+\overline{P}_\beta\widehat{A}-\overline{P}_\beta\overline{R}\,\overline{P}_\beta+\omega I\right)x(t).\tag{3.35}$$

Noticing $\overline{P}_\beta=X^{-1}$, and from (3.24) and using Lemma 2.3, we obtain

$$\overline{P}_\beta^{-1}\widehat{A}^T+\widehat{A}\overline{P}_\beta^{-1}-\overline{R}+\omega\overline{P}_\beta^{-1}\overline{P}_\beta^{-1}\le 0.\tag{3.36}$$

By pre-multiplying and post-multiplying the right-hand side of (3.36) with $\overline{P}_\beta$ it follows that

$$\widehat{A}^T\overline{P}_\beta+\overline{P}_\beta\widehat{A}-\overline{P}_\beta\overline{R}\,\overline{P}_\beta+\omega I\le 0.\tag{3.37}$$

So we have

$$\dot{V}(t,x_t)+2\alpha V(t,x_t)\le 0,\quad \forall t\ge 0.\tag{3.38}$$

It is obvious that

$$e^{2\alpha t}\dot{V}(t,x_t)+2\alpha e^{2\alpha t}V(t,x_t)\le 0.\tag{3.39}$$

Integrating both sides of (3.15) from 0 to t, we get

$$V(t,x_t)\le V(0,x_0)e^{-2\alpha t},\quad \forall t\ge 0.\tag{3.40}$$

On the other hand, from the expression of $V(t,x_t)$, it is easy to see that

$$\beta\|x(t)\|^2\le V(t,x_t),\quad \forall t\ge 0.\tag{3.41}$$

In addition, since

$$V_1(0,x_0) \leq (\lambda_{\max}(P)+\beta)\|\phi\|^2, \qquad V_2(0,x_0) \leq \varepsilon_1 h\|\phi\|^2, \qquad V_3(0,x_0) \leq \frac{1}{2}\varepsilon_2 h^2\|\phi\|^2, \tag{3.42}$$

we get $V(0,x_0) \leq (\lambda_{\max}(P)+\beta+h\varepsilon_1+(1/2)h^2\varepsilon_2)\|\phi\|^2$.

Hence we have

$$\|x(t,\phi)\| \leq \sqrt{\frac{\lambda_{\max}(P)+\beta+h\varepsilon_1+(1/2)h^2\varepsilon_2}{\beta}}\|\phi\|e^{-\alpha t}, \quad t \geq 0. \tag{3.43}$$

So the system (3.21) is robustly exponentially stabilizable. This completes the proof of Theorem 3.3. □

4. Illustrative Examples

In this section, we will give numerical examples to demonstrate the effective of the proposed methods.

Example 4.1. Consider system (2.1) with $h(t) = \cos^2 0.25t$ and

$$f(t,x(t),x(t-h(t)),u(t)) = \begin{pmatrix} \frac{1}{2}\cos t x_1(t) + \frac{3}{4}\sin t x_2(t-h(t)) \\ \frac{1}{2}\sin t x_2(t) + \frac{3}{4}\cos t u(t) \end{pmatrix},$$

$$A(t) = \begin{pmatrix} (-6e-4)\cos t - \frac{4-\sin t}{2\cos t+4} - 12e - \frac{17}{2} & (-4e-2)\cos t - 8e - 4 \\ (-4e-2)\cos t - 8e - 4 & (-6e-4)\cos t - \frac{4-\sin t}{2\cos t+4} - 12e - \frac{17}{2} \end{pmatrix},$$

$$A_1(t) = \frac{1}{2}\begin{pmatrix} e^{-0.5} & 0 \\ 0 & e^{-0.5} \end{pmatrix}, \qquad B(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$\Delta A(\upsilon,t) = \begin{pmatrix} \upsilon(t) & 0 \\ \upsilon(t) & 0 \end{pmatrix}, \qquad \Delta A_1(\xi,t) = \begin{pmatrix} 0 & 2\xi(t) \\ 0 & 2\xi(t) \end{pmatrix}, \qquad \Delta B(\varsigma,t) = \begin{pmatrix} \varsigma(t) \\ \varsigma(t) \end{pmatrix}, \tag{4.1}$$

where $\upsilon(t) = \eta_1$, $\xi(t) = \eta_2$, $\varsigma(t) = \eta_3$ with $|\eta_1| \leq 2$, $|\eta_2| \leq 0.5$, $|\eta_3| \leq 0.5$. It is easy to obtain that

$$\|f(t,x(t),x(t-h(t)),u(t))\| \leq \|x(t)\| + \|x(t-h(t))\| + \|u(t)\|, \tag{4.2}$$

that is, $a = b = d = 1$. We can get $h = 1$, $\delta = 0.5$. $\rho_\upsilon(t) = 2$, $\rho_\xi(t) = 1$, $\mu(t) = -0.5$, $\omega = 4$. For the positive numbers $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 1$, $\beta = 1$, $\alpha = 0.5$, we can verify that all the conditions of Theorem 3.1 are satisfied, and the solution of the Riccati differential equation (3.2) is given by

$$P(t)=\begin{pmatrix}\cos t+1 & 0\\ 0 & \cos t+1\end{pmatrix}. \tag{4.3}$$

Therefore, the system is robustly α-exponentially stabilizable with feedback control

$$u(t)=-(\cos t+1)x_1(t)-(\cos t+1)x_2(t). \tag{4.4}$$

Noting $p=\sup_{t\in R^+}\|P(t)\|=2$, the solution of the system satisfies

$$\|x(t,\phi)\|\le 2.1213\|\phi\|e^{-0.5t},\quad t\in R^+. \tag{4.5}$$

Remark 4.2. The systems in the examples that are dealt by [17] do not contain uncertainty in the linear part on state and control, and those systems are robustly stabilizable. But, our Example 4.1 contains uncertainty in the linear part on state and control, and the system in Example 4.1 is robustly α-exponentially stabilizable. Using the similar approach in [17], our results can be extended to the systems with multiple delays.

Example 4.3. Consider system (3.21) with $h(t)=0.5$ and

$$\begin{gathered}A=\begin{pmatrix}-25.5 & 0\\ 1 & -25.5\end{pmatrix},\qquad A_1=\frac{1}{2}\begin{pmatrix}e^{-0.5} & 0\\ 0 & e^{-0.5}\end{pmatrix},\qquad B=\begin{pmatrix}1\\1\end{pmatrix},\\ l_1=1,\qquad l_2=1,\qquad l_3=\sqrt{2},\qquad a=1,\qquad b=1.\end{gathered} \tag{4.6}$$

Taking $\varepsilon_1=\varepsilon_2=\varepsilon_3=\varepsilon_4=1$, $\beta=1$, $\alpha=0.5$, we can get

$$\begin{gathered}h=1,\qquad \delta=0.5,\qquad \widehat{A}=A+\alpha I=\begin{pmatrix}-25 & 0\\ 1 & -25\end{pmatrix},\quad \omega=4,\\ \overline{R}=-2I+BB^T-(1+1+8e+4e+2)I=-(6+12e)I+\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}.\end{gathered} \tag{4.7}$$

We can verify that all the conditions of Theorem 3.3 and LMI (3.24) and (3.25) are satisfied with

$$P=\begin{pmatrix}0.11111 & 0\\ 0 & 0.11111\end{pmatrix}. \tag{4.8}$$

By Theorem 3.3, the system is exponentially stabilizable with feedback control

$$u(t)=-0.555556x_1(t)-0.555556x_2(t), \tag{4.9}$$

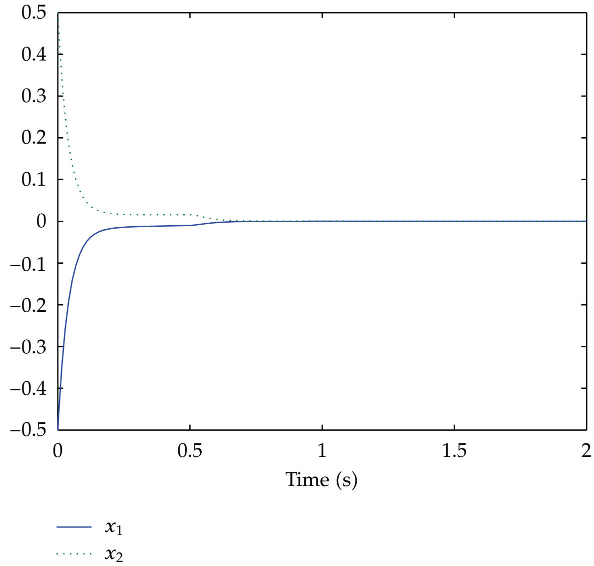

Figure 1: The state x_1 and x_2 of the closed-loop system in Example 4.3.

and the solution of the system satisfies

$$\|x(t,\phi)\| \leq 1.61589\|\phi\|e^{-0.5t}, \quad t \geq 0. \tag{4.10}$$

For

$$f(t,x(t),x(t-h(t))) = \begin{pmatrix} \frac{1}{2}\cos tx_1(t) + \frac{3}{4}\sin tx_2(t-0.5) \\ \frac{1}{2}\sin tx_2(t) \end{pmatrix},$$
$$\Delta A(\upsilon,t) = 0.5I, \qquad \Delta A_1(\xi,t) = 0.5I, \qquad \Delta B(\varsigma,t) = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}, \tag{4.11}$$

and the initial condition

$$\phi(t) = [-0.5 \ \ 0.5]^T, \quad \forall -0.5 \leq t < 0, \tag{4.12}$$

the simulation result is shown in Figure 1. It is seen from Figure 1 that the closed-loop system is exponentially stable.

5. Conclusions

The problem of robust stabilization for a class of dynamical nonlinear systems with uncertainties and time-varying delays has been considered. On condition that the derivative of time-varying delays has restriction, a novel stability criterion which can guarantee the exponential stabilization of the uncertain systems with time-varying delay and nonlinear perturbations has been established by using the Riccati differential equation. The continuous state feedback controller has been proposed. Furthermore, based on the Lyapunov method, a linear matrix inequality approach to robust exponentially stabilization for a class of uncertain systems with time-varying delays and nonlinear perturbations via linear memoryless state feedback has been proposed. Finally, two illustrative examples have been given to demonstrate the utilization of the results.

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