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Research Article

Extinction and Positivity of the Solutions for a p-Laplacian Equation with Absorption on Graphs

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We deal with the extinction of the solutions of the initial-boundary value problem of the discrete p-Laplacian equation with absorption $u_t = \Delta_{p,\omega}u - u^q$ with $p > 1$, $q > 0$, which is said to be the discrete p-Laplacian equation on weighted graphs. For $0 < q < 1$, we show that the nontrivial solution becomes extinction in finite time while it remains strictly positive for $p \geq 2$, $q \geq 1$ and $q \geq p - 1$. Finally, a numerical experiment on a simple graph with standard weight is given.

1. Introduction

The discrete analogue of the Laplacian on networks, the so-called discrete Laplacian, can be used in various areas, for example, modeling energy flows through a network or modeling vibration of molecules [1–4]. However, many phenomena on some cases cannot be expressed well by the discrete Laplacian. In view of this, a nonlinear operator, called the discrete p-Laplacian, has recently been studied by many researchers in various fields, such as dynamical systems and image processing [5–7].

Our interest in this work can be considered as a discrete analogue of the following initial boundary value problem for the p-Laplacian equation with absorption:

$$\begin{aligned} u_t(x,t) &= \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right) - \lambda |u|^{q-1}u, \quad (x,t) \in \Omega \times (0,\infty), \\ u(x,t) &= 0, \quad x \in \partial\Omega,\ t > 0, \\ u(x,0) &= u_0(x), \quad x \in \Omega, \end{aligned} \tag{1.1}$$

where Ω is a bounded domain in R^n. In fact, all the continuous regularization methods (local or nonlocal) with a given discretization scheme can be considered as particular cases of our proposed discrete regularization. Since the proposed framework is directly expressed in a discrete setting, no partial difference equations resolution is needed. Equation (1.1) has been extensively studied. In [8], the existence, uniqueness, regularity, and behavior of solutions to the initial-boundary value problem for (1.1) has been studied. Moreover, in [9], Gu proved that, for $\lambda > 0$, if $p \in (1, 2)$ or $q \in (0, 1)$ the solutions of the problem vanish in finite time, but if $p \geq 2$ and $q \geq 1$, there is nonextinction [9, Example 2 and Theorem 3.3]. In the absence of absorption (i.e., $\lambda = 0$), DiBenedetto [10] and Hongjun et al. [11] proved that the necessary and sufficient conditions for the extinction to occur is $p \in (1, 2)$.

The ω-heat equation $u_t(x,t) = \Delta_\omega u(x,t)$, which can be interpreted as a heat (or energy) diffusion equation on electric networks, has been studied by a number of authors such as [1, 3, 12–15] and so on, for example, the solvability of direct problems such as the Dirichlet and Neumann boundary value problems of the ω-Laplace equation, the global uniqueness of the inverse problem of the equation under the monotonicity condition, moreover, finding solutions to their initial and boundary problems, and representing them by means of their kernels have also been studied. Recently, in [16], Chung et al. considered the homogeneous Dirichlet boundary value problem for the ω -heat equation with absorption on a network:

$$\begin{aligned} u_t(x,t) &= \Delta_\omega u(x,t) - u^q, \quad (x,t) \in G \times (0, +\infty), \\ u(x,t) &= 0, \quad x \in \partial G, \ t > 0, \\ u(x,0) &= u_0(x), \quad x \in G. \end{aligned} \tag{1.2}$$

The absorption term denotes that the heat flows through networks are influenced by the reactive forces proportional to the power of their potentials. The authors proved that if $0 < q < 1$, a nontrivial solution of (1.2) becomes extinction in finite time, but if $q \geq 1$, it remains strictly positive. However, a lot of material usually have complicated interconnection governed by their intrinsic characteristics and to express such a feature, it needs to be a more complex systems than simple linear equations on networks. So many authors have adapted nonlinear operators which is useful to describe natures on networks and one of those operators is a discrete p-Laplacian which is a generalized nonlinear operator of the discrete Laplacian. And then, the present paper is devoted to the discrete analogues of (1.1) on networks, that is, we consider the following discrete p-Laplacian equation:

$$\begin{aligned} u_t(x,t) &= \Delta_{p,\omega} u(x,t) - u^q, \quad (x,t) \in G \times (0, +\infty), \\ u(x,t) &= 0, \quad x \in \partial G, \ t > 0, \\ u(x,0) &= u_0(x), \quad x \in G, \end{aligned} \tag{1.3}$$

where G is a finite simple graph, $p > 1$, $q > 0$, and $u_0(x)$ is a nonnegative function on graph G. The main work of this paper is to show, for $0 < q < 1$, the nontrivial solution becomes extinction in finite time while it remains strict positive in the case $p \geq 2$, $q \geq 1$ and $q \geq p - 1$.

2. Preliminary

In this section, we will begin with some definitions of graph theoretic notions, which are frequently used throughout this paper.

Let G be a finite simple, connected, and undirected graph, V_G and E_G denote its vertex set and its edge set, respectively. Two vertices x, y are adjacent if they are connected by an edge, in this case, we write $x \sim y$ or $(x, y) \in E$. Moreover, we also omit the subscript G in V_G, E_G, and so forth, if G is clear from context. In general, we can split the set of vertexes V into two disjoint subsets S and ∂S such that $V = S \bigcup \partial S$, which are called the interior and the boundary of V. A weight on a graph G is a function $\omega : V \times V \longrightarrow [0, \infty)$ satisfying

$$\begin{aligned} &(1)\ \omega(x,y) = \omega(y,x) > 0, \quad \text{if } x \sim y, \\ &(2)\ \omega(x,y) = 0, \quad \text{if } f(x,y) \notin E. \end{aligned} \tag{2.1}$$

Since the set of edge is uniquely determined by the weight, thus the simple weighted graph $G(V, E; \omega)$ can be simply denoted by $(V; \omega)$.

Throughout this paper, we consider the space of functions on vertex sets of the graph. The integration of function $f : V \longrightarrow R$ on a simple weighted graph $G(V; \omega)$ is defined as

$$\int_G f := \sum_{x \in V} f(x). \tag{2.2}$$

As usual, the set $C^1(V \times (0, \infty))$ consists of all functions u defined on $V \times (0, \infty)$ which is satisfy $u(x, t) \in C^1(0, \infty)$ for each $x \in V$. Further, for convenience, we denote

$$\begin{aligned} S_T &:= S \times (0, T), \\ V_T &:= V \times [0, T), \\ \Gamma_T &:= V_T - S_T = V \times \{t = 0\} \cup \partial S \times [0, T), \end{aligned} \tag{2.3}$$

where T is a fixed positive real number or ∞.

Finally, in the case of $p > 1$, for a function $f : V \longrightarrow R$, the graph p-directional derivative of f to the direction y for $x \in V$ is defined by

$$D_{p,\omega,y} \doteq |f(y) - f(x)|^{p-2}(f(y) - f(x))\sqrt{\omega(x,y)}, \tag{2.4}$$

and the graph p-Laplacian $\Delta_{p,\omega}$ of a function on $G(V; \omega)$ is defined as follows:

$$\Delta_{p,\omega} f(x) \doteq -\sum_{y \in V} D_{\omega,y}\left[D_{\omega,p,y} f(x)\right] = \sum_{y \in V} |f(y) - f(x)|^{p-2}(f(y) - f(x))\omega(x,y), \tag{2.5}$$

where

$$D_{\omega,y} f(x) = D_{2,\omega,y} f(x) = [f(x) - f(y)]\sqrt{\omega(x,y)}. \tag{2.6}$$

Note that, the p-Laplacian operator is nonlinear, with the exception of $p = 2$. For $p = 2$, it becomes the standard graph Laplacian.

3. p-Laplacian Equations with Absorption on Graphs

Before studying our problem, we will give a lemma, which will be frequently used in our later proofs.

Lemma 3.1 (see [17], Lemma 2.1). *For any $p \in (1, +\infty)$ and $a, b \in R$, one has*

$$\begin{gathered}\left(|a|^{p-2}a - |b|^{p-2}b\right) \leq C_1|a-b|(|a|+|b|)^{p-2}, \\ \left(|a|^{p-2}a - |b|^{p-2}b\right)(a-b) \geq C_2|a-b|^2(|a|+|b|)^{p-2},\end{gathered} \tag{3.1}$$

where C_1 and C_2 are positive constants depending only on p.

Theorem 3.2 (uniqueness for BVP). *Let Ψ be a continuous and increasing function, then the initial-boundary value problem*

$$\begin{gathered}u_t(x,t) - \Delta_{p,\omega}u(x,t) + \Psi(u) = f(x,t), \quad (x,t) \in S_T, \\ u(x,t) = h(x,t), \quad (x,t) \in \partial S \times [0,T), \\ u(x,0) = g(x), \quad x \in S,\end{gathered} \tag{3.2}$$

admits a unique solution in $C^1(V_T)$.

Proof. Suppose both u and $\tilde{u}$ are two solutions of (3.2) and let $v := u - \tilde{u}$. Next, we introduce an energy functional as the form

$$E(t) = \int_V v^2(x,t), \tag{3.3}$$

where $0 \leq t < T$. Taking the derivative of $E(t)$ with respect to t, and applying Lemma 3.1 and Fubini's theorem, we get

$$\begin{aligned}E'(t) &= 2\int_V vv_t = 2\int_V v(u_t - \tilde{u}_t) \\ &= 2\int_V (\Delta_{p,\omega}u - \Delta_{p,\omega}\tilde{u})(u-\tilde{u}) - 2\int_V (\Psi(u) - \Psi(\tilde{u}))(u-\tilde{u}) \\ &= -\sum_{x\in V}\sum_{y\in V}\omega(x,y) \\ &\quad \times \left[\left|u(y,t)-u(x,t)\right|^{p-2}(u(y,t)-u(x,t)) - \left|\tilde{u}(y,t)-\tilde{u}(x,t)\right|^{p-2}(\tilde{u}(y,t)-\tilde{u}(x,t))\right] \\ &\quad \times \left[(u(y,t)-u(x,t)) - (\tilde{u}(y,t)-\tilde{u}(x,t))\right] \\ &\quad - 2\int_V (\Psi(u)-\Psi(\tilde{u}))(u-\tilde{u})\end{aligned}$$

$$\begin{aligned}
&\le -C_2 \sum_{x\in V}\sum_{y\in V}\omega(x,y)\left|(u(y,t)-u(x,t))-(\tilde{u}(y,t)-\tilde{u}(x,t))\right|^2 \\
&\quad \times \left[\left|u(y,t)-u(x,t)\right|+\left|\tilde{u}(y,t)-\tilde{u}(x,t)\right|\right]^{p-2} \\
&\quad -2\int_V (\Psi(u)-\Psi(\tilde{u}))(u-\tilde{u}) \\
&\le 0,
\end{aligned} \tag{3.4}$$

which means $E(t) \le E(0) = 0$ for all $0 \le t < T$. Furthermore, we can conclude that $v \equiv 0$ in V_T. □

Remark 3.3. Similar to the process of the proof of Theorem 3.2, it is easy to prove that the uniqueness of the following initial value problem

$$\begin{gathered}
u_t(x,t) - \Delta_{p,\omega}u(x,t) + \Psi(u) = f(x,t), \quad (x,t) \in V_T, \\
u(x,0) = g(x), \quad x \in V,
\end{gathered} \tag{3.5}$$

holds in $C^1(V_T)$.

Now, we give a comparison principle.

Theorem 3.4. *Let Ψ be a continuous and increasing function, and suppose $u, \tilde{u} \in C^1(G_T)$ satisfy*

$$\begin{gathered}
u_t - \Delta_{p,\omega}u + \Psi(u) \le \tilde{u}_t - \Delta_{p,\omega}\tilde{u} + \Psi(\tilde{u}), \quad (x,t) \in S_T, \\
u(x,t) \le \tilde{u}(x,t), \quad (x,t) \in \partial S \times [0,T), \\
u(x,0) \le \tilde{u}(x,0), \quad x \in S,
\end{gathered} \tag{3.6}$$

then, $u \le \tilde{u}$ for all $(x,t) \in S_T$.

Proof. Letting

$$v := u - \tilde{u}, \tag{3.7}$$

then we have

$$v_t \le \left(\Delta_{p,\omega}u - \Delta_{p,\omega}\tilde{u}\right) - (\Psi(u) - \Psi(\tilde{u})), \quad (x,t) \in S_T. \tag{3.8}$$

Putting

$$v_+ := \max\{v, 0\}, \tag{3.9}$$

then it is obvious that $v_+ = 0$ on Γ_T. Now, multiplying both sides of (3.8) by v_+, and integrating over $(0,t)$, we obtain

$$\frac{1}{2}v_+^2(x,t) \le \int_0^t v_+\left[\left(\Delta_{p,\omega}u - \Delta_{p,\omega}\tilde{u}\right) - (\Psi(u) - \Psi(\tilde{u}))\right]dt, \quad (x,t) \in V_T. \tag{3.10}$$

Setting

$$J(t) = \{x \in V : u(x,t) > \tilde{u}(x,t)\}. \tag{3.11}$$

We will assume that $J(t) \neq \emptyset$ for each $t \in (0,T)$, and establish a contradiction. Integrating (3.8) over $J(t)$ and applying Fubini's theorem, we get

$$\frac{1}{2}\int_{J(t)} v_+^2(x,t) \le \int_0^t \left[\int_{J(t)} v(x,t)\left(\Delta_{p,\omega}u - \Delta_{p,\omega}\tilde{u}\right) - v(x,t)(\Psi(u) - \Psi(\tilde{u}))\right]dt. \tag{3.12}$$

Defining

$$\begin{aligned} F_{\omega,p}(u,\tilde{u})(x,y,t) = \omega(x,y)\Big[&\left|u(y,t) - u(x,t)\right|^{p-2}(u(y,t) - u(x,t)) \\ &-\left|\tilde{u}(y,t) - \tilde{u}(x,t)\right|^{p-2}(\tilde{u}(y,t) - \tilde{u}(x,t))\Big], \end{aligned} \tag{3.13}$$

and using Lemma 3.1, then for any (x,t) and $(y,t) \in V_T$, we arrive at

$$F_{\omega,p}(u,\tilde{u})(x,y,t)\left[v(y,t) - v(x,t)\right] \ge 0. \tag{3.14}$$

On the other hand, we have

$$\begin{aligned} \int_{J(t)} v(x,t)\left(\Delta_{p,\omega}u - \Delta_{p,\omega}\,\tilde{u}\right) &= \sum_{x\in J(t)}\sum_{y\in V} v(x,t)F_{\omega,p}(u,\tilde{u})(x,y,t) \\ &= \sum_{x\in J(t)}\sum_{y\in J(t)} v(x,t)F_{\omega,p}(u,\tilde{u})(x,y,t) \\ &\quad+ \sum_{x\in J(t)}\sum_{y\in V\setminus J(t)} v(x,t)F_{\omega,p}(u,\tilde{u})(x,y,t). \end{aligned} \tag{3.15}$$

Next, our goal is to estimate the two terms in the right side of the above equality. First, applying Fubini' theorem and (3.12), we have

$$\sum_{x\in J(t)}\sum_{y\in J(t)} v(x,t)F_{\omega,p}(u,\tilde{u})(x,y,t) = -\frac{1}{2}\sum_{x\in J(t)}\sum_{y\in J(t)}\left[v(y,t) - v(x,t)\right]F_{\omega,p}(u,\tilde{u})(x,y,t) \le 0. \tag{3.16}$$

On the other hand, if $x \in J(t)$ and $y \in V \setminus J(t)$, we then have $v(x,t) > 0$ and $v(y,t) < 0$. Furthermore, we get $v(y,t) - v(x,t) < 0$. Thus, by (3.14), we get

$$\begin{aligned}&\sum_{x\in J(t)}\sum_{y\in V\setminus J(t)} v(x,t)F_{\omega,p}(u,\widetilde{u})(x,y,t)\\&\qquad=\sum_{x\in J(t)}\sum_{y\in V\setminus J(t)} v(x,t)\frac{1}{v(y,t)-v(x,t)}(v(y,t)-v(x,t))F_{\omega,p}(u,\widetilde{u})(x,y,t)<0.\end{aligned} \tag{3.17}$$

In addition, noticing that

$$\int_{J(t)}(u-\widetilde{u})(\Psi(u)-\Psi(\widetilde{u}))\geq 0, \tag{3.18}$$

therefore, the right side of (3.12) is negative, which is a contradiction. The proof of Theorem 3.4 is complete. □

From the above theorem, we can obtain the following result which is similar to [16].

Corollary 3.5. *Assume that $u \in V_T$ satisfies*

$$\begin{aligned}u_t(x,t)-\Delta_{p,\omega}u(x,t)+|u|^{q-1}u=0,\quad&(x,t)\in S_T,\\u(x,t)=0,\quad&(x,t)\in\partial S\times[0,T),\\u(x,0)=u_0(x)\geq 0,\quad&x\in S.\end{aligned} \tag{3.19}$$

Then, $u \geq 0$ in S_T.

Remark 3.6. It follows from the above corollary that (3.19) is equivalent to the equation

$$\begin{aligned}u_t(x,t)-\Delta_{p,\omega}u(x,t)+u^q=0,\quad&(x,t)\in S_T,\\u(x,t)=0,\quad&(x,t)\in\partial S\times[0,T),\\u(x,0)=u_0(x)\geq 0,\quad&x\in S.\end{aligned} \tag{3.20}$$

4. Extinction and Positivity of the Solution

In this section, we investigate the extinction phenomenon and the positivity property of the solutions of the discrete p-Laplacian with absorption on graphs with boundary.

Theorem 4.1 (extinction). *Let $0 < q < 1$. Suppose $u \in C^1(V \times [0,\infty)$ satisfies*

$$\begin{aligned}u_t(x,t)-\Delta_{p,\omega}u(x,t)+u^q=0,\quad&(x,t)\in S\times(0,\infty),\\u(x,t)=0,\quad&(x,t)\in\partial S\times[0,\infty),\\u(x,0)=u_0(x)\geq 0,\quad&x\in S,\end{aligned} \tag{4.1}$$

there exists a finite time $T > 0$ such that $u(x,t) \equiv 0$ for all $(x,t) \in V \times [T, \infty)$.

Proof. The proof is similar to Theorem 4.3 in [16], we omit the details here. □

Theorem 4.2 (positivity). *Let $q \geq 1$, $p \geq 2$. Suppose $q+1 \geq p$ and $u \in C^1(V \times [0,\infty))$ satisfies*

$$\begin{gathered} u_t(x,t) - \Delta_{p,\omega} u(x,t) + u^q = 0, \quad (x,t) \in S \times (0,\infty), \\ u(x,t) = 0, \quad (x,t) \in \partial S \times [0,\infty), \\ u(x,0) = u_0(x) \geq 0, \quad x \in S. \end{gathered} \tag{4.2}$$

Then $u(x,t) > 0$ for all $(x,t) \in S \times (0,\infty)$. Moreover, one has

$$\int_S u^2 \geq c \exp\left(-\rho_0^{1-q} t\right), \tag{4.3}$$

for $t > T_1$, where c, ρ_0, T_1 is independent on $u(x,t)$.

Proof. In order to obtain the positivity of the solutions, we first introduce a transformation,

$$\begin{gathered} v(x,s) = \rho e^s u\left(x, \rho^{q-1} s\right), \\ s = \rho^{-(q-1)} t, \end{gathered} \tag{4.4}$$

then after a simple computation, it is easy to verify that v is the solution of the following problem:

$$\begin{gathered} v_s - \rho^{q-p+1} e^{-(p-2)s} \Delta_{p,\omega} v + e^{(1-q)s} v^q - v = 0, \quad (x,t) \in S \times (0,\infty), \\ v(x,t) = 0, \quad (x,t) \in \partial S \times [0,\infty), \\ v(x,0) = \rho u_0(x) \geq 0, \quad x \in S. \end{gathered} \tag{4.5}$$

Multiplying both sides of $(4.5)_1$ by v, and integrating on S, we have

$$\frac{1}{2} \frac{\partial}{\partial s} \int_S v^2 + \frac{1}{2} \rho^{q-p+1} e^{-(p-2)s} \sum_{x \in S} \sum_{y \in S} \omega(x,y) \left| v(y,s) - v(x,s) \right|^p + e^{(1-q)s} \int_S v^{q+1} - \int_S v^2 = 0. \tag{4.6}$$

Meanwhile, multiplying $(4.5)_1$ by v_s, and integrating on S, we get

$$\begin{aligned}\int_S v_s^2 + \frac{\partial}{\partial s}\left[\frac{1}{2p}\rho^{q-p+1}e^{-(p-2)s}\sum_{x\in S}\sum_{y\in S}\omega(x,y)|v(y,s)-v(x,s)|^p + \frac{1}{q+1}e^{(1-q)s}\int_S v^{q+1} - \frac{1}{2}\int_S v^2\right] \\ +\frac{p-2}{2p}\rho^{q-p+1}e^{-(p-2)s}\sum_{x\in S}\sum_{y\in S}\omega(x,y)|v(y,s)-v(x,s)|^p + \frac{q-1}{q+1}e^{(1-q)s}\int_S v^{q+1} = 0.\end{aligned} \tag{4.7}$$

Define

$$E(s) = \frac{1}{2p}\rho^{q-p+1}e^{-(p-2)s}\sum_{x\in S}\sum_{y\in S}\omega(x,y)|v(y,s)-v(x,s)|^p + \frac{1}{q+1}e^{(1-q)s}\int_S v^{q+1} - \frac{1}{2}\int_S v^2. \tag{4.8}$$

Noticing that $p \geq 2$ and $q \geq 1$, we have

$$E(s) \leq E(0) = \frac{1}{2p}\rho^{q+1}\sum_{x\in S}\sum_{y\in S}\omega(x,y)|u_0(y)-u_0(x)|^p + \frac{1}{q+1}\rho^{q+1}\int_S u_0^{q+1} - \frac{1}{2}\rho^2\int_S u_0^2. \tag{4.9}$$

Therefore, we can choose $\rho = \rho_0$ small enough such that $E(0) < 0$. Moreover, from (4.8) and (4.9), it follows that

$$-e^{(1-q)s}\int_S v^{q+1} \geq \frac{q+1}{2p}\rho_0^{q-p+1}e^{-(p-2)s}\sum_{x\in S}\sum_{y\in S}\omega(x,y)|v(y,s)-v(x,s)|^p - \frac{q+1}{2}\int_S v^2 - (q+1)E(0). \tag{4.10}$$

On the other hand, taking $\rho = \rho_0$ in (4.6), and combining (4.6) with (4.10), we get

$$\begin{aligned}\frac{1}{2}\frac{\partial}{\partial s}\int_S v^2 \geq \frac{q+1-p}{2p}\rho_0^{q-p+1}e^{-(p-2)s}\sum_{x\in S}\sum_{y\in S}\omega(x,y)|v(y,s)-v(x,s)|^p \\ -\frac{q-1}{2}\int_S v^2 - (q+1)E(0).\end{aligned} \tag{4.11}$$

Setting $y(s) = \int_S v^2$, since $q+1-p \geq 0$, we have

$$\frac{d}{ds}y(s) + (q-1)y(s) > 0. \tag{4.12}$$

Due to (4.12), there exists $s_0 > 0$, $c > 0$, such that $y(s) \geq c$ for $s > s_0$. By transformation (4.4), we get

$$\int_S u^2 \geq c\exp\left(-\rho_0^{1-q}t\right), \tag{4.13}$$

for $t > T_1$, where c, ρ_0, T_1 is independent on $u(x,t)$. $\square$

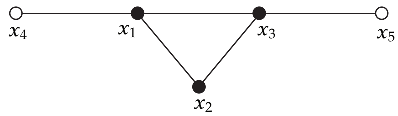

Figure 1: The graph.

Remark 4.3. By Theorem 4.2, when $p = 2$ and $q \geq 1$, the solution also remain strictly positive. In this case, our result is consistent with that in [16], but the method is very different from that previously used in [16].

5. Numerical Experiment

In this section, we consider a graph G whose vertices $S = \{x_1, x_2, x_3\}$ and $\partial S = \{x_4, x_5\}$ are linked as the above figure (see Figure 1) with the standard weight (i.e., $w(x, y) = 1$).

Let u be a solution of

$$u_t(x,t) = \Delta_{p,\omega} u(x,t) - u^q, \tag{5.1}$$

with the following initial condition $u_0(x_1) = 10, u_0(x_2) = 5, u_0(x_3) = 0.5$ and boundary condition $u(x_4, t) = 0, u(x_5, t) = 0$. Then, we obtain the following system of first order ordinary differential equations in terms of interior nodes:

$$\begin{aligned} u_t(x_1,t) &= \sum_{i=2,3,4} |u(x_i,t) - u(x_1,t)|^{p-2}(u(x_i,t) - u(x_1,t)) - u(x_1,t)^q, \\ u_t(x_2,t) &= \sum_{i=1,3} |u(x_i,t) - u(x_2,t)|^{p-2}(u(x_i,t) - u(x_2,t)) - u(x_2,t)^q, \\ u_t(x_3,t) &= \sum_{i=1,2,5} |u(x_i,t) - u(x_3,t)|^{p-2}(u(x_i,t) - u(x_3,t)) - u(x_3,t)^q. \end{aligned} \tag{5.2}$$

Since the above ordinary differential equations is nonlinear, we choose the following explicit difference scheme to compute the numerical solution:

$$\begin{aligned} \frac{u^{n+1}(x_1) - u^n(x_1)}{\Delta t} &= \sum_{i=2,3,4} |u^n(x_i) - u^n(x_1)|^{p-2}(u^n(x_i) - u^n(x_1)) - [u^n(x_1)]^q, \\ \frac{u^{n+1}(x_2) - u^n(x_2)}{\Delta t} &= \sum_{i=1,3} |u^n(x_i) - u^n(x_2)|^{p-2}(u^n(x_i) - u^n(x_2)) - [u^n(x_2)]^q, \\ \frac{u^{n+1}(x_3) - u^n(x_3)}{\Delta t} &= \sum_{i=1,2,5} |u^n(x_i) - u^n(x_3)|^{p-2}(u^n(x_i) - u^n(x_3)) - [u^n(x_3)]^q, \end{aligned} \tag{5.3}$$

where $u^n(x_i) = u(x_i, n\Delta t)$, for $i = 1, 2, \ldots, 5$ and $n = 0, 1, 2, \ldots$. We should point out that the time step must be set small enough, if not, the images of the function $u(x_i, t)$, $i = 1, 2, 3$ will

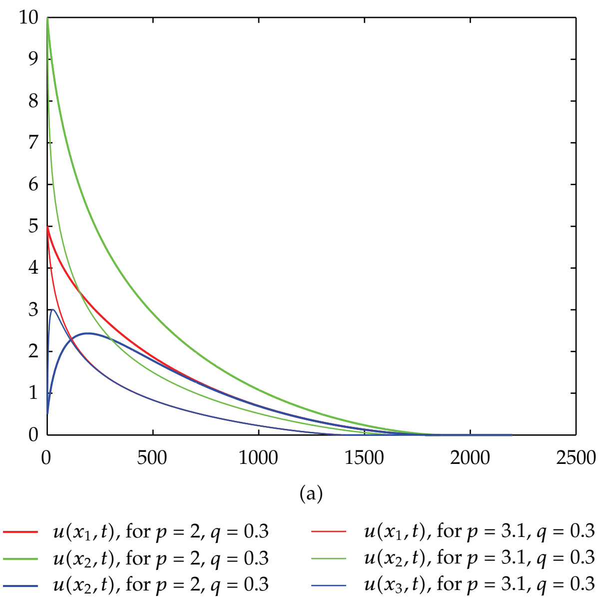

Figure 2: Extinction of the solutions.

appear oscillation phenomena near zero. Set $q = 0.3$, take $p = 3.1$ and $p = 2$, respectively. By Theorem 4.1, we have functions $u_i(x, t)$, $i = 1, 2, 3$ will vanish in finite time.

In the numerical experiment, the time step is chosen as 0.005, the numerical experiment result is shown in Figure 2, the solutions extinct after 1800 iterations, that is, $t = 9$. The green curve is the image of function $u(x_1, t)$, the image of function $u(x_2, t)$ is the red curve, and the image of function $u(x_3, t)$ is expressed by the blue one for $q = 3.1$ and $q = 2$, respectively.

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