Hindawi Publishing Corporation
Journal of Applied Mathematics
Volume 2011, Article ID 804150, 8 pages
doi:10.1155/2011/804150

Research Article

Starlikeness Properties of a New Integral Operator for Meromorphic Functions

Aabed Mohammed and Maslina Darus

School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 Bangi, Selangor D. Ehsan, Malaysia

Correspondence should be addressed to Maslina Darus, maslina@ukm.my

Received 3 March 2011; Revised 15 April 2011; Accepted 3 June 2011

Academic Editor: Pablo Gonza'lez-Vera

We define here an integral operator $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}$ for meromorphic functions in the punctured open unit disk. Several starlikeness conditions for the integral operator $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}$ are derived.

1. Introduction

Let Σ denotes the class of functions of the form

$$f(z) = \frac{1}{z} + \sum_{n=0}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the punctured open unit disk

$$\mathbb{U}^* = \{z \in \mathbb{C} : \ 0 < |z| < 1\} = \mathbb{U} \setminus \{0\}, \tag{1.2}$$

where $\mathbb{U}$ is the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$.

We say that a function $f \in \Sigma$ is meromorphic starlike of order α $(0 \le \alpha < 1)$, and belongs to the class $\Sigma^*(\alpha)$, if it satisfies the inequality

$$-\Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha. \tag{1.3}$$

A function $f \in \Sigma$ is a meromorphic convex function of order α $(0 \le \alpha < 1)$, if f satisfies the following inequality

$$-\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, \tag{1.4}$$

and we denote this class by $\Sigma_k(\alpha)$.

Analogous to the integral operator defined by Breaz et al. [1] on the normalized analytic functions, we now define the following integral operator on the space meromorphic functions in the class $\sum$.

Definition 1.1. Let $n \in \mathbb{N}$, $\gamma_i > 0$, $i \in \{1, 2, 3, \dots, n\}$. We define the integral operator $\mathscr{H}_{\gamma_1,\dots,\gamma_n}(f_1, f_2, \dots, f_n) : \Sigma^n \to \Sigma$ by

$$\mathscr{H}_{\gamma_1,\dots,\gamma_n}(f_1, \dots, f_n)(z) = \frac{1}{z^2}\int_0^z \left(-u^2 f_1'(u)\right)^{\gamma_1} \cdots \left(-u^2 f_n'(u)\right)^{\gamma_n} du. \tag{1.5}$$

For the sake of simplicity, from now on we will write $\mathscr{H}_{\gamma_1,\dots,\gamma_n}(z)$ instead of $\mathscr{H}_{\gamma_1,\dots,\gamma_n}(f_1, \dots, f_n)(z)$.

By $\Sigma_{k_p}(\beta)$ $(-1 \le \beta < 1)$, we denote the class of functions $f \in \Sigma$ such that

$$\left|\frac{zf''(z)}{f'(z)} + 2\right| < -\Re\left(\frac{zf''(z)}{f'(z)} + \beta\right) - 1. \tag{1.6}$$

In order to derive our main results, we have to recall here the following preliminary results.

Lemma 1.2 (see [2]). *Suppose that the function $\Psi : \mathbb{C}^2 \to \mathbb{C}$ satisfies the following condition:*

$$\Re\{\Psi(is, t)\} \le 0, \quad \left(s, t \in \mathbb{R};\ \ t \le \frac{-(1+s^2)}{2}\right). \tag{1.7}$$

If the function $p(z) = 1 + p_1 z + \cdots$ is analytic in $\mathbb{U}$ and

$$\Re\{\Psi(p(z), zp'(z))\} > 0, \quad (z \in \mathbb{U}), \tag{1.8}$$

then,

$$\Re\{p(z)\} > 0 \quad (z \in \mathbb{U}). \tag{1.9}$$

Proposition 1.3 (see [3]). *If $f \in \Sigma$ satisfying*

$$-\Re\left\{\frac{z(zf''(z) + 3f'(z))}{zf'(z) + 2f(z)}\right\} > \alpha, \quad 0 \le \alpha < 1,$$
$$\left|\frac{zf'(z)}{f(z)} + 1\right| < 1, \tag{1.10}$$

then,

$$-\Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha. \tag{1.11}$$

2. Starlikeness of the Operator $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$

In this section, we investigate sufficient conditions for the integral operator $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$ which is defined in Definition 1.1, to be in the class $\Sigma^*(\alpha)$, $0 \leq \alpha < 1$.

Theorem 2.1. *Let $f_i \in \sum$, $\gamma_i > 0$ for all $i \in \{1,\ldots,n\}$. If*

$$-\Re\left(\frac{zf_i''(z)}{f_i'(z)}\right) > \frac{-1}{n\gamma_i} + 2, \tag{2.1}$$

then $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$ belongs to $\Sigma^*(0)$.

Proof. On successive differentiation of $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$, which is defined in (1.5), we get

$$\begin{aligned}
&2z\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z) + z^2\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z) = \left(-z^2f_1'(z)\right)^{\gamma_1}\cdots\left(-z^2f_n'(z)\right)^{\gamma_n},\\
&z^2\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z) + 4z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z) + 2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)\\
&\quad= \sum_{i=1}^{n}\gamma_i\left(-z^2f_i'(z)\right)^{\gamma_i-1}\left(-z^2f_i''(z) - 2zf_i'(z)\right)\prod_{j=1,j\neq i}^{n}\left(-z^2f_j'(z)\right)^{\gamma_j}.
\end{aligned} \tag{2.2}$$

Then from (2.2), we obtain

$$\frac{z^2\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z) + 4z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z) + 2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}{z^2\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z) + 2z\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)} = \sum_{i=1}^{n}\gamma_i\left(\frac{f_i''(z)}{f_i'(z)} + \frac{2}{z}\right). \tag{2.3}$$

By multiplying (2.3) with z yield,

$$\frac{z^2\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z) + 4z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z) + 2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z) + 2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)} = \sum_{i=1}^{n}\gamma_i\left(\frac{zf_i''(z)}{f_i'(z)} + 2\right). \tag{2.4}$$

That is equivalent to

$$\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z) + 3\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z) + 2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}\right\} + 1 = \sum_{i=1}^{n}\gamma_i\left(\frac{zf_i''(z)}{f_i'(z)} + 2\right). \tag{2.5}$$

Or

$$-\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z)+3\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)+2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}\right\}=\sum_{i=1}^{n}\gamma_i\left(-\frac{zf_i''(z)}{f_i'(z)}\right)-2\sum_{i=1}^{n}\gamma_i+1. \tag{2.6}$$

We can write the left-hand side of (2.6), as the following:

$$\begin{aligned}&\frac{-\left(z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)/\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)\right)\left(\left(z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z)/\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)\right)+3\right)}{\left(z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)/\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)\right)+2}\\&\quad=\sum_{i=1}^{n}\gamma_i\left(-\frac{zf_i''(z)}{f_i'(z)}\right)-2\sum_{i=1}^{n}\gamma_i+1.\end{aligned} \tag{2.7}$$

We define the regular function p in $\mathbb{U}$ by

$$p(z)=-\frac{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)}{\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}, \tag{2.8}$$

and $p(0)=1$. Differentiating $p(z)$ logarithmically, we obtain

$$-p(z)+\frac{zp'(z)}{p(z)}=1+\frac{z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z)}{\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)}. \tag{2.9}$$

From (2.7), (2.8), and (2.9), we obtain

$$p(z)+\frac{zp'(z)}{-p(z)+2}=\sum_{i=1}^{n}\gamma_i\left(-\frac{zf_i''(z)}{f_i'(z)}\right)-2\sum_{i=1}^{n}\gamma_i+1. \tag{2.10}$$

Let us put

$$\Psi(u,v)=u+\frac{v}{-u+2}. \tag{2.11}$$

From (2.1), (2.10), and (2.11), we obtain

$$\begin{aligned}\Re\{\Psi(p(z),zp'(z))\}&=\gamma_1\left(-\Re\frac{zf_1''(z)}{f_1'(z)}\right)+\cdots+\left(-\Re\frac{zf_n''(z)}{f_n'(z)}\right)-2(\gamma_1+\cdots+\gamma_n)+1\\&>\gamma_1\left(\frac{-1}{n\gamma_1}+2\right)+\cdots+\gamma_n\left(\frac{-1}{n\gamma_n}+2\right)-2(\gamma_1+\cdots+\gamma_n)+1=0.\end{aligned} \tag{2.12}$$

Now, we proceed to show that

$$\Re\{\Psi(is,t)\} \leq 0, \quad \left(s,t \in \mathbb{R}; t \leq \frac{-(1+s^2)}{2}\right). \tag{2.13}$$

Indeed, from (2.11), we have

$$\Re\{\Psi(is,t)\} = \Re\left\{is + \frac{t}{-is+2}\right\} = \frac{2t}{4+s^2} \leq -\frac{1+s^2}{4+s^2} < 0. \tag{2.14}$$

Thus, from (2.12), (2.14), and by using Lemma 1.2, we conclude that $\Re\{p(z)\} > 0$, and so

$$-\Re\left\{\frac{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)}{\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}\right\} > 0 \tag{2.15}$$

that is, $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$ is starlike of order 0. □

Theorem 2.2. *For $i \in \{1,\ldots,n\}$, let $\gamma_i > 0$ and $f_i \in \Sigma_k(\alpha_i)$ $(0 \leq \alpha_i < 1)$. If $0 < \sum_{i=1}^{n} \gamma_i(1-\alpha_i) \leq 1$, $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$ be the integral operator given by* (1.5) *and*

$$\left|\frac{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)}{\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)} + 1\right| < 1. \tag{2.16}$$

Then $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$ belong to $\Sigma^*(\mu)$, where $\mu = 1 - \sum_{i=1}^{n} \gamma_i(1-\alpha_i)$.

Proof. Following the same steps as in Theorem 2.1, we obtain

$$-\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z) + 3\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z) + 2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}\right\} = \sum_{i=1}^{n}\gamma_i\left\{-\left(\frac{zf''_i(z)}{f'_i(z)} + 1\right)\right\} + 1 - \sum_{i=1}^{n}\gamma_i. \tag{2.17}$$

Taking the real part of both terms of the last expression, we have

$$-\Re\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z) + 3\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z) + 2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}\right\} = \sum_{i=1}^{n}\gamma_i\left\{-\Re\left(\frac{zf''_i(z)}{f'_i(z)} + 1\right)\right\} + 1 - \sum_{i=1}^{n}\gamma_i. \tag{2.18}$$

Since $f_i \in \Sigma_k(\alpha_i)$, for $i \in \{1, \ldots, n\}$, we receive

$$-\Re\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z) + 3\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z) + 2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}\right\} > \sum_{i=1}^{n}\gamma_i\alpha_i + 1 - \sum_{i=1}^{n}\gamma_i. \tag{2.19}$$

Therefore,

$$-\Re\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z) + 3\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z) + 2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}\right\} > 1 - \sum_{i=1}^{n}\gamma_i(1 - \alpha_i). \tag{2.20}$$

Using (2.16), (2.20), and applying Proposition 1.3, we get $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z) \in \Sigma^*(\mu)$, where $\mu = 1 - \sum_{i=1}^{n}\gamma_i(1 - \alpha_i)$. □

Letting $\alpha_i = \alpha$, $i \in \{1, \ldots, n\}$ in Theorem 2.2, we get the following.

Corollary 2.3. *For $i \in \{1, \ldots, n\}$, let $\gamma_i > 0$ and $f_i \in \Sigma_k(\alpha)$ $(0 \le \alpha < 1)$. If*

$$0 < \sum_{i=1}^{n}\gamma_i \le \frac{1}{1 - \alpha}, \tag{2.21}$$

$\mathcal{H}_{\gamma_1,\ldots,\gamma_n}$ be the integral operator given by (1.5) and

$$\left|\frac{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)}{\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)} + 1\right| < 1. \tag{2.22}$$

Then $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$ is starlike of order $1 - (1 - \alpha)\sum_{i=1}^{n}\gamma_i$.

Theorem 2.4. *For $i \in \{1, \ldots, n\}$, let $\gamma_i > 0$ and $f_i \in \Sigma_{k_p}(\beta_i)$ $(-1 \le \beta_i < 1)$. If*

$$0 < \sum_{i=1}^{n}\gamma_i(1 - \beta_i) \le 1, \tag{2.23}$$

$\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$ be the integral operator given by (1.5) and

$$\left|\frac{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)}{\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)} + 1\right| < 1. \tag{2.24}$$

Then $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$ is starlike of order $1 - \sum_{i=1}^{n}\gamma_i(1 - \beta_i)$.

Proof. Following the same steps as in Theorem 2.1, we obtain

$$-\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z)+3\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)+2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}\right\} = -\sum_{i=1}^{n}\gamma_i\left(\frac{zf_i''(z)}{f_i'(z)}+2\right)+1$$
$$=\sum_{i=1}^{n}\gamma_i\left\{-\left(\frac{zf_i''(z)}{f_i'(z)}+\beta_i\right)-1\right\}+1-\sum_{i=1}^{n}\gamma_i+\sum_{i=1}^{n}\gamma_i\beta_i$$
$$=\sum_{i=1}^{n}\gamma_i\left\{-\left(\frac{zf_i''(z)}{f_i'(z)}+\beta_i\right)-1\right\}+1-\sum_{i=1}^{n}\gamma_i(1-\beta_i). \tag{2.25}$$

We calculate the real part from both terms of the above equality and obtain

$$-\Re\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z)+3\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)+2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}\right\}$$
$$=\sum_{i=1}^{n}\gamma_i\left\{-\Re\left(\frac{zf_i''(z)}{f_i'(z)}+\beta_i\right)-1\right\}+1-\sum_{i=1}^{n}\gamma_i(1-\beta_i). \tag{2.26}$$

Since $f_i \in \Sigma_{k_p}(\beta_i)$ for all $i \in \{1,\ldots,n\}$, the above relation then yields

$$-\Re\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z)+3\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)+2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}\right\}$$
$$>\sum_{i=1}^{n}\gamma_i\left|\frac{zf_i''(z)}{f_i'(z)}+2\right|+1-\sum_{i=1}^{n}\gamma_i(1-\beta_i). \tag{2.27}$$

Because $\sum_{i=1}^{n}\gamma_i|zf_i''(z)/f_i'(z)+2|\geq 0$, we obtain that

$$-\Re\left\{\frac{z\left(z\mathcal{H}''_{\gamma_1,\ldots,\gamma_n}(z)+3\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)\right)}{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)+2\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}\right\}>1-\sum_{i=1}^{n}\gamma_i(1-\beta_i). \tag{2.28}$$

Using (2.24), (2.28) and applying Proposition 1.3, we get $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$ is a starlike function of order $1-\sum_{i=1}^{n}\gamma_i(1-\beta_i)$. □

Letting $\beta_i=\beta, i\in\{1,\ldots,n\}$ in Theorem 2.4, we get the following.

Corollary 2.5. *For $i\in\{1,\ldots,n\}$, let $\gamma_i>0$ and $f_i\in\Sigma_{k_p}(\beta)$ $(-1\leq\beta<1)$. If*

$$0<\sum_{i=1}^{n}\gamma_i\leq\frac{1}{1-\beta}, \tag{2.29}$$

$\mathcal{H}_{\gamma_1,\ldots,\gamma_n}$ *be the integral operator given by* (1.5) *and*

$$\left|\frac{z\mathcal{H}'_{\gamma_1,\ldots,\gamma_n}(z)}{\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)}+1\right|<1. \tag{2.30}$$

Then $\mathcal{H}_{\gamma_1,\ldots,\gamma_n}(z)$ *is starlike of order* $1-(1-\beta)\sum_{i=1}^{n}\gamma_i$.

Letting $n=1$, $\gamma_1=\gamma$ and $f_1=f$ in Corollary 2.5, we get the following.

Corollary 2.6. *Let* $\gamma>0$, *and* $f\in\Sigma_{k_p}(\beta)$ $(-1\le\beta<1)$. *If*

$$0<\gamma\le\frac{1}{1-\beta}, \tag{2.31}$$

$\mathcal{H}_{\gamma}(z)$ *be the integral operator,*

$$\mathcal{H}_{\gamma}(z)=\frac{1}{z^2}\int_0^z\left(-u^2f'(u)\right)^{\gamma}du, \qquad \left|\frac{z\mathcal{H}'_{\gamma}(z)}{\mathcal{H}_{\gamma}(z)}+1\right|<1. \tag{2.32}$$

Then $\mathcal{H}_{\gamma}(z)$ *is starlike of order* $1-(1-\beta)\gamma$.

Other work related to integral operator for different studies can also be found in [4–6].

Acknowledgments

The work here was supported by MOHE Grant: UKM-ST-06-FRGS0244-2010. The authors also would like to thank the referee for his/her careful reading and making some valuable comments which have improved the presentation of this paper.

References

[1] D. Breaz, S. Owa, and N. Breaz, "A new integral univalent operator," *Acta Universitatis Apulensis*, no. 16, pp. 11–16, 2008.
[2] S. S. Miller and P. T. Mocanu, "Differential subordinations and univalent functions," *The Michigan Mathematical Journal*, vol. 28, no. 2, pp. 157–172, 1981.
[3] A. Mohammed and M. Darus, "New criteria for meromorphic functions," Submitted.
[4] A. Mohammed and M. Darus, "New properties for certain integral operators," *International Journal of Mathematical Analysis*, vol. 4, no. 41–44, pp. 2101–2109, 2010.
[5] A. Mohammed and M. Darus, "A new integral operator for meromorphic functions," *Acta Universitatis Apulensis*, no. 24, pp. 231–238, 2010.
[6] N. Breaz, D. Breaz, and M. Darus, "Convexity properties for some general integral operators on uniformly analytic functions classes," *Computers and Mathematics with Applications*, vol. 60, no. 12, pp. 3105–3107, 2010.