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Research Article

Positive Almost Periodic Solutions for a Time-Varying Fishing Model with Delay

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This paper is concerned with a time-varying fishing model with delay. By means of the continuation theorem of coincidence degree theory, we prove that it has at least one positive almost periodic solution.

1. Introduction

Consider the following differential equation which is widely used in fisheries [1–4]:

$$\dot{N} = N[L(t,N) - M(t,N)] - NF(t), \tag{1.1}$$

where $N = N(t)$ is the population biomass, $L(t,N)$ is the per capita fecundity rate, $M(t,N)$ is the per capita mortality rate, and $F(t)$ is the harvesting rate per capita.

In (1.1), let $L(t,N)$ be a Hills' type function ([1, 2])

$$L(t,N) = \frac{a}{1 + (N/K)^{\gamma}} \tag{1.2}$$

and take into account the delay and the varying environments; Berezansky and Idels [5] proposed the following time-lag model based on (1.1) [1–6]

$$\dot{N}(t) = N(t)\left[\frac{a(t)}{1 + (N(\theta(t))/K(t))^{\gamma}} - b(t)\right], \tag{1.3}$$

where $b(t) = M(t,N) + F(t)$.

The model (1.3) has recently attracted the attention of many mathematicians and biologists; see the differential equations which are widely used in fisheries [1, 2]. However, one can easily see that all equations considered in the above-mentioned papers are subject to periodic assumptions, and the authors, in particular, studied the existence of their periodic solutions. On the other hand, ecosystem effects and environmental variability are very important factors and mathematical models cannot ignore, for example, reproduction rates, resource regeneration, habitat destruction and exploitation, the expanding food surplus, and other factors that affect the population growth. Therefore it is reasonable to consider the various parameters of models to be changing almost periodically rather than periodically with a common period. Thus, the investigation of almost periodic behavior is considered to be more accordant with reality. Although it has widespread applications in real life, the generalization to the notion of almost periodicity is not as developed as that of periodic solutions; we refer the reader to [7–18].

Recently, the authors of [19] proved the persistence and almost periodic solutions for a discrete fishing model with feedback control. In [20, 21], the contraction mapping principle and the continuation theorem of coincidence degree have been employed to prove the existence of positive almost periodic exponential stable solutions for logarithmic population model, respectively. A primary purpose of this paper, nevertheless, is to utilize the continuation theorem of coincidence degree for this purpose. To the best of the authors' observation, there exists no paper dealing with the proof of the existence of positive almost periodic solutions for (1.3) using the continuation theorem of coincidence degree. Therefore, our result is completely different and presents a new approach.

2. Preliminaries

Our first observation is that under the invariant transformation $N(t) = e^{y(t)}$, (1.3) reduces to

$$\dot{y}(t) = \frac{a(t)}{1 + \left(e^{y(\theta(t))}/K(t)\right)^{\gamma}} - b(t) \tag{2.1}$$

for $\gamma > 0$, with the initial function and the initial value

$$y(t) = \phi(t), \quad y(0) = y_0, \ t \in (-\infty, 0). \tag{2.2}$$

For (2.1) and (2.2), we assume the following conditions:

(A1) $a(t), b(t) \in C([0, +\infty), [0, +\infty))$ and $K(t) \in C([0, +\infty), (0, +\infty))$;

(A2) $\theta(t)$ is a continuous function on $[0, +\infty)$ that satisfies $\theta(t) \leq t$;

(A3) $\phi(t) : (-\infty, 0) \to [0, \infty)$ is a continuous bounded function, $\phi(t) \geq 0, y_0 > 0$.

By a solution of (2.1) and (2.2) we mean an absolutely continuous function $y(t)$ defined on $(-\infty, +\infty)$ satisfying (2.1) almost everywhere for $t \geq 0$ and (2.2). As we are interested in solutions of biological significance, we restrict our attention to positive ones.

According to [22], the initial value problem (2.1) and (2.2) has a unique solution defined on $(-\infty, \infty)$.

Let X, Y be normed vector spaces, $L : \text{Dom}\, L \subset X \to Y$ be a linear mapping, and $N : X \to Y$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\ \text{Ker}\, L = \text{codim}\ \text{Im}\, L < +\infty$ and $\text{Im}\ L$ is closed in Y. If L is a Fredholm mapping of index zero and there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that $\text{Im}\ P = \text{Ker}\ L$, $\text{Ker}\, Q = \text{Im}\, L = \text{Im}(I - Q)$, it follows that the mapping $L|_{\text{Dom}\ L \cap \text{Ker}\, P} : (I - P)X \to \text{Im}\ L$ is invertible. We denote the inverse of that mapping by K_P. If Ω is an open bounded subset of X, then the mapping N will be called L-compact on $\overline{\Omega}$, if $QN(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \to X$ is compact. Since $\text{Im}\, Q$ is isomorphic to $\text{Ker}\, L$, there exists an isomorphism $J : \text{Im}\, Q \to \text{Ker}\, L$.

Theorem 2.1 (see [19]). *Let $\Omega \subset X$ be an open bounded set and let $N : X \to Y$ be a continuous operator which is L-compact on $\overline{\Omega}$. Assume that*

(1) *$Ly \neq \lambda Ny$ for every $y \in \partial\Omega \cap \text{Dom}\ L$ and $\lambda \in (0, 1)$;*

(2) *$QNy \neq 0$ for every $y \in \partial\Omega \cap \text{Ker}\ L$;*

(3) *the Brouwer degree $\deg\{JQN, \Omega \cap \text{Ker}\ L, 0\} \neq 0$.*

Then $Ly = Ny$ has at least one solution in $\text{Dom}\ L \cap \overline{\Omega}$.

3. Existence of Almost Periodic Solutions

Let $AP(\mathbb{R})$ denote the set of all real valued almost periodic functions on $\mathbb{R}$, for $f \in AP(\mathbb{R})$ we denote by

$$\Lambda(f) = \left\{ \tilde{\lambda} \in \mathbb{R} : \lim_{T \to \infty} \frac{1}{T} \int_0^T f(s) e^{-i\tilde{\lambda} s} \mathrm{d}s \neq 0 \right\},$$
$$\text{mod}(f) = \left\{ \sum_{j=1}^{m} n_j \tilde{\lambda}_j : n_j \in \mathbb{Z}, m \in \mathbb{N}, \tilde{\lambda}_j \in \Lambda(f),\ j = 1, 2, \ldots, m \right\}, \tag{3.1}$$

the set of Fourier exponents and the module of f, respectively. Let $K(f, \varepsilon, S)$ denote the set of ε-almost periods for f with respect to $S \subset C((-\infty, 0], \mathbb{R})$, $l(\varepsilon)$ denote the length of the inclusion interval, and $m(f) = \lim_{T \to \infty} (1/T) \int_0^T f(s)\ \mathrm{d}s$ denote the mean value of f.

Definition 3.1. $y(t) \in C(\mathbb{R}, \mathbb{R})$ is said to be almost periodic on $\mathbb{R}$ if for any $\varepsilon > 0$ the set $K(y, \varepsilon) = \{\delta : |y(t + \delta) - y(t)| < \varepsilon, \forall t \in \mathbb{R}\}$ is relatively dense; that is, for any $\varepsilon > 0$ it is possible to find a real number $l(\varepsilon) > 0$ for any interval with length $l(\varepsilon)$; there exists a number $\delta = \delta(\varepsilon)$ in this interval such that $|y(t + \delta) - y(t)| < \varepsilon$ for any $t \in \mathbb{R}$.

Throughout the rest of the paper we assume the following condition for (2.1):

(H) $a(t), b(t), K(t), t - \theta(t) \in AP(\mathbb{R}), m(\mathrm{b}/K^{\gamma}) \neq 0$ and $m(a) \neq m(b)$.

In our case, we set

$$X = Y = V_1 \oplus V_2, \tag{3.2}$$

where

$$V_1 = \{y \in AP(\mathbb{R}) : \operatorname{mod}(y) \subseteq \operatorname{mod}(F),\ \forall \mu \in \Lambda(y) \text{ satisfies } |\mu| > \alpha\},$$
$$V_2 = \{y(t) \equiv k, k \in \mathbb{R}\}, \tag{3.3}$$

where

$$F = F(t,\phi) = \frac{a(t)}{1 + \left(e^{\phi(\theta(0))}/K(t)\right)^{\gamma}} - b(t), \quad \phi \in C([-\infty,0],\mathbb{R}) \tag{3.4}$$

and α is a given constant; define the norm

$$\|y\| = \sup_{t\in\mathbb{R}}|y(t)|, \quad y \in \mathrm{X} \text{ (or Y)}. \tag{3.5}$$

Remark 3.2. If f is ε-almost periodic function, then $\int^t f(s)\mathrm{d}s$ is ε-almost periodic if and only if $m(f) = 0$. Whereas $f \in AP(\mathbb{R})$ does not necessarily have an almost periodic primitive, $m(f) = 0$. That is why we can not make $V_1 = \{z \in AP(\mathbb{R}) : m(z) = 0\}$ and let $V_1 = \{y \in AP(\mathbb{R}) : \operatorname{mod}(y) \subset \operatorname{mod}(F), \forall \mu \in \Lambda(y)$ satisfy $|\mu| > \alpha\}$.

We start with the following lemmas.

Lemma 3.3. *X and Y are Banach spaces endowed with the norm $\|\cdot\|$.*

Proof. If $y_n \in \mathrm{V}_1$ and y_n converge to y_0, then it is easy to show that $y_0 \in AP(\mathbb{R})$ with mod $(y_0) \subset \operatorname{mod}(F)$. Indeed, for all $|\tilde{\lambda}| \leq \alpha$ we have

$$\lim_{T\to\infty} \frac{1}{T}\int_0^t y_n(s)e^{-i\tilde{\lambda}s}\mathrm{d}s = 0. \tag{3.6}$$

Thus

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T y_0(s)e^{-i\tilde{\lambda}s}\mathrm{d}s = 0, \tag{3.7}$$

which implies that $y_0 \in V_1$. One can easily see that V_1 is a Banach space endowed with the norm $\|\cdot\|$. The same can be concluded for the spaces X and Y. The proof is complete. □

Lemma 3.4. *Let $L : X \to Y$ and*

$$Ly = \frac{a(t)}{1 + \left(e^{y(\theta(t))}/K(t)\right)^{\gamma}} - b(t), \tag{3.8}$$

where $Ly = y' = \mathrm{d}y/\mathrm{d}t$. Then L is a Fredholmmapping of index zero.

Proof. It is obvious that L is a linear operator and $\operatorname{Ker} L = V_2$. It remains to prove that $\operatorname{Im} L = V_1$. Suppose that $\phi(t) \in \operatorname{Im} L \subset Y$. Then, there exist $\phi_1 \in V_1$ and $\phi_2 \in V_2$ such that

$$\phi = \phi_1 + \phi_2. \tag{3.9}$$

From the definitions of $\phi(t)$ and $\phi_1(t)$, one can deduce that $\int^t \phi(s)\mathrm{d}s$ and $\int^t \phi_1(s)\mathrm{d}s$ are almost periodic functions and thus $\phi_2(t) \equiv 0$, which implies that $\phi(t) \in V_1$. This tells us that

$$\operatorname{Im} L \subset V_1. \tag{3.10}$$

On the other hand, if $\varphi(t) \in V_1 \setminus \{0\}$ then we have $\int_0^t \varphi(s)\mathrm{d}s \in AP(\mathbb{R})$. Indeed, if $\tilde{\lambda} \neq 0$ then we obtain

$$\lim_{T\to\infty} \frac{1}{T} \int_0^T \left[\int_0^t \varphi(s)\mathrm{d}s\right] e^{-i\tilde{\lambda}t}\mathrm{d}t = \frac{1}{i\tilde{\lambda}} \lim_{T\to\infty} \frac{1}{T} \int_0^T \varphi(t) e^{-i\tilde{\lambda}t}\, \mathrm{d}t. \tag{3.11}$$

It follows that

$$\Lambda\left[\int_0^t \varphi(s)\mathrm{d}s - m\left(\int_0^t \varphi(s)\mathrm{d}s\right)\right] = \Lambda(\varphi(t)). \tag{3.12}$$

Thus

$$\int_0^t \varphi(s)\mathrm{d}s - m\left(\int_0^t \varphi(s)\mathrm{d}s\right) \in V_1 \subset X. \tag{3.13}$$

Note that $\int_0^t \varphi(s)\mathrm{d}s - m(\int_0^t \varphi(s)\mathrm{d}s)$ is the primitive of φ in X; therefore we have $\varphi \in \operatorname{Im} L$. Hence, we deduce that

$$V_1 \subset \operatorname{Im} L, \tag{3.14}$$

which completes the proof of our claim. Therefore,

$$\operatorname{Im} L = V_1. \tag{3.15}$$

Furthermore, one can easily show that $\operatorname{Im} L$ is closed in Y and

$$\dim \operatorname{Ker} L = 1 = \operatorname{codim} \operatorname{Im} L. \tag{3.16}$$

Therefore, L is a Fredholm mapping of index zero. □

Lemma 3.5. *Let $N : X \to Y$, $P : X \to X$, and $Q : Y \to Y$ such that*

$$\begin{gathered} Ny = \frac{a(t)}{1 + \left(e^{y(\theta(t))}/K(t)\right)^{\gamma}} - b(t), \quad y \in X, \\ Py = m(y), \quad y \in X, \; Qz = m(z), \; z \in Y. \end{gathered} \tag{3.17}$$

Then, N is L-compact on $\overline{\Omega}$ (Ω is an open and bounded subset of X).

Proof. The projections P and Q are continuous such that

$$\operatorname{Im} P = \operatorname{Ker} L, \qquad \operatorname{Im} L = \operatorname{Ker} Q. \tag{3.18}$$

It is clear that

$$\begin{gathered} (I - Q)V_2 = \{0\}, \\ (I - Q)V_1 = V_1. \end{gathered} \tag{3.19}$$

Therefore

$$\operatorname{Im}(I - Q) = V_1 = \operatorname{Im} L. \tag{3.20}$$

In view of

$$\begin{gathered} \operatorname{Im} P = \operatorname{Ker} L, \\ \operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q), \end{gathered} \tag{3.21}$$

we can conclude that the generalized inverse (of L) $K_P : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ exists and is given by

$$K_P(z) = \int_0^t z(s)\mathrm{d}s - m\left[\int_0^t z(s)\mathrm{d}s\right]. \tag{3.22}$$

Thus

$$\begin{gathered} QNy = m\left[\frac{a(t)}{1 + \left(e^{y(\theta(t))}/K(t)\right)^{\gamma}} - b(t)\right], \\ K_P(I - Q)Ny = f[y(t)] - Qf[y(t)], \end{gathered} \tag{3.23}$$

where $f[y(t)]$ is defined by

$$f[y(t)] = \int_0^t [Ny(s) - QNy(s)]\mathrm{d}s. \tag{3.24}$$

The integral form of the terms of both QN and $(I-Q)N$ implies that they are continuous. We claim that K_P is also continuous. By our hypothesis, for any $\varepsilon<1$ and any compact set $S\subset C((-\infty,0],\mathbb{R})$, let $l(\varepsilon,S)$ be the inclusion interval of $K(F,\varepsilon,S)$. Suppose that $\{z_n(t)\}\subset \operatorname{Im} L=V_1$ and $z_n(t)$ uniformly converges to $z_0(t)$. Because $\int_0^t z_n(s)\mathrm{d}s\in Y(n=0,1,2,3,\ldots)$, there exists $\rho(0<\rho<\varepsilon)$ such that $K(F,\rho,S)\subset K(\int_o^t z_n(s)\ \mathrm{d}s,\varepsilon)$. Let $l(\rho,S)$ be the inclusion interval of $K(F,\rho,S)$ and

$$l=\max\{l(\rho,S),l(\varepsilon,S)\}. \tag{3.25}$$

It is easy to see that l is the inclusion interval of both $K(F,\varepsilon,S)$ and $K(F,\rho,S)$. Hence, for all $t\notin[0,l]$, there exists $\mu_t\in K(F,\rho,\mathrm{S})\subset K(\int_0^t z_n(s)\mathrm{d}s,\varepsilon)$ such that $t+\mu_t\in[0,l]$. Therefore, by the definition of almost periodic functions we observe that

$$\begin{aligned}\left\|\int_0^t z_n(s)\mathrm{d}s\right\| &= \sup_{t\in\mathbb{R}}\left|\int_0^t z_n(s)\mathrm{d}s\right|\\ &\le \sup_{t\in[0,l]}\left|\int_0^t z_n(s)\mathrm{d}s\right|+\sup_{t\notin[0,l]}\left|\left(\int_0^t z_n(s)\mathrm{d}s-\int_0^{t+\mu_t} z_n(s)\mathrm{d}s\right)+\int_0^{t+\mu_t} z_n(s)\mathrm{d}s\right|\\ &\le 2\sup_{t\in[0,l]}\left|\int_0^t z_n(s)\mathrm{d}s\right|+\sup_{t\notin[0,l]}\left|\int_0^t z_n(s)\mathrm{d}s-\int_0^{t+\mu_t} z_n(s)\mathrm{d}s\right|\\ &\le 2\int_0^l |z_n(s)|\mathrm{d}s+\varepsilon.\end{aligned} \tag{3.26}$$

By applying (3.26), we conclude that $\int_0^t z(s)\mathrm{d}s(z\in \operatorname{Im}\ L)$ is continuous and consequently K_P and $K_P(I-Q)Ny$ are also continuous.

From (3.26), we also have that $\int_0^t z(s)\mathrm{d}s$ and $K_P(I-Q)Ny$ are uniformly bounded in $\overline{\Omega}$. In addition, it is not difficult to verify that $QN(\overline{\Omega})$ is bounded and $K_P(I-Q)Ny$ is equicontinuous in $\overline{\Omega}$. Hence by the Arzelà-Ascoli theorem, we can immediately conclude that $K_P(I-Q)N(\overline{\Omega})$ is compact. Thus N is L-compact on $\overline{\Omega}$. □

Theorem 3.6. *Let condition (H) hold. Then* (2.1) *has at least one positive almost periodic solution.*

Proof. It is easy to see that if (2.1) has one almost periodic solution $\overline{y}$, then $\overline{N}=e^{\overline{y}}$ is a positive almost periodic solution of (1.3). Therefore, to complete the proof it suffices to show that (2.1) has one almost periodic solution.

In order to use the continuation theorem of coincidence degree theory, we set the Banach spaces X and Y the same as those in Lemma 3.3 and the mappings L, N, P, Q the same as those defined in Lemmas 3.4 and 3.5, respectively. Thus, we can obtain that L is a Fredholm mapping of index zero and N is a continuous operator which is L-compact on $\overline{\Omega}$.

It remains to search for an appropriate open and bounded subset Ω. Corresponding to the operator equation

$$Ly = \lambda Ny, \qquad \lambda \in (0,1), \tag{3.27}$$

we may write

$$\dot{y}(t) = \lambda\left[\frac{a(t)}{1+\left(e^{y(\theta(t))}/K(t)\right)^{\gamma}} - b(t)\right]. \tag{3.28}$$

Assume that $y = y(t) \in X$ is a solution of (3.28) for a certain $\lambda \in (0,1)$. Denote

$$y^* = \sup_{t\in\mathbb{R}} y(t), \qquad y_* = \inf_{t\in\mathbb{R}} y(t). \tag{3.29}$$

In view of (3.28), we obtain

$$m(a(t) - b(t)) = m\left(\frac{b(t)}{K^{\gamma}(t)}\left(e^{y(\theta(t))}\right)^{\gamma}\right) \tag{3.30}$$

and consequently,

$$m(a(t) - b(t)) \geq m\left(\frac{b(t)}{K^{\gamma}(t)}\right)e^{\gamma y_*}, \tag{3.31}$$

which implies from (H) that

$$y_* \leq \gamma^{-1}\ln\left(\frac{m[a(t) - b(t)]}{m(b(t)/K^{\gamma}(t))}\right). \tag{3.32}$$

Similarly, we can get

$$y^* \geq \gamma^{-1}\ln\left(\frac{m(a(t) - b(t))}{m(b(t)/K^{\gamma}(t))}\right). \tag{3.33}$$

By inequalities (3.32) and (3.33), we can find that there exists $t_1 \in \mathbb{R}$ such that

$$\left|y(t_1)\right| \leq M, \tag{3.34}$$

where

$$M = \left|\gamma^{-1}\ln\left(\frac{m(a(t) - b(t))}{m(b(t)/K^{\gamma}(t))}\right)\right| + 1. \tag{3.35}$$

Then from (3.26), we have

$$\|y(t)\| \leq |y(t_1)| + \sup_{t\in\mathbb{R}}\left|\int_{t_2}^{t} y'(s)\mathrm{d}s\right| \leq M + 2\sup_{t\in[t_2,t_2+l]}\int_{t_2}^{t}|y'(s)|\mathrm{d}s + \varepsilon \tag{3.36}$$

or

$$\|y(t)\| \leq M + 2\int_{t_2}^{t_2+l}|y'(s)|\mathrm{d}s + 1. \tag{3.37}$$

Choose the point $\nu - t_2 \in [l, 2l] \ \cap K(F,\rho,S)$, where $\rho(0<\rho<\varepsilon)$ satisfies $K(F,\rho) \subset K(z,\varepsilon)$. Integrating (3.28) from t_2 to ν, we get

$$\begin{aligned}\lambda\int_{t_2}^{\nu}\frac{a(t)}{1+[K(s)]^{-\gamma}e^{\gamma y(\theta(s))}}\mathrm{d}s &= \lambda\int_{t_2}^{\nu}|b(s)|\mathrm{d}s + \int_{t_2}^{\nu}y'(s)\mathrm{d}s\\ &\leq \lambda\int_{t_2}^{\nu}|b(s)|\mathrm{d}s + \varepsilon.\end{aligned} \tag{3.38}$$

However, from (3.28) and (3.38), we obtain

$$\begin{aligned}\int_{t_2}^{\nu}|y'(s)|\mathrm{d}s &\leq \lambda\int_{t_2}^{\nu}|b(s)|\mathrm{d}s + \lambda\int_{t_2}^{\nu}\frac{a(t)}{1+[K(s)]^{-\gamma}e^{\gamma y(\theta(s))}}\mathrm{d}s,\\ &\leq 2\int_{t_2}^{\nu}|b(s)|\mathrm{d}s + \varepsilon\\ &\leq 2\int_{t_2}^{\nu}|b(s)|\mathrm{d}s + 1.\end{aligned} \tag{3.39}$$

Substituting back in (3.37) and for $\nu \geq t_2 + l$, we have

$$\|y(t)\| \leq M', \tag{3.40}$$

where

$$M' = M + 4\int_{0}^{\nu}|b(s)|\mathrm{d}s + 3. \tag{3.41}$$

Let $\widetilde{M} = M + M'$. Obviously, it is independent of λ. Take

$$\Omega = \left\{y \in X : \|y\| < \widetilde{M}\right\}. \tag{3.42}$$

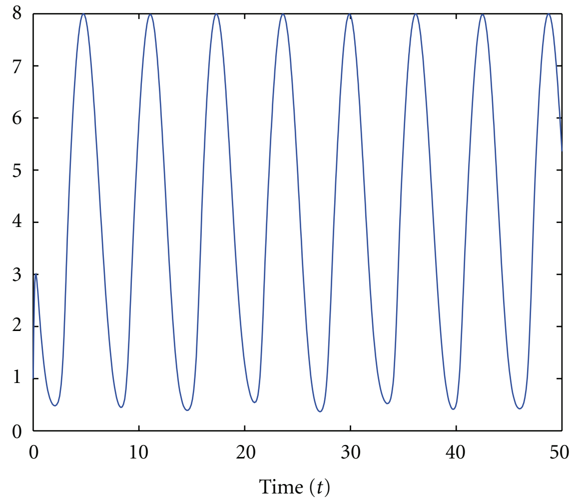

Figure 1: Transient response of state $N(t)$ when $\gamma = 2$.

It is clear that Ω satisfies assumption (1) of Theorem 2.1. If $y \in \partial\Omega \cap \operatorname{Ker} L$, then y is a constant with $\|y\| = \widetilde{M}$. It follows that

$$QNy = m\left(\frac{a(t)}{1 + \left(e^{y(\theta(t))}/K(t)\right)^{\gamma}} - b(t)\right) \neq 0, \tag{3.43}$$

which implies that assumption (2) of Theorem 2.1 is satisfied. The isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$ is defined by $J(z) = z$ for $z \in \mathbb{R}$. Thus, $JQNy \neq 0$. In order to compute the Brouwer degree, we consider the homotopy

$$H(y,s) = -sy + (1-s)JQNy, \quad 0 \leq s \leq 1. \tag{3.44}$$

For any $y \in \partial\Omega \cap \operatorname{Ker} L$, $s \in [0,1]$, we have $H(y,s) \neq 0$. By the homotopic invariance of topological degree, we get

$$\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} = \deg\{-y, \Omega \cap \operatorname{Ker} L, 0\} \neq 0. \tag{3.45}$$

Therefore, assumption (3) of Theorem 2.1 holds. Hence, $Ly = Ny$ has at least one solution in $\operatorname{Dom} L \cap \overline{\Omega}$. In other words, (2.1) has at least one positive almost periodic solution. Therefore, (1.3) has at least one positive almost periodic solution. The proof is complete. $\square$

4. An Example

Let $a(t) = e^{\pi}(3 + \cos\sqrt{2}t), b(t) = (1/2)e^{\pi}(3 + \cos\sqrt{2}t), K(t) = 4 + \sin\sqrt{t}, \gamma > 0, \theta(t) = t - 2 - \sin\sqrt{3}t$. Then (1.3) has the form

$$\dot{N}(t) = N(t)\left[\frac{e^{\pi}\left(3 + \cos\sqrt{2}t\right)}{1 + \left(N\left(t - 2 - \sin\sqrt{3}t\right)/\left(4 + \sin\sqrt{t}\right)\right)^{\gamma}} - \frac{1}{2}e^{\pi}\left(3 + \cos\sqrt{2}t\right)\right]. \tag{4.1}$$

One can easily realize that $m(b(t)/[K(t)]^{\gamma}) > 0$ and $m(a(t)) > m(b(t))$; thus condition (H) holds. Therefore, by the consequence of Theorem 3.6, (4.1) has at least one positive almost periodic solution (Figure 1).

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