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Research Article

The Dimensional Fluxes of the Hypercylindrical Function

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We present the results of the theoretical researches of some characteristics of the generalized hyperspherical function with two degrees as independent dimensions. Primarily are given the answers on the quantification of the dimensional potentials (fluxes) of this function in the domain of the integer natural numbers. Beside them, we have got the solutions for some continual fluxes of the contour hypercylindrical (HY) functions on the basis. The symbolical evaluation and numerical verification of the series values and integrals are realized through the program packages Mathcad Professional and Mathematica.

1. Introduction

Hypercylinder function is the hypothetical function connected to multidimensional space. The most significant value of this function is in the fact that it originated [1] on the properties of the cylindrical entities: point, diameter, square, surface, and volume of cylinder. Another property is generalizing of these functions from discretion to continuum. It belongs to the group of special functions, so its testing is performed on the basis of known functions of these types: gamma (Γ), psi (ψ_0), error function (erf), and the like.

Definition 1.1. The generalized hypercylindrical function is defined by equality [2]

$$\mathrm{HY}(k,n,r) = \frac{2\sqrt{\pi^{k-1}}r^{k+n-3}\Gamma(k+1)}{\Gamma(k+n-2)\Gamma((k+1)/2)} \quad (k,n \in \Re,\ r \in N), \tag{1.1}$$

where $\Gamma(z)$ is the gamma function.

Thanks to the interpolation properties of the gamma function, we can analytically pass from the field of the natural integer values on the set of real and noninteger values with which there is concurrence of events both for its graphic interpretation and more concise mathematical analysis. It is developed on the basis of the two freedom degrees k and n, as special (vectors) dimensions, besides r radius, as implicitly included freedom degree for every hypercylinder (Figures 1 and 2). The dominant theorem that is set is the one that refers to the recurrent property of this function (when the height is $h = 2r$). It implicitly includes that the left vectors ($n = 2, 1, 0, -1, -2, \ldots$) of the $M[\mathrm{HY}]_{kxn}$ matrix columns in Figure 1 we get on the basis of the reverent vector ($n = 3$) deduction, and the right vectors ($n = 4, 5, 6, 7, 8, \ldots$) on the basis of integrals on r radius [2]

$$\frac{\partial}{\partial r}\mathrm{HY}(k,n,r) = \mathrm{HY}(k,n-1,r), \qquad \mathrm{HY}(k,n+1,r) = \int_0^r \mathrm{HY}(k,n,r)dr. \tag{1.2}$$

To the development of the theory of the multidimensional objects, especially have ontributed: Conway and Sloane [3], Gwak et al. [4], Hinton [5], Hocking and Young [6], Manning [7], Maunder [8], Neville [9], Von R. Rucker [10], Sommerville [11], Sun and Bowman [12], and the others, and to its testing, Ramanujan and Hardy [16, 17]. Today the researches of the hypercylindrical function are represented both in Euclid's and Riemann's geometry (molecular dynamics, neural networks, hypercylindrical black holes and the like).

2. Dimensional Potentials: The Fluxes of the HY Function

2.1. Vertical Dimensional Flux of the Hypercylindrical Function

The discrete dimensional potential or *flux* of the hypercylindrical function presents the total of all single functions in the (sub)matrix of this function that develops for the integer natural freedom degrees. Formally, flux can be quantification by twofold series that covers this area of HY function. The first step is to define the value of the infinite succession of functions ordered in columns (vectors) of the submatrix $M[\mathrm{HY}]_{kxn}$ $(k, n \in N)$. This is at the same time as well the definition of the vertical dimensional fluxes of HY function. The first value that is being calculated, refers to the fourth column ($n = 3$) of the submatrix $M[\mathrm{HY}]_{kxn}$ in Figure 1. In this case, it follows that the flux is equal

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,3,r) = \frac{2}{\pi} + 2r + 4r^2 + 2\pi r^3 + \frac{8}{3}\pi r^4 + \pi^2 r^4 + \cdots + \varepsilon\left(\frac{2\sqrt{\pi^{k-1}}r^k}{\Gamma((k+1)/2)}\right). \tag{2.1}$$

The analytical value of this series is

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,3,r) = \sum_{k=0}^{\infty}\frac{2\sqrt{\pi^{k-1}}r^k}{\Gamma((k+1)/2)} = 2\left(\frac{1}{\pi} + re^{\pi r^2}\operatorname{erfc}\left(-r\sqrt{\pi}\right)\right). \tag{2.2}$$

$$
\mathrm{M[HY]}_{kxn} =
\begin{array}{cc}
\begin{matrix} -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} & \begin{matrix} n \\ k \end{matrix} \\
\begin{bmatrix}
\frac{1260}{\pi^3 r^8} & -\frac{180}{\pi^3 r^7} & \frac{30}{\pi^3 r^6} & -\frac{6}{\pi^3 r^5} & \frac{3}{2\pi^3 r^4} & -\frac{1}{2\pi^3 r^3} & \frac{1}{4\pi^3 r^2} & -\frac{1}{4\pi^3 r} & \text{undef.} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\pi^2} & -\frac{r}{2\pi^2} \\
-\frac{120}{\pi^2 r^6} & \frac{24}{\pi^2 r^5} & -\frac{6}{\pi^2 r^4} & \frac{2}{\pi^2 r^3} & -\frac{1}{\pi^2 r^2} & \frac{1}{\pi^2 r} & \text{undef.} & \text{undef.} & \text{undef.} \\
0 & 0 & 0 & 0 & 0 & \frac{1}{\pi} & \frac{r}{2\pi} & \frac{r^2}{2\pi} & \frac{r^3}{6\pi} \\
0 & 0 & 0 & 0 & \frac{2}{\pi} & \frac{2r}{\pi} & \frac{r^2}{\pi} & \frac{r^3}{3\pi} & \frac{r^4}{12\pi} \\
0 & 0 & 0 & 2 & 2r & r^2 & \frac{r^3}{3} & \frac{r^4}{12} & \frac{r^5}{60} \\
0 & 0 & 8 & 8r & 4r^2 & \frac{4r^3}{3} & \frac{r^4}{3} & \frac{r^5}{15} & \frac{r^6}{90} \\
0 & 12\pi & 12\pi r & 6\pi r^2 & 2\pi r^3 & \frac{\pi r^4}{2} & \frac{\pi r^5}{10} & \frac{\pi r^6}{60} & \frac{\pi r^7}{420} \\
64\pi & 64\pi r & 32\pi r^2 & \frac{32\pi r^3}{3} & \frac{8\pi r^4}{3} & \frac{8\pi r^5}{15} & \frac{4\pi r^6}{45} & \frac{4\pi r^7}{315} & \frac{\pi r^8}{630}
\end{bmatrix}
&
\begin{matrix} -4 \\ -3 \\ -2 \\ -1 \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix}
\end{array}
$$

$n \in N$

$K \in N$

Figure 1: The functional submatrix $M[\mathrm{HY}(k,n,r)]$ for the freedom degrees $k \in -4,-3,\ldots,4$ and $n \in -1,0,\ldots,7$ and with six characteristics hypercylindrical functions (undef. are nondefined, most often singular values of this function).

In the paper are used three known error functions as follows: $\mathrm{erf}(z)$-basic, $\mathrm{erfc}(z)$-cumulative and $\mathrm{erfi}(z)$-imaginary. When k values are even $(0,2,4,\ldots)$, in other words odd ones $(1,3,5,\ldots)$, the series can be divided as dichotomous, so we can now obtain two complementary series

$$
\begin{aligned}
\sum_{k=0}^{\infty} \mathrm{HY}(k,3,1) &= \sum_{k=0,2,4,\ldots}^{\infty} \mathrm{HY}(k,3,r) + \sum_{k=1,3,5,\ldots}^{\infty} \mathrm{HY}(k,3,r) \\
&= 2\left(\frac{1}{\pi} + re^{\pi r^2}\,\mathrm{erf}\left(r\sqrt{\pi}\right)\right) + 2re^{\pi r^2}.
\end{aligned}
\tag{2.3}
$$

Well, the result (2.3) can be presented in the from of series with even $(k = 2b)$ and odd members $(k = 2b+1)$ that complement (one another). In that sense follows

$$
\sum_{b=0}^{\infty} \frac{2\pi^{b-1}(2r)^{2b} b!}{(2b)!} = 2\left(\frac{1}{\pi} + re^{\pi r^2}\,\mathrm{erf}\left(r\sqrt{\pi}\right)\right), \qquad \sum_{b=0}^{\infty} \frac{2\pi^{b} r^{2b+1}}{b!} = 2re^{\pi r^2}. \tag{2.4}
$$

On the basis of the solution (2.3), as the starting and reference (one), and applying the reference relations (1.2), we can obtain the series values for lower freedom degrees $(n < 3)$.

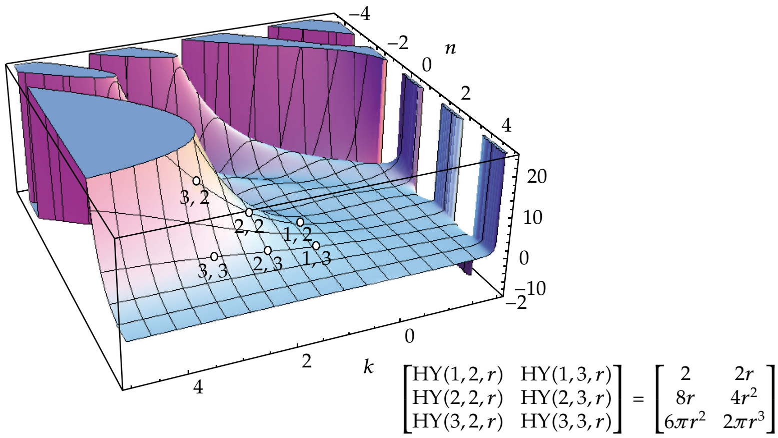

Figure 2: The surface graphic of the hypercylindrical function with the constant radius $r = 1$ and the selected fields of freedom degrees $(-4 < n \leq 5)$ and $(-2 < k \leq 5)$ (using the software *Mathematica*).

So, we establish the connection of cylinder hypervolume ($n = 3$) with its hypersurface ($n = 2$). In that sense follows a new vector flux for $n = 2$:

$$\frac{\partial}{\partial r}\sum_{k=0}^{\infty}\mathrm{HY}(k,3,r) = \sum_{k=0}^{\infty}\mathrm{HY}(k,2,r), \tag{2.5}$$

so that

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,2,r) = 2\left[2r + e^{\pi r^2}\operatorname{erfc}\left(-r\sqrt{\pi}\right)\left(1 + 2\pi r^2\right)\right]. \tag{2.6}$$

Further, for the hypercylinder ($n = 1$), we obtain a series on the basis of the derivative of the previous series, so that

$$\frac{\partial}{\partial r}\sum_{k=0}^{\infty}\mathrm{HY}(k,2,r) = \sum_{k=0}^{\infty}\mathrm{HY}(k,1,r) = 4\left[2\left(1 + \pi r^2\right) + \pi r e^{\pi r^2}\operatorname{erfc}\left(-r\sqrt{\pi}\right)\left(3 + 2\pi r^2\right)\right]. \tag{2.7}$$

For the zero dimension $n = 0$, the series value is as well located on the derivative basis, so it follows that

$$\begin{aligned}\sum_{k=0}^{\infty}\mathrm{HY}(k,0,r) &= \sum_{k=0}^{\infty}\frac{2r^{k-3}\sqrt{\pi^{k-1}}\,\Gamma(k+1)}{\Gamma(k-2)\Gamma((k+1)/2)} \\ &= 4\pi\left[r\left(10 + 4\pi r^2\right) + e^{\pi r^2}\operatorname{erfc}\left(-r\sqrt{\pi}\right)\left[3 + 4\pi r^2\left(3 + \pi r^2\right)\right]\right].\end{aligned} \tag{2.8}$$

For greater freedom degree that $n = 3$, series are found through an inverse operation, that is, by recurrent relation on the basis of integrating on the radius r. Well,

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,n+1,r) = \int_0^r \left(\sum_{k=0}^{\infty}\mathrm{HY}(k,n,r)\right)dr, \tag{2.9}$$

and for the fourth dimension is valid the next integral form

$$\begin{aligned}\sum_{k=0}^{\infty}\mathrm{HY}(k,4,r) &= \int_0^r \left(\sum_{k=0}^{\infty}\mathrm{HY}(k,3,r)\right)dr = \int_0^r 2\left[\frac{1}{\pi} + re^{\pi r^2}\operatorname{erf}\left(r\sqrt{\pi}\right)\right]dr \\ &= \frac{1}{\pi}\left[e^{\pi r^2}\operatorname{erfc}\left(-r\sqrt{\pi}\right) - 1\right].\end{aligned} \tag{2.10}$$

For greater freedom degree than $n = 3$, series are found through a recurrent relation (1.2), by integrating on radius

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,4,r) = \int_0^r \left(\sum_{k=1,3,5,\ldots}^{\infty}\mathrm{HY}(k,3,r)\right)dr + \int_0^r \left(\sum_{k=0,2,4,\ldots}^{\infty}\mathrm{HY}(k,3,r)\right)dr. \tag{2.11}$$

So, we come to the expression

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,4,r) = \int_0^r 2re^{\pi r^2}dr + \int_0^r \left(\sum_{b=0}^{\infty}\frac{2^{2b+1}\pi^{b-1}r^{2b}b!}{(2b)!}\right)dr. \tag{2.12}$$

Solving the integrals and using the relation on dichotomous series [2], we obtain the dimension flux of the fourth dimension

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,4,r) = \frac{e^{\pi r^2}-1}{\pi} + \sum_{b=0}^{\infty}\frac{2^{2b+1}\pi^{b-1}r^{2b+1}b!}{(2b+1)!}. \tag{2.13}$$

For the freedom degree of $n = 5$, series can be found through the integrating of the obtained solution (2.10)

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,5,r) = \int_0^r \frac{1}{\pi}\left[e^{\pi r^2}\operatorname{erfc}\left(-r\sqrt{\pi}\right) - 1\right]dr. \tag{2.14}$$

The general solution is known, and it is

$$\int e^{bz^2}\operatorname{erfc}(az)dz = \frac{\sqrt{\pi}\operatorname{erf}\left(z\sqrt{b}\right)}{2\sqrt{b}} - \frac{1}{b\sqrt{\pi}}\sum_{k=0}^{\aleph}\frac{a^{2k+1}\Gamma\left(k+1,-bz^2\right)}{b^k k!(2k+1)} + C. \tag{2.15}$$

After settling the expression, we get the concrete solution

$$\begin{aligned}\sum_{k=0}^{\infty}\mathrm{HY}(k,5,r) &= \sum_{k=0}^{\infty}\frac{2r^{k+2}\sqrt{\pi^{k-1}}\Gamma(k+1)}{\Gamma(k+3)\Gamma((k+1)/2)}\\ &= \frac{1}{\pi}\left[\frac{\operatorname{erfi}\left(r\sqrt{\pi}\right)}{2} - r + \frac{1}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^{2k+1}}{2k+1}\left(1-\frac{\Gamma\left(k+1,-\pi r^2\right)}{k!}\right)\right].\end{aligned} \tag{2.16}$$

For the same freedom degree, the series can be found with the integrating of the dichotomous expression

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,5,r) = \int_0^r \frac{e^{\pi r^2}-1}{\pi}dr + \int_0^r\left(\sum_{b=0}^{\infty}\frac{2^{2b+1}\pi^{b-1}r^{2b+1}b!}{(2b+1)!}\right)dr. \tag{2.17}$$

So that

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,5,r) = \frac{1}{\pi}\left(\frac{\operatorname{erfi}\left(r\sqrt{\pi}\right)}{2} - r\right) + \sum_{b=0}^{\infty}\frac{2^{2b+1}\pi^{b-1}r^{2(b+1)}b!}{(2b+2)!}. \tag{2.18}$$

The same result is obtained as well on the basis of the complementary dichotomous series

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,5,r) = \sum_{b=0}^{\infty}\frac{2\pi^b r^{2b+3}}{(2b+2)(2b+3)b!} + \sum_{b=0}^{\infty}\frac{2^{2b+1}\pi^{b-1}r^{2(b+1)}b!}{(2b+2)!}. \tag{2.19}$$

Here, we use the imaginary error function that is equal to [14]

$$\operatorname{erfi}(z) = -i\cdot\operatorname{erf}(i\cdot z) = \frac{2}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{z^{2k+1}}{k!(2k+1)}. \tag{2.20}$$

For the last analysed vector flux, through similar procedures follow:

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,6,r) = \int_0^r\left(\sum_{b=0}^{\infty}\frac{2\pi^b r^{2b+3}}{(2b+2)(2b+3)b!}\right)dr + \int_0^r\left(\sum_{b=0}^{\infty}\frac{2^{2b+1}\pi^{b-1}r^{2(b+1)}b!}{(2b+2)!}\right)dr. \tag{2.21}$$

It can be presented that the first subintegral member of the dichotomous series (2.19) is equal to

$$\sum_{b=0}^{\infty}\frac{2\pi^b r^{2b+3}}{(2b+2)(2b+3)b!} = \frac{1}{\pi}\left(\frac{\operatorname{erfi}\left(r\sqrt{\pi}\right)}{2} - r\right). \tag{2.22}$$

So, its integral is

$$\int_0^r \frac{1}{\pi}\left(\frac{\operatorname{erfi}\left(r\sqrt{\pi}\right)}{2} - r\right)dr = \frac{1}{2\pi}\left[\frac{1-e^{\pi r^2}}{\pi} + r\operatorname{erfi}\left(r\sqrt{\pi}\right) - r^2\right]. \tag{2.23}$$

The dimensional flux for the sixth freedom degree is now

$$\sum_{k=0}^{\infty}\frac{2r^{k+3}\sqrt{\pi^{k-1}}\Gamma(k+1)}{\Gamma(k+4)\Gamma((k+1)/2)}=\frac{1}{2\pi}\left[\frac{1-e^{\pi r^2}}{\pi}+r\,\mathrm{erfi}\left(r\sqrt{\pi}\right)-r^2\right]+\sum_{b=0}^{\infty}\frac{2^{2b+1}\pi^{b-1}r^{2b+3}b!}{(2b+3)!}. \tag{2.24}$$

For each freedom degree the recurrent relation would be formulated as

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,n,r)=\int_0^r\left(\sum_{k=0,2,\ldots}^{\infty}\mathrm{HY}(k,n-1,r)+\sum_{k=1,3,\ldots}^{\infty}\mathrm{HY}(k,n-1,r)\right)dr. \tag{2.25}$$

The similar formulation would be related as well to the recurrent relation of the type

$$\sum_{k=0}^{\infty}\mathrm{HY}(k,n,r)=\frac{\partial}{\partial r}\left(\sum_{k=0,2,\ldots}^{\infty}\mathrm{HY}(k,n+1,r)+\sum_{k=1,3,\ldots}^{\infty}\mathrm{HY}(k,n+1,r)\right). \tag{2.26}$$

It can be supposed that the values of the vector fluxes are less and less with the increasing of the freedom degree n, so that the limit values are equal

$$\lim_{n\to\infty}\sum_{k=0}^{\infty}\mathrm{HY}(k,n,r)=0,\quad \text{respectively,}\quad \lim_{n\to\infty}\int_0^{\infty}\mathrm{HY}(k,n,r)dk=0. \tag{2.27}$$

The systematized numerical values of the discrete and continual fluxes, (for $r=1$), are given in Table 1.

The dimensional fluxes can be studied as well for the complex part. So, for example with recurrence we get the series value for negative freedom degrees, and so for $n=-1$, follows [2]

$$\begin{aligned}\sum_{k=0}^{\infty}\mathrm{HY}(k,-1,r)&=\sum_{k=0}^{\infty}\frac{2r^{k-4}\sqrt{\pi^{k-1}}\Gamma(k+1)}{\Gamma(k-3)\Gamma((k+1)/2)}\\&=8\pi\left\{2\left(1+2\pi r^2\right)\left(4+\pi r^2\right)+\pi r e^{\pi r^2}\,\mathrm{erfc}\left(-r\sqrt{\pi}\right)\left[15+4\pi r^2\left(5+\pi r^2\right)\right]\right\}.\end{aligned} \tag{2.28}$$

For more lower freedom degree, that is $n=-2$, the flux is more complex and is defined by the next analytical value

$$\begin{aligned}\sum_{k=0}^{\infty}\mathrm{HY}(k,-2,r)&=\frac{\partial}{\partial r}\sum_{k=0}^{\infty}\mathrm{HY}(k,-1,r)=\sum_{k=0}^{\infty}\frac{2\,r^{k-5}\sqrt{\pi^{k-1}}\Gamma(k+1)}{\Gamma(k-4)\Gamma((k+1)/2)}\\&=8\pi^2\left\{2r\left(3+2\pi r^2\right)\left(11+2\pi r^2\right)+e^{\pi r^2}\,\mathrm{erfc}\left(-r\sqrt{\pi}\right)\right.\\&\qquad\left.\times\left[2\pi r^2\left(2\pi r^2\left(15+2\pi r^2\right)+45\right)+15\right]\right\}.\end{aligned} \tag{2.29}$$

Table 1

The freedom degree n	$\sum_{k=0}^{\infty} \mathrm{HY}(k,n,1)$	$\int_0^{\infty} \mathrm{HY}(k,n,1)dk$
0	46629.763683374723	46629.884250891254
1	5399.2276899298969	5399.1696270574682
2	674.04323182871168	674.09309651202785
3	92.635271951133292	92.399315367283533
4	14.323730365858739	14.034919970188039
5	2.4571998816772904	2.2995389510681934
6	0.4379835171696227	0.3825093919178916
7	0.0753101246771644	0.0609006925490481
8	0.0119175529305197	0.0089502805193842
9	0.0017014681733111	0.0011952249074813
10	0.0002182341952046	0.0001445005596724
11	0.0000252196190879	0.0000158503684769
12	0.0000026406525791	0.0000015847488146
$\vdots$	$\vdots$	$\vdots$
30	$6.610193523821e-29$	$2.6540266176391e-29$
$\vdots$	$\vdots$	$\vdots$
∞	$\lim_{n\to\infty} \sum_{k=0}^{\infty} \mathrm{HY}(k,n,r) = 0$	$\lim_{n\to\infty} \int_0^{\infty} \mathrm{HY}(k,n,r)dk = 0$
$\sum_n \sum_k$	52812.977966365550	—

So, for example, the flux for radius $r = 1/2$ is $\sum_{k=0}^{\infty} \mathrm{HY}(k,-2,1/2) \approx 43770.014449969689$. The other values, also can be established on the basis of the recurrent relations (2.25), in other words (2.26).

2.2. The Fluxes on the Basis of the Series of the Hypercylindrical Functional Matrix

The discrete dimensional fluxes can be calculated as well on the "horizontal line", that is, adding functions values on the submatrix series $M[\mathrm{HY}]_{k,n}$ (Figure 1). For example, through the series development for $k = 3$ the flux would contain the next members:

$$\sum_{n=0}^{\infty} \mathrm{HY}(3,n,r) = 12\pi + 12\pi r + 6\pi r^2 + 2\pi r^3 + \frac{1}{2}\pi r^4 + \cdots + \varepsilon\left(\frac{12\pi r^n}{\Gamma(n+1)}\right). \tag{2.30}$$

Some values of discrete and continual fluxes (for $r = 1$) are given in Table 2.

Table 2

Freedom degree k	$\sum_{n=0}^{\infty} \mathrm{HY}(k,n,r)$	$\sum_{n=0}^{\infty} \mathrm{HY}(k,n,1)$	$\int_0^{+\infty} \mathrm{HY}(k,n,1)dn$
0	$2e^r/\pi$	1.730511958864530	1.846084380068597
1	$2e^r$	5.43656365691809	5.248099060246887
2	$8e^r$	21.7462546276723	22.46216193622815
3	$12\pi e^r$	102.4768106720828	85.44633790197124
4	$64\pi(e^r-1)$	345.4810604213614	237.5329239075405
5	$120\pi^2(e^r-r-1)$	850.6988994458283	508.7293133444046
6	$384\pi^2[2e^r-(r+1)^2-1]$	1654.544866434988	892.2064355912792
7	$\sum_{n=0}^{\infty}(1680\pi^3 r^{n+4}/\Gamma(n+5))$	2688.661898887322	1338.143855891663
8	$\sum_{n=0}^{\infty}(12288\pi^3 r^{n+5}/\Gamma(n+6))$	3790.427657262334	1768.586344896620
9	$\sum_{n=0}^{\infty}(30240\pi^4 r^{n+6}/\Gamma(n+7))$	4757.702808149823	2104.228782270664
10	$\sum_{n=0}^{\infty}(245760\pi^4 r^{n+7}/\Gamma(n+8))$	5416.805463135256	2289.343605902835
11	$\sum_{n=0}^{\infty}(665280\pi^5 r^{n+8}/\Gamma(n+9))$	5672.025047777118	2304.879158127108
12	$\sum_{n=0}^{\infty}(5898240\pi^5 r^{n+9}/\Gamma(n+10))$	5520.736405122306	2167.383511182325
13	$\sum_{n=0}^{\infty}(17297280\pi^6 r^{n+10}/\Gamma(n+11))$	5036.815477026034	1917.751163982797
$\vdots$	$\vdots$	$\vdots$	$\vdots$
50	$\sum_{n=0}^{\infty} \mathrm{HY}(50,n,r)$	0.000000117826838	0.000000029841827
$\vdots$	$\vdots$	$\vdots$	$\vdots$
∞	$\lim_{k\to\infty}\sum_{n=0}^{\infty} \mathrm{HY}(k,n,r)=0$	0	$\lim_{k\to\infty}\int_0^{\infty} \mathrm{HY}(k,n,r)dn=0$
—	$\sum_k \sum_n$	52812.977966365550	—

2.3. Some Continual Fluxes of the Hypercylindrical Function

The trend of the distribution of the vector fluxes is increasing, and then asymptotically falling, with the linear growth of the freedom degree n. From the standpoint of the functional analysis the most interesting series of the matrix $M[\mathrm{HY}]_{k,n}$ are the ones referring to the freedom degrees $k = 2$ and $k = 3$. The first series ($k = 2$) covers the known functions for the square size ($8r$) and surface ($4r^2$). The members of the following series are, among the others, the cylinder functions of the surface ($6\pi r^2$) and volume ($2\pi r^3$) (Figure 1). The same series are interesting as well for calculatiing continual fluxes. So the continual natural flux for the hypercylinder surface is analysed in view of integrals, instead of series. This integral is specific, because its subintegral function is the reciprocal *gamma* function. Its value, as it is known, is equal to the value of Fransen-Robinson's constant [15]

$$F=\int_0^{\infty}\frac{1}{\Gamma(x)}dx=e+\int_0^{\infty}\frac{e^{-n}}{\pi^2+\ln^2 n}dn\approx 2.8077702420285. \qquad (2.31)$$

The integral values of the wanted flux is now $\int_0^\infty \mathrm{HY}(2,n,1)dn = \sum_{n=0}^\infty \mathrm{HY}(2,n,1) + \int_0^\infty (8e^{-n}/(\pi^2+\ln^2 n))dn$, what concretely for the unit radius is

$$\int_0^\infty \frac{8}{\Gamma(n)}dn = 8\left(e + \int_0^\infty \frac{e^{-n}}{\pi^2+\ln^2 n}dn\right) = 8F \approx 22.462161936228. \tag{2.32}$$

In respect to the continual dimension n, the more generalized cylinder volume flux follows in view of Ramanujan-Hardy's integral [16, 17]:

$$\int_0^\infty \frac{y^x}{\Gamma(x+1)}dx = e^y - \int_0^\infty \frac{e^{-xy}}{x\left(\pi^2+\ln^2 x\right)}dx. \tag{2.33}$$

Ramanujan defined this integral, and it was analytically intensified by Hardy. In that sense, the previous expression can be applied in calculating the hypercylindrical function flux, when $k=3$, as

$$\int_0^\infty \mathrm{HY}(3,n,1)dn = \int_0^\infty \frac{12\pi r^n}{\Gamma(n+1)}dn = 12\pi\left(e^r - \int_0^\infty \frac{e^{-nr}}{n\left(\pi^2+\ln^2 n\right)}dn\right) \quad (\text{for } x=n,\ y=r). \tag{2.34}$$

The integral can be calculated as well as the difference between the series and the integral with the value (for $r=1$)

$$\int_0^\infty \frac{12\pi r^n}{\Gamma(n+1)}dn = \sum_{n=0}^\infty \frac{12\pi r^n}{\Gamma(n+1)} - \int_0^\infty \frac{12\pi e^{-nr}}{n(\pi^2+\ln^2 n)}dn\bigg|_{r=1} \approx 278.6085228836. \tag{2.35}$$

2.4. The Progressions of the Vector Fluxes

The whole dimension flux in the freedom degree domain with the natural numbers, is obtained in the result of double amount with which are considered the integer values of the hypercylindrical function $\mathrm{HY}(k,n,r)$, for all $k,n,r \geq 0$. This double series must be convergent, and this characteristic is in the function of hypercylinder radius. The flux can be watched as well for every column $M[\mathrm{HY}]_{k,n}$ of the matrix individually. So, there is for the nth column (in the mark $\langle n\rangle$), the flux in the form of series

$$\Phi_{\mathrm{HY}}^{\langle n\rangle}(k,n,r) = \sum_{k=0}^\infty \mathrm{HY}(k,n,r). \tag{2.36}$$

2.5. The Orthogonal Dimensional Fluxes

These fluxes are the fluxes of the all columns or of all series of the matrix $M[\mathrm{HY}]_{k,n}$. As the number of columns, that is of the series infinite, the total flux is as follows.

Definition 2.1. The dimensional flux (potentional) of the functional matrix with two freedom degrees k and n is defined as a double series

$$\Phi_{\mathrm{HY}}(k,n,r)=\sum_{n=0}^{\infty}\Phi_{\mathrm{HY}}^{\langle n\rangle}(k,n,r)=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\mathrm{HY}(k,n,r). \tag{2.37}$$

When the certain number of members is calculated, the flux has the next form:

$$\begin{aligned}
\Phi_{\mathrm{HY}}(k,n,r)&=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\mathrm{HY}(k,n,r)\\
&=4\pi\left[r\left(10+4\pi r^2\right)+e^{\pi r^2}\operatorname{erfc}(-r\sqrt{\pi})\left[3+4\pi r^2\left(3+\pi r^2\right)\right]\right]\\
&\quad+4\left[2\left(1+\pi r^2\right)+\pi r e^{\pi r^2}\operatorname{erfc}(-r\sqrt{\pi})\left(3+2\pi r^2\right)\right]\\
&\quad+2\left[2r+e^{\pi r^2}\operatorname{erfc}(-r\sqrt{\pi})\left(1+2\pi r^2\right)\right]\\
&\quad+2\left(\frac{1}{\pi}+re^{\pi r^2}\operatorname{erfc}(-r\sqrt{\pi})\right)+\frac{1}{\pi}\left[e^{\pi r^2}\operatorname{erfc}(-r\sqrt{\pi})-1\right]\\
&\quad+\frac{1}{\pi}\left(\frac{\operatorname{erfi}(r\sqrt{\pi})}{2}-r\right)+\frac{1}{\pi^2}\sum_{k=0}^{\infty}\frac{(-1)^{2k+1}}{2k+1}\left(1-\frac{\Gamma(k+1,-\pi r^2)}{k!}\right)\\
&\quad+\frac{1}{2\pi}\left[\frac{1-e^{\pi r^2}}{\pi}+r\operatorname{erfi}(r\sqrt{\pi})-r^2\right]+\cdots.
\end{aligned} \tag{2.38}$$

The flux on the matrix series in the domain of the natural number is defined as a double series, but with the changed sequence of summing. This dimensional flux is, thus, defined as

$$\Omega_{\mathrm{HY}}(k,n,r)=\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}\mathrm{HY}(k,n,r). \tag{2.39}$$

In view of previously established members, the matrix flux has the following form:

$$\begin{aligned}
\Omega_{\mathrm{HY}}(k,n,r)&=\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}\mathrm{HY}(k,n,r)\\
&=\frac{2e^r}{\pi}+2e^r+8e^r+12\pi e^r+64\pi(e^r-1)+120\pi^2(e^r-r-1)\\
&\quad+768\pi^2\left(e^r+r-\frac{r^2}{2}+1\right)+\sum_{n=0}^{\infty}\frac{1680\pi^3r^{n+4}}{\Gamma(n+5)}+\sum_{n=0}^{\infty}\frac{12288\pi^3r^{n+5}}{\Gamma(n+6)}+\cdots.
\end{aligned} \tag{2.40}$$

The equivalence of the orthogonal dimensional fluxes means the equality of the double series

$$\Phi_{\mathrm{HY}}(k,n,r)=\Omega_{\mathrm{HY}}(k,n,r). \tag{2.41}$$

So, for example, for $r = 1$ the dimensional fluxes have the unambiguous numerical value that is:

$$\Phi_{\mathrm{HY}}(k, n, 1) = \Omega_{\mathrm{HY}}(k, n, 1) = 52812.977966365550. \tag{2.42}$$

2.6. The Application of the Recurrent Operators with Defining the Diagonal Dimensional Fluxes of the HY Function

In the previous analyses the defining of the dimensional fluxes of the matrix $M[\mathrm{HY}]_{k,n}$ is performed in view of the adding up of the HY function value for (on) the columns, in other words, the series of this matrix. The more detailed analysis would be very ample including the exponential function, then error functions $\mathrm{erf}(z)$, $\mathrm{erfc}(z)$, $\mathrm{erfi}(z)$, the incomplete gamma function $\Gamma(a, z)$ and the like. When we use the ideas of transition operators from the reference function into the destined HY function, in the functional hypercylindrical matrix $M[\mathrm{HY}]_{k,n}$ we can also establish the dimensional fluxes values on the diagonals (2.44), whose total would present the whole flux for the matrix in the field of freedom degrees for natural numbers, that is, $k, n \in N$. Such matrix contains infinite number of elements. For reference functions we take HY functions on the positions of the first series of the matrix, and they are so-called zero HY functions: $\mathrm{HY}(0, 0, r), \mathrm{HY}(0, 1, r), \ldots, \mathrm{HY}(0, n, r), \ldots$ The destined functions are arranged on the "gradual" growth law $(+\Delta k)$ and decrease $(-\Delta n)$ of increments.

Definition 2.2. The flux series operator $\vartheta(\Delta k, \Delta n, 0)$ is defined by the relation

$$\begin{aligned} \vartheta(\Delta k, \Delta n, 0) &= \frac{\mathrm{HY}(k + \Delta k, n + \Delta n, r)}{\mathrm{HY}(k, n, r)} \\ &= \frac{\pi^{\Delta k/2} r^{\Delta k + \Delta n} \Gamma(k + n - 2) \Gamma(k + \Delta k + 1)}{\Gamma(k + 1) \Gamma(k + n + \Delta k + \Delta n - 2) \Gamma((k + \Delta k + 1)/2)} \Gamma\left(\frac{k + 1}{2}\right). \end{aligned} \tag{2.43}$$

$$\begin{array}{c} \frac{n}{k} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ \vdots \end{array} \begin{array}{c} \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & \cdots \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & \dfrac{2}{\pi} & \dfrac{2}{\pi} r & \dfrac{1}{\pi} r^2 & \dfrac{1}{3\pi} r^3 & \cdots \\ 0 & 0 & 2 & 2r & r^2 & \dfrac{1}{3} r^3 & \dfrac{1}{12} r^4 & \cdots \\ 0 & 8 & 8r & 4r^2 & \dfrac{4}{3} r^3 & \dfrac{1}{3} r^4 & \dfrac{1}{15} r^5 & \cdots \\ 12\pi & 12\pi r & 6\pi r^2 & 2\pi r^3 & \dfrac{\pi}{2} r^4 & \dfrac{\pi}{10} r^5 & \dfrac{\pi}{60} r^6 & \cdots \\ 64\pi r & 32\pi r^2 & \dfrac{32\pi}{3} r^3 & \dfrac{8\pi}{2} r^4 & \dfrac{8\pi}{15} r^5 & \dfrac{4\pi}{45} r^6 & \dfrac{4\pi}{315} r^7 & \cdots \\ 60\pi^2 r^2 & 20\pi^2 r^3 & 5\pi^2 r^4 & \pi^2 r^5 & \dfrac{\pi^2}{6} r^6 & \dfrac{4\pi^2}{42} r^7 & \dfrac{\pi^2}{336} r^8 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \end{array} \tag{2.44}$$

Meanwhile, as increments for the absolute value are mutually equal and unit ones, that is, $|\Delta k| = |-\Delta n| = 1$, to them is assigned a new joint argument u ($\Delta k = \Delta n = u$). In addition, the starting value of the k-og freedom degree states $k = 0$, so the theta operator gets the form

$$\theta(u, -u, 0) = 2^u \sqrt{\pi^u}\Gamma\left(\frac{u}{2} + 1\right). \tag{2.45}$$

The destined function is now calculated as

$$\mathrm{HY}(u, n-u, r) = \theta(u, -u, 0) \cdot \mathrm{HY}(0, n, r) = \frac{2^{u+1}\sqrt{\pi^{u-2}}r^{n-3}}{\Gamma(n-2)}\Gamma\left(\frac{u}{2} + 1\right). \tag{2.46}$$

The dimensional flux on the diagonal presents the total of ist particular nembers. So, for the first diagonal (with the mark $\langle 0 \rangle$) flux is equal

$$\Pi^{\langle 0 \rangle}(k, n, r) = \mathrm{HY}(0, 0, r) = 0, \tag{2.47}$$

then for the second,

$$\Pi^{\langle 1 \rangle}(k, n, r) = \mathrm{HY}(0, 1, r) + \mathrm{HY}(1, 0, r) = 0. \tag{2.48}$$

For the third,

$$\Pi^{\langle 2 \rangle}(k, n, r) = \mathrm{HY}(0, 2, r) + \mathrm{HY}(1, 1, r) + \mathrm{HY}(2, 0, r) = 0. \tag{2.49}$$

Or for the fourth,

$$\Pi^{\langle 3 \rangle}(k, n, r) = \mathrm{HY}(0, 3, r) + \mathrm{HY}(1, 2, r) + \mathrm{HY}(2, 1, r) + \mathrm{HY}(3, 0, r) = 10 + \frac{2}{\pi} + 12\pi. \tag{2.50}$$

The flux in the nth diagonal would be reckoned in the sum form

$$\Pi^{\langle n \rangle}(k, n, r) = \sum_{u=0}^{n} \mathrm{HY}(u, n-u, r) = \frac{r^{n-3}}{\Gamma(n-2)} \sum_{u=0}^{n} 2^{u+1}\sqrt{\pi^{u-2}}\Gamma\left(\frac{u}{2} + 1\right) \quad (n \neq 2). \tag{2.51}$$

The flux for the value $n = 2$ is calculated on the basis of the highest function value. Considering that the equivalence of the double factorial and trigonometric functions [18] is known

$$u!! = 2^{(1/4)[1+2u-\cos(\pi u)]}\pi^{(1/4)[\cos(\pi u)-1]}\Gamma\left(\frac{u}{2} + 1\right). \tag{2.52}$$

The expression (2.19) can after settling be written as well in the equivalent form

$$\Pi^{\langle n \rangle}(k, n, r) = \frac{r^{n-3}}{(n-3)!} \sum_{u=0}^{n} u!! \sqrt[4]{(2\pi)^{2u}\left(\frac{2}{\pi}\right)^{3+\cos(\pi u)}}. \tag{2.53}$$

So, we get for the fifth diagonal ($n = 4$), using the expressions (2.51) or (2.53)

$$\Pi^{\langle 4\rangle}(k,n,r) = \sum_{u=0}^{n} \mathrm{HY}(u,n-u,r) = \frac{2r}{\pi}\left(5\pi + 38\pi^2 + 1\right). \tag{2.54}$$

For the sixth diagonal ($n = 5$) follows:

$$\Pi^{\langle 5\rangle}(k,n,r) = \sum_{u=0}^{n=5} \mathrm{HY}(u,n-u,r) = \frac{r^2}{\pi}\left(5\pi + 38\pi^2 + 60\pi^3 + 1\right). \tag{2.55}$$

As the number of diagonals is infinite, the total flux is formed as the series of all diagonal fluxes

$$\begin{gathered} \Pi_{\mathrm{HY}}(k,n,r) = \sum_{n=0}^{\infty} \Pi^{\langle n\rangle}(k,n,r), \quad \text{or concretely,} \\ \Pi_{\mathrm{HY}}(k,n,r) = \sum_{n=0}^{\infty}\sum_{u=0}^{n} \mathrm{HY}(u,n-u,r) = \sum_{n=0}^{\infty}\left(\frac{r^{n-3}}{\Gamma(n-2)}\sum_{u=0}^{n} 2^{u+1}\sqrt{\pi^{u-1}}\,\Gamma\left(\frac{u+1}{2}\right)\right). \end{gathered} \tag{2.56}$$

For example, approximately, the flux for $r = 1$ and $n = 11$ it concretely is

$$\Pi_{\mathrm{HY}}(k,11,1) = \frac{109601}{20160\pi} + \frac{109601}{4032} + \frac{1437299}{10080}\pi + \frac{426437}{1680}\pi^2 + \frac{58497}{280}\pi^3 + \frac{2901}{28}\pi^4 + \frac{33}{2}\pi^5. \tag{2.57}$$

In the developed, form the total flux has the polynominal structure of members

$$\begin{aligned} \Pi_{\mathrm{HY}}(k,n,r) = 10 &+ \frac{2}{\pi} + 12\pi + r\left(10 + \frac{2}{\pi} + 76\pi\right) + r^2\left(5 + \frac{1}{\pi} + 38\pi + 60\pi^2\right) \\ &+ r^3\left(\frac{5}{3} + \frac{1}{3\pi} + \frac{38}{3}\pi + 148\pi^2\right) + r^4\left(\frac{5}{12} + \frac{1}{12\pi} + 37\pi^2 + 70\pi^3\right) \\ &+ r^5\left(\frac{1}{12} + \frac{1}{60\pi} + \frac{19}{30}\pi + \frac{37}{5}\pi^2 + \frac{582}{5}\pi^3 + \frac{1}{12}\right) \\ &+ r^6\left(\frac{1}{72} + \frac{1}{360\pi} + \frac{19}{180}\pi + \frac{37}{30}\pi^2 + \frac{97}{5}\pi^3 + 42\pi^4\right) + \cdots. \end{aligned} \tag{2.58}$$

The diagonal flux of the hypercylindrical function can be expressed by the series of general form

$$\Pi_{\mathrm{HY}}(v,r) = \sum_{v=0}^{\infty} a_v r^v. \tag{2.59}$$

Here, r is the summation index with which we take into consideration the order of elements from left to right and from top to bottom on (along) the diagonal (Figure 3). The polynomial

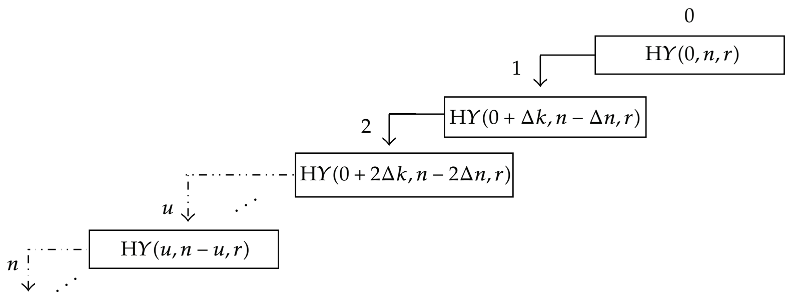

Figure 3: The representative submatrix (above) and the addition principle of the matrix members on the diagonal.

coefficients contain the rational numbers and the graded constant π (Table 3). The first three coefficients are equal to zero, so they are not put in the summation sequence. Its other values are ($v = 0, 1, \ldots, 12$) given in Table 3.

The series approximation with 13 coefficients in the decimal notation is in the form

$$\sum_{v=0}^{12} a_v r^v \approx 48,33 + 249,4r + 716,87r^2 + 1502,27r^3 + 2546,01r^4 + 3684,24r^5 + 4705,22r^6 + 5422,03r^7 + 5727,08r^8 + 5610,38r^9 + 5143,66r^{10} + 4445,23r^{11} + 3642,42r^{12}. \tag{2.60}$$

Approximately, the double series leads us to the solution that is very near to the correct one. Namely, for the unit radius and reducing on $\infty \sim n = 30$ the double series of the diagonal flux develops the following structure:

$$\begin{aligned}
\Pi_{\mathrm{HY}}(u, n, 1) &\approx \sum_{n=0}^{n=30} \sum_{u=0}^{n} \mathrm{HY}(u, n-u, r) \\
&= \frac{739975398988375932899873137}{272221736260458804019200000} + \frac{739975398988375932899873137}{1361108681302294020096000000}\pi^{-1} \\
&\quad + \frac{510736042137463255831073137}{35818649507955105792000000}\pi \\
&\quad + \frac{238514305877004451811873137}{11342572344185783500800000}\pi^2 + \frac{395025082463267992919694289}{18904287240309639168000000}\pi^3 \\
&\quad + \frac{103021449478899595399537}{986310638624850739200}\pi^4 \\
&\quad + \frac{3457149785418899743999223}{94521436201548195840000}\pi^5 + \frac{2633045028485350315237}{270061246290137702400}\pi^6 \\
&\quad + \frac{7044091324892514823027}{3375765578626721280000}\pi^7
\end{aligned}$$

$$
\begin{aligned}
&+\frac{9964355179641216157}{2679179030656128000}\pi^{8}+\frac{354387723451762783}{625141773819763200 0}\pi^{9}\\
&+\frac{1285336197080807}{170493211041753600}\pi^{10}+\frac{14026248671711}{15786408429792000}\pi^{11}\\
&+\frac{513480360467}{5464525994928000}\pi^{12}+\frac{3516397543}{390323285352000}\pi^{13}+\frac{101418347}{130107761784000}\pi^{14}\\
&\approx 52810,5103716.
\end{aligned}
\tag{2.61}
$$

The diagonal dimension flux is characteristic with its coefficients that contain the π^n constant in the graded series numbers, as distinguished from the vertical fluxes with the destination of the errors functions and π and e constants. The horizontal fluxes, as it is presented (2.40) contain the exponential functions. Meanwhile, the total flux for the unit radius is convergent and it can be calculated with considerably greater value. The value aberration (2.61) of the accurate value is only (or 0,00467%)

$$
\Delta\Pi = 52812,97796636 - 52810,5103716 \approx 2,467595. \tag{2.62}
$$

The whole dimensional continual flux ($k, n \in N$) of the unit hypercylindrical function $\mathrm{HY}(k, n, 1)$ is equal to the value of the twofold integral

$$
\iint_0^\infty \frac{2\sqrt{\pi^{k-1}}\Gamma(k+1)}{\Gamma(k+n-2)\Gamma((k+1)/2)}\,dk\,dn, \tag{2.63}
$$

and we find its solution on the analytical and numerical bases.

3. Conclusion

On view of the supposition of recurrent relations (1.2), namely, (2.25) and (2.26) that exist in the scope of the hypercylindrical function, we can calculate the discrete dimension flux of this function in the domain of integer freedom degrees. The quantitative flux value at the most depends of the formulated value of the hypercylinder radius. Meanwhile, as the function $\mathrm{HY}(k, n, r)$ is the three variables function, its dependence is by all means restricted by the values of k variables, namely n. In the paper are calculated several continual fluxes for the contour hypercylindrical functions, in view of Ramanujan-Hardy's integrals. With the continual flux in the domain $k, n \in \overline{0, \infty}$ the problem is more complex, because we must, for its defining, to perform the twofold integration (2.63). Although it is not yet calculated, we suppose that its value is very close to the discrete flux, obtained in view of the double series.

Calculating the dimensional flux by the *diagonal algorithm* is much simpler and faster by computer, because the total flux is now defined as the convergent graded series and does not contain as components the special functions. In any case its value is identical with the

Table 3

v	a_v- polynomial coefficient $\sum_v a_v r^v$
0	$10+\frac{2}{\pi}+12\pi$
1	$\frac{2}{\pi}+10+76\pi$
2	$\frac{1}{\pi}+5+38\pi+60\pi^2$
3	$\frac{1}{3\pi}+\frac{5}{3}+\frac{38\pi}{3}+148\pi^2$
4	$\frac{1}{12\pi}+\frac{5}{12}+\frac{19\pi}{6}+37\pi^2+70\pi^3$
5	$\frac{1}{60\pi}+\frac{1}{12}+\frac{19\pi}{30}+\frac{37\pi^2}{5}+\frac{582\pi^3}{5}$
6	$\frac{1}{360\pi}+\frac{1}{72}+\frac{19\pi}{180}+\frac{37\pi^2}{30}+\frac{97\pi^3}{5}+42\pi^4$
7	$\frac{1}{2520\pi}+\frac{1}{504}+\frac{19\pi}{1260}+\frac{37\pi^2}{210}+\frac{97\pi^3}{35}+\frac{1150\pi^4}{21}$
8	$\frac{1}{20160\pi}+\frac{1}{4032}+\frac{19\pi}{10080}+\frac{37\pi^2}{1680}+\frac{97\pi^3}{280}+\frac{575\pi^4}{84}+\frac{33\pi^5}{5}$
9	$\frac{1}{181440\pi}+\frac{1}{36288}+\frac{19\pi}{90720}+\frac{37\pi^2}{15120}+\frac{97\pi^3}{2520}+\frac{575\pi^4}{756}+\frac{2279\pi^5}{126}$
10	$\frac{1}{1814400\pi}+\frac{1}{362880}+\frac{19\pi}{907200}+\frac{37\pi^2}{151200}+\frac{97\pi^3}{25200}+\frac{115\pi^4}{1512}+\frac{2279\pi^5}{1260}+\frac{143\pi^6}{30}$
11	$\frac{1}{19958400\pi}+\frac{1}{3991680}+\frac{19\pi}{9979200}+\frac{37\pi^2}{1663200}+\frac{97\pi^3}{277200}+\frac{115\pi^4}{16632}+\frac{2279\pi^5}{13860}+\frac{905\pi^6}{198}$
12	$\frac{1}{47900160}+\frac{1}{239500800\pi}+\frac{19\pi}{119750400}+\frac{37\pi^2}{19958400}+\frac{97\pi^3}{3326400}+\frac{115\pi^4}{199584}+\frac{2279\pi^5}{166320}+\frac{905\pi^6}{2376}+\frac{13\pi^7}{12}$

fluxes that are calculated on the basis of the series, relatively to HY-matrix columns, so there is valid the numerically verified statement that

$$\Phi_{\mathrm{HY}}(k,n,r)=\Omega_{\mathrm{HY}}(k,n,r)=\Pi_{\mathrm{HY}}(k,n,r)\big|_{r=1}=52812.977966365550. \tag{3.1}$$

In any case, this solution is initial for solving the other dimensional fluxes, both for hypercylindrical and hyperspherical [3, 19], in other words hypercube function [20].

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