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On bounded analytic functions and closure problems

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Introduction

1. Let us denote by H^p, $p \geq 1$, the space of functions $f(z)$ holomorphic in $|z| < 1$ and such that

$$N_p(f) = \lim_{r \to 1} \left\{ \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta \right\}^{\frac{1}{p}} = \left\{ \frac{1}{2\pi} \int_0^{2\pi} |f(e^{i\theta})|^p \, d\theta \right\}^{\frac{1}{p}} < \infty,$$

where $f(e^{i\theta}) = \lim_{r \to 1} f(re^{i\theta})$ a. e. It is obvious that H^p is a Banach space under the norm N_p. If we combine the wellknown representation of a linear functional on $L^p(0, 2\pi)$ with a theorem of M. Riesz on conjugate functions, we find that the general linear functional on H^p, $p > 1$, has the form

$$L(f) = \int_0^{2\pi} f(e^{i\theta}) \overline{g(e^{i\theta})} \, d\theta, \quad g \in H^q, \quad p^{-1} + q^{-1} = 1. \tag{1}$$

The simple structure of the general linear functional on H^p is the key to a great number of results for these spaces.

The "limit space" as $p \to \infty$ is the space B of bounded analytic functions in $|z| < 1$ with the uniform norm

$$\|f\| = \sup_{|z|<1} |f(z)|. \tag{2}$$

Although this space has a simpler function-theoretic nature than H^p, its theory as a Banachspace is extremely complicated. This fact depends to a great extent on the absence of a simple representation for linear functionals. On the other hand, B is not only a Banach space, but also a Banach algebra.

If one seeks results for B which for H^p depend on the formula (1), the following question should be asked: how shall we weaken the norm (2) in B in order to ensure that the functionals have a representation of type (1)? In the first section we shall treat this problem by introducing certain weight functions. The method will also be used to find a function-theoretic correspondance to weak convergence on a finite interval.

In the second section, we shall consider a closure problem for B where the relation between B and its subring C of uniformly continuous functions will be of importance. Finally, we shall make an application of the results to the Pick-Nevanlinna interpolation problem.

Section I

2. When we look for functionals on B analogous to (1), there are two essentially different possibilities: we may take for g a function in H^1 or in $L^1(0, 2\pi)$. These representations are fundamentally different since the above-mentioned theorem on conjugate functions fails for $p = 1$. Let us call the two types of representations (A) and (B):

$$\text{(A)} \qquad L(f) = \int_0^{2\pi} f(e^{i\theta}) \overline{g(e^{i\theta})} \, d\theta, \quad g \in H^1;$$

$$\text{(B)} \qquad L(f) = \int_0^{2\pi} f(e^{i\theta}) K(\theta) \, d\theta, \quad K \in L^1(0, 2\pi).$$

In the sequel, let $\mu(r)$ denote a continuous function on $0 \le r \le 1$ such that $0 \le \mu(r) \le 1$ and $\mu(1) = 0$. Furthermore, let C_μ be the space of functions analytic in $|z| < 1$ such that $\lim\limits_{r \to 1} f(re^{i\theta}) \mu(r) = 0$ uniformly in θ. If we introduce the norm

$$\|f\|_\mu = \sup_{r<1} \mu(r) |f(re^{i\theta})|,$$

C_μ becomes a Banach space.

By means of the weight functions μ, we can now solve our problem. Let us first consider functionals of type (A).

Theorem 1. *Every linear functional on C_μ has on its subspace B a representation of the form (A) if and only if*

$$(3) \qquad \overline{\lim_{r \to 1}} \, \mu(r) \log \frac{1}{1-r} < \infty.$$

Let us first assume that condition (3) is satisfied, and let $L(f)$ be a linear functional on C_μ. By a theorem of Riesz-Banach, there exists a function $\sigma(z)$ of bounded variation in $|z| < 1$ such that

$$L(f) = \iint_{|z|<1} \mu(|z|) f(z) \, d\sigma(z), \quad f \in C_\mu.$$

If now $f \in B$ and is represented by its Cauchy integral, we find that if

$$g_\varrho(\zeta) = \frac{1}{2\pi} \iint_{|z|<\varrho} \frac{\mu(|z|)}{1 - \bar{z}\zeta} \, \overline{d\sigma(z)}$$

and if $L_\varrho(f)$ is the functional of type (A) defined by this function, then $\lim\limits_{\varrho \to 1} L_\varrho(f) = L(f)$. It follows for $\varrho < \varrho' < 1$

$$\overline{\lim_{\varrho,\varrho'\to 1}} N_1(g_\varrho - g_{\varrho'}) \leqq \text{Const.} \sup_{r<1} \mu(r) \log \frac{1}{1-r} \overline{\lim_{\varrho,\varrho'\to 1}} \iint_{\varrho\leq|z|\leq\varrho'} |d\sigma(z)| = 0.$$

Hence $g \in H^1$ exists such that $N_1(g - g_\varrho) \to 0$, $\varrho \to 1$, and the functional of type (A) defined by this function g coincides with L on the space B. The first part of the theorem is consequently proved.

If, on the other hand, (3) does not hold, there exists a sequence $\{r_\nu\}_1^\infty$, $r_\nu \uparrow 1$, such that

$$\mu_\nu = \mu(r_\nu) \log \frac{1}{1-r_\nu} \to \infty, \quad \nu \to \infty.$$

We then form the following expression, which is easily seen to be a linear functional on C_μ:

$$L(f) = \sum_{\nu=1}^{\infty} f(r_\nu)\mu(r_\nu)\lambda_\nu.$$

$\{\lambda_\nu\}_1^\infty$ is here a sequence of positive numbers such that

$$\sum_1^\infty \lambda_\nu < \infty \text{ and } \sum_{\nu=1}^\infty \lambda_\nu \mu_\nu = \infty.$$

For f belonging to B, we have $L(f) = \lim_{n\to\infty} L_n(f)$, where L_n is the functional of type (A) defined by the function

$$g_n(z) = \frac{1}{2\pi} \sum_{\nu=1}^{n} \frac{\lambda_\nu \mu(r_\nu)}{1 - r_\nu z}.$$

Let us now assume that the functional defined above has a representation of type (A) on B. If $h(\theta)$ is an arbitrary function with continuous derivative and period 2π, then

$$\lim_{n\to\infty} \int_0^{2\pi} h(\theta)\overline{g_n(e^{i\theta})}\,d\theta = \int_0^{2\pi} h(\theta)\overline{g(e^{i\theta})}\,d\theta. \tag{4}$$

Namely, if $\tilde{h}(\theta)$ is the conjugate function of h, neither side changes its value if we add $i\tilde{h}$ to h. But for $h + i\tilde{h}$, (4) holds by assumption. For $\delta > 0$, we choose a non-negative periodic function $h_\delta(\theta)$ with continuous derivative such that $h_\delta = 0$ for $-c < \theta < 0$, $h_\delta \leq 1$ for all θ and $h_\delta = 1$ for $\delta \leq \theta < c < \pi$. By (4), we have

$$\overline{\lim_{\delta\to 0}} \left| \lim_{n\to\infty} \int_0^{2\pi} h_\delta(\theta)\overline{g_n(e^{i\theta})}\,d\theta \right| < \infty.$$

Going back to the expression for g_n, however, we find

$$\overline{\lim_{\delta\to 0}} \left| \lim_{n\to\infty} \operatorname{Im} \left\{ \int_0^{2\pi} h_\delta(\theta)\overline{g_n(e^{i\theta})}\,d\theta \right\} \right| \geqq$$

$$\geqq \text{Const.} \lim_{\delta\to 0} \sum_{\nu=1}^{\infty} \lambda_\nu \mu(r_\nu) \int_\delta^\pi \frac{\theta\, d\theta}{|1 - r_\nu \theta|^2} \geqq \text{Const.} \sum_1^\infty \lambda_\nu \mu_\nu = \infty.$$

This contradiction proves the theorem.

3. It is very easy to see — in the same way as above, if we represent the functions in B by Poisson's integral instead of Cauchy's — that the following theorem is true for functionals of type (B).

Theorem 2. *The functionals on C_μ have a representation of type (B) on B for every choice of the weight function μ.*

This theorem is a particular case of a more general result which we shall now briefly discuss.

Let D be the Banach space of bounded functions $\varphi(x)$ on $(0, 2\pi)$, where we have introduced the uniform norm $\|\varphi\|$. With every function in D, we associate the corresponding harmonic function in the unit circle

$$u_\varphi = u(z;\varphi) = \frac{1}{2\pi}\int_0^{2\pi} \frac{1-r^2}{1+r^2-2r\cos(\theta-x)}\varphi(x)\,dx, \quad z = re^{i\theta}.$$

Let D^* be the space of these functions u.

Suppose now that S is a linear subset of D and that S^* is the corresponding subset of D^*. If $\bar{S}$ is the weak closure of S, the following theorem can be proved.

Theorem 3. *A function $\psi(x) \in D$ belongs to $\bar{S}$ if and only if, for every weight function $\mu(r)$ and every $\varepsilon > 0$, a function $\varphi \in S$ exists such that*

$$|u(z;\psi) - u(z;\varphi)| < \varepsilon\mu(|z|)^{-1}, \quad 0 \le |z| < 1. \tag{5}$$

Let us first assume that the above approximation is impossible for some $\mu(r)$. Then u_ψ does not belong to the closure of S^* in the metric of D_μ^*, where D_μ^* is formed by harmonic functions in the same way as C_μ was formed by analytic functions. We conclude that a functional L^* on D_μ^* exists which vanishes on S^* and does not vanish for u_ψ. As before, we have a representation on D^*

$$L^*(u_\lambda) = \iint_{|z|<1} u(z;\lambda)\mu(|z|)\,d\sigma(z), \tag{6}$$

where σ is of bounded variation in $|z| < 1$. If we insert the Poisson integral for u_λ and change the order of integration, we get

$$L^*(u_\lambda) = \int_0^{2\pi} \lambda(x)K(x)\,dx,$$

where $K(x)$ belongs to $L^1(0, 2\pi)$. Hence ψ does not belong to $\bar{S}$.

In the proof of the converse we shall use the following lemmas.

Lemma 1. *If $K(x)$ belongs to $L^1(0, 2\pi)$ and*

$$K(x) \sim \frac{a_0}{2} + \sum_1^\infty (a_n\cos nx + b_n\sin nx), \tag{7}$$

there exists a sequence of positive numbers A_n *with* $\lim_{n\to\infty} A_n = \infty$, *such that for every sequence* $\{\lambda_n\}_1^\infty$ *of positive numbers which is increasing, concave and satisfies* $\lambda_n \leq A_n$

$$\frac{a_0}{2} + \sum_1^\infty \lambda_n (a_n \cos nx + b_n \sin nx) \tag{8}$$

is a Fourier-Stieltjes series.

Lemma 2. *Given a sequence of positive numbers* a_n, $\lim_{n\to\infty} a_n = 0$, *there exists a non-negative function* $h(t)$ *in* $L^1(0, 1)$ *such that*

$$a_n \leq b_n = \int_0^1 t^n h(t)\,dt;$$

$h(t)$ *can furthermore be chosen so that* $\{b_n^{-1}\}$ *is a concave sequence.*

To prove lemma 1, we need only observe that the Cesaro mean of (7) converges in mean to $K(x)$ and make repeated use of partial summations in the series (7) and (8). The proof of lemma 2 is completely straight forward.

We return to the proof of theorem 3 and assume that there exists a function $K(x)$ in $L^1(0, 2\pi)$ so that

$$\int_0^{2\pi} K(x)\varphi(x)\,dx = 0, \quad \varphi \in S,$$

while the corresponding integral for ψ is different from zero. From lemmas 1 and 2 we deduce that $K(x)$ has a representation

$$K(x) = \underset{\varrho\to 1}{\overset{(1)}{\text{l.i.m.}}} \int_0^\varrho h(r)\,dr \int_0^{2\pi} \frac{1 - r^2}{1 + r^2 - 2r\cos(x - \theta)}\,d\tau(\theta),$$

where τ is of bounded variation and $h(r)$ belongs to $L^1(0, 1)$. A weight function $\mu(r)$ can now be chosen so that $f(r) = h(r)\mu(r)^{-1}$ belongs to $L^1(0, 1)$. If we define σ by $d\sigma(z) = d\tau(\theta) f(r)\,dr$, $z = re^{i\theta}$, it follows by absolute convergence from our assumption on K that

$$L^*(u_\varphi) = \iint_{|z|<1} u(z; \varphi)\mu(|z|)\,d\sigma(z) = 0, \quad \varphi \in S,$$

while $L^*(u_\psi) \neq 0$. This means that the approximation (5) is not possible for the weight function we have just defined. The proof of theorem 3 is thus complete.

As an illustration of the significance of the functionals (A), let us mention the following result: if $|a_\nu| < 1$ and $\sum_1^\infty (1 - |a_\nu|)$ diverges, then for every bounded analytic function $f(z)$ in $|z| < 1$ and every $\varepsilon > 0$ constants $\{c_\nu\}_1^n$ exist so that

$$\left| \sum_1^n c_\nu \frac{a_\nu - z}{1 - z\bar{a}_\nu} - f(z) \right| < \varepsilon \log \frac{1}{1 - |z|}, \quad 0 \leq |z| < 1.$$

Section II

4. We shall in this section study a problem which is connected with the fact that we can multiply two elements in B, i.e. that B is a Banach algebra. It should be stressed that some of our results are easy consequences of a general result on Banach algebras — this is in particular true of theorem 5 — but it is necessary for applications to have proofs of classical nature.[1]

We start with the following closure theorem for the subspace C of B of uniformly continuous functions.

Theorem 4. *If $f_1, f_2, \dots, f_n$ belong to C, then $\{z^k f_m(z)\}$, $m = 1, 2, \dots, n$; $k = 0, 1, \dots,$ is fundamental on C if and only if*

$$|f_1(z)| + |f_2(z)| + \cdots + |f_n(z)| \neq 0, \quad |z| \leq 1. \tag{9}$$

We shall prove the theorem in the case $n = 2$; the general case is treated quite similarly.

Let $L(f)$ be a linear functional on C which vanishes on the subspace E spanned by the given functions. The representation

$$L(f) = \int_0^{2\pi} f(e^{i\theta})\, d\mu(\theta)$$

follows from the corresponding result for the space of continuous functions on $(0, 2\pi)$ without difficulty, since the spaces are separable. We thus have

$$\begin{aligned} \int_0^{2\pi} e^{ik\theta} f_1(e^{i\theta})\, d\mu(\theta) &= \int_0^{\pi} e^{ik\theta}\, d\mu_1(\theta) = 0 \\ \int_0^{2\pi} e^{ik\theta} f_2(e^{i\theta})\, d\mu(\theta) &= \int_0^{2\pi} e^{ik\theta}\, d\mu_2(\theta) = 0 \end{aligned} \qquad k = 0, \quad 1, \dots .$$

From these relations it follows by a theorem of F. and M. RIESZ[2] that μ_1 and μ_2 are absolutely continuous functions. We then immediately infer from our assumption (9) that also $\mu(\theta)$ is absolutely continuous. Hence $K(\theta) \in L^1(0, 2\pi)$ exists so that

$$\int_0^{2\pi} K(\theta) f_m(e^{i\theta}) e^{ik\theta}\, d\theta = 0, \quad m = 1,\ 2; \quad k = 0,\ 1, \dots .$$[3]

This means that $K(\theta) f_m(e^{i\theta})$ is the boundary function of an analytic function $F_m(z)$ which belongs to H^1 and satisfies $F_m(0) = 0$. Furthermore,

$$K(\theta) = \lim_{r \to 1} \frac{F_m(re^{i\theta})}{f_m(re^{i\theta})} \text{ a.e.}, \quad m = 1,\ 2.$$

[1] See I. GELFAND and G. SILOV: Über verschiedene Methoden der Einführung der Topologie in die Menge der maximalen Ideale eines normierten Ringes. Mat. Sbornik 9 (1941).

[2] See e.g. ZYGMUND: Trigonometrical series, Warszawa-Lwów, 1935, p. 158.

[3] For the following, see BEURLING, A.: On two problems concerning linear transformations in Hilbert space. Acta Math. 81 (1949).

It follows that $F_1(z)/f_1(z)$ and $F_2(z)/f_2(z)$ in $|z|<1$ are two different representations of one and the same meromorphic function $H(z)$. By assumption (9), $H(z)$ is holomorphic and belongs to H^1 and its boundary function coincides a.e. with $K(\theta)$. For an arbitrary function f in C we thus have

$$L(f) = \int_0^{2\pi} H(e^{i\theta}) f(e^{i\theta})\, d\theta = 2\pi H(0) f(0) = 0$$

since $H(0) = 0$, and we can conclude that $E = C$.

If, on the other hand, (9) does not hold, the functions $f_m(z)$ must have a common zero in $|z| \leq 1$ and only functions which vanish at this point can belong to E.

As an immediate consequence of theorem 4 we get the following result.

Theorem 5. *If (9) holds, then for every $g \in C$, functions $p_1, p_2, \ldots, p_n$ in C exist such that*

$$\sum_{\nu=1}^{n} p_\nu(z) f_\nu(z) \equiv g(z). \tag{10}$$

It is clearly sufficient to prove the theorem for $g(z) \equiv 1$. By the theorem above, polynomials $P_\nu(z)$ can be chosen such that $\|F+1\| < \frac{1}{2}$, where

$$F(z) = \sum_{\nu=1}^{n} P_\nu(z) f_\nu(z).$$

In particular, $|F(z)| > \frac{1}{2}$ in $|z|<1$. We see that our relation is satisfied if we choose $p_\nu(z) = P_\nu(z)/F(z)$.

For later applications, we observe that the result holds for an arbitrary simply connected domain bounded by a Jordan curve — the analytic function which maps such a domain onto the unit circle is continuous on the boundary — and also that even in this more general case, polynomials $P_\nu(z)$ exist such that $|\sum_1^n P_\nu(z) f_\nu(z)| \geq \delta > 0$ — this follows from a known approximation theorem of Walsh.

5. We shall now use theorem 5 to prove an analogous result for the space B. We must in this case replace condition (9) by a stronger assumption and we introduce the following notation. For a given function f in B and an arbitrary a in $|a| \leq 1$, let us use the notation

$$\mu_f(a) = \varliminf_{z \to a} |f(z)|,$$

where, for $|a| = 1$, we have to approach a from inside the unit circle.

Theorem 6. *If E is a subfamily of B such that for every a, $|a| \leq 1$, $f \in E$ exists such that $\mu_f(a) \neq 0$, then any function g in B has a representation (10), where f_ν belongs to E and p_ν belongs to B.*

For every a, $|a| = 1$, there is closed interval A around a and a function f in E such that $\mu_f(\zeta) > 0$ for $\zeta \in A$. We cover $|z| = 1$ by a finite number of these intervals $A_1, A_2, \ldots, A_m$, $A_\nu = (e^{i\alpha_\nu}, e^{i\beta_\nu})$, and assume that

$$0 < \alpha_1 < \beta_m < \alpha_2 < \beta_1 < \alpha_3 < \beta_2 < \dots \qquad (\text{mod } 2\pi).$$

Let f_ν be the function in E which corresponds to A_ν.

Since $|f_\nu(z)| \geq \delta > 0$ in a neighbourhood of A_ν, there are functions w_ν in B such that $g_\nu(z) = w_\nu(z) \cdot f_\nu(z)$ are analytic and $\neq 0$ on A_ν. Let us now choose a number γ so that $\alpha_2 < \gamma < \beta_1$. We construct a function $q_1(z) \in C$ such that

$$\begin{cases} |q_1(e^{i\theta})| = 1 \text{ on } \alpha_1 \leq \theta \leq \gamma \\ |q_1(e^{i\theta})| \leq 1 \text{ everywhere} \\ |q_1(e^{i\theta})| \leq \varepsilon \text{ on } \theta \leq \alpha_1 - \varepsilon \text{ and } \theta \geq \beta_1 \end{cases} \qquad (\text{mod } 2\pi),$$

where $\varepsilon > 0$ will be determined later. We may furthermore assume that $q_1(z) \neq 0$. A similar function $q_2(z)$ is constructed for A_2 with $|q_2(e^{i\theta})| = 1$ on (γ, β_2). We next consider the functions

$$\begin{matrix} F_1(z) \\ F_2(z) \end{matrix} = q_1(z) \cdot g_1(z) \pm q_2(z) \cdot g_2(z).$$

For ε sufficiently small we obviously have $\mu_{F_1}(\zeta) \neq 0$ on (α_1, α_2) and on (β_1, β_2) and similarly for F_2. On the rest of the interval (α_1, β_2), i.e. on (α_2, β_1), F_1 and F_2 are continuous and have no common zero, since such a zero would be a zero for $F_1 \pm F_2$. We can as before multiply F_i by a function H_i, which belongs to B, so that $G_i(z) = F_i(z) H_i(z)$, $i = 1, 2$, is continuous on (α_1, β_2). We may also assume that $H_i(z) \neq 0$. The new functions G_i are continuous in a domain D: $r_0 \leq r \leq 1$, $\alpha_1 \leq \arg z \leq \beta_2$ and in D we have

$$|G_1(z)| + |G_2(z)| \neq 0.$$

By the remark of theorem 5, polynomials P_1 and P_2 exist such that

$$\varphi_1(z) = P_1(z) \cdot G_1(z) + P_2(z) \cdot G_2(z)$$

does not vanish on (α_1, β_2).

In our original situation we can thus use φ_1 instead of f_1 and f_2, and A_1' instead of A_1 and A_2. We continue the same process, which only consists in the forming of linear combinations, and we finally obtain a function $\varphi \in C$ which is of the form (10) and does not vanish for $|z| \geq \varrho$.

In the same way as above, we construct a function ψ of the form (10) such that $\psi(z) \neq 0$ in $|z| \leq \varrho'$, $\varrho' > \varrho$. Finally, we consider the functions

$$\begin{matrix} \phi_1(z) \\ \phi_2(z) \end{matrix} = \psi(z) \pm K \varphi(z).$$

If the constant K is sufficiently large, then $\mu_{\phi_i}(a) \neq 0$ for $|a| \geq \varrho$, and we see as before that ϕ_1 and ϕ_2 have no common zeros. After multiplication by suitable, non-vanishing functions in B, we obtain two functions $\psi_i(z)$ in C which are of the form (10) and have no common zeros. With the aid of these functions, we get by theorem 5 a linear representation of any function in B. The proof of theorem 6 is thus complete.

6. As an illustration of the function-theoretic significance of theorem 6, we make an application to the Pick-Nevanlinna interpolation problem.

Let $S = \{a_\nu\}$ be an infinite sequence in $|z| < 1$. It is well-known that if there exists an analytic function $F(z)$ in $|z| < 1$ such that

$$\int_0^{2\pi} \log^+ |F(re^{i\theta})| \, d\theta = O(1), \quad r \to 1, \tag{11}$$

which vanishes on S without vanishing identically, then there exists a function of the same kind in B; the condition on S is given by

$$\sum_1^{\infty} (1 - |a_\nu|) < \infty. \tag{12}$$

We can then ask the following more general question: given a function F and a set S, when does there exist a bounded function which takes the same values as F on S? Unless F is bounded, it is evidently necessary that (12) converge. We must also introduce some condition which ensures that F is bounded on S. We shall here prove the following theorem.

Theorem 7. *Let S be a given set such that (12) holds and suppose that $\arg a_\nu$ belong to a closed set E. If the function $F(z)$ satisfies (11) and if furthermore*

$$\overline{\lim_{z \to e^{i\theta}}} |F(z)| < \infty, \quad \theta \in E,$$

then the interpolation $f(a_\nu) = F(a_\nu)$, $f \in B$, is possible.

$F(z)$ can be represented as the quotient of two bounded functions $\varphi(z)$ and $\psi(z)$, where $\psi(z)$ has no zeros. We may furthermore assume that $\mu_\psi(\zeta) > 0$ on E, since $F(z)$ is bounded in a neighbourhood of E. If now

$$\pi(z) = z \prod_1^{\infty} \frac{a_\nu - z}{1 - z\bar{a}_\nu} \frac{\bar{a}_\nu}{|a_\nu|}, \quad a_\nu \neq 0,$$

then $\mu_\psi + \mu_\pi \neq 0$ in $|a| \leq 1$. By theorem 6, functions p and q belonging to B exist such that

$$p(z)\psi(z) + q(z)\pi(z) \equiv \varphi(z).$$

For $z = a_\nu$ we have

$$p(a_\nu) = \frac{\varphi(a_\nu)}{\psi(a_\nu)} = F(a_\nu);$$

$p(z)$ is hence a solution of the interpolation problem.

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