QUERIES

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IS EVERY METRIC ON THE CANTOR SET σ-MONOTONE?

Definition 1. Let (X, d) be a metric space. X is said to be *c-monotone* if

(i) there is a linear order "$<$" on X such that whenever $x < y < z$, then $d(x, y) \leq c \cdot d(x, z)$, and

(ii) open intervals $(a, b) \equiv \{x : a < x < b\}$ are open in X.

X is said to be *monotone* if X is c-monotone for some $c \in \mathbb{R}$, and *σ-monotone* if X is the countable union of monotone spaces.

The notions have applications in fractal geometry, see [2]. The following is proved in [1]. A metric space is monotone if and only if it is bi-Lipschitz equivalent to a 1-monotone space. A metric space with a dense monotone subspace is monotone. σ-monotone spaces have low topological dimension: If X is monotone and separable, then X (topologically) embeds into $\mathbb{R}$ and if X is σ-monotone, then its topological dimension is at most 1. But there are spaces with low dimension that are not σ-monotone: There exists a compact set $X \subset \mathbb{R}^2$ homeomorphic to $[0, 1]$ that is not σ-monotone; in fact, each monotone subset of X is nowhere dense in X. It follows that X has a countable subspace that is not monotone, and a completely metrizable null-dimensional subspace that is not σ-monotone (recall that a topological space is *null-dimensional* if it has a base consisting of clopen sets). However, no example of a null-dimensional compact space that is not σ-monotone is known.

Question 1. Is there a compatible metric on the Cantor Ternary Set that is not σ-monotone?

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