

ON THE LOGIC OF "FEW", "MANY", AND "MOST"

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1 *Introduction* The traditional relations of contradictoriness, contrari-ness, and entailment (of universal to particular) hold for the 'square of opposition' in (1), where (2) shows how "few" comes in.

(1)	affirmatives	negatives
nearly universal	A Most S are P	E Most S are not P
more than particular	I Many S are P	O Many S are not P

- (2) (Most S are P) if & only if (Few S are not-P)
 (Most S are not-P) if & only if (Few S are P)

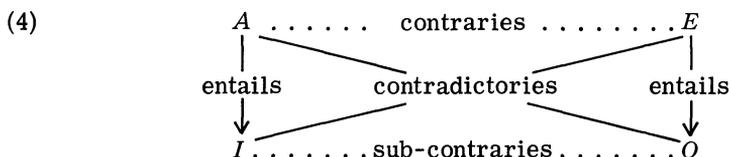
"Little" and "much" should be substituted for "few" and "many" respectively, when the S-term is a mass noun or abstract singular rather than a count noun. The relationships portrayed hold for "most" taken in the sense of "nearly all" (i.e., the sense for which (2) does hold). The relationships do not hold for the generic sense of "most", that wherein "Most S are P" means nothing more specific than that the number (or amount) of S that are (is) P is greater than the number (or amount) of S that are (is) not P.

These facts constitute the basic logic for "few", "many", and "most" in English. (I presume these facts will also obtain, with appropriate adjustments, in all other natural languages. It would be most surprising if they did not.) By 'basic logic' I mean the absolutely ground-level or baseline phenomena that any *further* logical or empirical account—such as an abstract formal system for these quantifiers with explicit (logical) semantics, or a method of natural language description incorporating semantic representations of these quantifiers—must build upon. The only purpose of this paper is to explain and defend the claim that (1) and (2) do present the basic facts. I shall do this by describing the genesis of the claim (my actual line of reasoning that led to these results) and detailing the qualifications necessary to defend it. (The important qualification,

of the square format is that the *corners contradict*. Contradiction is the strongest 'opposition' *qua* negation or denial that two propositions can stand in. A proposition *p* contradicts a proposition *q* if (and only if) they *must* have opposite truth values. If the one is true the other is false and *vice versa*. One alternate way of saying the same thing is that (i) they cannot both be true *and* (ii) they cannot both be false. In this way of looking at it, two propositions contradict each other if *two* conditions are met. If only *one* of the two conditions is met, the propositions do *not* contradict but they are *contraries*.² If two of Aristotle's categoricals cannot both be true but can both be false, they are deemed *contraries*. If two of them cannot both be false but can both be true, they are deemed *sub-contraries*. The *A* and *E* forms are contraries. The *I* and *O* forms are sub-contraries.

Two other features of Aristotle's square bear reviewing in order to be clear about my application of the ideas to "few", "many", and "most". First is existential import. It is necessary that the universal forms carry a *presupposition* of existential import for the entailments to their respective particular forms to hold. For example, "All Greeks are sailors" entails "Some Greeks are sailors" only if the *truth* of the first presupposes that there are Greeks. (In quantification theory, this presupposition of existential import is dropped and the entailments do *not* hold. For the kind of preliminary account of "few", "many", and "most" that seems required today it happens that following Aristotle is more fruitful than following quantification theory. This result may have some retroactive effect in serving to recommend Aristotle's approach *vis à vis* quantification theory. Of course in other matters—e.g., relations—Aristotle's methods fail miserably.) Secondly, the entailments from universal to particular only hold if the quantifier of the particular forms is interpreted broadly or liberally. That is, "some" must be interpreted as "some or *more*". It carries that presupposition for the entailments to hold. (The recognition of this in quantification theory rests in the interpretation of the existential quantifier to mean "at least one, possibly more".) "All politicians are liars" does *not* entail "Some politicians are liars" when the latter proposition is interpreted to mean what "Some, but *not more*, politicians are liars" or "Only a few politicians are liars" do. But when "some" is interpreted liberally to mean "some (a few) or more", then the entailments from universal to particular hold.

To sum up, the relationships of Aristotle's square I am concerned with are the following³:



Finally, remember that the *E* form categorical (in English) is expressible by "No *S* are *P*" (and close variants utilizing "no" and "none") *and* by "All *S* are not *P*", where the former is unambiguous and

the latter ambiguous (at least in written form). Ordinary English intonations serve to (or help to) disambiguate "All S is not P"—e.g., "All Greeks are not sailors" is *not* an English expression of an *E* form, but rather negation—negation *qua* contradiction—of the *A* form. Whereas, "All Greeks are *not* sailors" does express the *E* form (what "No Greeks are sailors" means). (The traditional logic-class example for the ambiguity is "All that glitters is not gold", which read with Shakespearean intonation is the denial of the *A* form—"All that glitters is not gold". To get the *E* form requires primary intonation on "not"—"All that glitters is *not* gold".) There is a lesson in this phenomenon. The disambiguation of a form like "All S is not P" clearly makes us aware of what the two forms of negation at issue—contradiction and contrariness—are. One is negation of the *full sentence*—to yield contradiction. The other—contrariness—is *predicate* negation. In some forms (disregarding intonation clues), it is not necessarily clear which negation "not" is signifying. A corollary of this is that some useful equivalences of phrases—such as "not all" with "some . . . not"—may be misleading. The fact that "not all senators are honest" entails and is entailed by "some senators are not honest" does not mean that the application of "not" in the former sentence is primarily an application of it to the quantifier. For it is not. (In quantification theory one correlate of this—viz., " $\neg(x)Fx$ " is logically equivalent to " $(\exists x) \neg Fx$ "—can also be misleading in this regard. Taking a negation symbol 'through' a quantifier by changing it does not presuppose that full-sentence negation in fact reduces to negation of the quantifier. For it does not. Quantifier negation is another matter. I return to that below with respect to integrating all of these quantifiers.)

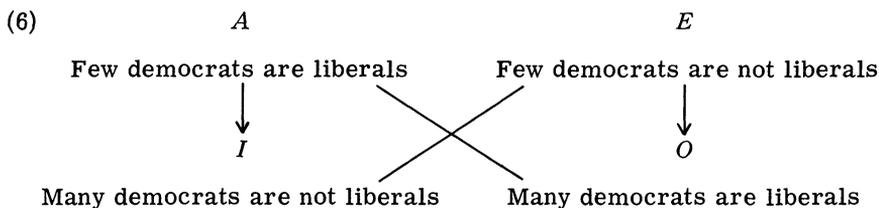
Negation, "Few", and "Many"

My reasoning, which led to the proposal of (1) and (2), began with simple sentences containing "few" and "many". I first treated "few" as exactly paralleling "all" in Aristotle's analysis. What is it to deny "Few S are P" in the sense of *contradicting* it? Particularly, contradicting it as "Some S are not P" contradicts "All S are P". It seems obvious that "Many S are P" contradicts it. Or does it? Certainly "Few children are generous" opposes in some way "Many children are generous", but is it contradiction? Remembering that negation-*qua*-contrariness can be conceived of as predicate negation, we might retreat from seeking the contradictory of "Few S are P" and recall that the contraries in Aristotle's square derive from predicate negation. So, "Few S are not-P" must (might) be the contrary of "Few S are P". (I will, hereafter, indicate predicate negation by explicitly eliding "not" with the predicate term—e.g., "not-P". When no such elision is indicated then the whole form *may* be ambiguous—though not always, e.g., "Some presidential assistant is not a liar" just does *not* seem ambiguous between "Some presidential assistant is a non-liar" and "It is not the case that some presidential assistant is a liar".) Then we could consider these to be parallel to the universal forms of Aristotle—*A*: Few S are P; and *E*: Few S are not-P—and the entailments to be the *I* and

O forms. What does "Few S are P" entail? I would propose it entails "Many S are not-P", if only because that fits with the previously hypothesized contradictory of "Few S are P" to yield the following 'square of opposition':

- (5) A: Few S are P E: Few S are not-P
 I: Many S are not P O: Many S are P

The point of schematizing these forms in this way is just to construct something that might have all the relationships of (4)—contradiction, contraries, and entailments. (Note that this would probably mean that the "not" in the I form is to be taken as predicate negation.) An example does confirm (5) to some degree:



In one particular sense of "few", the A and E sentences of (6) cannot both be true. If *few* democrats are liberals—not "a few", but "just a few", "only a few", or "not many"—then it cannot *also* be true that few are not. For if liberals and non-liberals are exhaustive categories (i.e., a democrat is either liberal or non-liberal (= not-liberal))—which I believe they are (in contrast, say, to liberal and conservative, which might not be exhaustive)—then it cannot be the case (semantically speaking) that *just a few* (i.e., 'few') democrats are liberals and just a few are not liberals. If that were the case it would permit lots of other democrats ('the rest', as we say) to be *neither* liberal *nor* not-liberal. And maybe it could be the case that few democrats are liberals and few democrats are conservative (because, say, most are something in between, called 'moderates'), but that possibility does not carry over for genuine predicate negation which, I believe, is intended to express exhaustive classification.

Do the entailments hold—A to I, E to O? If few democrats are liberals, then are many (maybe most) not liberals? (Leave "most" aside, momentarily.) It seems so. Try other examples: if few politicians are honest, are many not-honest? If few citizens are well-informed, are many not well-informed? (Similarly, for the E-O entailments: if few republicans are not conservatives, are many conservatives; or, if few oranges are not yellow, are many oranges yellow?) To me, the answer is yes for all these cases. Could the I and O forms of (6) both be true, for that would suggest they are true sub-contraries? I believe they could. That is the point of disregarding "most". When "many" does *not* mean "most", then they could. If there are *enough* democrats (not a small number, like three or four, but a larger number, 100s or 1000s), then many could be liberals and many could not be without contradiction. What permits (5) and

proposition q if and only if there is no possibly way that (sense that, or possible world in which) p could be true and q false. But there is such a way for $p =$ "Many students are confused" and $q =$ "Few students are not confused". (There is another way too. "Many students are confused" might be true because "All students are confused" is true. But then "Few students are not confused" could not be true, since *none* would not be confused.)

Secondly, a consequence of (5) is that the phenomenon represented *contrasts* with the way negation operates with "all" and "some". Interpreting those quantifier words as is typical in the predicate logic amounts to bringing a negation 'through' a quantifier by changing it; viz.,

- (11) "Not all . . ." if and only if "Some . . . not . . ."
 or: $\neg(x)Fx \equiv (Ex) - Fx$
 or: $\neg(x)(Sx \supset Px) \equiv (Ex) - (Sx \supset Px)$
 $\equiv (Ex) (Sx \ \& \ -Px)$

e.g., "Not all politicians are liars" if and only if "Some politicians are not liars" (Similarly, for "Not some . . ." to "All . . . not . . ."; e.g., "It is false that some orators are intelligent" if and only if "All orators are not intelligent".) However, 'bringing a negation through' for "few" and "many" conceived as quantifiers analogous to "all" and "some" does *not* both change the quantifier to its complementary one and leave the negation to follow the changed quantifier. Rather, it *only* changes the quantifier. No negation is placed after the changed quantifier.

"*Most*" Where does "most" come in? At first (even, perhaps, before proposing (5)), it might be thought that there is a square of opposition for "few" and "most" sentences, say:

- (12) A: Few S are P E: Few S are not-P
 I: Most S are not-P O: Most S are P

For "Few S are P" does seem to entail "Most S are not P" (similarly, "Few are not" entails "Most are") *and also* "Most S are P" seems to contradict "Few S are P" in a way that "Few S are not P" does not (i.e., maybe the distinction between contrary and contradiction is thereby captured for "few" and "most"). This is wrong. The clue to the mistake rests in noticing that the *I* and *O* forms of (12) *also entail*, respectively, the *A* and *E* forms. Thus, the distinction between contradictoriness and contrariety implied in (12) collapses! What we are left with, however, is significant. It is the *equivalence* between "Few S are P" and "Most S are not P" and between "Few S are not P" and "Most S are not P"—for *one* sense (or use) of "most" (not the generic sense).⁷ For example:

- (13) "Few democrats are liberals" is equivalent to
 "Most democrats are not liberals"
 "Few democrats are not liberals" is equivalent to
 "Most democrats are liberals"

A consequence of the equivalence of "few" and "most-not" (and "few-not" and "most") structures is that a new rendition of (5) is derivable, substituting "few" with "most" structures:

- (14) A: Most S are not P E: Most S are P
 (from: Few S are P) (from: Few S are not P)
 I: Many S are not P O: Many S are P

Making this substitution provides a crucial prediction that is then confirmed—at least it is for my dialect and/or rational faculty. To me it was problematic what the negation-*qua*-contradiction of "Most S are P" (or "Most S are not P") was supposed to be. What is it to deny "Most southerners are democrats" for example? In general, of course, it may be any number of things. One might deny it in order to assert that *all* southerners are democrats. But what would denial amount to where exact contradiction is in mind? Intuitions failed me. (This is no surprise. They often do on logically interesting sentences—e.g., on whether or not "it must be necessary that 2 plus 2 is 4" intuitively follows from "2 plus 2 must be 4"). But the square of opposition of (14) *predicts* that "Most S are P" will have as its contradiction—what "-(Most S are P)" amounts to logically speaking—the sentence structure "Many S are not P". So test it with some examples. Could the contradictory sort of denial of "Most southerners are democrats" be "Many southerners are not democrats"? I believe now that it is. The 'flatest' sort of denial (negation-*qua*-contradiction) of "Most are" is "Many are not". Similarly, in the other direction. If it is false that many southerners are not democrats, then most of them are democrats.

Now it is possible to bring in a missing elegance that Aristotle's square of opposition for "all" and "some" contains. (This detail is not nearly so important, I believe, as those discussed so far.) In Aristotle's square the members of the first column, A and I, were both affirmatives (no explicit negative present) and the members of the second column, E and O, were both negatives. And this regularity permitted the useful classification of the members of the square as universal affirmative, universal negative, particular affirmative, and particular negative. The same can be done now by *interchanging* the columns of (14)—which has no effect on the crucial relations displayed (contradictions, contraries, sub-contraries, and entailments). And, as a final bit of terminological simulation, I label the "Most" sentences "nearly universal" (in respect for Aristotle's "universal") and the "Many" sentences "more than particular" (respecting Aristotle's "particular"):

(15)

	affirmatives	negatives
nearly universal	A Most S are P	E Most S are not P
more than particular	I Many S are P	O Many S are not P

(15) is preferable to both (5) and (14) for a number of reasons. First, it exactly emulates Aristotle's square for "all" and "some". Second, it permits the construction of a square like (5)—but with the columns reversed (an unimportant modification)—by recognition of the equivalences between "Few S are P" statements and "Most S are not P" statements (and, similarly, "Few are not" and "Most are"). Finally, it also permits a closer comparison of two of the three problematic quantifiers (viz., "most" and "many") with "all" and "some". For the way negation operates 'across' such quantifiers can be partially formalized in direct parallel to universal and existential quantification; viz.,

$$(16) \quad \begin{aligned} (\text{Most } x) \phi x &\equiv -(\text{Many } x) \neg \phi x \\ (\text{Many } x) \phi x &\equiv -(\text{Most } x) \neg \phi x \end{aligned}$$

2 *Some presuppositions* In addition to the train of thought that led to my claims in (1) and (2) (which I have just described above), there are two matters of immediate interest that ought to be dealt with in order to complete this account of the most basic logical features of "few", "many", and "most". First, there is the specification of some of the obvious presuppositions required to be in effect when statements are made that have the logical features I have indicated. Secondly, there is the question of the relationships of these quantifiers to others—particularly, "all" and "some". Two presuppositions from Aristotle's square carry over for (1). First is existential import. There can be some debate about Aristotle's assumption of existential import for universal categoricals just because there are two rather successful logical systems that work as well as they do precisely through dropping this presupposition. These are the method of Venn diagrams and, more importantly, first-order quantification theory (predicate logic). Remembering the Venn diagram techniques is particularly helpful, I believe, in making clear what is done in the predicate calculus. The representation of universal propositions on Venn diagrams is most easily grasped, I think, by conceiving of it as actually a representation of the denial of particular propositions (*with* existential import). One shades out a specific area to represent precisely that there is nothing 'in' it (no members of the class that the area can be taken to represent). But this is just representing a universal proposition by representing what is semantically equivalent to it—viz., the denial of the particular (particularly taken as an existential). "All S is P" represented on Venn diagrams is really a *direct* representation of "It is false some S is not P"—particularly when the latter is taken to be the same as "It is false there *exists* an S which is P". (Similarly, for the *E* and *O* forms.) This same (albeit slight) prejudice for the particular form (or existential) over the universal persists in quantification theory. The *reason* universal forms have no existential import is that we take them to be denials of the particular forms, particularly where the latter are interpreted to *have* existential import. The lack of existential import for universals, as should be well recognized, also turns on the connectives utilized (particularly, the material conditional).

But if Aristotle's *A* and *E* forms can be debated with respect to

existential import, the 'nearly universal' forms of (1) and (15) do not seem to be open at all to the same discussions. For all forms appear to clearly carry existential import. "Most soldiers are (not) heroes" must presuppose or imply that *there exist* soldiers. The rather tenuous arguments that the ordinary usages of "All S are P" do not carry existential import do not apply.⁸ For there is not any plausible paraphrase, subjunctive or not, of "Most S are (not) P" (or of "Few" sentences) which succeeds in even hinting at lack of presupposition of existential import. What would it be? "If there were any S, then most of them would be P"? But in such a structure, the whole matter is shifted to the antecedent of the subjunctive conditional. This tends to confirm, rather than suggest an alternative to, the presupposition of existential import for "Most" sentences. (Similarly, an allegedly Belnap-inspired paraphrase, "Of the S, most of them are P", does not seem any better at loosening the obvious existential presupposition of the whole.) Indeed, both "Many" and "Most" propositions have as much (or the same) existential import as particular or existential propositions. For this reason, among others (such as confusing "few" with "a few"), there seems to have been a tendency to lump all of these quantifiers ("few", "many", and "most") with the existential quantifier "some". This is particularly true with regard to the *second* presumption that makes Aristotle's square work—which is as follows. For "All S are P" to entail "Some S are P", the "some" must be interpreted liberally (or loosely, or 'inclusively')—to mean (approximately) "some or more", or "at least one, possibly more". If "some" is interpreted as "exactly some"—or, better (in English), "one, some, or a few, but no more"—then "All S are P" would *not* entail "Some are P". But in *one* sense it does. So, "some" *must* have a reading (or use, or role) distinct from meaning "exactly a few" or "some but no more". (Notice that it is the looseness of interpretation of "some" that makes Aristotle's square work out in another connection. A forms entail corresponding I forms, when "some" is loosely interpreted (similarly, E and O). But when sub-contrariness is in question, then the liberality of "some" cannot extend too far. For "some or more" might extend beyond "many" to "most" or "all"; i.e., "Some S are P" being true because "All S are P" is (because "some" permits "some or more") *cannot* be compatible with "Some S is not-P" (since that contradicts "All S is P"). So, the circumstances that do permit both "Some S are P" and "Some S are not-P" to both be true simply are not those that permit "All S is P" to be true.) Now the same holds for (1) with respect to "Most" forms entailing corresponding "Many"'s. The latter quantifiers must be liberally (loosely, or 'inclusively') interpreted to permit the entailments. "Most democrats are liberals" entails "Many democrats are liberals" just in the sense in which "many" of the latter form is taken to mean "many or more". For if it is interpreted as "many, but no more" (i.e., neither 'most' nor 'all'—for what is more than many except 'most' or 'all', short of counting or measuring?), then "Most S are P" cannot entail "Many S are P" (nor can the entailments hold for the negative versions). But the solution—or liberality—required is no more than what is

required to defend Aristotle's entailments for "all" and "some". So, invoking the same presumption again is no departure (and what is useful and valuable for one—i.e., for Aristotle's square, which I believe is obviously useful and valuable—is useful and valuable for the other, (1)). (I return below to this topic of 'inclusiveness' of the entailed quantifier in relating Aristotle's quantifiers to those of (1) and (2).)

A final presumption that is required to support (1) and (2) is *new*—not found in Aristotle's square. It is most easily presented in answer to an objection—as follows. Take a particular example, say, "Most soldiers are abroad". Its contradictory is said to be "Many soldiers are *not* abroad". Well, let us test that. Imagine that we are talking of soldiers of the U.S. Army. Also, imagine that the total number of them is 1 million. To fit the example, let us stipulate that 900 thousand (give or take a few thousand, it does not matter) *are* abroad, and that that fact is what supports the claim expressed by "Most soldiers (of the U.S. Army) are abroad". (That is, nine out of ten are.) By (1), then, "Many soldiers (the U.S. ones) are not abroad" cannot also be true, because it is supposed to *contradict* "Most soldiers are abroad". But—and this is the objection—this is wrong. It certainly *can* be true, for it *is* in this case! For 100,000 soldiers are not abroad (according to the example) and even though 9 out of 10 of the soldiers are abroad, still 100 thousand are *too many* soldiers (*viz.*, more than I have ever seen or can imagine viewing in a short time, say) to prohibit "Many soldiers are not abroad" from being true. For *many*—*viz.*, 100 thousand—*are not* abroad. It might be that (1) holds for some collections (not too small and not too large, eggs in dozens for example), but it does not hold for large numbers. (E.g., if ten of the 12 eggs I bought today are rotten, then "Most of the eggs I bought are rotten" is true and prohibits the truth of "Many of the eggs I bought are not rotten". For ten are rotten, and that is most; but, only two are not rotten and that is *not* many.)

The objection can be answered, however. And, it is false that the objection shows that the square of (1) only holds for non-large numbers. What is required to defend (1) is the recognition of an as yet unmentioned presupposition. I shall call this the 'presupposition of a constant reference class'. It parallels in importance, I believe, the presupposition of existential import in Aristotle's square (to support the universal to particular entailments). Not only does any proposition of the form "Most S is P" presuppose that there do exist some S's—e.g., for "Most soldiers are abroad" to be true, there must *be* some existing soldiers—but further, the immediate inferences portrayed in (1) and (2) only hold if it is *presupposed* that the reference class remain the same and, thereby, *be* the size which any comparisons (tacit or not so tacit) refer to. For the example constructed it is the number of U.S. soldiers altogether—the class *with* 1 million members—that is the reference class. Nine out of ten are abroad. So, most of them are. But there are a *lot* of soldiers that are not abroad. (So, many are not?) The question is "A lot *compared to what?*"

Compared to the number of U.S. soldiers that I have ever seen, 100 thousand *are* a lot. But that I submit, is evasion at best, and equivocation at worst. (Albeit a hidden equivocation, *not* perhaps ever noticed before by anybody *if*, which I doubt, what I am presenting herein is unprecedented.) For the point of supporting a declaration of "Most soldiers are abroad" by citing the 900 thousand out of a million that are is that the sentence makes clear what the reference class is and the citation of the evidence gives us its size. Now compared to 1 million anythings (soldiers, pencils, or dollars), 100,000 *is not* many. So, if 1 million are abroad and 100 thousand are not, then it is simply *not* true that many of them are not abroad. For compared to the total, one-tenth of it is *not many of the total*. And this goes for *any* size class.

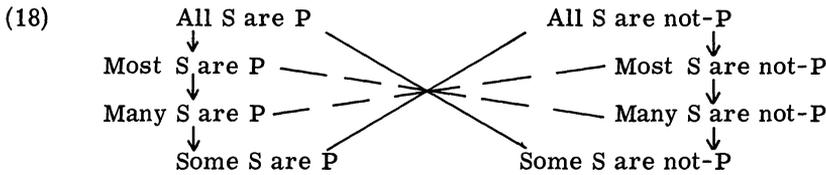
There is, of course, another sense in which "Many soldiers are not abroad" *is* true. This is when we *switch* reference classes—e.g., from all the soldiers there are, say, to the ones I have viewed in my life until now. Switching to that reference class does *support* the truth of "Many soldiers are not abroad", but in that case we are *not* considering *exactly* the same proposition, since we have changed a crucial presupposition. (That is, I will in the end maintain that propositions are distinct from sentence types and even from sentence meanings in that propositions 'contain' (*inter alia*) their presuppositions. We might have the same sentence type, with the same sentence meaning, but if presuppositions vary in the use of that sentence (*with* its meaning), then the same proposition is not necessarily expressed.) "Many soldiers—compared to all those I have ever seen—are not abroad" is simply *not* the same proposition as "Many soldiers—compared to all U.S. ones there are—are not abroad". And it is the latter, *not* the former which is at issue in (1) and (2). So, a crucial presupposition of the greatest importance to support (1) has been revealed.

The easier way to defend against this objection would have been to move to (5) via (2). That is, in light of the arguments given previously "Most soldiers are abroad" is logically equivalent to "Few soldiers are not abroad" (as (2) represents). The drawback with this course is that it is too quick. It changes the issue, since it is less objectionable (it seems to me) that "Few soldiers are not abroad" contradicts (or, at least conflicts importantly with) "Many soldiers are not abroad". For either few are not or many are not, it cannot be both. But then the objection (if it is not, thereby, completely suppressed) *should* arise in the application of (2) here. Can it be true that few soldiers are not abroad (i.e., 100 thousand are not), if most *are* abroad (i.e., 900 thousand)? Well, by switching reference class—from all there are to, say, the number you have ever seen—one might well maintain (2) cannot apply. Since 100 thousand is very many more soldiers than you have ever seen (say), realizing that might cause one to balk at the truth of "Few soldiers are not abroad" (since 100 thousand is so many). But of course 100 thousand *is not* so many soldiers compared to all there are, so the application of (2) to "Most soldiers are abroad" is legitimate when the reference class is constant (not switched).⁹

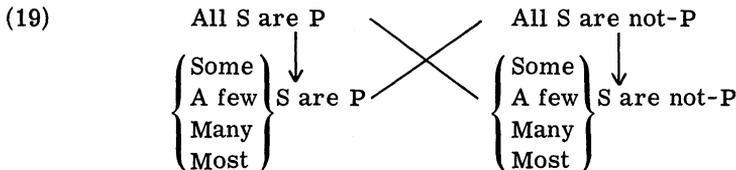
3 Integration How do the relationships in (1) relate to Aristotle's relationships portrayed in (3)? Can the two squares be integrated? In one way, the answer is easy. For the liberality of interpretation that permits the universal to particular entailments to hold in (3) and which carries over into (1), *also* permits the following entailments:

- (17) "All S are P" entails "Most S are P" which entails
 "Many S are P" which entails "Some S are P"

(Similarly, for the negative forms of (3) and (1).) The "all" forms entail the "most" forms when "most" is taken as "most, or more". Similarly for from "Most" to "Many" (= "many or more") and "Many" to "Some" (= "some or more" or "a few or more").¹⁰ This means that (1) and (3) can be combined as follows:



So, the square of opposition of (1) occurs 'inside of' Aristotle's. Or, Aristotle's universal forms occur 'above' the 'nearly universal' forms of (1) and his particular forms 'below' the more than 'particular' forms. None of this is surprising, especially in light of the entailments (as in (17)). What *is* troublesome is that (17) and (18) (or (1)) do *not* in any obvious way easily combine with what has been the traditional (or, 'logic-class') way of treating "many" and "most". Along with "a few" all of these quantifier words (except "all" and other genuine universal quantifiers) would have been grouped *together* and Aristotle's square would have merely been modified to appear:



This rendition of the relationships would be particularly well supported by the liberal (or 'inclusive') interpretation of the existential (particular) quantifier "some". The universal forms entail their corresponding particulars just because "some" is taken as "some *or more*". And the latter liberality includes, thereby, other non-universal quantifiers like "a few", "many", and "most". And since that is the case, not only is "Some S are not-P" the contradictory of "All S are P", but so (e.g.) is "Most S are P".

How can the obvious incompatibilities of (18) and (19) be resolved? Can, for example, "Most atheists are not fools" contradict "All atheists

are fools” (via (19)) and also contradict “Many atheists are fools” (via (1))? Before directly addressing this issue, a different source of perplexities should be noted. What can confuse the discussion at this point (and may have already above somewhere) is an application of negation or denial to quantified propositions that is *neither* full sentence negation (yielding contradiction) *nor* predicate negation (producing contraries). And an important case of this kind I have in mind now I shall call ‘*quantifier negation*’. The phenomenon is most easily revealed in considering negative replies to relevant declarations—e.g., “No!” said in response to any of the following:

- (20) “Many democrats are liberal”
- (21) “Most politicians are liars”
- (22) “All citizens are well-informed.”

One might, by disagreeing through saying “No” to the first, follow with the supporting declaration that is contradictory—viz., “Most democrats are not liberals” (or, equivalently, “Few democrats are liberals” or “Not many democrats are liberals”). Or, similarly one might follow it with a supporting contrary like “Many democrats are not liberals” (which, since it is a *subcontrary* is a rather weak denial, a weaker way of supporting “No”). But there is a *third* possibility. One *might* say “All democrats are liberals”. Notice that in what I have said only the quantifier has been changed to “all” in supporting the denial. This is *quite* different from saying “All democrats are *not* liberals”. Sometimes denial pertains to the *quantifier itself* (*not* to the whole sentence as when contradiction is at issue, nor to the predicate as when contrariness is raised). So, one might *object* negatively to these kinds of declarations with the intention of offering an alternative quantifier. So, for (20), one might respond with any of the following:

- (23) “No, all democrats are liberals”
- “No, most democrats are liberals”
- “No, some (or, a few) democrats are liberals”

Analogously, for (21) and (22), one might respond:

- (24) “No, all politicians are liars”
- “No, many politicians are liars”
- “No, some (a few) politicians are liars”
- (25) “No, most citizens are well-informed.”
- “No, many citizens are well-informed.”
- “No, some (or, a few) citizens are well-informed.”

So, in addition to negation *qua* contradiction and negation *qua* contrary, there is quantifier negation. The rule appears to be as follows. To negate (or deny), *qua* quantifier negation, a proposition of the form

$$(26) \quad \left\{ \begin{array}{c} \text{All} \\ \text{Most} \\ \text{Many} \\ \text{A few} \\ \text{Some} \end{array} \right\} \text{S are P}$$

replace its quantifier with a distinct one chosen from the set of quantifiers, "all", "most", "many", "some", and "a few". In other words, there exist equivalences of the following sort (where "NOT_Q-" represents quantifier negation):

$$(27) \quad \text{NOT}_Q\text{-(All S are P)} \equiv \left\{ \begin{array}{c} \text{Most} \\ \text{Many} \\ \text{A few} \\ \text{Some} \end{array} \right\} \text{S are P}$$

$$\text{NOT}_Q\text{-(Most S are P)} \equiv \left\{ \begin{array}{c} \text{All} \\ \text{Many} \\ \text{A few} \\ \text{Some} \end{array} \right\} \text{S are P}$$

Etc.

One should have noted that "few" has been left out of the list of quantifiers available for quantifier negation. It does not belong. Not realizing this is probably the source of many confusions about these quantifiers. To see this, pretend that it did belong. Then, a 'quantifier negation' (or 'quantifier denial') of "Many politicians are liars" could be "Few politicians are liars". But the latter, via (2), is the same (semantically) as "Most politicians are *not* liars". But *that* is no mere quantifier negation. It is the contradictory (according to (1)). And remembering (2), we can see (suppose?) that the *reason* "few" cannot be in the list of available quantifiers for this kind of negation is that it amounts to "most are not". So, the replacement of "Many" by "few" is no mildmannered denial like quantifier-negation, but modification of more complicated structure. It is changing the form "Many S are P" into the form "Most S are not-P", the more complicated matter that (1) deals with.

Another way to make this clear is to recognize that quantifier negation (denial) is very *weak*. The quantifier-denial of p can be quite compatible with p (like sub-contraries), but so can their denials. For example, "All politicians are liars" and "Some politicians are liars". They conflict quantifier-wise, but they can both be true and they can both be false. This appears to be the case for all combinations of the quantifiers (pair-wise) of those mentioned in (26) and (27). But addition of "few" to that list destroys the weakness of the negations (denials) and introduces the stronger denials tantamount to contradictions and contraries.

It does not appear to me that all uses of "not" of the type found in Aristotle's square or (1) and (2) can permit interpretations that indicate quantifier negation. (That is, perhaps few of those structures are

ambiguous between the previous forms and quantifier negation.) Certainly, one type of English expression for denial does express quantifier negation (beyond the discourse methods of (20)-(25)), viz., "it is not the case that". "It is not the case that all politicians are liars" can certainly be ambiguous between a flat denial of "All politicians are liars" (\equiv "Some politicians are not liars") and some quantifier negation—say, "Most politicians are liars". (Also, of course, it might be interpreted as the contrary of "All politicians are liars", so that "It is not the case that all politicians are liars" is at least three-ways ambiguous.) However, it seems to me that there are at least a few (difficult to specify) intonations of sentences containing "not" that admit of a quantifier-negation interpretation—say, "*Many* politicians are not liars" (distinct from "*Many* politicians *are* not liars"), such as in "*Many* politicians are not liars, *all* of them are liars".¹¹ Now, with this interesting sidelight clarified, how can (18) be resolved with (19)? To shorten the discussion, I will just treat the question of "All S are P" in (19) contradicting (apparently) "Some (a few, many, or most) S are not-P" in light of the complexities of the A to O sorts of contradictions portrayed in (18). (The relevant E to I contradictions will submit to exactly the same analysis.) Further, some abbreviations will be quite useful, since so many sentence structures are considered at one time. As follows:

(28)	ALL = All S are P	ALL-not = All S are not-P
	MOST = Most S are P	MOST-not = Most S are not-P
	MANY = Many S are P	MANY-not = Many S are not-P
	SOME = Some S are P	SOME-not = Some S are not-P

(And I will presume that "a few", as distinct from "few", is close enough in function to "some" to not make any relevant difference.)

(19) appears to conflict with (1) and (18) because (19) says that the relevant denials of ALL forms entail any of the following—SOME-not (the obvious entailment), *or* MANY-not, *or* MOST-not.¹² And (18) does not represent such facts. Further, the reverse seems to hold—that the denial of any of those listed (SOME-not, MANY-not, MOST-not) entails ALL. In other words

$$(29) \quad \text{not-ALL} \supset \left\{ \begin{array}{l} \text{SOME} \\ \text{MANY} \\ \text{MOST} \end{array} \right\} \text{not}$$

(where " \supset " represents mutual entailments between noted forms and full sentence denial or negation is indicated by the prefix of "not-", a convention I will continue below.) There are four cases to be examined as a result:

$$(30) \quad \text{not-ALL} \rightarrow \left\{ \begin{array}{l} \text{SOME} \\ \text{MANY} \\ \text{MOST} \end{array} \right\} \text{not}$$

$$(31) \quad \text{not} \left\{ \begin{array}{c} \text{SOME} \\ \text{MANY} \\ \text{MOST} \end{array} \right\} \text{not} \rightarrow \text{ALL}$$

$$(32) \quad \text{ALL} \rightarrow \text{not} \left\{ \begin{array}{c} \text{SOME} \\ \text{MANY} \\ \text{MOST} \end{array} \right\} \text{not}$$

$$(33) \quad \left\{ \begin{array}{c} \text{SOME} \\ \text{MANY} \\ \text{MOST} \end{array} \right\} \text{not} \rightarrow \text{not-ALL}$$

(where (31) is the 'contrapositive' variant of (30) as is (33) of (32)).

Consider (30) with respect to the MOST alternative. The full-sentence denial of ALL is certainly *compatible* with MOST-not, for the denial of ALL forms entail SOME-not and the latter could be true because MOST-not is (remember (17)). However, it does not in fact *entail* it. For there could be circumstances in which a particular proposition of the form of a denial of ALL is true and the appropriate MOST-not is false. For example, consider the not-ALL form exemplified in "It is not the case that all politicians are liars" when that proposition is true only because there is only *one* lonely politician who is not a liar (say, the mayor of Tully, N.Y.—who is not, by the way, the mayor of Cicero, N.Y.). In these circumstances "It is not the case that all politicians are liars" (on the intended interpretation, not predicate negation nor quantifier negation) is true, but "Most politicians are not liars" would be false since no more than one is not a liar. So, not-ALL forms do not *entail* MOST-not forms. (The same example yields the same result substituting "MANY" for "MOST".)

Similarly for (31), using the same example. A not-MOST-not form can be true (viz., "It is not the case that most politicians are not-liars" true, which is equivalent to "Most politicians are not-liars" false—which presume) when the relevant ALL form is false (viz., "All politicians are liars", false because of the single exception in Tully). (And the same goes through substituting "MANY" for "MOST".) So, neither (30) nor (31) are entailments for the MANY and MANY forms. Only the SOME forms hold. But the latter are simple components of the Aristotelian square already incorporated in (18). (19) is misleading to the extent that the extractions from (30) and (31) utilizing MOST and MANY do not produce the indicated entailments.

What of (32) and (33)? Consider, for example, ALL forms entailing not-MOST-not forms—say, "All politicians are liars" entailing "It is not the case that most politicians are not liars". Is there any way circumstances permit such an ALL form to be true and the relevant not-MOST-not form to be false? For the not-MOST-not to be false, the MOST-not form would have to be true. So, could (32) be falsified via some circumstances in which ALL is true and MOST-not true? There are no such circumstances. If all my coffee is stale, then it can in no way be also true that most of it is not. Likewise, if all politicians are liars then it cannot be true that most of them are not. (And not even equivocating, by taking the first

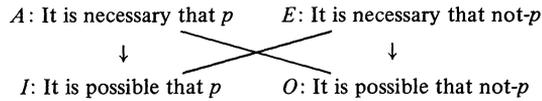
"not" of "not-MOST-not" as a predicate denial to produce the relevant contrary "MOST", is of any avail. For then not-MOST-not would be equivalent to MOST. And could ALL be true while MOST is false? Not under the 'liberal' interpretation of quantifiers where "most" means "most or more", and where quantifier negation is excluded.)

Similarly for (33). MOST-not cannot be true where not-ALL is false—e.g., that most mushrooms are not-yellow while it is false that not all of them are yellow. That would require the joint truth of "Most mushrooms are not yellow" and "All of them are yellow", which is impossible if anything is. The same reasoning holds substituting "MANY" for "MOST" in both the analyses given for (32) and (33). Again, of course, substituting "SOME" (either "some is" or "some are") affirms (32) and (33). For that result is just a part of Aristotle's square—viz., ALL entail SOME-not, and SOME-not entail not-ALL.

Should (32) and (33), as extracted from (19), be at least partial support of (19) and evidence for some shortcoming of (18)? No, for (32) and (33)—and, therefore, analogues of these I did not consider (like ALL-not entailing not-MOST)—are *already* accounted for in (18). Consider, for example, ALL entailing not-MOST-not. By (18) ALL entails MANY (via the intermediate MOST). But also, via (18), MANY entails not-MOST-not. Thus, by transitivity of entailment, ALL entails not-MOST-not. (Similarly, ALL entails MOST which, via (18) still, entails not-MANY-not.) Finally, ALL entails not-SOME-not via the original Aristotelian square embedded in (18). So, at last, (19) should be completely dispensed with. It is mostly misleading and where it is correct the phenomena in question are adequately captured by (18).^{13,14}

NOTES

1. The first-order functional calculus is not, of course, an unqualified success, but it does explain some things to some degree of satisfactoriness. And, more importantly, it serves as both (i) the main model of how logical inquiry should proceed (resting as it does on the even more satisfactory propositional calculus) and (ii) the main starting point for investigating the same or related logical phenomena.
2. Also note, then, that only contradiction satisfies the truth-functional concept of denial. Negation in the propositional calculus always signifies the contradictory (not the contrary) of what is negated (i.e., oppositeness in truth-value). The contrariness sort of oppositeness can be discovered in the propositional calculus, but it is not of much explanatory value—e.g., " $p \ \& \ q$ " is contrary to " $\neg p$ " (they cannot both be true, though they can both be false).
3. The importance of utilizing the Aristotelian concepts of contradiction and contrariety cannot be overemphasized. For example, the same phenomena are at the bottom of the initial definitions for modal logic—e.g., " $\Box p$ " =_{Df} " $\neg \Diamond \neg p$ " even though it is not usually introduced this way. Consider necessity and possibility as operators on sentences (or, as 'higher predicates') expressible, respectively, "it is necessary that . . ." (as the form revealing what some other occurrences of "necessarily" express) and "it is possible that . . ." (the revealing form for some other occurrences of "possibly"). (These uses give rise to the *de dicto* alethic modalities. Other uses of "possibly" and "necessarily" can give rise to the *de re* forms, which are best thought of, I believe, as species of predicate modifiers.) For any proposition " p ":



These A and E forms cannot both be true, but can both be false (because their falsities are the truths of I and O). The contradictions represented in the corners support the axioms (or definitions) " $\Box p = -\Diamond -p$ " and " $\Diamond p = -\Box -p$ ". The entailments support the axioms " $\Box p \supset p$ " and " $p \supset \Diamond p$ ".

4. I am indebted to Guy Carden for making this point clear to me. Without the distinction between "few" and "a few", it seems impossible to construct a cogent square of opposition. The nature of these quantifiers is not much discussed in the logical literature; e.g., Keynes (in his *Formal Logic*, Macmillan, 1887, pp. 61-62) correctly distinguishes "few" from "a few" and goes on to relate "few" to "most" as I do below, but he does not take the crucial step of relating them to "many".
5. This relatively simple point—that "Few S are P" is logically equivalent to (in a mutual entailment with) "Not many S are P" (and, similarly, that "Not a few S are P" is equivalent to "Many S are P")—is *very* basic. I claim that each of the following entails its partner:

Few democrats are liberals . . . Not many democrats are liberals
 Few soldiers are not abroad . . . Not many soldiers are not abroad
 Not a few democrats are liberals . . . Many democrats are liberals
 Not a few soldiers are not abroad . . . Many soldiers are not abroad

In sum, applying "not" to "few" produces a quantifier equivalent to "many" and applying "not" to "many" produces a quantifier equivalent to "few". I emphasize this fact for two reasons. First, on the basis of this phenomenon, I propose further that the application of "not" to either "few" or "many" *not* be taken simply as some sort of modification of the quantifier (analogous, perhaps, to modification of head nouns by attributive adjectives) —which might easily and plausibly be thought to be the point. Rather, the square of opposition of (5) above results from taking the application of "not" to be a negation of the whole sentence. Taking it this way is what permits (indeed requires) the interpretation of the diagonal corners of (1) to be contradictories.

The second reason for emphasizing the equivalences of "not a few" and "many", and "not many" and "few", is that one can with troublesome cases (see below) fall back on this phenomenon as basic. If a troublesome example can be translated into terms simply with "few" and "many", then (often) this basic phenomenon can be used as indirect evidence to help decide the case. In fact, this *will* happen with respect to "most" and "many". A result will be that an important presupposition about such squares will be brought to light that is fairly submerged in (1).

6. As I mentioned, statements in English of the form "All S are not P" can sometimes be ambiguous between "No S is P" and "Not all S are P"; e.g., "All that glitters is not gold". It is not clear to me that either "Few S are not P" or "Many S are not P" can have usages that produce an analogous ambiguity. But even if they did, that would not serve to refute the analysis I offer, but rather to confirm it (since ways of specifying the ambiguities are herein provided). Try "*Much* that glitters *is* not gold". Does that differ from "Much that glitters is *not* gold"? Is the former equivalent to "not much that glitters is gold" (= "Little that glitters is gold")? Maybe. Also, note that these sentences make it clear that the criterion for use of "little"- "much" needs expanding at least slightly, since "gold" (the mass noun) is *not* the S-term.
7. Another way of looking at this is that the use of "most" under consideration is just that which can be stipulated via (2) and illustrated via examples like (13). Label this use "most_f" —the 'few-not' sense of "most". Label the more generic use (or sense) "most_g" —the 'generic'

sense of “most”. The generic sense in “Most_g S are P” is true under the interpretation whereby the quantity (number or amount) of S that are (is) P *exceeds* the quantity of S that are not-P. The few-not sense is more specific. “Most_f S are P” must be interpreted as:

The quantity (number or amount) of S that are (is) P *greatly exceeds* the quantity of S that are (is) not-P.

Only in this way will “Many S are not-P” be a defensible contradictory of “Most_f S are P.” Let “Q(SP)” abbreviate “the quantity (number or amount) of S that are (is) P” and “Q(S \bar{P})” abbreviate “the quantity of S that are not-P”. Then, the denial of “Most_f S are P” requires one of the following conditions to be true:

- (i) that $Q(SP) = Q(S\bar{P})$, or
- (ii) that $Q(SP) < Q(S\bar{P})$, or
- (iii) that $Q(SP) > Q(S\bar{P})$.

But (iii) can hold (as the condition for the relevant denial) *only if* $Q(SP)$ is *not very much* greater than $Q(S\bar{P})$. If “Most_f democrats are liberals” is contradicted, then more democrats *can* be liberals than not liberals only so long as it is not *too* many more. Otherwise, where “Most S are P” is taken to be “Most_g S are P”, a counter example to (1), (14), and (15) can be easily devised. Of the 5 million democrats (say), let one more than half of them (viz., 2,500,001) be liberals. This would not entail that it is false that many are not (what (1), (14), and (15) require). *But* if one more than half is *not* taken as enough to make “Most_f S are P” true, then this kind of counter example is avoided. (This point may be clearer in retrospect, after reading the following sections. See note 9 for additional discussion of the interplay between the two “most”s.)

8. Especially tenuous (to me) is the part of such arguments to the effect that indicatives like “All S are P” can express something common to statement with existential import and to others without it, *because* (would you believe?) what the subjunctive “If it (he, she, etc.) were S, then it (he, she, etc.) would be P” expresses is what the indicative “All S are P” can sometimes be used to express. So, I tend to side with Aristotle. This need not, however, be any criticism of quantification theory, for its defense comes mainly from another direction—viz., that of its abstract and theoretical utility. Its connection to English surface structures should not be taken to be direct, but to be quite indirect—such as furnishing abstract components which together with others combine to generate (not to *be*) surface structures. (*Cf. my Concepts and Language* (Mouton, 1973), Chapter 1 and the Appendix.)
9. For the solution to this objection I am indebted to Anne Sullivan Peterson who pointed out to me that *switching* reference classes was the source of the trouble. An objection somewhat analogous to this one occurs for definition between “most_f” and “most_g”. First, adopt the following definitions (based on notations introduced in note 7 above):

- (i) Most_f S are P if and only if $Q(SP) \gg Q(S\bar{P})$
- (ii) Most_g S are P if and only if $Q(SP) > Q(S\bar{P})$
- (iii) Most_f S are P *entails* Most_g S are P
- (iv) Most_g S are P *does not entail* Most_f S are P

Now notice that *if* $Q(SP) > Q(S\bar{P})$ *and if* the difference between $Q(SP)$ and $Q(S\bar{P})$ is *not small* (i.e., there is not just a bare majority of S that are P), then that alone is sufficient to make “Most_f S are P”—or, $Q(SP) \gg Q(S\bar{P})$ —*plausible*. The objection to (1) in this connection is that there are circumstances in which “Most_f S are P” could be true but that that would not entail that “Many S are not P” is false. For example, presume that there are 700 faculty members voting in an election for collective bargaining and suppose that 400 vote in favor of it and 300 against. The difference between those faculty members in favor of it and those not is not small, viz., 100. (Also, presume that no one abstains.) So, “Most_f of the faculty are in favor of collective bargaining” is plausible. So, if it is plausible, there is some sense to

admitting its truth (or an interpretation, circumstance, or possible world in which it is true). But then its truth in those circumstances would not seem to entail the falsity of "Many members are not in favor". For it is true (not false) that many members are not in favor. 300 of them are not in favor.

The solution to this objection is not so striking as that which prompted the 'presupposition of a constant reference class'. The solution is simply an appeal to borderline cases. In the example given, the borderline between an application of "most_f" and an application of only "most_g" has just not quite been reached. Out of 700 voting, a greater difference than 100 is required before "most_f" applies. And the test is via (2). If it is not the case that few members are not in favor, then it is not the case that most_f members are in favor. If the vote were 550 in favor and 150 against, then relative to 700, 150 (by some) would be regarded as few. Then, in that case "Most_f of the faculty members are in favor" would be true and would entail the falsity of "Many faculty members are not in favor" (for only 150 of 700 are not in favor and, relatively speaking, that is not many). In other words, there are two ways borderline cases about "most_f" use can arise. The first is via (2). If the S that are P exceed the S that are not, then most_f S are P if and only if few S are not. But how few (or little) is few (little) enough? The second way of looking at it is that the S that are P must exceed the S that are not by more than a slight amount. Then the question is how much more than a slight amount is required. If *both* criteria are used, there results an appropriate continuum of borderline cases. For the lower end of the first criterion (when "Most_f S are P" is in question because there are enough, or almost enough, S that are not-P to make "Few S are not-P" doubtful) appears to merge with the upper end of the second (when S that are P exceed those that are non-P by somewhat more than a small amount). For example, consider 25 pencils of which 15 are new and 10 not. Then, are most_f of the pencils new? By (2), the test is that most_f of the pencils are new if 10 of 25 count as *few*. (I do not think it does; i.e., "Few of the pencils are not new" is not true if it is 10 of the 25 that are not new.) By the second test we can ask if 5 pencils—the difference between the new ones and the not-new—is 'small' relative to the whole set (25). That is, does that excess amount to only a 'bare majority', or is it somewhat larger than that? (I think it *is* small, or at least not clearly not-small—5, relative to 25, is less than a fourth of them.) Change the proportions. Let 16 be new and 9 not. Is 9 'few' compared to 25? (Still I think not, but this is closer to 'few'.) Now the difference between new and not-new pencils is 7. And 7 seems (to me) even more clearly not-small relative to 25 than 5 does. So, by the second test "most_f" might apply, but not by the first (9 is not 'few' enough). The clear case (for my intuitions) is reached when we change the proportions to 17 new and 8 not. By the second test, the difference is 9. And 9 is just not small relative to 25. The other test is via (2), that few are not-new—viz., 8. Is 8 'few' relative to 25? Well, it is less than one-third. By the second test (for my intuitions) the borderline has been crossed (9 is not small relative to 25). By the first, it is almost crossed. That is 8 is 'few' enough to (perhaps) count as few of 25. (So, "Few of the 25 pencils are not-new" is plausibly true because 8 are not-new.) Where the second test gives a clear answer (a not-small difference, or no longer a bare majority), the first very nearly gives a clear answer. Therefore, the combination of *both* tests gives an appropriate spread of borderline cases (with varying assessments possible, doubtless a function of other interests and purposes) which, to my intuitions, accurately represents the facts about the two uses of "most"—i.e., that there is a complex series of borderline cases between (i) "Most_g S are P" where "Few S are not-P" is *false*, and (ii) "Most_f S are P".

10. The difference between "some" and "a few" is very slight. My intuitions are that "some" indicates more than one when the subject term is plural and one when the subject term is singular. So it is hard in English to get a form that is generic for, or ambiguous between, "some" meaning one and "some" meaning more than one. You cannot leave it open in English; "Some men were late" means more than one was and "Some man was late" means one was. However, "a few" (to me) signifies three or more. If exactly two men were late, then though "Some men were late" is appropriate, "A few men were late" is not. But "a

few” can permit *more* than three. To clearly limit the number requires “few”, I believe (though some of the connotation of “few” certainly transfers to “a few”). This has clear ramifications for the analysis of “only” in English, since “few” seems to function like “only a few”. But this issue is a larger one than might be supposed, I believe, since accounting for “only” ought to begin through examination of its contradictories and contraries. And examination of that raises interesting perplexities revealing that “only” is not quite like other quantifiers (if it is, indeed, one at all) due to its very clear relations to the connective “only if” (as in “if and only if”). (Also, see note 11.)

11. It should be noted that quantifier-negation added to the other sorts discussed probably does *not* exhaust the kinds of negation or denial that such propositions can be subject to. There are probably additional types—as many as there are semantic and pragmatic features of such sentences. The next thing I would look at is the possibility that implicit logical connectives might be the subject of negation; i.e., in addition to sentence negation, predicate negation, and quantifier negation, there might be ‘connective negation’. I would begin the analysis by examining “only”, which functions very much like the quantifiers under discussion but which also may not be one. (For the same reason cited above—its close relation to “only if” (= “if-then”).)
12. (19) could be thought of as the result of inferring (ii) from (i):
- (i) (All S are P) if and only if not-(Some, or many, or most S are not-P)
 - (ii) [(All S are P) if and only if not-(Some S are not-P)] & [(All S are P) if and only if not-(Many S are not-P)] & [(All S are P) if and only if not-(Most S are not-P)]

When viewed this way, (19) certainly appears *unpromising*, since the inference is fallacious.

13. In the earlier stages of the inquiry reported herein I thought there might be something to the fact that (i) could be paraphrased by (ii), but (iii) could not be paraphrased by (iv)—since (iv) consists of ungrammatical forms:
- (i) Some (one, few, a few, many, two, three, etc.) S are (is) P
 - (ii) There are some (is one, few, a few, many, two, three, etc.) S that are (is) P
 - (iii) All (Most, Every, Each, etc.) S are (is) P
 - (iv) *There are all (most, every, each, etc.) S that are (is) P

This grammatical fact makes it appear that “most” alone (of the non-universal quantifiers) gravitates towards “all” in some important ways (e.g., as a modifier of it perhaps). And if so, maybe there is something logically or semantically important signified by it. If there is, I do not yet see it. For one thing, the ungrammaticality of (iv) may not be so striking as it first seems, since a very slight syntactical variation *does* permit paraphrases of (iii) that *do* contain “there is (are)”; e.g.,

- (v) All (Most, Every, Each, etc.) S that *there are (is)* are (is) P

So, maybe the ungrammaticality of (iv) is merely a superficial phenomenon with no particularly deep consequences semantically. Concerning these phenomena, Helen Cartwright has suggested to me that

- (vi) Most soldiers are abroad, but there are not many of them

could possibly be true. I take (vi) to be an only slightly elliptical version of (vii):

- (vii) Most soldiers are abroad, but there are not many of them that are

Taken in this way (about the shorter way, see the last paragraph of this note), (vi) would appear to be a counter example to (1) at least with respect to the universal-to-particular entailments. Worse yet, it suggests that

- (viii) Most soldiers are abroad, but many of them are not

might be non-contradictory (thereby, disconfirming (1) at its heart). (I speak here of “most” in the sense of “most_f”, not in the sense of “most_g”. (viii) could easily be seen to be non-contradictory for “most_g”. See notes 7 and 9 above.) The way out (also suggested by H. Cartwright) is to note that sentence forms (ix) and (x) need not have all the same uses:

- (ix) Most S are P
- (x) Most *of the* S are P (or: Most S *that there are* are P)

The form illustrated in (x) is more to the point of what is accounted for in (1) and (2). That is, the “of the” (like “that there are” of (v)) *expresses* what is a presumption on the shorter forms in (1) and (2) (the presupposition of existential import—which, together with the presumption that S-terms be ‘distributed’, permits the relationships of (1) to hold). *But* the shorter forms might also be the (surface) forms of some distinctly different kinds of sentences (ones not permitting synonymous “of the” paraphrases). And *that* kind of form is what must be involved when (vii) is deemed possibly true (non-contradictory).

One way of explaining this is to suppose that a non-contradictory use of (vii) would involve reference to the ‘kind’ soldiers. This reference would be similar to the way one sort of use of (xi) can (or might) involve reference to a ‘kind’—in particular, when (xi) can be accurately paraphrased by (xii):

- (xi) All soldiers are aggressive.
- (xii) Soldiers (by nature, by definition) are aggressive.

(This proposal results from another idea of H. Cartwright, though the original context for it was slightly different from this one; *cf.* her “Some Remarks About Mass Nouns and Plurals”, abstract in *Journal of Philosophy*, vol. LXIX(19), 1972, p. 680.) (vii) might be plausible, self-consistent, and consistent with (1) in roughly the same way (xiii) might be possibly true (non-contradictory):

- (xiii) All soldiers are aggressive, but there are not any of them who are.

(xiii) could be possible (non-contradictory) when it expresses exactly what (xiii)’ does:

- (xiii)’ Soldiers (by nature) are aggressive, but there are not any of them who are.

(Try “Dinosaurs are the largest mammals, but there are not any of them that are” and “Cheetahs run faster than men, but there are not any of them who do (have)”). In a similar fashion, “Most soldiers are aggressive” might also have a use wherein the subject term is not ‘distributed’ (and existential import not presumed)—roughly paraphrased (say) by “Soldiers (by nature, definition, etc.) are aggressive *for the most part*”. Then (vii) could be true (and not in conflict with (1)), if taken as expressing the complex claim that

- (a) soldiers are the *kind* of things which by nature or definition are abroad (not in their home country) *for the most part* (i.e., a few may be at home, such as in training and in ‘home’ offices, but if most are home and are acting in typically soldierly ways then they are not ‘soldiers’ but police, revolutionaries, or something else)

and

- (b) *not many* of the soldiers that *there are* are aggressive because (say) there do not exist many (or any) soldiers altogether.

That is, there is a superficial sense in which the contrariness relation of *A* and *E* forms in (1) can be disconfirmed by the ‘counter example’

- (xiv) Most soldiers are abroad & Most soldiers are not abroad (or: Not many soldiers are abroad)

when the conditions making it true are all and only those that would make true

- (xv) Soldiers (by nature, etc.) are mostly abroad & Most of the soldiers that there are, are not abroad.

But that is no real disconfirmation, since it is based on an equivocation. Sentences that do not have ‘distributed’ subject terms (nor existential import)—those not permitting relevant “of the” paraphrases—do not disconfirm (1) since they are simply *not* the sentence forms (1) is intended to cover. (Whether or not those *other* forms, like (vii) or (xiii)’, can be explained with some notion of reference to kinds, as I suggested, is not necessary to defending (1). I only mention the speculation in order to help convey the contrasts that the linguistic data seem to me to manifest.)

The *other* interpretation of (vi) is that shorter one in which the final clause merely asserts that not many soldiers—abroad or not—exist. On that interpretation (vi) is not even superficially at odds with (1); i.e., both “Most of the soldiers are abroad” and “Not many soldiers exist” could be true, for “Not many soldiers exist” is perfectly compatible with “Many of the soldiers are abroad”. The ‘many’ and ‘most’ relations captured by (1) are all ‘relative’ quantifications. “Not many soldiers exist” is either ‘absolute’ or else ‘relative’ compared to some tacit standard—e.g., the number of presently existing humans. But if “not many soldiers exist” *is* true because the number is small relative to (say) the number of humans, then this proposition would be more revealingly expressed by “Not many (of the) humans (that there are) are soldiers”.

14. This paper is an extension and revision of an earlier paper of the same title, parts of which were read to the Linguistic Society of America Summer Meetings, University of Michigan, Ann Arbor, August 5, 1973.

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