

Intermediate Quantifiers Versus Percentages

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Abstract In his 1986 paper (*Notre Dame Journal of Formal Logic*) Thompson offers rules for determining validity and invalidity of so-called “statistical syllogisms” (syllogisms with percentages replacing the traditional quantities of universal and particular) which are both *unsound* and *incomplete*. As a result, his claim that the genuine 5-quantity syllogistic (the traditional syllogistic with the three “intermediate” quantities added, expressible by “few”, “many”, and “most”) is included in his system is trivial, *if* true at all. It turns out not to be even true, as revealed by detailed examination of distribution, Thompson’s rules, and his claims for equivalences.

In his “Syllogisms with Statistical Quantifiers” [12], Thompson has used and abused our unpublished manuscript “The Compleat Syllogistic: Including Rules and Venn Diagrams for Five Quantities”. He used it as a basis for developing his own system of so-called “statistical syllogisms” and he abused it through insufficient acknowledgement, by misrepresenting our extension of the classical syllogistic, and by claiming that our extension was included in his inadequate system. Our aim herein is *not* to present and discuss our results, for that is what is done in our manuscript.¹ Rather, our aim is to expose the inadequacy of Thompson’s allegedly syllogistic system for evaluating certain arguments—those arguments that resemble genuine syllogisms but which contain percentages instead of ordinary quantifiers. In Section 1, we demonstrate the unsoundness and incompleteness of Thompson’s system and give rejoinders to likely replies by Thompson. In Section 2, we present important simplifications of Thompson’s notations and rules, discuss distribution, and reveal further difficulties. In Section 3, we give the underlying reasons for Thompson’s basic misapprehensions of the relation between his “statistical” quantifiers—viz., percentages—and genuinely intermediate quantifiers.

1 Thompson offers some rules for determining the validity of syllogistic-like argument forms—forms which are just like Aristotelian syllogisms with regard

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- (3) The syllogism is therefore valid for any settings in which $\sigma \leq 50$. It is difficult to imagine any occasions, even in ordinary unscientific arguments, where the value of σ would go much over 5, σ greater than 50 is virtually inconceivable . . . (p. 101).

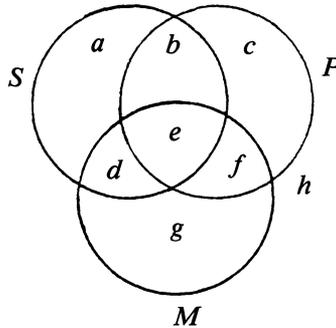
But is it inconceivable? Thompson has just explained what the conditions are for his rules to make this syllogism *fail* to preserve truth from premises-to-conclusion. But if there are such conditions, then *by the definition* of “validity”, the form itself is *invalid*. Validity requires that there *be* no such circumstances or interpretations. Thompson has just given them for this syllogism. Why is this a defect? Because this syllogism is *provably* valid (truth-preserving). So, Thompson’s rules call a form invalid (though he *disguises* this by saying it is ambiguously valid), even though it *is* valid. So, Thompson’s rules are *incomplete*.

Here is the proof of the syllogism’s validity (very similar to the one in [6], at the bottom of p. 358):

- (4) *Proof*: BPO-III, proved by *reductio ad absurdum* (using the notations for sub-classes in (5) below)

- | | | |
|----|----------------------------|--|
| 1. | Few M are P | $d + g \gg e + f$. . . premise |
| 2. | Few M are not S : | $d + e \gg g + f$. . . premise |
| 3. | -(Some S are not- P): | $a = 0$ and $d = 0$. . . denial of conclusion |
| 4. | | $g \gg e + f$ from steps 1 and 3 |
| 5. | | $e \gg g + f$ from steps 2 and 3 |
| 6. | | $g \gg e$ from step 4 |
| 7. | contradiction | $e \gg g$ from step 5 |

- (5)

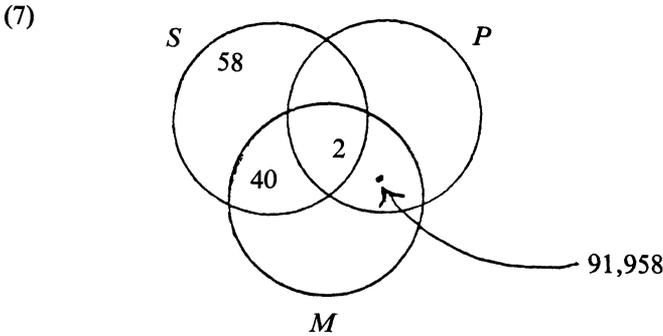


Now concerning soundness—i.e., whether every form which Thompson’s rules deem valid really *is* truth-preserving-premises-to-conclusion on every interpretation—consider the “syllogism” that Thompson gives on the top of p. 102:

- (6) Many more than 97% of M are P
 so, $\frac{42\% \text{ of } S \text{ are } M}{42\% \text{ of } S \text{ are } P}$

Again this is a case that Thompson says is “ambiguously valid”. With the previous example, we concluded that an ambiguously valid syllogism was actually *invalid* according to Thompson (since what he *said* about truth-conditions *forced* that conclusion). Here we conclude the *opposite*! We make this conclusion be-

cause Thompson brings in *another* peculiar notion (after ambiguous validity) to *explain* the alleged validity. He says that it is valid, but *not sound* (top, p. 102). Applying soundness to forms rather than to instances is *very* peculiar.⁵ Nevertheless, we take it that Thompson expresses his very strong desire that this syllogism be regarded as valid by calling it “valid but not sound”. The form, however, is *flatly invalid*. It is invalid because there are counterexamples—cases where the conclusion is false when the premises are both true. One of these is the case wherein there are 58 *S*s which are non-*P* and non-*M*, 40 *S*s that are *M* but non-*P*, 2 *S*s that are *M* and *P*, and 91,958 *P*s that are *M* and non-*S*. A Venn diagram helps to clarify the example



In this case, 91,960 out of 92,000 *M* are *P*; i.e., nearly 100% (i.e., 99.9%) of the *M*s are *P* (so the first premise is true, *if* it makes sense at all). The second premise is true because 42 out of 100 *S* are *M*. However, the conclusion is false, since only 2 out of 100 *S* are *P*. So, *very* many fewer than 42% of the *S* are *P*. A syllogism cannot be valid (or “ambiguously valid” or “valid but not sound”), *if* there can be circumstances in which the premises could be true and the conclusion false. So there is at least one syllogistic form Thompson claims is valid, which is provably invalid. Thus, Thompson’s rules are provably *unsound*.

Thompson’s rules for so-called statistical syllogisms are neither sound nor complete. Nothing Thompson claims about the “inclusion” of the 5-quantity syllogistic (from our manuscript or from [6]) within his system is other than trivial. We have proved the 5-quantity syllogistic to be both sound and complete. To embed it in any such entirely *inadequate* system is of no interest. (Assuming that it really *is* so embedded. We raise strong doubts below in Section 3.)

Now there are some obvious replies that Thompson might make. He might say that the trouble is with “ambiguously valid”. He can admit that it should be dropped (since it permits *inter alia* the opposite interpretations we have just given, one valid, the other invalid). Further, Thompson could say, the first kind of case—wherein values for σ s are involved—must be revised so that there is no possibility of interpreting such a form as invalid (so that no one can think, as we did, that the form is really taken to be invalid). That is, limit the value of σ appropriately. Then with the appropriate σ ’s all such forms are valid (as the proof of BPO-III in [4] confirms for one). Secondly, “valid but not sound” should also be dropped, Thompson might say, and this sort of case should simply be called *invalid*, not only because of counterexamples like that just given, but because the motivations for calling it “unsound” *are* reasons (when properly understood)

for calling it *invalid*. So, here an “ambiguously valid” form *is* invalid—whereas for the first “ambiguous validity” it is valid (when appropriate limits to σ are added).

Here is our rejoinder. First, such replies are unpromising to start with because they are *ad hoc*. But beyond that, they consist of terminological adjustments and promises. Will such adjustments work? Thompson could say he will adjust terms so as to call all valid forms “valid” and all invalid ones “invalid”, but will such adjustments be compatible with his rules? Won’t he have to adjust the rules too? For example, what will the limits on σ be? That Thompson could be *successful* in adjusting terms, rules, and constraints on rules is not a possibility we would bet on. For we predict that any merely revised system will be susceptible to new proofs of its inadequacy (incompleteness and unsoundness) analogous to those we have just given. What Thompson must do is present a proof of the adequacy of such a system (that it *is* sound and complete). (Concerning consistency, see Section 4.2 of our manuscript.) At the very least, he must *argue* for its adequacy.

2 And now for something completely different—distribution. Thompson (cf. [12]) developed his rules by basing them on the facts about distribution (what it *is* in Aristotle’s system and how to extend it to syllogisms with intermediate quantifiers) discovered by Carnes. (Thompson has not sufficiently acknowledged this.) Although these rules have been referred to in the publications cited in note 1 (and explicitly stated in the oral presentations, e.g., in Peterson [3] *before* Thompson [11] was in print), they have *not* appeared in print before. Here are Carnes’ rules for the 5-quantity syllogistic:

- (8) Rules of Distribution: R1. In a valid syllogism, the sum of the distribution indices (DIs) for the middle term must exceed 5.
- R2. No term may be more nearly distributed in the conclusion than it is in the premises (i.e., no term may bear a higher *DI* in the conclusion than it bears in the premises).
- Rules of Quality: R3. At least one premise must be affirmative.
- R4. The conclusion is negative if and only if one of the premises is negative.
- Rules of Quantity: R6. At least one premise must have a quantity of majority (*T* or *D*) or higher.
- R7. If any premise is nonuniversal, then the conclusion must have a quantity that is less than or equal to that premise.

(where $DI = 5$, for subjects of universals and predicates of negatives
 $DI = 4$, for subjects of predominants (*P* and *B* forms)
 $DI = 3$, for subjects of majorities (*T* and *D* forms)
 $DI = 2$, for subjects of commons (*K* and *G* forms)
 $DI = 1$, for subjects of particulars and predicates of affirmatives;
 cf. Section 2, “Rules” of our ms. for a complete discussion of these rules.)

As with the traditional system, these Rules of Quantity are dispensable (R6 and R7 being derivable from R1–R4). Notice that the Rules of Quality are simply the same as the traditional rules. So, the basic difference between these rules and the traditional ones is R1 and R2, concerning *distribution*.

To discuss Thompson’s rules it is *very* helpful to introduce some simplifications. First, Thompson really has two *basic* quantifier forms (9), and he defines the three others in terms of them (10).

- (9) $n\%$ of S ; Almost $n\%$ of S .
 (10) “More than $n\%$ of S are P ” =_{def.} “It is not the case that $(100 - n)\%$ of S are not P ”
 “Many more than $n\%$ of S are P ” =_{def.} “It is not the case that almost $(100 - n)\%$ of S are not P ”
 “Less than $n\%$ of S are P ” =_{def.} “More than $(100 - n)\%$ of S are not P ”.

Thompson assigns distribution indices which are analogous to those assigned in (8) above. (He got the idea from our manuscript, for notice that in his original publication on intermediate syllogisms, [11], he only used the classical concept of distribution—which turned out to be demonstrably inadequate.) However, his assignments are complicated (varying on two dimensions, represented by “ n ” and “ σ ”, with an infinitesimal “ ι ” thrown in). Here is a major simplification. Let his quantifiers other than the simple “ $n\%$ of S ” be defined (or interpreted) as follows (where “ ι ” is Thompson’s infinitesimal and “ σ ” is his so-called significance level):

- (11) “Almost $n\%$ of S are P ” =_{def.} “ $(n + (\iota - \sigma))\%$ of S are P ”
 “More than $n\%$ of S are P ” =_{def.} “ $(n + \iota)\%$ of S are P ”
 “Many more than $n\%$ of S are P ” =_{def.} “ $(n + \sigma)\%$ of S are P ”
 “Less than $n\%$ of S are P ” =_{def.} “ $(n - \iota)\%$ of S are P ”.

“ ι ” for Thompson represents an infinitesimal amount, so that $(0 + \iota)\%$ of something is enough (though just infinitesimally enough) to *be* some of it.⁶ Thompson’s significance level, represented by “ σ ”, is some amount which must be specified for each instantiation of an argument form. Evidently this will not be the same in every case, but will vary (a *very* troublesome idea as it turns out). For example, how much *more* than 50% is required for it to be true that “*Many* (or *much*) more than 50% of S are P ”? Similarly, how much *less* than 50% can it be that is permitted for “*Almost* 50% of S are P ” to be true? These required amounts are more or less represented by Thompson’s “ σ ”.

The advantage of these definitions or interpretations (exactly equivalent to Thompson’s notations) is that *five* of Thompson’s distribution rules (2a–e, p. 97) can be reduced to one—viz.:

- (12) Any (percentage) categorical statement distributes its subject to a degree *equal* to that of its quantifier. (Universal distribution is, therefore, 100 and particular distribution is $0 + \iota$.)

In other words, the distribution indices for Thompson's quantifiers are read off the definitions in (11). Thompson's rules for testing validity (R1–R3 concerning distribution) remain the same as he states them, but are now much easier to use.

Consider the example he gives on p. 100:

- (13)
$$\frac{\text{Almost } 27\% \text{ of } M \text{ are not } P}{\text{Many more than } 73\% \text{ } M \text{ are } S}$$
so, $\text{Some } S \text{ are not } P.$

Applying the definitions in (11), produces this interpretation of (13):

- (14)
$$\frac{(27 + (\iota - \sigma))\% \text{ of } M \text{ are not } P}{(73 + \sigma)\% \text{ of } M \text{ are } S}$$
so, $(0 + \iota)\% \text{ of } S \text{ are not } P.$

Notice that the sum of the Distribution Indices (*DI*s) of the middle term is $(27 + (\iota - \sigma)) + (73 + \sigma) = 100 + \iota$. This satisfies Thompson's first validity rule, that the middle term must be *more* than maximally distributed in a valid syllogism (a central idea in his system, borrowed from Carnes' rules stated above). Also, the minor term is equal in distribution to the minor term's distribution in the conclusion (same *DI* in both), satisfying Thompson's second validity rule. And the major term is equal in distribution to the major term's distribution in the conclusion (same *DI* in both), satisfying Thompson's third validity rule.⁷ (And his Rule 4 is satisfied.) This example illustrates the advantage of the simplified notation. For there is no longer any need to carry subscripts on *DI*s as Thompson does (this is eliminated via (11) above), and there is no need to add, subtract, and compare them (as on p. 97).⁸

Although this example seems to work smoothly, there is a difficulty with it which Thompson has not noted. When we added the *DI*s of *M* of (14) we got " $27 + (\iota - \sigma) + 73 + \sigma$ ", which Thompson sums to " $100 + \iota$ ". But this result assumes that the value of " σ " is the *same* in both occurrences. Remember that " σ " is a variable. Why should one assume that the amount of slack needed to be *almost* 27% is the same slack which, when added to 73% produces *many* more than 73%? Assuming the two values to be the same causes no problems (apparently) for *this* syllogism. For the σ needed to produce many more than 73% is (apparently) *greater* than the σ by which one can fall short of 27% and still have *almost* 27%. In this case, the combined *DI*s of *M* would then *exceed* $100 + \iota$, so that Rule 1 would be met. However, evaluations with " σ " will not always work so smoothly.

For example, how much more is required to have *many* more than 29% of a class? How much less than 70% still counts as *almost* 70%? Someone can plausibly propose that the former is *larger* than the latter. Consider this syllogism:

- (15)
$$\frac{\text{Many more than } 29\% \text{ of } M \text{ are not } P \dots (29 + \sigma)\% \text{ of } M \text{ are not } P}{\text{Almost } 70\% \text{ of } M \text{ are } S \dots (70 + (\iota - \sigma))\% \text{ of } M \text{ are } S}$$
so, $\text{Some } S \text{ are not } P \dots (0 + \iota)\% \text{ of } S \text{ are not } P.$

If we combine the *DI*s of *M* *without regard* for the possible difference in values of σ , we will obtain " $29 + \sigma + 70 + (\iota - \sigma) = 99 + \iota$ "—which clearly violates Rule 1. Yet, if we take account of the fact that the value of the first σ is *larger* (by, say, at least 2%) than that of the second one, we exceed maximal distribu-

tion for *M*. And, then, the form is valid. So, is this form *really* valid? Or really invalid? Intuitively, it seems valid. But without specific instructions that the first “ σ ” is larger by at least one than the second “ σ ”, the argument will be invalid by Thompson’s rules.

Consider the immediate entailment Thompson offers on p. 97 – viz.,

$$(16) \quad \frac{47\% \text{ of } S \text{ are } P}{\text{so, Almost } 50\% \text{ of } S \text{ are } P \dots (50 + (\iota - \sigma))\% \text{ of } S \text{ are } P.}$$

The *DI* of *S* in the premise is 47%, while the *DI* of *S* in the conclusion is $50 + (\iota - \sigma) = 50 - (\sigma - \iota)$. For values of σ greater than or equal to 3, the *DI* of *S* in the premise is greater than or equal to the *DI* of *S* in the conclusion, and then the inference is valid (according to Thompson). But whether to count percentages *within* 3% of *n* as almost *n*% is the question here. If it is plausible to entertain *only* values of less than 3 (for σ) here, then this inference is invalid. Well, maybe restricting σ to less than 3 here would be unreasonable. But now consider a similar inference:

$$(17) \quad \frac{\text{Many more than } 42\% \text{ of } S \text{ are } P \dots (42 + \sigma)\% \text{ of } S \text{ are } P}{\text{Almost } 50\% \text{ of } S \text{ are } P \dots (50 + (\iota - \sigma))\% \text{ of } S \text{ are } P.}$$

Someone might propose a value of 5 for σ in the premise, but 2 or less for σ in the conclusion. (Would that be very implausible?) If so, then *S* has a *DI* of 47 in the premise and of $48 + \iota$ in the conclusion – which means the inference is invalid.

What these examples show is that Thompson’s significance levels can vary in ways he did not adequately discuss and that validity and invalidity turn on them crucially (which was what we took advantage of in Section 1 above in arguing the total inadequacy of Thompson’s system). The imprecision introduced with significance levels makes a mockery of the interesting and fruitful extension of traditional distribution introduced in Carnes’ validity rules in (8) above. Thompson’s squares of opposition (on pp. 98–99) derive their absurd appearances from the *un*Aristotelian interactions of Thompson’s values for *n* and σ (where some *I*-forms entail corresponding *A*-forms, a travesty on the traditional labelings and the cautious extensions *we* introduced). According to his footnote 6, under certain conditions statements of the form “*Many* more than *n*% of *S* are *P*” can even coincide (truth-functionally) with statements of the form “*n*% of *S* are *P*”!

3 In order to approach the most basic feature which undermines Thompson’s system, consider again Thompson’s example (from p. 102) already presented above in (6). Here it is in our simplification of Thompson’s notation:

$$(18) \quad \frac{\text{Many more than } 97\% \text{ of the } M \text{ are } P \dots (97 + \sigma)\% \text{ of } M \text{ are } P}{\frac{42\% \text{ of the } S \text{ are } M}{42\% \text{ of the } S \text{ are } P.}}$$

In this form, the relevant *DI*s can be read off the formulas. In Section 1, we showed that there was a counterexample to this form – given in (7) – and argued that this proves Thompson’s system to be unsound (since his rules count the form

as valid – “valid but not sound”, Thompson said). But there are further difficulties. In order to make the first premise of (6) (= (18)) *true*, we had to assume that 99.9% of the *S* were *enough* more than 97% (viz., 2.9 more) to be considered to be *Many* more than 97%. (Grammatically speaking, it would appear to be more correct to *say* here “*much* more”, rather than “many more”.) However, *if* that is true, then the form must come out clearly *invalid* by Thompson’s own rules. For in this case, it would make the value of σ for the first premise 2.9. And any value *less* than 3 will make the form invalid by violating Thompson’s Rule 1 – viz., that the middle term must be *more* than maximally distributed. For since the predicate of any affirmative statement has the *DI* of $0 + \iota$ (Thompson’s 1(a) on p. 97), the combined *DI* for *M* is $97 + 2.9 + 0 + \iota$, which is *less* than 100. Thus, (18) would be *invalid* for Thompson, even though he called it valid but unsound.

Someone might propose that the trouble is simply with the first premise of (18), the problem being that 97% is so close to 100% that *much* more than 97% (say, a *whole lot* more) cannot exist. For the proportion required to be *much* more than *any* percentage (dictated by the connotation of “much more” in English and its cognates in other languages) forces the total percentage to exceed 100%. And in that case, the premise cannot be true at all, since there cannot be more than 100% of anything.⁹ So, under some interpretations or understandings, the first premise cannot *ever* be true. Thus, it is *akin* to a contradictory premise (false in every possible world, situation, or circumstance), which *may* be why Thompson advanced the infelicity of “valid, but not sound”.

Pondering the troubles with (6) (= (18)) – that *if* its first premise can be true at all then it is invalid via counterexample (and Thompson’s rules would say it is invalid) *and* that *if* its first premise cannot be true at all (as just suggested) then it is (trivially) valid – brings us back to exactly *how* Thompson claims our system is “included” in his. ALL that he said on the topic (without discussion) is contained in the equivalences he asserts on the bottom half of p. 95. Consider just one of them:

(19) G: “Many *S* are not *P*” is equivalent to “Many more than 0% of *S* are not *P*”.

It is entirely *through* such equivalences that Thompson claims that our 5-quantity syllogistic is “included” in his system; i.e., setting *n* at zero is evidently supposed to produce a subsystem equivalent to the 5-quantity syllogistic. This implication is all that Thompson offers on the subject. If it is true, it is trivial (as we argued in Section 1 above). *But* is it *even true*? If (19) is false, then his claim cannot be true (whether trivial or not). (19) *is* false, for reasons that begin to show in analyzing (6) via (18).

Keeping in mind the perplexities with “Many more than 97%”, consider “Many more than 0%”. How much *is* that? Recall from Peterson ([2] pp. 166–167, an article cited by Thompson), the example “Many soldiers are not abroad”. Is this statement equivalent to “Many more than 0% of the soldiers are not abroad” and/or to “Many more than no soldiers are abroad” (since 0% is none)? We find it very hard to tell. Evidently, Thompson thinks it is obvious. To show what the problem is, consider the conditions previously discussed (in [2]): 1 million soldiers, 100,000 at home and 900,000 abroad. Is 100,000 soldiers *many*

more than 0% of them? Well, yes, but so, as a matter of fact, is 16 soldiers. Don't we have to ask "compared to *what*?"? Even though 100,000 soldiers is a lot of soldiers (compared to all I have ever seen), it is *not* so many compared to all that there are, for it is *only* one-tenth of them. So, maybe "many soldiers are not abroad" is *false* (relative to all the soldiers there are). Do the same thing for "Many more than 0% of the soldiers are not abroad" or "Many more than no soldiers are not abroad" and our intuitions desert us. Here is a possible aid. Replace 0% or none with a *very* small percentage that is very close to 0% or none. Consider "Many more than 2% of the soldiers are not abroad" or even "Many more than 1/10th of a percent of the soldiers are not abroad". Many more than some very small percentage ought to mean *almost* [*sic*] the same thing—have almost the same truth conditions—as many more than 0% (or none). But now notice that it is does *not*—at least not in this case. 100,000 is a whole lot more than 2% of the soldiers (even more than 1/10th of a percent of them). So, "Many more than 2% of the soldiers are not abroad" would be *true* under exactly the same conditions as "Many soldiers are not abroad" (when 10% are since 100,000 are not abroad) is *false*. So, if "many more than 2%" is *very* similar to "many more than 0%", the latter *cannot* be equivalent to simply "many". This begins to show what is the matter with Thompson's equivalences. Speaking of many more percentage points (what Thompson's forms express, in effect) is quite a bit different from speaking of many soldiers (relative to some standard—such as all that there are, or all you have ever seen). In fact, this is the fundamental error at the bottom of Thompson's system.

Many soldiers is a proportion of *soldiers*; many more than a certain *proportion* of soldiers is not. It is a proportion of a proportion. In sum, Thompson's use of genuinely *intermediate* quantifiers is an application of them to proportions themselves. "Few", "many", and "most" are applied by Thompson to fractions or proportions of a number of items (or quantity of stuff), not to the items or stuff itself. Thompson has *iterated* quantifiers. "Many more than 50%" is *three* quantifiers iterated, "50%" preceded by "more than", and that preceded by "many". (And they are *not* iterated in the simplest way—as in "All of 30% of half of S".) Our 5-quantity syllogistic—the 5-quantity syllogistic which we discovered by extending Aristotle's one step—is the *basic* account of intermediate quantifiers. If Thompson's system shows anything, it shows that it is not easy (and perhaps not even possible) to give an account of the basic intermediate quantifiers by considering them to be limiting cases of *iterated* quantifiers. For in no clear or reasonable sense is "many more than 0%" identical in meaning or logical function to simply "many".

We have shown the connecting point between genuinely intermediate quantifiers—"few", "many", and "most"—and fraction-like quantifiers ("fractionals") to be the relation between "most" (in the *generic* sense distinct in meaning and logic from "almost-all") and "half". "Most S are P" means exactly what "More than half of the S are P" does (or, in Thompson's percentages, "More than 50% of the S are P"). Thus, "Half (or 50%) of the S are P" is exactly equivalent to (entailing and entailed by) "Its false that more than half the S are not-P"—where *each* such quantified statement is interpreted "liberally" to carry the *tacit* proviso expressed by appending "or more" (cf. p. 157 of [2] and p. 351

of [6]). Fractional quantifiers—which are *not* just ordinary fractions or percentages but a carefully defined kind of fraction-like quantifier—can be completely integrated with the 5-quantity syllogistic quantifiers, as has been detailed in [6] (based on Section 5.2 of our manuscript). Within this integration, however, it is necessary to *identify* “many” and “almost-all” quantifiers with specific fractionals (cf. [6] p. 355–356).

As far as we can tell, Thompson’s approach to percentage quantifiers (which he fancifully deems “percentiles” and “statistical quantifiers”) will never work *because* of his introduction of the variable “ σ ”. Thus, the only *genuine* innovation offered by Thompson simply doesn’t work. We have already shown (in Section 5.4 of our manuscript) that a system of percentage or proportional quantifiers like that introduced by Finch [1] can be accounted for by an *extension* of our algebraic method for higher-quantity (i.e., fractional quantifier) syllogisms. Perhaps, someday someone can carry out an adequate account of *iterations* of intermediate, fractional, and simple proportional (Finch-type) quantifiers.¹⁰ The chances that Thompson’s approach to it will succeed, however, are almost zero [*sic*]—if not entirely nonexistent.

NOTES

1. We hope that someday the whole of this manuscript will be published. However, our results *have* been made public in orthodox ways. Cf. [2]–[9]. Thompson appears to be *unfamiliar* with all of these except [2], relying on the earliest version of our unpublished manuscript.
2. Some typographical errors occurred in [6]. On pp. 358–359, the numeral “2” should be deleted in three proofs in which it occurs: (i) of EKG-2 in line (6), (ii) of PKI-3 in line (6), (iii) of AFK-I in line (5). Also, concerning the discussion on p. 355, considerations not introduced there will require that $i + j$ cannot be greater than 100. So, $i + j = 100$ (as explained in footnote 7 of our ms.).
3. Thompson says that his “system of categorical syllogisms includes both the complete Aristotelian system and the complete system of intermediate syllogisms developed by myself [5] and by Peterson and Carnes [3]” (p. 95). The implication that his system is complete, properly speaking, is refuted in his footnote to this passage wherein he admits that *his* system was *incomplete* because of the results of our ms. He should have said “whole” or “entire”, instead of “complete”.
4. The appropriate modification of the concept of validity for formal systems (in contemporary inquiries in logic)—viz., true on every interpretation (so that a formal system is deemed complete when every formula that is true on every interpretation is a theorem)—is truth-*preservation*-from-premises-to-conclusion. Then so-called “rules” of the syllogism are adequate *only* if every form that *is* truth-preserving-premises-to-conclusion-on-every-interpretation is deemed valid by the rules (completeness). Similarly, a set of syllogistic rules is adequate *only* if every form deemed valid by the rules really is truth-preserving on every interpretation (soundness). (This is pursued in detail for the 5-quantity syllogistic in Section 4 of our manuscript, “The Compleat Syllogistic”.)
5. We could have an *instance* of a valid form in an unsound argument—say, the form “All M are P , all S are M , so all S are P ” (Barbara), when S = singers, M = males,

and $P =$ poets. The instance is valid because the form is, but the premises are not true, but false. Thus, the instance is UNSound. note, however, that the form itself is *not* thereby unsound. It is not clear what sense could be attached to a *form* itself being unsound—other, that is, than the system or rules which selected the form as valid, being unsound. Now Thompson might reply that a *form* could have contradictory premises. And then no instance could be sound, since contradictory premises cannot be true. Okay, but the usual appellation for this kind of case is not that it is valid and unsound, but rather that it is *trivially* valid (not violating truth-preservation-premises-to-conclusion because the premises cannot ever be true).

6. There are inherent difficulties with Thompson's treatment of particular categoricals, "Some S are (not) P " forms. Since he adopts a framework of percentages for treating categorical quantity (a framework far short of being sophisticated enough to be deemed "statistical", as in his title), the universal quantity must be identified with 100%. But this forces infinitesimals upon Thompson, since there seems to be no smallest percentage greater than zero which is insufficient to make it true that "Some S is P ", where "some" is interpreted to be *anything* greater than zero. However, in any particular case it will not come out that way. In fact, for any finite total number of (countable) S s, "some S s" will always be considerably more than an infinitesimal—viz., a definite (whole) number of them (viz., at least one). .05% of 2000 men is still one whole man, but .05% of 200 men is none. (An infinitesimal percentage of any finite number of men is also none.) The "at least one" characteristic is lost on Thompson's approach. Indeed, it would appear that Thompson's thinking here would apply better when the subject terms are mass terms like "water", "earth", "salt", or (abstractly) "humility". (The last example is best, for it may well be that the amount of humility needed—and all that is actually found—to make the statement "Some humility is found among politicians" true is merely an infinitesimal amount.) For at least the possibility of infinite divisibility (and so an infinitesimal amount that does count as more than none) applies to their referents. (Or *seems* to apply. For, of course, at a certain microscopic point, subdividing some water ends in destroying it—e.g., reaching single water molecules which, of course, individually have none of the properties of water as a kind of stuff. An individual water molecule is not wet and is not a quantity of stuff at all, but a countable thing.)
7. A further, less important, simplification can be introduced with Thompson's second and third rules (p. 99). Collapse them to one—viz., each nonmiddle term in the premises must be distributed to at least the same degree as it is in the conclusion (i.e., have the same *DI* in the premises that it does in the conclusion, or a greater one). And, of course, Thompson's fourth rule is simply the classical second rule of quality, R4 above in (8).
8. Some typographical errors in Thompson's paper: the formula on the third line from the bottom of p. 97 should read " $(47 - 50) = -3 > - (5 - \iota) = (\iota - 5) - 0$ ". The last line on p. 99 should have " P " where there is an " M ".
9. Caution is advisable here *vis à vis* English. For even though you cannot have a portion, or even all, of a quantity of stuff or number of things which exceeds 100% of it, you can, of course, add to some quantity or number so that the result *increases* the quantity or number by, say 150%. Percentage *increases* are not, however, what is at issue in these examples.
10. But see Peterson's "Complexly Fractionated Syllogistic Quantifiers" forthcoming in *Journal of Philosophical Logic*.

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