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Why is Conjunctive Simplification Invalid?

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Abstract Connexive logic accepts as tautologous the principle that no statement may be directly inferred from its own denial. This principle is logically inconsistent with the principle of Conjunctive Simplification, that from 'p and q' we may infer 'p'. Connexive logicians generally reject Conjunctive Simplification on the grounds that some substitution instances for 'q' might countermand the otherwise valid inference from 'p' to 'p'. Under the 'subtraction' theory of negation '$\sim p$' would be such a substitution instance, since according to the subtraction theory of negation, nothing follows from a contradiction. However, this paper argues that logicians need not necessarily adopt the subtraction theory of negation in order to find reasons to reject Conjunctive Simplification.

Introduction In any generation there are a certain number of logicians who find the so-called Paradoxes of Material Implication to be an intolerable embarrassment. As a result, there are, by now, quite a variety of formal logics that eliminate the so-called Paradoxes, in one way or another. Regrettably any attempt to reform the material conditional inevitably requires the rejection of one or another of the apparently desirable formulas of the truth-functional calculus. Relevance logic, for example, typically rejects Disjunctive Syllogism. Connexive logic, on the other hand, typically rejects Conjunctive Simplification, the principle that from 'p and q' we may infer 'p'.[1] In my opinion it is connexive logic which takes the correct approach. Conjunctive Simplification is an invalid formula which ought to be rejected.

Naturally, if we are going to advocate the rejection of such an apparently valid and undeniably useful formula, we must be able to give an intelligible account of our reasons. My purpose in this paper is to explain the connexivists' rejection of Conjunctive Simplification. In Routley et al. [6], the authors have argued that connexive logic rejects Conjunctive Simplification because connexivism is committed to a view of negation which they call subtraction negation. I shall argue that connexive logic is *not* in fact committed to subtraction nega-

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tion, but that even without the subtraction theory of negation, there are good reasons why Conjunctive Simplification should be rejected.

I shall explain connexive logic only enough to put the debate in context. My main interest here is in exploring the reasons that any logician—connexivist or otherwise—might give for rejecting the principle of Simplification. If one is a connexivist, Conjunctive Simplification *must* be rejected, since it is inconsistent with principles which a connexivist specifically wishes to include. But given the close formal connection between Conjunctive Simplification and the classical Paradox principles of C. I. Lewis, it is likely that any logician who wishes to exclude the Paradox principles might reasonably contemplate rejecting Conjunctive Simplification as well.

Connexive logic and conjunctive simplification The characteristic theorem of connexive logic is the principle which has come to be known as Aristotle's Thesis. Aristotle's Thesis denies that a statement may be inferred directly from its own denial. Thus Aristotle's Thesis may be written as

(1) $\sim(\sim p \rightarrow p)$

or as

(2) $\sim(p \rightarrow \sim p)$.

(1) is the one properly designated "Aristotle's Thesis," but (2) obviously follows from (1)—with a little help from Double Negation—and may be regarded as an alternative form of Aristotle's Thesis. (1) is known as Aristotle's Thesis because of a passage in the *Prior Analytics* in which Aristotle actually employs this formula in a proof.[2]

Any connexive logic—that is, any logic that accepts Aristotle's Thesis—must also reject Conjunctive Simplification. R. B. Angell [1] offers the following proof:

(3)	1.	$((p \cdot q) \rightarrow q)$	principle of Simplification
	2.	$((p \cdot \sim p) \rightarrow (\sim p \cdot p))$	principle of Commutation
	3.	$((\sim p \cdot p) \rightarrow p)$	1 $p/\sim p$ q/p
	4.	$((p \cdot \sim p) \rightarrow p)$	2, 3, Hypothetical Syllogism
	5.	$(\sim p \rightarrow \sim(p \cdot \sim p))$	4, Transposition
	6.	$((p \cdot \sim p) \rightarrow \sim p)$	1 $q/\sim p$
	7.	$((p \cdot \sim p) \rightarrow \sim(p \cdot \sim p))$	5, 6, Hypothetical Syllogism

But, of course,

(4)	1.	$\sim(p \rightarrow \sim p)$	Aristotle's Thesis
	2.	$\sim((p \cdot \sim p) \rightarrow \sim(p \cdot \sim p))$	1$p/(p \cdot \sim p)$.

Line 7 of (3) contradicts line 2 of (4). Thus a system cannot consistently accept both Conjunctive Simplification and Aristotle's Thesis, unless of course it rejects Commutation, Hypothetical Syllogism, or Transposition. Angell's proof also shows that the two formulas

(5) $((p \cdot \sim p) \rightarrow p)$
(6) $((p \cdot \sim p) \rightarrow \sim p)$

must be rejected for the sake of consistency. Indeed they appear to be among the substitution instances in virtue of which Conjunctive Simplification must be rejected.

Since, in connexive logic at least, Conjunctive Simplification must be rejected as incompatible with other highly desirable principles, let us consider the case against Conjunctive Simplification, to show that indeed it *should* be rejected. Of course, when I say that Conjunctive Simplification should be rejected, I don't mean that every *instance* of it should be rejected. No doubt it validly follows from "There are a book and a pencil on the table," that "There is a book on the table." I mean to argue only that Conjunctive Simplification does not have the status of a *tautology*. If even a single instance of Conjunctive Simplification can be found in which the conclusion fails to follow from the premises, then that will be sufficient to show that Conjunctive Simplification is not a valid *Principle*. In fact I shall show that there is a whole class of such instances.

Conjunctive simplification and premise enhancement According to some logicians, the crucial difference between deduction and induction is that the validity of a deduction cannot be countermanded by the addition of further premises, while the validity of an induction *can* be. That is, let $\{P_1, \ldots P_n\}$ be a set of premises for an argument, and let Q be the conclusion. Given that the argument is a valid deduction, the argument is presumed to remain valid even if we add P_{n+1} to the premise-set, no matter what P_{n+1} asserts. On the other hand, given that the argument is a valid induction, we cannot presume that the addition of P_{n+1} will be validity-preserving.

The reason that an induction cannot be presumed to remain valid in the face of an enhanced premise-set is that new premises may provide further information that would undercut the assumptions upon which the original inference was based. For example, suppose a Martian, recently landed on Earth, takes what it has good reason to believe is a fair and impartial sample of the first several hundred human beings that it meets, and argues in the following (valid) manner:

(7) 63% of humans so far encountered have pink hair.

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Therefore, roughly 63% of humans have pink hair.

But the validity of the argument is entirely undercut if we expand the premise-set to include the information that our Martian has landed in the middle of a punk rock concert.

It is sometimes held that deductive inference is nothing more than the limiting case of inductive inference. From such a perspective it would seem reasonable to suppose that a deductive inference might also be undercut by the addition of countervening premises. For example, the statement,

(8) If I drop this stone, then it will fall.

states the warrant for drawing an inference from "I drop this stone" to "It will fall". The law of gravity being what it is, this inference must be presumed to be deductive. Yet the premise-set can be enhanced in such a way that the inference becomes invalid. For example,

(9) If I drop this stone and gravity fails to function, it will fall.

Another even more clearly deductive example,

(10) If John is a bachelor, then John is unmarried.

with an enhanced premise-set can be made into,

(11) If John is a bachelor and all bachelors are married, then John is not married.

It is an admittedly valid principle of nearly any system of logic that any statement implies itself. Thus '$(p \rightarrow p)$' is a valid principle even of connexive logic. But if premise enhancement is permitted, then we may derive '$((p \cdot q) \rightarrow p)$'. We might reasonably object to the principle of Conjunctive Simplification if it could be shown that Conjunctive Simplification permits pernicious instances of premise enhancement, i.e. if it could be shown that 'q' is capable of asserting something that would countermand the otherwise valid inference from 'p' to 'p'.

Theories of negation Routley et al. [6] attribute the connexivist rejection of Conjunctive Simplification to a view of negation that is clearly at odds with the classical view. They label the classical view 'complementation' negation, while the view which is attributed to connexivism is labeled 'subtraction' negation.

According to the complementation view, an assertion says one part of everything that can be said, and its denial (in some sense) says everything else. Thus everything follows from a contradiction, because a contradiction says everything. It completely fills logical space.

According to subtraction negation, however, an assertion says one thing, and its denial withdraws it. A contradiction cancels itself out and leaves nothing behind. Therefore, nothing follows from a contradiction because a contradiction leaves logical space completely empty. Routley et al. describe subtraction negation using the following images.

> p as running up a flag [which states the condition given in p], $\sim p$ as running it down again; p as writing something on a board, $\sim p$ as rubbing it out again, or putting a line through it, cancelling it out; p as recording a message, $\sim p$ as erasing it; p as stating something, $\sim p$ as withdrawing it. ([6], p. 89)

The main idea is that '$\sim p$' does not really make a separate assertion, but merely retracts or withdraws 'p'.

The view that connexivism is committed to the subtraction theory of negation receives encouragement from Strawson [7], whose logical theory is consistent with weak connexivism. Strawson specifically endorses the subtraction account, and offers images similar to those offered by Routley et al.: assertion as walking somewhere, negation as walking back again; assertion as offering a gift, negation as taking it away or revoking the offer. Strawson concludes that a contradiction "cancels itself and leaves nothing" ([7], pp. 2–3).

In any case, Strawson aside, it is easy to see why Routley et al. attribute the subtraction view of negation to connexivism. Connexivism rejects the formula '$((p \cdot q) \rightarrow p)$' on the grounds that '$q$' might assert something that would countermand the inference from 'p' to 'p'. According to the subtraction view of negation, replacing 'q' with '$\sim p$' would countermand the inference by revoking 'p'. With nothing asserted in the premises, 'p' in the conclusion would naturally fail to follow.

Even so, I think Routley and his associates are wrong in claiming that connexivism is committed to the subtraction theory. Between the view that everything follows from a contradiction and the view that nothing follows from a contradiction there is still room for the view that some things follow from a contradiction while others do not. Let me label this middle-ground view of negation the 'reversal' theory of negation.

If the idea is to offer images, then consider something like this. Suppose that we have a card-table which represents logical space, and precisely enough cards to entirely fill the top of the table. Assertion is represented by laying a card on the table. Denial is represented differently by each of the theories of negation. In subtraction negation, denial is represented by picking up a card which had previously been put down. In complementation negation, denial is represented by not putting the card down in the first place. The effect of this is that anything not explicitly asserted is assumed to be denied. In reversal negation, denial is represented by laying a card down *upside down*. Contradiction is represented by tearing the card in half and laying it down half right-side up and half upside down.

I think that the reversal theory is obviously the common sense view. We normally do not think that everything follows from a contradiction. We do not think, for example, that a person who holds contradictory beliefs thereby holds all beliefs. But on the other hand we normally do not think that nothing follows from a contradiction. We normally expect that a contradiction is likely to lead, at the very least, to further contradictions.

If connexivism were indeed committed to the subtraction theory, then I would be among those willing to reject connexivism. The subtraction theory strikes me as a rather bizarre parody of the ordinary notion of negation, though it is no worse, and indeed probably better, than the complementation theory. But, as we shall see, certain things follow from a contradiction even in connexive logic, while certain other things do not. Thus in connexive logic, it is not the case that everything follows from a contradiction; but it is also not the case that nothing follows from a contradiction. Hence connexivism is *really* committed to the reversal theory of negation.

The rejection of simplification without the subtraction theory While the subtraction theory does provide a rationale for rejecting Conjunctive Simplification, it does not offer the only possible rationale. It does not even offer the rationale that best captures the spirit of the usual examples. Consider again the previous examples. "If I drop this stone and gravity fails to function, then it will fall" is an invalid inference, not because the additional premise contradicts or removes some of the force from the other premises, but because the additional premise disputes the validity of the very principle by which the conclusion is supposed to be drawn. The same is true of "If John is a bachelor and all bachelors are married, then John is not married". The claim "All bachelors are married" takes nothing away from the claim that John is a bachelor. But it does deny the presumed justification for inferring "John is not married" *from* "John is a bachelor."

Hence connexivists reject Conjunctive Simplification, not primarily because they fear that '$\sim p$' will be substituted for 'q', but because they fear that '$\sim(p \rightarrow p)$' will be. The formula that we must guard against, and by virtue of which connexivism rejects Conjunctive Simplification, is,

(12) $((p \cdot \sim(p \rightarrow p)) \rightarrow p)$

for this formula asserts that 'p' follows from 'p', even under the condition that it does not!

Why, then, should formulas (5) and (6) also be objectionable? Even without the subtraction theory of negation, it turns out that these are among the formulas that motivate the rejection of Conjunctive Simplification. James R. Bode [3] notes that from

(13) If I strike this match, it will light.

in classical logic, we get the following alarming result.

(14) If I strike this match and it doesn't light, it will light.

Connexive implication is stronger than material implication but, like material implication, cannot be true when its antecedent is true and its consequent false. That is, from '$(p \cdot \sim q)$' it follows that '$(p \rightarrow q)$' is false. What this shows is that when the premise-set (i.e., the antecedent) of an argument is enhanced with the denial of the conclusion (i.e., the denial of the consequent), the premises then generally imply the denial of the validity of the inference by which the conclusion was to be drawn.[3]

The problem with (5) and (6) is therefore not that the premises are contradictory, but that the premises include the denial of the conclusion. 'p' fails to follow from '$(p \cdot \sim p)$', not because of the subtraction theory of negation, but because '$(p \cdot \sim p)$' undermines the principle of inference by which the inference from 'p' to 'p' is supposed to follow. And this is true, not because '$(p \cdot \sim p)$' implies nothing at all (as the subtraction theory asserts), but rather because '$(p \cdot \sim p)$' implies '$\sim(p \rightarrow p)$'. I said earlier that certain things follow from a contradiction even in connexive logic. Specifically, what follows from a contradiction is the denial of the principle of identity, and possibly the denial of other basic principles of inference as well.

Routley and Montgomery [5] point out the curious fact that

(15) $(((p \rightarrow p) \cdot \sim(p \rightarrow p)) \rightarrow \sim(p \rightarrow p))$

is a thesis of even the weakest type of connexive system, while

(16) $(((p \rightarrow p) \cdot \sim(p \rightarrow p)) \rightarrow (p \rightarrow p))$

is provably excluded from any connexive system. So long as one thinks of these as nearly identical instances of the principle of Conjunctive Simplification, this fact does indeed appear anomalous. But observe that (15) is simply another case in which a contradiction leads to the denial of the principle of identity. (16), on the other hand, says that the very same contradiction can simultaneously lead to an affirmation of the principle of identity. Given this understanding of the two formulas, the anomaly vanishes. It makes perfect sense that (15) be included but (16) excluded from a connexive logic.

Naturally, when we add the denial of a principle of inference to the premises of an argument which is supposed to follow by that very principle, we render the argument invalid. But that is precisely what the principle of Conjunctive Simplification permits. Hence the principle of Conjunctive Simplification is invalid, even under the reversal theory of negation.

NOTES

1. For a discussion of the various types of nonstandard, non-truth-functional logics, see [6].
2. See Aristotle [2], ii 4.57b3. Discussion of this passage, and of Aristotle's acceptance of 'Aristotle's Thesis' may be found in Łukasiewicz [4], p. 50.
3. There is, however, one type of situation in which the denial of the conclusion appearing in the premises does not countermand the validity of the inference. That occurs when one of the other premises explicitly states the principle that a statement *may* be inferred from its own denial. For example,

$$((\sim p \cdot (\sim p \rightarrow p)) \rightarrow p)$$

is a valid formula of connexive logic. However, while it is *valid*, it is never *sound* since the second of the two premises contradicts a fundamental tautology of connexive logic, namely, Aristotle's Thesis!

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