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ON THE DIVISIBILITY OF THE CLASS NUMBER OF $Q(\sqrt{-pq})$ BY 16

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ABSTRACT. Let h denote the class number of the imaginary quadratic field of discriminant $d = -pq$, where p and q are primes of the form $4s + 1, 4t - 1$, respectively. According to P. Kaplan (J. Math. Soc. Japan **25** (1973), 596–608), h is divisible by 8 precisely when $-q$ is a biquadratic residue modulo p. Assuming that 8 divides h, the authors give a necessary and sufficient condition for the divisibility of h by 16, in terms of quadratic and biquadratic residuacity symbols related to Legendre's equation $px^2 + qy^2 - z^2 = 0$. If x, y, z are coprime positive integers satisfying this equation, with x odd, y even and $z = 4n + 1$, they show that h is divisible by 16 if, and only if, $(z/p)_4 = (2x/z)$. Conditional results on this problem, e.g., when one can take $x = 1$ above, were obtained by E. Brown (Houston J. Math. 7 (1981), 497–505). The corresponding problem for the discriminants $d = -p$ and $d = -2p$ was also treated by the authors (Canad. Math. Bull. **25** (1982), 200–206).

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