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CORRECTION TO "ON THE EXISTENCE OF A MINIMAL SUFFICIENT SUBFIELD"

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We are grateful to Professor Harald Luschgy for pointing out an inaccuracy in the above paper [10]. Theorem 3.1 of [10] is an incorrect translation of Theorem 2.2 of Krickeberg (1960). Theorem 3.1 of [10] becomes correct under the additional assumption that each subfield U_τ contains all μ-null sets (this follows from Theorem 6.1 of Hunt [11] and the null set assumption; see also the discussion by Luschgy [12]). The auxiliary Theorem 3.2 of [10], whose proof depended on the incorrect version of Theorem 3.1, is also incorrect; Example 3 of Burkholder (1961) gives a counterexample.

Fortunately the main result, Theorem 2.3 of [10], is correct and can be proved without appealing to Theorem 3.2. The second and third paragraphs of the proof of Theorem 2.3 in Section 4 of [10] should be replaced by the following three paragraphs. The notation used here is that used in [10].

Since each U_τ is sufficient for M, there exists a U_τ-measurable function h_τ in B such that h_τ is a version of $P[A \mid U_\tau]$ for each P in M. For fixed P, the collection of subfields $\{U_\tau \vee N_p\}$ is directed downward by inclusion, and the collection $\{(h_\tau, U_\tau \vee N_P)\}$ is a P-uniformly integrable martingale relative to P: if $U_\tau \subseteqq U_\rho$ then

$$\begin{aligned} E_P[h_\rho \mid U_\tau \vee N_P] &= E_P\{P[A \mid U_\rho] \mid U_\tau \vee N_P\} \\ &= P[A \mid U_\tau \vee N_P] \quad \text{a.e.} \quad [P] \\ &= h_\tau \quad \text{a.e.} \quad [P] \end{aligned}$$

by Lemma 4.8 of Bahadur (1954). Since $U_\tau \vee N_P \supseteqq N_P$ for each τ, the corrected statement of Theorem 3.1 of [10] implies that there exists a function f_P, measurable with respect to the subfield $\bigcap_\tau (U_\tau \vee N_P)$, such that $\lim_\tau \|h_\tau - f_P\|_P = 0$, where $\|\cdot\|_P$ denotes the $L_1(P)$-norm. Truncating if necessary, we take each f_P to satisfy $0 \leqq f_P \leqq 1$ on X, i.e., f_P is in B.

We now will apply Lemma 1.2 of Pitcher (1965) to show that the element (f_P) of $\prod B(P)$ is countably coherent. For any fixed $1 < r < \infty$ and P in M, let $W_r(P)$ denote the weakly topologized unit ball in $L_r(P)$, let $\prod_{P \text{ in } M} W_r(P)$ denote the Cartesian product space endowed with the Tychonoff (product) topology, and let $B_r = \bigcap_{P \text{ in } M} W_r(P)$, and let $i_r : B_r \to \prod W_r(P)$ be defined by $i_r(h) = (h, h, \cdots)$ (see the bottom of page 598 of Pitcher (1965)). For h in $L_s(P)$ (where $r^{-1} + s^{-1} = 1$) we have

$$\begin{aligned} |\textstyle\int h_\tau h \, dP - \int f_P h \, dP| &\leqq (\textstyle\int |h_\tau - f_P|^r \, dP)^{1/r} (\int |h|^s \, dP)^{1/s} \\ &\leqq \|h_\tau - f_P\|_P^{1/r} (\textstyle\int |h|^s \, dP)^{1/s} \end{aligned}$$

since $|h_\tau - f_P| \leqq 1$. Therefore the net $\{i_r(h_\tau)\}$ converges to (f_P) in the product topology. Since each h_τ is in $B \subseteq B_r$, (f_P) must be in the closure of $i_{\dot{r}}(B_r)$, so Lemma 1.2 of Pitcher (1965) implies that (f_P) is countably coherent.

By hypothesis (X, S, M) is coherent, so there exists a function f in B such that $f_P = f$ a.e. $[P]$ for each P in M. For every τ, f_P is $U_\tau \vee N_P$ measurable for all P in M, so f is measurable with respect to $\tilde{U}_\tau \equiv \bigcap_{P \text{ in } M} (U_\tau \vee N_P)$. However, U_τ is sufficient for M and contains N, so Lemma 3.3 of [10] implies that $\tilde{U}_\tau = U_\tau$. Thus f is measurable with respect to $\bigcap_\tau U_\tau \equiv U_{-\infty}$. For S in $U_{-\infty}$ and P in M,

$$\int_S f\, dP = \int_S f_P\, dP = \lim_\tau \int_S h_\tau\, dP = P(A \cap S)\,,$$

so f is a version of $P[A \mid U_{-\infty}]$ for each P in M. Since A was an arbitrary member of S, this shows that $U_{-\infty}$ is sufficient for M, so the proof is complete.

ADDITIONAL REFERENCES

[10] HASEGAWA, M. and PERLMAN, M. D. (1974). On the existence of a minimal sufficient subfield. *Ann. Statist.* **2** 1049–1055.

[11] HUNT, G. A. (1966). *Martingales et Processus de Markov*. Dunod, Paris.

[12] LUSCHGY, H. (1974). Anmerkung zu einem Konvergensatz für Martingale mit nach links filtrierender Indexmenge. *Z. Wahrscheinlichkeitstheorie und Verw. Gebiete* **30** 171–172.

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