

We assume that the  $x'_i$  are independently distributed and each  $x'_i$  is distributed according to the same law of distribution, whence we find that the characteristic function for the law of distribution of harmonic means of samples of  $n$  is

$$\phi(t) = \left\{ \int_0^\alpha \frac{k}{\sigma} e^{it|x| - \frac{|x|}{\sigma} x^2} dx \right\}^n, \quad (42)$$

from which, after simplification, we find that

$$\phi(t) = \frac{k^n 2^n \sigma^{2n}}{(1 - \sigma it)^{3n}}. \quad (43)$$

We now find that the law of distribution for  $u$  is

$$P(u) = \frac{2^n k^n \sigma^{2n}}{2\pi} \int_{-\alpha}^{\alpha} \frac{e^{-itu}}{(1 - \sigma it)^{3n}} dt,$$

which, after evaluation and simplification, becomes

$$P(u) = \frac{2^n k^n}{\sigma^n \Gamma(3n)} u^{3n-1} e^{-\frac{u}{\sigma}}. \quad (44)$$

Recalling that in this case,  $u = 1/|x_1| + 1/|x_2| + \cdots + 1/|x_n|$ , we make the transformation  $u = n/H$ , where  $H$  is the harmonic mean; whence, from (44), we find that the desired law of distribution of harmonic means of samples of  $n$  is given by

$$P(H) = \frac{2^n k^n n^{3n-1}}{\sigma^n \Gamma(3n)} H^{1-3n} e^{-\frac{n}{\sigma H}}. \quad (45)$$

**7. Conclusions.** We have shown that the same analysis is applicable to find the explicit expression for all the distribution laws we have discussed in this paper.

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#### ERRATA

In my paper\* there appear two blunders which were called to my attention by A. T. Craig.

In section 4, pages 107-108, headed "The distribution of variances and standard deviations," I have obtained the distribution function of the sum of the squares of  $n-1$  independent values of  $x$  and not the distribution function of the sum of the squares of the deviations from the sample mean of the  $n$  independent values of  $x$ .

In section 2, pages 104-105, headed "The distribution of differences," I have obtained the distribution function of the differences of *absolute values* and not the distribution function of the actual differences.

\* Weida, F. M., "On Certain Distribution Functions when the Law of the Universe is Poisson's First Law of Error," *Annals of Mathematical Statistics*, Vol. VI, No. 2, June, 1935, pp. 102-110.