

Using this result and inequality (2), which was established in Section 4, we have $m_1^2 < m_2 < 2m_1^2$, and the required result follows immediately on dividing by m_1^2 . We also note that $\lim_{h \rightarrow -\infty} \psi(h) = 1$, and $\lim_{h \rightarrow \infty} \psi(h) = 2$. Thus no narrower limits can be found. To obtain these limits, we use the result, $\lim_{h \rightarrow -\infty} Z/h = 0$, which follows from $\lim_{h \rightarrow -\infty} Z e^{h^2/2} = \left[\int_{-\infty}^{\infty} e^{-t^2/2} dt \right]^{-1} = (\sqrt{2\pi})^{-1}$. Thereby we have

$$\lim_{h \rightarrow -\infty} \psi(h) = \lim_{h \rightarrow -\infty} \frac{1/h^2 - Z/h + 1}{(Z/h - 1)^2} = \frac{0 - 0 + 1}{(0 - 1)^2} = 1,$$

and

$$\lim_{h \rightarrow \infty} \psi(h) = \lim_{h \rightarrow \infty} \frac{e^{\omega(h)}[1 - h(Z - h)]}{e^{\omega(h)}(Z - h)^2}$$

which is indeterminate of the form 0/0 as given. Using L'Hospital's rule and making certain obvious simplifications, we obtain

$$\lim_{h \rightarrow \infty} \psi(h) = \lim_{h \rightarrow \infty} \frac{-2}{Z(Z - h) - 2} = \frac{-2}{1 - 2} = 2.$$

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ABSTRACTS OF PAPERS

(Abstracts of papers presented at the Ithaca meeting of the Institute, March 18-20, 1954)

1. Confidence Region Procedures Based on the Logarithm of the Likelihood. CARL R. OHMAN, Princeton University.

Let $f(x, \theta_0)$ be a probability function where θ_0 is one of a set of permissible parameter points $\theta = (\theta_1, \dots, \theta_h)$ contained in some subspace of R_h . A sample (x_1, \dots, x_n) of size n is observed and a set of h functions, $\varphi_j = (1/\sqrt{n}) \sum_{i=1}^n k_{ij} L_i$, $j = 1, \dots, h < n$, computed, where $L_i = \partial \log f / \partial \theta_i$, $f = \prod_{i=1}^n f(x_i, \theta)$, and the k_{ij} are chosen so that $E(\varphi_j) = 0$, $E(\varphi_i \varphi_j) = \delta_{ij}$. For a given sample, the φ_j are functions of θ , and $(\varphi_1(\theta), \dots, \varphi_h(\theta))$ is a point in the pivotal space $\Phi \subseteq R_h$. If a region W can be constructed in Φ so that $\Pr\{(\varphi_1, \dots, \varphi_h) \in W\} = \alpha$ independently of θ_0 , the corresponding region in the parameter

space will be a 100α per cent confidence region for θ_0 . If the φ_j are normally distributed, the sphere $W = \{\sum \varphi_j^2 \leq \chi^2\}$ is suitable. Otherwise, an approximate pivotal region can be constructed using one of several modified Cornish-Fisher procedures. The details of one such procedure are given and several examples are discussed. (This procedure differs somewhat from that described by M. S. Bartlett, *Biometrika*, Vol. 40, (1953) pp. 12, 306.) The remainder of the paper discusses (i) the regularity conditions under which these procedures are valid, (ii) the large and small sample properties of the resulting regions, (iii) the possibility of improvement using higher order derivatives of f , and (iv) the problem of nuisance parameters.

2. A One-Sided Confidence Interval for an Unknown Distribution Function.

HERBERT ROBBINS, Columbia University.

Theorem. Let x_1, \dots, x_n be independent with common continuous c.d.f. $F(x)$, let $F_n(x)$ be the sample c.d.f. = (number of $x_i \leq x$)/ n , and let t be any constant between 0 and 1. Then $\Pr [F(x) \geq tF_n(x) \text{ for all } -\infty < x < \infty] = 1 - t$. *Proof.* By the usual transformation, $x'_i = F(x_i)$, the assertion need only be established when the x_i are uniformly distributed on $[0, 1]$. In that case we have $\Pr [F_n(x) \leq x/t \text{ for all } -\infty < x < \infty] = n! \int_0^1 \int_{(n-1)/n}^{x/n} \dots \int_{2/n}^{x/n} \int_{1/n}^{x/n} dx_1 \dots dx_n = n! [x^n/n! - tx^{n-1}/n!]_0^1 = 1 - t$.

3. The Mean Successive Difference in Samples from an Exponential Population.

P. G. MOORE, University College, London, and Princeton University.

A random sample of size n is drawn from the exponential population having probability density function $p(x) = \theta^{-1} \exp \{- (x - A)/\theta\}$ for $x \geq A$ and zero elsewhere. Let x_1, x_2, \dots, x_n be the n observations in their correct temporal order. The mean successive difference is defined as $d = \sum_{i=1}^{n-1} |x_i - x_{i+1}| / (n - 1)$. The first four moments of this expression are found in order to obtain approximate significance points for d . These may be used, if θ is known, to test the hypothesis of homogeneity in the original sequence of observations. The properties of d for the cases where A , or θ , is not constant from observation to observation but varies in some way are also investigated. Application of the foregoing suggests the use of the statistic $\Lambda = d/\bar{x}$ as a test for homogeneity which is independent of the population parameter θ in the case where A is known. In the final part of the paper, alternative statistics based on $|x_i - x_{i+r}|$ or $|\Delta^r x_i|$ for $r = 2, 3, \dots$ are discussed, and also the properties of the mean successive difference when the sampling is not from an exponential but the general χ^2 or Pearson Type III population.

4. Application of the Duality Theorem of Linear Programming to Testing Hypotheses. HOWARD RAIFFA, Columbia University.

Consider a finite sample space and finite parameter space. Let $\omega_0 = \{\omega_{01}, \omega_{02}, \dots, \omega_{0r}\}$ and $\omega_1 = \{\omega_{11}, \omega_{12}, \dots, \omega_{1s}\}$ be disjoint subsets of the parameter space. For any randomized test, ϕ , of ω_0 against ω_1 let $\alpha_i(\phi) = E(\phi | \omega_{0i})$ for $i = 1, 2, \dots, r$, and $\beta_j(\phi) = E(1 - \phi | \omega_{1j})$ for $j = 1, 2, \dots, s$. We apply the duality theorem of linear programming to find ϕ which, subject to the condition that $\alpha_i(\phi) \leq \alpha_0$ for $i = 1, 2, \dots, r$, is a minimizer of $\max_j \beta_j(\phi)$. The results lead naturally to the notion of least favorable a priori distributions over ω_0 and ω_1 ; the results are interpreted geometrically. The value, $\min_\phi \max_j \beta_j(\phi)$ has an interpretation as a generalized distance in $E^{(n)}$ from the origin to a displaced positive orthant. The distance is the shortest path constrained to follow a ray of a cone (which ray is associated with the notion of the least favorable case) and directions parallel to the axis. Existence results, approximation results based on gaps, and an algorithm for solving such problems are considered.

5. **Multiple Points of Paths of Brownian Motion in the Plane.** ARYEH DVORETZKY, Columbia University; P. ERDÖS, Notre Dame University; and S. KAKUTANI, Yale University.

The main result established here is that almost all two-dimensional (mathematical) Brownian motion paths in the plane possess multiple points of arbitrary high finite multiplicity. The method of proof is similar in part to that of a previous paper (*Acta Sci. Math. Szeged.*, T. 12 (1950), pp. 75–81). Combining the results of the two papers it is known that, with probability 1: Brownian paths in four-dimensional or higher space have no double points; Brownian paths in three-dimensional space have double points; Brownian paths in two-dimensional space have points of arbitrary high finite multiplicity. The problems of the existence of multiple (in particular triple) points in three-dimensional Brownian paths and of points of infinite multiplicity in two-dimensional paths have not been settled yet. Another unsolved problem is that of the existence of points of uncountable multiplicity in one-dimensional Brownian motion.

6. **On the Distribution of the Largest and Smallest Roots of a Matrix in Multivariate Analysis.** K. C. S. PILLAI, University of North Carolina and University of Travancore.

This paper presents in a more convenient and usable form than before the general expression for the exact c.d.f. of the largest root (from which that of the smallest can be easily derived) of certain sample ($p \times p$) matrices (positive definite or positive semi-definite with s non-null roots, $s \leq p$) arising in connection with different tests of hypotheses on p -variate normal populations. The exact c.d.f. is obtained for number of roots going up to eight (the expressions for $s = 6, 7$ and 8 being given for the first time). Approximations to the c.d.f. are given for number of roots up to five which are useful for computing percentage points (upper 5 per cent or less in the case of the largest root and lower 5 per cent or less in the case of the smallest root) for small integral values of one parameter connected with the sample. To illustrate the use of the approximations, exhaustive tables of upper 5 and 1 per cent points for the largest root, in the case of two roots, have been computed; the error of approximation has been shown to be negligible.

7. **A Problem in Two-Stage Decision Theory. (Preliminary Report.)** MORRIS SKIBINSKY, University of North Carolina.

Let D_m be the class of two-stage decision rules with first sample, X , of given size m , and second sample size which may depend on X . For a Bayes solution, this second sample size is given by the integral value of ν which minimizes a certain function, $G_\nu(X)$. This paper is concerned with the case of independent normal observations having unit variance. It is required to decide between means θ_0, θ_1 having arbitrary but fixed a priori probabilities (positive and adding to 1), where cost per observation is constant and the loss functions are simple. The nature of the second sample size function for a Bayes test in D_m is determined, and theorems which demonstrate its fundamental properties are proved. Results, leading to explicit formulas for this function, for the probabilities of wrong decisions, and for the expected size of the second sample, are obtained, by assuming the ratio, Z (= minimum wrong decision loss, over cost per observation) to be large. Comparisons, analytically for large Z , and tabularly for certain fixed values of the parameters, are made with analogous one-stage and sequential procedures, in terms of error probabilities and expected simple size.

8. Some Simple Sequential Tests and Estimates for Comparing Variances.

ALLAN BIRNBAUM, Columbia University.

Let $x = (x_1, x_2, \dots)$, $y = (y_1, y_2, \dots)$ be sequences of independent observations from normal distributions with means 0 and variances σ_x^2 for each x_i , σ_y^2 for y_i . Let $u = (u_1, u_2, \dots) = (x_1 + x_2, x_3 + x_4, \dots)$, $v = (v_1, v_2, \dots) = (y_1 + y_2, y_3 + y_4, \dots)$. Let $R_i = \sum_1^i u_j$, $S_i = \sum_1^i v_j$. Let $T = (T_1, T_2, \dots)$ be the sequence of all R_i 's and S_i 's in increasing order. Let $B = (b_1, b_2, \dots)$ where $b_i = 1$ if $T_i = \text{some } R_j$, $b_i = 0$ if $T_i = \text{some } S_j$. Then B is a sequence of independent Bernoulli trials with $p = \text{Pr}\{b_i = 1\} = (1 + \sigma_x^2/\sigma_y^2)^{-1}$. Sequential or nonsequential methods for tests and interval estimates for p give corresponding tests and estimates for σ_x^2/σ_y^2 . In certain variance-components experiments, each replication generates a set of n_x transformed observations x_i , n_y transformed observations y_i . Then by use of a guessed value of σ_x^2/σ_y^2 a modified procedure will tend to utilize x_i 's and y_i 's at the rate at which they are generated by successive replications. The method generalizes to give comparisons of 3 or more variances, with B a sequence of multinomial observations.

9. A Minimal Sequence of Statistics. R. R. BAHADUR, Columbia University

Let $x = (x_1, x_2, \dots)$ be a sequence of real valued random variables, and suppose that x is distributed according to some unknown one of a given set P of probability measures p . For each m , let $x_{(m)} = (x_1, x_2, \dots, x_m)$. It is assumed that for each m the set of possible distributions of $x_{(m)}$ is dominated. For each m let $y_m = T_m(x_{(m)})$ be a statistic on the sample space of $x_{(m)}$. The sequence $\{y_m\}$ is said to be sufficient if for each m , y_m is a sufficient statistic for P when the sample point is $x_{(m)}$; $\{y_m\}$ is said to be transitive if, for each m and each p in P , the conditional distribution of y_{m+1} given $x_{(m)}$ depends on the condition only through T_m . The author has shown elsewhere ("Sufficiency and statistical decision functions," *Ann. Math. Stat.*, (1954)) that sequences which are sufficient and transitive play an important role in the reduction of sequential decision problems. It is shown in this note that there exists a sufficient and transitive sequence $\{y_m^*\}$ such that, corresponding to any sequence $\{y_m\}$ which is also sufficient and transitive, there exists a sequence F_1, F_2, \dots of functions such that, except on a set which is of p -measure zero for each p in P , $y_m^* = F_m(y_m)$ for each m . The result has application to the problem of determining the maximum possible reduction of sequential decision problems by the principle of sufficiency.

10. Strong Convergence of Stochastic Approximation Methods of Robbins-Monro and Wolfowitz-Kiefer. M. N. GHOSH, University of North Carolina.

The Robbins-Monro scheme of stochastic approximation of the root θ of the regression equation $M(x) = \alpha$ has been shown to converge strongly to θ under the following conditions: 1) $\limsup |m(x) - \alpha|/(x - \theta) < k$ as $|x| \rightarrow \infty$, and $\int_{-\infty}^{\infty} [y - m(x)]^2 dH(y|x) \leq \sigma^2$; 2) $M(x) \leq \alpha - \varphi(\delta)$ for $x < \theta - \delta$, $M(x) \geq \alpha - \varphi(\delta)$ for $x > \theta + \delta$, where $\varphi(\delta) > 0$; and 3) $\sum a_n$ is divergent, $\sum a_n^2$ is convergent. The Wolfowitz-Kiefer process for estimating the maximum of the regression function has also been shown to be strongly convergent to θ , the point at which $M(x)$ is maximum, under nearly same assumptions as in K and W except that we do not need $M(x)$ to satisfy the Lipschitz condition, that is (2.8) in K and W .

11. An Ergodic Property of the Brownian Motion Process. (Preliminary Report.)

CYRUS DERMAN, Columbia University.

Let $X(t)$ $0 \leq t < \infty$ be a one-dimensional separable Brownian Motion Process. The following theorem is proved. If $f(x)$ and $g(x)$ are any real-valued Borel-measurable functions, summable in the line $-\infty < x < \infty$, then with probability one

$$\lim T \rightarrow \infty \int_0^T f(x(t)) dt / \int_0^T g(x(t)) dt = \bar{f} / \bar{g}$$

provided that $\bar{g} \neq 0$ where $\bar{f} = \int_{-\infty}^{\infty} f(x) dx$ and $\bar{g} = \int_{-\infty}^{\infty} g(x) dx$. This theorem is a probability one version of a theorem proved by G. Kallianpur and H. Robbins, "Ergodic property of the Brownian motion process," *Proc. Nat. Acad. Sci.*, Vol. 39 (1953), pp. 525-533. A method first used by Doebelin (*Bull. Soc. Math. France*, 1938) and later exploited more fully by Chung (to appear in the *Trans. Amer. Math. Soc.*) was used to prove the theorem.

12. On the Distribution of Hotelling's Generalized T Test. K. C. S. PILLAI
University of North Carolina and University of Travancore.

Let S^* and S be two independent sample covariance matrices belonging to two p -variate normal populations with m and n degrees of freedom respectively. Hotelling defines a measure of multivariate dispersion T_0^2 , given by $T_0^2/m = \text{trace } S^{-1}S^*$. According to this definition S and S^* are positive definite ($p \times p$) matrices. Assume that S^* can also be positive semi-definite with s nonnull characteristic roots, $s \leq p$, and denote by $U^{(s)}$ the trace of $(m/n)S^{-1}S^*$. Establishing certain recurrence relations between the moment generating functions of $U^{(s)}$ and $U^{(s-2)}$, the lower order moments of $U^{(s)}$ are obtained. These suggest an approximation to the p.d.f. of $U^{(s)}$ in the form of an F distribution, where $U^{(s)}/s$ is distributed as $\nu_1 F/\nu_2$ with $\nu_1 = s(2m' + s + 1)$ and $\nu_2 = 2(sn' + 1)$; m' and n' being functions of m , n and s . For $s = 1$, the approximate p.d.f. reduces to that of Hotelling's T and is exact. For $s = 2$, the accuracy of approximation has been discussed by comparison with the exact c.d.f. obtained by Hotelling. The approximate distribution guarantees sufficient accuracy for practical use.

13. Power and Sample Size for Small Samples on Testing Hypotheses Concerning a Bernoulli Variable. HOWARD RAIFFA, Columbia University.

The problem of testing a simple hypothesis versus a single alternative concerning the parameter p of a Bernoulli variable is considered. By example, it is shown that if the number of successes is used as the sample point: a) a decision rule which is admissible among the nonrandomized rules is not necessarily admissible; b) if we confine ourselves to non-randomized strategies then, for a given significance level, increasing the sample size might decrease the power of the most powerful test; and c) among the class of randomized tests, for a given significance level, increasing the sample size does not necessarily increase the power of the most powerful test. Assertion c), which is not generally realized, is illustrated for the case where the type II error is not zero. This is easily explained by returning to the sample space comprising sequences of successes and failures. Emphasis is laid on the fact that the desirability of increasing power by increasing sample size is intimately related to the actual significance level of the tests. Attention is drawn to similar examples involving the multinomial distribution.

(Abstracts of papers presented at the Gainesville meeting of the Institute, March 18, 1954)

14. On the Central Limit Theorem for m -Dependent Variables. P. H. DIANANDA,
University of North Carolina and University of Malaya.

Let X_1, X_2, \dots be a sequence of m -dependent random variables with zero means and finite variances. Let $S_n = X_1 + \dots + X_n$ and $s_n = \sqrt{E(S_n^2)}$. Suppose that, as $n \rightarrow \infty$, (1) $\liminf (s_n^2/n) > 0$, (2) $\limsup E(X_n^2) < \infty$, and (3) for every fixed $\epsilon > 0$,

$$s_n^{-2} \sum_{i=1}^n \int_{|x_i| > \epsilon s_n} x^2 dF_i(x) \rightarrow 0,$$

where $F_i(x)$ is the distribution function of X_i ($i = 1, 2, \dots$). Then S_n/s_n is asymptotically distributed as a standardized normal variable. If, further, $F_i(x)$ is independent of i ($i = 1, 2, \dots$) then (2) and (3) are automatically satisfied and (1) is a sufficient condition for the result. These results and their analogues in the vector case generalize results given

by the author in an earlier paper, "The central limit theorem for m -dependent variables asymptotically stationary to second order," (to appear in *Proc. Camb. Phil. Soc.*). A property of joint distribution functions of m -dependent variables with finite variances, given in the earlier paper, is used in an improved form in proving the results of this paper

15. On a Property of a Class of Decision Procedures for Ranking Means of Normal Populations. (Preliminary Report.) K. C. SEAL, University of North Carolina.

Suppose there are $(n + 1)$ normal populations $N(\mu_i, \sigma^2), i = 0, 1, 2, \dots, n$, with unknown means and a common but unknown variance, and that one random observation from each of these $(n + 1)$ populations is given. It is desired to choose the smallest group of populations which includes the population with greatest mean. Suppose an estimate s^2 of σ^2 is known which is independent of the given observations $x_i (i = 0, 1, \dots, n)$. Let $1 - \alpha (0 < \alpha < 1)$ be the g.l.b. of the desired probability of correct choice, whatever may be μ_i 's ($i = 0, 1, \dots, n$). The class (corresponding to different sets of c 's) of decision rules given below has the property that probability of incorrect choice never exceeds that of correct choice. Let $t_\alpha^{(c_1, \dots, c_n)} (c_i \geq 0; i = 1, \dots, n; \sum_{i=1}^n c_i = 1)$ denote the upper α per cent point in the p.d.f. of $t^{(c_1, \dots, c_n)} = [\sum_{i=1}^n c_i y_{(i)} - y_{(0)}] / s$, where $y_i (i = 0, 1, \dots, n)$ are $(n + 1)$ random observations from $N(0, \sigma^2)$ and $y_{(1)} \leq y_{(2)} \leq \dots \leq y_{(n)}$ are n ranked observations among y_1, \dots, y_n . The class of decision rules is defined as follows: "Reject any observation x_0 from the given observations $x_i (i = 0, 1, \dots, n)$ if $\sum_{i=1}^n c_i x_{(i)} - x_0 \geq st_\alpha^{(c_1, \dots, c_n)} (c_i \geq 0; i = 1, \dots, n; \sum_{i=1}^n c_i = 1)$, and accept otherwise; $x_{(i)}$ stands for the i th ranked observation among x_i 's ($i = 1, \dots, n$). Proceed as above for each of $(n + 1)$ observations separately." Other properties of this class of decision procedures and the selection of an optimum rule from this class are under investigation.

16. Simultaneous Confidence Bounds on Canonical Regressions. S. N. ROY, University of North Carolina.

In an earlier paper ("Simultaneous confidence interval estimation" by S. N. Roy and R. C. Bose, *Ann. Math. Stat.*, Vol. 24 (1953), pp. 513-536) simultaneous confidence bounds on canonical regression coefficients were given with an exact joint confidence coefficient. The confidence statement itself was, however, quite complicated and not of much direct physical use. The present paper uses a technique recently developed by the author (and reported at the last meetings of the Institute of Mathematical Statistics) to obtain a set of confidence bounds, much simpler and physically more usable, but with a joint confidence coefficient greater than or equal to a pre-assigned level, the level being one that is also actually attained.

17. A New Test of Compound Symmetry. S. N. ROY, University of North Carolina.

It is well known that if x_1 and x_2 have a bivariate normal distribution with variances σ_1^2 and σ_2^2 and correlation coefficient ρ , then $x_1 + x_2$ and $x_1 - x_2$ has zero correlation provided that $\sigma_1 = \sigma_2$. It is also well known that this fact and the central distribution of the correlation coefficient are used to test the hypothesis $\sigma_1 = \sigma_2$, which is the hypothesis of compound symmetry for a bivariate normal population. For an $N(\xi, \Sigma)$, where ξ is $p \times 1$ and Σ is $p \times p$, the corresponding hypothesis is that all the diagonal elements of Σ are equal, and so also all the nondiagonal ones. Starting from the bivariate compound symmetry test and using a technique discussed in an earlier paper ("On a heuristic method of test construction and its use in multivariate analysis" by S. N. Roy, *Ann. Math. Stat.*, Vol. 24 (1953), pp. 220-238) a test of this hypothesis is obtained in terms of the largest characteristic root of a matrix and its distribution.

18. A 2×2 Factorial with Paired Comparisons. ROBERT M. ABELSON, AND RALPH ALLAN BRADLEY, Virginia Polytechnic Institute.

The parameters previously specified for a method of paired comparisons are redefined in such a way as to permit the use of treatments in factorial array. The algebraic procedure is shown in general but the normal equations resulting from the use of maximum likelihood are nonlinear and difficult to solve. Easy solution of the normal equations seems to be limited to the 2×2 factorial and an explicit solution is given for that case. The method of paired comparisons presented for 2×2 factorial treatments permits most of the comparisons available through usual analysis of variance. It is possible to test for the presence of both main effects and their interaction. A numerical example is included.

19. On Wald's Confidence Interval for the Ratio of Variances in a Variance Components Model. W. A. THOMPSON, JR., Virginia Polytechnic Institute and University of North Carolina.

Wald's confidence interval ("A note on regression analysis," *Ann. Math. Stat.*, Vol. 18 (1947), p. 586) is specialized to the case of incomplete block designs with random block effects. A theorem concerning the multiplicity of the characteristic roots of the variance-covariance matrix of the adjusted yields is discussed and applied to Wald's confidence interval. A practical example is discussed. This work was done under contracts with the Air Force and the Quartermaster Corps.

NEWS AND NOTICES

Readers are invited to submit to the Secretary of the Institute news items of interest

Personal Items

Paul M. Blunk has accepted the position of Operations Analyst with the Consolidated Vultee Aircraft Corporation at Fort Worth, Texas.

Dr. R. S. Burlington, Chief Mathematician of the Bureau of Ordnance, Navy Department, and head of the Evaluation and Analysis Group of the Bureau of the Ordnance, has been named Special Assistant to the Director of Research and Development, Bureau of Ordnance, Navy Department, Washington, D. C.

Visiting Associate Professor Kai Lai Chung of Cornell University has been appointed Associate Professor at Syracuse University. He is in charge of an ARDC Research project on probability and statistics there.

Charles W. Dunnett, formerly Biometrician for the Food and Drug Laboratory, Ottawa, Canada, is now on the statistical staff of the Lederle Laboratories Division of the American Cyanamid Company located in Pearl River, New York.

Edward A. Fay, formerly a graduate student at the University of California, has been employed since September 1950 as a statistician with the United States Naval Ordnance Test Station, China Lake, California.

Professor E. J. Gumbel, Columbia University, has been appointed Visiting Professor for Statistics at the Free University, Berlin (West) for the summer term 1954. Professor Gumbel has also been elected a member of the International Statistical Institute at The Hague.

Stuart T. Hadden, formerly Chemical Engineer with the Research & Develop-